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PREFACE

This issue, Number 3 of Volume 49, contains those papers presented at the Winter Convention, New York City, January 27-31, 1930, which were not included in the April Quarterly, as well as the papers of the North Eastern District Meeting held in Springfield, Massachusetts, May 7-10, 1930.

The report of the Board of Directors for the fiscal year ending April 30, 1930 concludes this number.

Controlling Power Flow with Phase-Shifting Equipment*

BY W. J. LYMAN†
Associate, A. I. E. E.

Synopsis.—In the operation of an electric power distributing system involving various kinds of networks, a fundamental necessity is that each transforming and transmitting unit carry a reasonable share of the total load. In the case of a single system that has been developed in a coordinated manner the problem of load division has not in general been troublesome and has been solved by judicious selection of parallel circuits, use of reactors, proper system set-up, etc. The advent of inter-company connections has brought together systems of varying characteristics, changed the relation between the basic elements involved, and in some cases has made difficult the control of power flow over tie lines small in comparison to the systems which they unite.

It is the purpose of this paper to discuss the case of load control where closed rings or parallel path circuits are involved. After briefly reviewing the principles of voltage phase angle and power flow, a description is given of some actual tests performed on a very large interconnected system with a 250-mi. transmission loop, involving five power companies and a total generator capacity well over 1,000,000 kw. The results of these measurements are analyzed for the purpose of checking theoretical against actual values and forming a basis for the solution of problems of a similar nature.

* * * * *

INTRODUCTION

It is a generally recognized fact that one of the most important and sometimes the most perplexing problems encountered in the operation of inter-company connections is that of securing accurate and flexible means of regulating power flow. This is a problem which rapidly becomes more complex with the increase in the number of systems combined through interconnection into one group. Inter-company contracts, tie line capacity, and internal system limitations often require that the load exchanged at the various points be regulated within rather definite limits. Conditions have arisen where the maintenance of proper load division has been very difficult if not impossible and attention has already been directed to the solution of this important problem. Although efforts are being made to broaden interchange contract requirements it is sometimes difficult to avoid exceeding physical limitations and to secure the most efficient over-all operation.

It is the purpose of this paper to discuss experimental and analytical methods by which the fundamental problem of load control may be attacked.

At this point it may be well to mention that the problems of inter-company connection differ only in magnitude from conditions existing within most systems and cases undoubtedly exist where the same treatment might be applied to secure more flexible and economical inter-company operation.

THE PROBLEM

When two companies are tied together at one point and have no other interconnection, the problem of load control is relatively simple. Should Company I generate say 10,000 kw. in excess of its own load, there can be but one result; the 10,000 kw. must go over the tie line to Company II. Obviously, if this exchange is to be secured without frequency variation, the increase in generator output by Company I must be carefully coordinated with a corresponding decrease in output of prime movers by Company II. The maintenance of adequate system stability, power factor control, etc., are then the chief problems. If Company II is connected in turn to a third Company III, the three not forming a closed loop, the problem of load control is very similar but somewhat more involved. Proper apportionment can be secured through governor speed adjustments for any number of series connections, the regulation becoming more difficult as the number of companies increases.

If, however, the interconnected systems form a closed loop, an entirely new element enters the problem and results in conditions which cannot be corrected by governor adjustment. In other words, if the three companies in the preceding example are connected to form a triangle, an adjustment of load on the tie between I and II would result in power flow in the other two lines. An investigation soon reveals the fact that the three exchanges cannot be independently controlled and that an analysis and solution is best facilitated by the conception of voltage-phase relations.

PHASE ANGLES

Considering once more the case of two interconnected companies, the conditions which exist are practically identical with those which accompany the transmission of power over any typical circuit. This is illustrated by the single-phase vector diagram of Fig. 1. It is apparent that the transmission of power from I to II results not only in a difference in magnitude between terminal voltages (in this case V_{II} is less than V_I), but also a shift in phase angle denoted by the angle θ. In actual practise the phase shift is not always apparent, but the drop in voltage magnitude is often objectionable

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†Electrical Planning Engineer, Duquesne Light Company, Pittsburgh, Pa.

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and can be readily corrected by transformer taps, voltage regulators, or power factor control.

The addition of a third system to form a loop is illustrated in Fig. 2. With the same interchange of 10,000 kw. from I to II and with the tie line 2-3 open, voltage relations will be the same as in Fig. 1. With no transfer between I and III, their voltages will be practically equal and in phase. By means of regulator adjustments, it would be possible to secure voltages at the open circuit, equal in magnitude but not in the same phase position. Reference to Fig. 2 illustrates this condition and shows that there would be across the open switch a voltage about 90 deg. out of phase with the line voltage. A convenient method of attack is to consider that closing the switch will superimpose on the existing loadings around the loop a circulating power determined by the magnitude of the quadrature voltage and the total effective impedance, the direction of flow being from leading to lagging voltage.

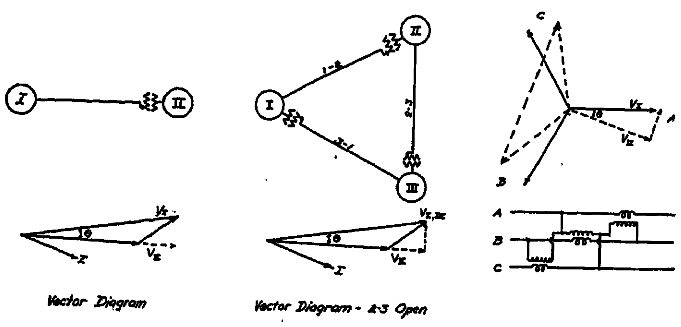

FIG. 1 FIG. 2 FIG. 3

FIGS. 1, 2, 3—VOLTAGE-PHASE DISPLACEMENT AND CORRECTION

Fig. 1—Two systems
Fig. 2—Three systems
Fig. 3—Phase rotator

Conditions similar to that just outlined often arise in the operation of actual systems. Sometimes the differences in phase angle at a tie point are such as to make satisfactory operation impossible. These variations are produced as a result of conditions of unsymmetrical system set-up and loading such as: difference in per cent reactance of cable and open wire lines, unequal number of transformations, location of power plants near the end of the transmission system, and non-uniform loading.

In order to prevent this circulating power which might upset exchange values, voltage conditions, and loading around the circuit, it would be necessary to introduce a quadrature voltage any place in the loop equal to the inherent displacement produced by circuit loading. A simple method of producing this correction would be to connect transformers as in Fig. 3. By connecting the secondary of a regulating transformer in one phase and the primary across the other two phases, a voltage at 90 deg. may be obtained. It can be shown that for ordinary transmission circuits, the power flow is nearly proportional to the phase displacement of the voltage. It is feasible, therefore, to obtain accurate and flexible control of power exchange by supplying some equipment which will produce the required quadrature voltage in adjustable amounts and direction.

Adding a voltage at right angles to the line voltage begins to change the magnitude as well as the direction for the larger angles. Because of this fact it may be economical to combine voltage control with phase shifting in one equipment. This can readily be achieved by one tap changing transformer and two sets of series transformers using star and delta connections. The basic principle is illustrated in Fig. 4.

By laying out the transmission ring with all impedances and given simultaneous loads it is possible to calculate the required amount of phase shift for any assumed set of conditions. In this connection it should be emphasized that the so-called "velocity of electricity" has no effect on the phase displacement even for very long lines. The "velocity" of 186,000 miles per second applies only to transients and not to steady-state conditions.

Obviously, the success of any such installation depends on the accuracy and foresight represented in the phase-angle determination. It was for the purpose of establishing a workable method of making such calculations and checking theoretical and actual values that a test was performed and analyzed on a high-tension transmission ring about 250 mi. around and involving five systems.

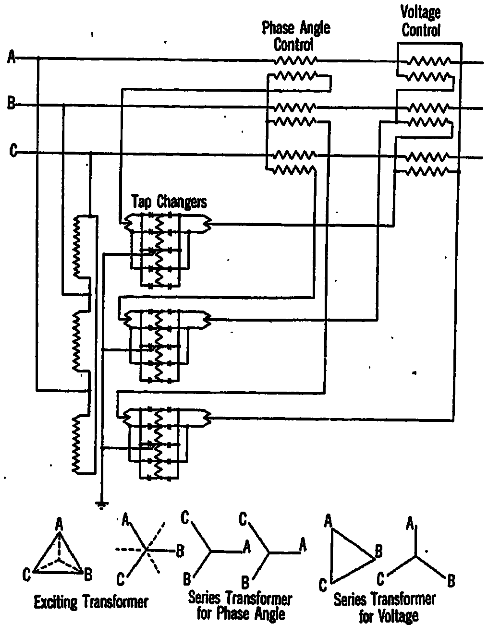

FIG. 4—DIAGRAM OF EQUIPMENT FOR SHIFTING PHASE-ANGLE AND REGULATING VOLTAGE

The following discussion, it should be remembered, pertains only to cases where a closed loop or parallel path network exists.

DESCRIPTION OF TESTS

The high-tension transmission ring which was investigated on this test is shown simplified on the diagram of Fig. 5. The systems of the West Penn Power Company, Ohio Power Company, Ohio Public Service, Pennsylvania and Ohio Power and Light Company, and the Duquesne Light Company were involved. Occasional operation with this ring closed on itself had demonstrated the inflexibility of interchange control.

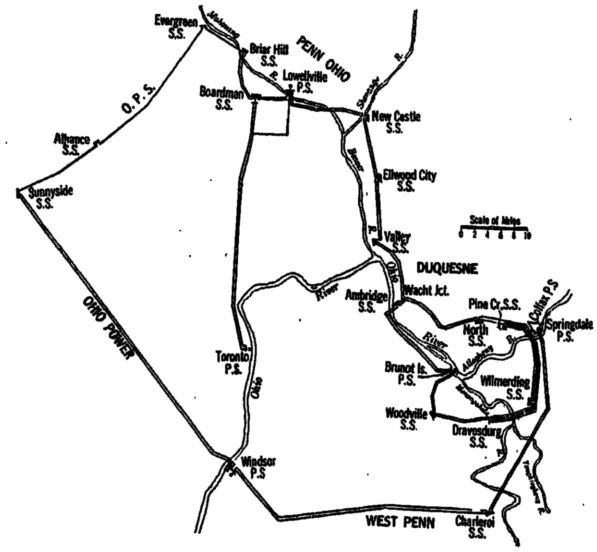

FIG. 5—MAP SHOWING TRANSMISSION LINES DEVOLVED IN INTERCONNECTION TEST OF APRIL 16, 1928

The method of procedure followed to obtain the desired readings was:

1. With the tie line open at Valley Substation of Duquesne Light Company, all other connections being closed, measure the phase displacement at this point and at the same time secure readings of magnitude and direction of power flow on all parts of the ring, together with existing voltage conditions.

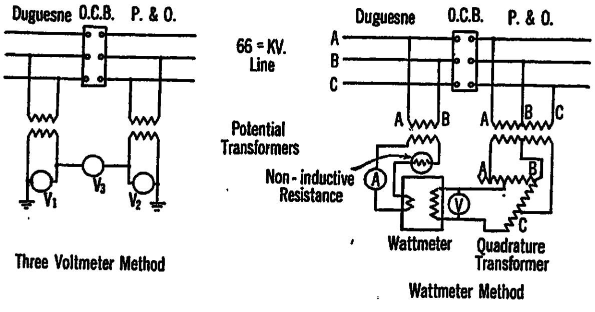

FIG. 6—PHASE-ANGLE MEASUREMENT

2. Close the tie breaker at Valley Substation and determine the amount of power flow corresponding to the measured angle. A record was also made of power station and circuit loading all around the ring.

3. Open the tie breaker at Valley and repeat the readings of No. 1.

4. Secure all these readings for various values of initial phase angle. The variations are obtained by shifting generation between power stations with resulting changes in tie line loads as illustrated on Fig. 8.

The success of the test hinged largely on accurate measurements of phase angle, and consequently two methods were developed for measuring this value and in addition, a rough check was secured from the synchroscope. Probably the simplest means of determining the angle is by the three voltmeter method as shown on Fig. 6. Here the magnitude of the two system voltages is determined and the vector difference measured directly on the third voltmeter. Obviously, this scheme cannot be applied accurately to small angles and does not indicate lag or lead so apparatus was set up to use the wattmeter method. Briefly, this method consists of sending current through the current coil of a single-phase wattmeter supplied through a non-inductive resistance from the potential transformer on one

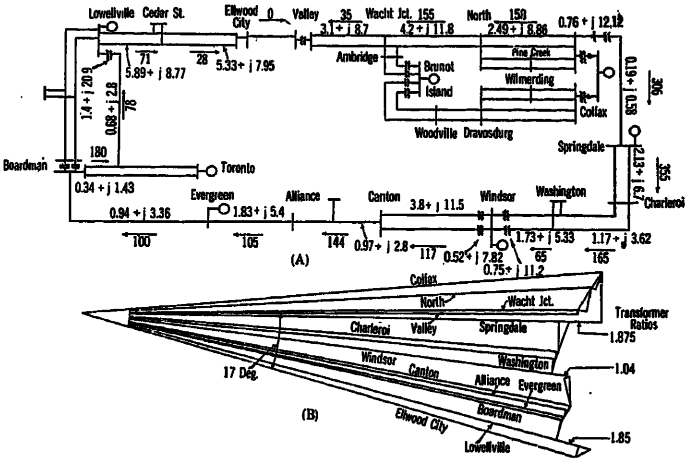

FIG. 7—PHASE-ANGLE CALCULATIONS FOR READINGS TAKEN AT 6.00 P. M. ON APRIL 16, 1928

A. Schematic circuit diagram showing amperes and impedance per circuit in 66-kv. terms

B. Voltage vector diagram derived from the conditions in (A)

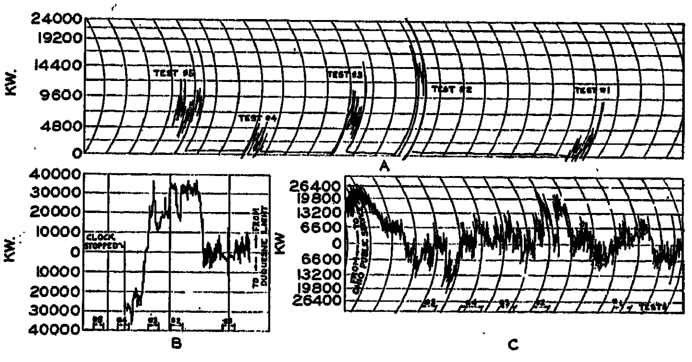

FIG. 8—RECORDS OF POWER FLOW OVER TIE LINES

A. Kw. chart at Valley Substation on tie between Duquesne Light Co. and Pennsylvania & Ohio Power & Light Co.

B. Kw. chart at Colfax Station on tie between Duquesne Light Co. and West Penn Power Co.

C. Kw. chart on Alliance circuit at Canton side of the breaker while the potential for the voltage coil was secured through a 90 deg. displacement transformer from a potential on the other side of the line breaker. Thus when the two line voltages are in phase, currents in the two elements of the wattmeter will be 90 deg. out of phase and the reading will be zero. This point can be accurately checked with the circuit breaker closed and the method is very sensitive for small angles. Fig. 6 shows the diagram of connections for the wattmeter method. An expression for determining the angles from the wattmeter, voltmeter, and ammeter readings is:

$$\text{Sin } \theta = \frac{W}{V\,I}$$

ANALYSIS OF RESULTS

Phase Angle. The first comparison made was between the theoretical phase angle and the measured value. Accurate descriptions were secured of transmission line lengths, conductors, and spacing and transformer capacity and percent reactance. From this information, the ohmic values of impedance were derived for each section of circuit. In this way it was possible to construct a complete vector diagram for the loop and show the various voltage relations. Fig. 7 shows the results of one of the sets of readings for which the calculations show an angle of 17 deg. as compared with a measured angle of 19.6 deg.

On Fig. 8 is a reproduction of the recording wattmeter chart of interchange power at Valley Substation. There was considerable fluctuation and it is obviously impossible to get readings taken a few minutes apart to check accurately.

Equivalent Reactance. Another check was secured by comparing the actual total effective impedance around the ring as determined from a physical description of the apparatus with the effective impedance calculated from the readings on the test. Thus, on this test, one of the voltmeters indicated a vector difference of 22,800 volts with the breaker opened, and the wattmeter indicated 16,300 kw. resulting power flow when the breaker was closed. These quantities result in an effective impedance of 90.5 ohms as compared with a figure of 82.7 ohms derived from transformer and line characteristics.

These values of so-called "effective" impedance may be expressed as $\frac{Z^2}{X}$ and thus vary slightly from the true impedance. The reason for this is readily apparent when we consider that although the current caused by the quadrature vector voltage difference due to the phase displacement is equal to that voltage divided by the true impedance $\left(i.\,e., I = \frac{V}{Z}\right)$, this current is out of phase with the line voltage and must be multiplied by $\frac{X}{Z}$ to get the true power component.

Thus:

$$P = \sqrt{3}\,E\,I\cos\theta$$

$$I = \frac{V}{Z}$$

$$\cos\theta = \frac{X}{Z}$$

P = Watts
E = Line voltage
V = Quadrature voltage (neutral terms)
X = Total circuit reactance
Z = Total circuit impedance

Hence:

$$P = \sqrt{3}\,E\,V\left(\frac{X}{Z^2}\right)$$

And:

$$\frac{Z^2}{X} = \text{Effective impedance}$$

The use of pure impedance to determine kv-a. would probably lead to error since any generator voltage regulators in operation would tend to maintain constant voltage magnitudes. Phase displacements are little affected by this action, however, and it is interesting to note that the closing of the tie breaker at Valley Substation and the resulting shifting of power flow had no effect on any of the power plant kw. outputs.

In a sense, the introduction of a quadrature voltage in a closed loop causes a circulation of power through each section, increasing or decreasing each of the previous loads by the same amount. This is illustrated in Fig. 8 where it is apparent that the same change in power appeared in the Valley-Ellwood tie, the Colfax-Springdale tie, and the Canton-Alliance circuit. In this manner it is possible to determine the voltage that must be introduced in a loop to produce any desired change in loading.

Table I was prepared as a summary of the results of the five tests showing the comparisons between calculated and actual values and other important features to be noted. Complete readings were not secured with which to calculate the phase angle for the first and last tests. The agreement between measured and calculated angles in the other three cases is quite close and the average effective impedance from test is only 7 per cent above the value derived from circuit characteristics. The variation in figures for kw. per degree is about 25 per cent above and below the average. These inconsistencies are not unreasonable when allowance is made for the fluctuating nature of the interchange power.

APPLICATION

Where systems of the same frequency are interconnected directly or through transformers to form a closed loop, the load control would probably take the form of a special type of regulator equipped for tap changing under load and connected for quadrature

TABLE I

EXHIBIT OF RESULTS DERIVED FROM INTERCONNECTION TEST

	Phase angle (degrees)				Total effective impedance $\frac{Z^2}{X}$, ohms		Interchange at Valley S. S.		Kilowatts per degree
	Measured								
Test No.	Vm.	Wm.	Average	Calculated	Circuit characteristics	From test	Kw.	To	(metered)
1	4.0	4.3	4.15	..	82.7	87.9	4,800	P & O	1157
2	20.3	18.8	19.65	17	82.7	90.5	16,300	P & O	830
3	8.0	7.5	7.75	9	82.7	80.9	7,200	P & O	930
4	4.3	3.2	3.75	5	82.7	86.0	3,600	D. L.	960
5	11.3	10.3	10.80	..	82.7	96.8	8,400	D. L.	778
				Ave.	82.7	88.4			931

action. Since the addition of a voltage at right angles makes a slight change in resulting magnitude, the power control could economically be combined with the ordinary voltage regulator to give complete flexibility. This apparatus would be very similar in construction and operation to the transformers which are designed for tap-changing under load and which have been developed in the last few years to a high degree of reliability. Thus the phase shifting regulator or transformer does not represent an entirely new piece of equipment but can be built and applied without extensive development work.

If voltage and reactive conditions are undesirable on either or both systems indicating the possible necessity for synchronous condensers, a motor generator set of the synchronous-synchronous frame shifting type might be used which would at the same time control power interchange. However, conditions would have to be unusual to warrant such an expense.

It is a well-known fact that a three-phase voltage regulator incidentally produces a phase shift and the application of two such devices in series will make possible the control of power and wattless kv-a. This scheme could be applied to low-voltage, medium capacity tie lines but probably is not economical in large applications. An installation of this type has been reported in operation on an extensive power system in Texas.

Where the interconnection is between systems of different frequency a motor-generator set is required, and power control can be secured by a frame shifter for the synchronous type or by use of the asynchronous machine if it is not necessary to maintain rigid phase relations.

In any event, the necessary range of phase shift can be readily determined by the method outlined and it should be remembered that this method of analyzing power flow problems and selecting equipment characteristics applies equally well to conditions within a system itself. Diversity of supply and differences in circuit characteristics may result in non-uniform loading and inefficient use of existing facilities. Conditions very likely exist where the installation of additional capacity could be obviated if inexpensive phase shifters could be applied.

Discussion

C. J. Sittinger: The operating requirements of phase-shifting equipment for purposes of controlling power flow over looped parallel circuits as encountered quite generally in interconnected systems are dependent primarily on three factors, *viz:*

1. The total impedance of the circuit to be controlled and ratio of the total impedances of the parallel circuits.

2. The range and direction of power flow and closeness of control desired.

3. The geographical and seasonal distribution of load throughout that portion of the interconnected systems involved in the loop.

For purposes of illustrating the first condition, assume that the total impedance of main branch of the loop over which a definite amount of power is to be sent is $Z\,\underline{/\varphi}$ expressed as a per cent of the voltage drop corresponding to the kv-a. to be transmitted, and includes the impedance of the phase-shifting equipment.

Let $m\,\underline{/(n-1)\,\varphi}$ be the ratio total impedance of the other branch of the loop to the impedance of the main branch, so that its impedance will be $m\,Z\,\underline{/n\,\varphi}$

$$\text{Parallel impedance}\ \frac{Z\,\underline{/\varphi} \times m\,Z\,\underline{/n\,\varphi}}{Z\,\underline{/\varphi} + m\,Z\,\underline{/n\,\varphi}} = \frac{m\,Z\,\underline{/n\,\varphi}}{1 + m\,\underline{/(n-1)\,\varphi}}$$

$$\text{Series impedance}\quad Z\,\underline{/\varphi} + m\,Z\,\underline{/n\,\varphi} = Z\,\underline{/\varphi}\,(1 + m\,\underline{/(n-1)\,\varphi})$$

Without adjustment of the phase-shifter, the transmission of the given base load at a given power factor, cos θ, will cause a voltage drop between the two ends of the loop of

$$1.00\,\overline{\backslash\theta}\;\frac{m\,Z\,\underline{/n\,\varphi}}{1 + m\,\underline{/(n-1)\,\varphi}}\ \text{per cent}$$

The amount of power diverted from the main branch to the other branch of the loop would then be

$$\frac{1.00\,\overline{\backslash\theta}}{m\,Z\,\underline{/n\,\varphi}} \times \frac{m\,Z\,\underline{|\,n\,\varphi}}{1 + m\,\underline{/(n-1)\,\varphi}} = \frac{1.00\,\overline{\backslash\theta}}{1 + m\,\underline{/(n-1)\,\varphi}}$$

The voltage which must be inserted in the series circuit of the loop in order to neutralize this power flow would be

$$\frac{1.00\,\overline{\backslash\theta}}{1 + m\,\underline{/(n-1)\,\varphi}} \times Z\,\underline{/\varphi}\,(1 + m\,\underline{/(n-1)\,\varphi}) = 1.00\,\overline{/\theta} \times Z\,\underline{/\varphi}$$

This voltage is just equal and opposite to the drop in the main branch due to the entire load, and results in equal voltages at both ends of the loop, which, of course, is the condition necessary for no diversion of power, or zero flow in the other branch. The only factors, therefore, which need be taken into account in a simple loop of fixed impedance without intermediate loads, are the amount of load, the power factor, and the total impedance of the main branch.

Assuming a specific case of load at unity power factor, 10 per cent impedance and a line angle of 75°, which would not be uncommon if transformers were included in the circuit, the per-cent corrective voltage would be $10\ \underline{/75^\circ}$ and the phase angle

$$\text{shift would be } \tan^{-1}\frac{10\sin 75^\circ}{100+10\cos 75^\circ}=\tan^{-1}\ 0.0922=5^\circ-16'$$

For power transmitted in the opposite direction, the same corrective voltage would be necessary, but the phase angle shift

$$\text{would be } \tan^{-1}\frac{10\sin 75^\circ}{100-10\cos 75^\circ}=\tan^{-1}0.0982=5^\circ-36'$$

For half the power flow the per cent corrective voltage would be $5\ \underline{/75^\circ}$ and the phase-shift $\tan^{-1}0.0469$ or $2^\circ - 41'$.

It is evident that, aside from other considerations, the total phase-shift for a reasonable power flow is relatively small and if this were the only factor, the specification of phase-shifting equipment would be a simple matter.

Referring to the amount of power diverted when operating without phase-shift control, which was found to be

$$\frac{1.00\ \overline{\backslash\,\theta}}{1+m\ \underline{/(n-1)\ \varphi}},$$ it will be seen that the amount of diversion

is dependent only upon the ratios of the impedances of the branches and the total power flow. If $\varphi = 75$ deg. as in the preceding example and the impedance in the diverting branch is twice that of the main branch with a line angle of 60 deg., so that $m = 2$ and $n = 0.8$, then for a unity power factor load, the power diversion will be

$$\frac{1.00\ \overline{\backslash\,\theta}}{1+2\ \underline{/0.2\times 75^\circ}}=1.00/2.93+j\,0.52=0.330-j\,0.059$$

$$=0.335\ \underline{/10^\circ-9'}$$

Similarly if the impedance in the diverting branch is one-half that of the main branch with a line angle of $82^\circ - 30'$ so that $m = 1/2$ and $n = 1.1$ then for unity power factor load the diversion will be

$$1.00/1+1/2\ \underline{/-0.1\times 75^\circ}=1.00/1.496-j\,0.065$$

$$=0.676+j\,0.029=0.677\ \underline{/2^\circ-27'}$$

With a 17-tap phase-shifter of the type outlined by Mr. Lyman, employing reactors in the tap changing equipment, a change of one position with a 50,000 kw. load would cause an average shift of 1030 kw. in the first case and 2110 kw. in the second case. For other total loads, the amount of load shift would be proportionate. It is obvious that fine control with this type of equipment is not possible and must be supplemented by some readjustment of voltages throughout the system if no power is to be diverted from the main branch. It is also evident that the impedance ratio of the two branches of the load and the amount of power to be delivered have a marked effect on the closeness of control possible.

As pointed out, the discussion up to now has been confined to a simple loop without intermediate loads. The more usual case consists of a continuous line of one or more circuits between two points, with or without one or more transformations in voltage, and numerous points along the line where power is supplied to the system or loads are taken off. From each end of this line there may be other lines running toward each other over another route and which could form the nucleus of a second connection between the two points in question. Due to distribution of loads and generating stations, the open ends of the lines will as a rule have considerable inherent phase displacement as well as differing in voltage. If the interconnection between the two points joins a hydroelectric district with a steam generating district, the inherent phase displacement will vary considerably from season to season depending upon the amount of water available for hydroelectric generation.

To complete the loop by closing the open ends of these lines and subsequently control the flow of power over this loop by means of phase-shifting equipment, there would be required, in addition to the phase-shift necessary for load control, a further phase-shift to take care of system differences. In other words, it must supply additional voltage at the proper angle so that the open ends of the lines will match, thus permitting closing of the loop without disturbing the system balance. This phase-shift may, and usually will be of greater magnitude than the phase-shift necessary for load control and it may be positive or negative with respect to the latter. Fig. 7 of Mr. Lyman's paper illustrates this condition very clearly, and further indicates that lengthy and laborious computations may be required in order to determine the probable necessary range of operation of phase-shifting equipment. The difference between the calculated and the actual phase displacement is due primarily to the fact that the various impedances cannot be determined accurately. The reactance of a conductor is affected by proximity to conductors of another circuit. The reactance of a transformer depends upon the particular taps used. This latter reactance is especially subject to wide variation on different taps in the case of auto-transformers and phase-shifters. On a particular case which the writer has been studying, the reactance of the phase-shifting equipment varied from 3.4 per cent to 5.5 per cent.

Referring again to the question of closeness of control it was pointed out that the angular shift necessary to obtain proper power control is ordinarily quite small,—on the order of 3 deg. to 5 deg. The phase-shifting equipment must, however, supply the necessary voltage component to permit the connected systems to match. With an inherent difference of 15 deg., a total phase-shift of 20 deg. may be necessary in one direction, and possibly 10 deg. in the opposite direction when seasonal load change reverses the inherent phase difference. If the tap changing equipment is connected to take the full range of the phase-shifter or 30°, then control of load will be six times as great. For the typical cases given previously a change of one position on the tap changer would cause average load shifts of 6200 kw. and 12,70 kw. respectively for a 50,000 kw. load, compared to 1030 kw. and 2110 kw. in the case of the simple loop without intermediate loads.

It is hoped that this discussion will give a clearer picture of the conditions under which phase-shifting equipment must operate. Two induction regulators in series will give excellent control inasmuch as an infinite number of steps is possible. However, if large changes of in-phase voltage are necessary the capacity for phase-shift is appreciably reduced. Indicators would be necessary in order to show the position of the moving coils, otherwise it would be difficult to determine what changes have been made. The cost of such equipment, especially for higher voltages, is usually prohibitive. The regulating transformer type of phase-shifter is mechanically a much simpler piece of equipment, having no moving parts except the tap changer. The ability to shift phase is only affected in a small way for wide changes of in-phase voltage and the cost of such equipment is relatively small, since it consists essentially of transformers and their appurtenances. For interconnections involving appreciable inherent phase differences, however, this equipment is open to the objection that close control of load is not possible with the designs thus far offered.

It appears that three methods of overcoming this difficulty are possible. The first would be to provide the exciting transformer with from fifty to a hundred or more taps, or three to six times that now offered, but only the phase-shift tap changers would be involved. These should be brought to the stationary member of some contact making device such as a drum controller. The seventeen taps of the tap changer and the neutral point would be brought to the moving member so as to permit manual selection of any seventeen successive taps, their range being selected to allow sufficiently close control of the load.

Instead of a multiplicity of taps, the standard seventeen taps could be provided except at the center where fourteen more taps would be added, which, together with the three standard center taps would span the proper portion of the regulating transformer to give the fineness of control desired. The load tap changer would then be permanently connected to the fine taps. To take care of the wider inherent phase displacements due to system differences, the neutral or zero point would be connected to the movable arm of a manual tap changer to which the standard taps would be connected. A combination of this scheme with the first would make possible double the phase shift with a given regulating transformer.

As another alternative scheme, a second phase shifting series transformer with tap changer equipment is suggested supplementing the main transformer, but of such a ratio that its voltage would be one-third to one-sixth (or perhaps even as low as the voltage obtained in the main phase-shifting transformer when spanning three taps). This would require three sets of tap changers all working on the same taps of the exciting transformer; but this does not seem to involve any serious complication or much added expense, and would certainly add much to the flexibility required of such equipment in order to make it of practical use.

C. A. Powel: Mr. Lyman shows in his paper that if the voltage and power readings are carefully made, the power flow around a loop can be readily calculated and the results depended upon. Now that a-c. calculating boards are available, the large amount of arithmetical work involved in such calculations can be eliminated.

Mr. Lyman also shows that 17 deg. phase displacement existed across the breaker at Valley substation, and that when the breaker was closed, this same angle existed internally in the loop, causing a circulating current largely in phase with the voltage. This can perhaps be likened to a short-circuited synchronous generator where the terminal voltage disappears and is replaced by the short-circuit current. As Mr. Lyman points out, if the circulating current, in seeking the path of least impedance, leads to undesirable results, such as overloading cables or upsetting contractual exchange values, the situation can be easily remedied by phase angle control equipment.

W. J. Lyman: One thing which was not mentioned in presenting the paper is the possible combination of this scheme of phase angle control with voltage control as is being used to a very large extent at the present time. In other words, the equipment necessary to get this power control as described is not radically new. It is simply an alteration in the apparatus which is already quite standard. This is probably one of the main elements of value in the scheme.

Another important fact is that only one device of this nature is required for any one loop. In the system shown in the paper, although there are three companies and three tie lines, only one regulator is necessary.

The last thing I want to mention is that the increments of angular displacement provided for by the equipment must be in accordance with the total impedances involved in the loop.

Electrically Welded Structures
Under Dynamic Stress

BY MORRIS STONE* and J. G. RITTER†
Non-member Non-member

Synopsis.—In the general application of welding to various types of constructions, those structures carrying variable stress have been least adapted to fabricated construction. In the present paper it is shown that by properly considering such factors as maximum working stress, residual welding stresses, and stress concentration, it is indeed possible to weld any type of structure without fear of trouble. Many various cases of such structures are described as to their method of fabrication and success of operation.

THE general application of arc welding to all types of construction has extended rapidly during the past few years, but not in the field where dynamic stresses were imposed because of the lack of information on the fatigue resistance characteristics of such welded structures. Even as far back as 1911, Stanton[1] and Abell[2] after making a series of Wohler tests of welded material, advised against welds for variable stress applications. However, recent investigations such as those described in this paper, justify a more general increase in the use of welding under these conditions.

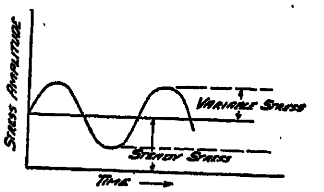

FIG. 1—DIAGRAM ILLUSTRATING COMBINED STEADY AND VARIABLE STRESS

The general nature of the action of weld metal under fatigue is much the same as for virgin metal, except that lower test values are obtained, and less consistency of results. If we consider, first, the pure weld metal, we find endurance limit values varying from ± 9000 lb. per sq. in. to ± 22,000 lb. per sq. in., depending on the care in making the weld, its porosity, size of weld, etc. Dustin,[3] commenting on the fatigue of welds, states the difficulty in obtaining consistent values, due to blow holes, slag inclusion, etc. This aspect of testing has forced the use of large size test pieces for obtaining fatigue values on weld material.[4] The reason for such a necessity can be appreciated when one considers the very nature of a weld, which is porous and non-homogeneous. The effect of a small surface hole will be much more serious on a small specimen than on a large—so that in general, one should expect higher and more consistent results from large size fatigue specimens.

An active comparison of endurance limits as determined for varying sizes of welds is the object of a present research, and we need not be surprised to find the larger size welds showing lower endurance limits. Such an effect can be explained by the injurious influence of residual stresses in the larger welds, which do not occur in the smaller welds, where they can be relieved by the heat of subsequent welding. Such results would indicate opposite tendencies to those published[5] for tensile strengths, yield points, etc.

From the widely differing values of endurance limit, it is entirely feasible to count on the value of ± 15,000 lb/in^2. This, it will be realized, must be compared with endurance limits of ± 25,000 lb. per sq. in. for mild carbon steel to as high as ± 60,000 lb. per sq. in. for some alloy steels.

Most structures are rarely subjected to purely variable stress—alternating between equal values of tension and compression—but rather combinations of steady and variable components as illustrated in Fig. 1. The failure of metals under such conditions is still a matter of investigation, and is only now becoming clarified from the point of view of design utility.

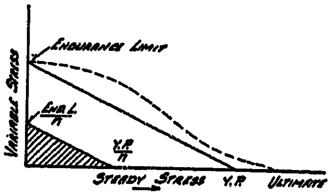

FIG. 2—CURVES SHOWING RELATION BETWEEN VARIABLE AND STEADY STRESSES

One of the most common stress conditions, and probably the most elementary for a material study, is that of simple tension or compression superimposed by a variable stress of smaller magnitude. Experiments, investigating failure under various ratios of steady to variable stress, have been carried out for many years, with certain characteristic relationships becoming evident. If we plot steady stress as abscissas versus variable stress as ordinates,[6] the experimental curve of

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*Mechanical Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

†Welding Engineer, Westinghouse, Elec. & Mfg. Co., East Pittsburgh, Pa.

[1] T. E. Stanton and J. R. Pannell, I. C. E., Dec. 1911.

[2] W. S. Abell, I. C. E., March 1919, in an investigation for Lloyd's.

[3] H. Dustin, *Revue Universelle des Mines*, Dec. 1926.

[4] R. E. Peterson, "Fatigue Tests of Large Specimens." A. S. T. M., June 1929.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 27-31, 1930.

[5] G. Lobo, "Strength of Butt Welds," *Journal of Amer. Welding Society*, April 1929.

[6] *Recent Improvements in Turbine Generators*, S. L. Henderson and C. R. Soderberg, A. I. E. E. Quarterly TRANS., Vol. 47, April 1928, p. 549.

"Stress Analysis in Electrical Rotating Machinery," *A. S. M. E. Trans.*, 1928, M. Stone.

failure follows somewhat that shown by the dotted line in Fig. 2. This has been approximated to by Gerber as a parabola, and by Goodman as a straight line connecting endurance limit and ultimate stress, but it is more conservative and easier of application to use the straight line approximation shown connecting endurance limit and yield point.

The relation between allowable variable stress, S_v, and corresponding steady stress, S_s, as defined by the straight line is, where

S_e = Endurance limit
S_y = Yield point

$$\frac{S_v}{S_e} + \frac{S_s}{S_y} = 1$$

Now in the actual applications of arc welded structures, we do not permit ourselves to operate close to failure stresses; therefore the idea of factor of safety must be introduced.

Due to the very many exigencies that may occur in welding, such as poor fusion, impurities, porosity, etc., we, in our design practise, have set a factor of safety of 3 to be used both on the yield point in steady stress applications and on the endurance limit for fatigue. This confines our stresses to the shaded area shown in Fig. 2 and our stress relation now is expressed by

$$\frac{S_v}{S_e} + \frac{S_s}{S_y} = \frac{1}{n}\left(n = \text{factor of safety}\right)$$

For welds, this relation takes on the quantitative aspect of

$$\frac{S_v}{15{,}000} + \frac{S_s}{30{,}000} = \frac{1}{3}$$

The endurance limit and yield point values being taken here are average figures.

To illustrate its use: Suppose we have a pure variable stress, then $S_s = o$, and the allowable S_v is given by

$$S_v = \frac{15{,}000}{3} = 5000 \text{ lb. per sq. in.}$$

On the other hand, suppose we consider a static application, where $S_v = o$, then the allowable stress, S_s is

$$S_s = \frac{30{,}000}{3} = 10{,}000 \text{ lb. per sq. in.}$$

These are the values adhered to in all designs where welds have to withstand live stress. An intermediate case may be: If $S_s = 5000$ lb. per sq. in., what is the allowable variable stress that may be superimposed? Thus

$$\text{Allowable } S_v = \left(\frac{1}{3} - \frac{5000}{30{,}000}\right) 15{,}000 = 2500 \text{ lb.}$$

per sq. in.

The design situation becomes more complex where welds are to withstand combined shear and direct stress,[7] or direct stress on planes at right angles to each other. These fields have not as yet been fully investigated experimentally even in the realm of metals themselves, and even less for welds. This leaves us with the necessity of relying on the well established rules for stress combinations as calculated from the ordinary theory of elasticity, and interpreted as we have discussed above.

At this point, we must inject an idea extraneous to fatigue considerations for pure metals, but one which occurs when heat treated alloy steels are welded together or to ordinary steels. Tests that have recently been made consisted of welding a ring of metal around a heat treated ni-chrome steel (0.56 per cent Cr, 1.17 per cent Ni.) test bar, and then after machining off the bead, subjecting the bar to an alternating stress. The endurance limit dropped from 66,000 lb./in.^{2} (for the alloy steel) to 16,200 lb./in.^{2} Like reductions were noted in ductility and impact resistance. This important result emphasizes the point that the fatigue strength of welded joints cannot be increased by using high grade parent metal—the effect of welding is to reduce the endurance limit to only average values for mild carbon steel. This fact is important, particularly, where joints are designed to fail in the parent metal and not in the weld. This result, however, is not to be interpreted to apply in the case of static stress, where tensile properties do not appreciably change.

As soon as we begin to extend our ideas to welded joints, much of our knowledge becomes inapplicable.

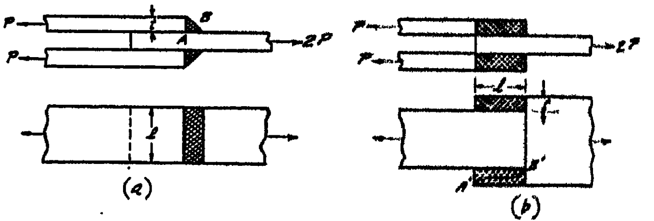

FIG. 3—TWO TYPES OF WELDED JOINTS

To interpret our informaion on weld and parent metal for use in welded joints is to assume that the stress distribution is known. This unfortunately is the stumbling block in most cases.

Even in the simplest of welds—fillets, butt, side, or end welds—actual tests must be carried out to determine the upper limit for stress application. And for the great majority of joints, which are made up of combinations of the above, there exists little chance of being able to predict stress distribution.

To examine the simplest of joints, let us compare the two types of joints shown in Fig. 3, the first, a joint of front welds and the second, a joint of longitudinal or side welds, as to their suitability to withstand dynamic stress.

From tests that have been made in the past, notably by Dustin,[8] the strength of joint (a) is best predicted by considering the section $A\,B$ to be subjected to pure tension and we must associate the stress given by

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[7] F. C. Lea, *Engineering*, Aug. 20, 1926.

[8] H. Dustin, Congres International pour l'Essai des Materiaux, Amsterdam, 1927.

$$f = \frac{\sqrt{2}\,P}{t\,l} \text{ with failure.}$$

Failure usually takes place at this critical section. For type (*b*) welds, the breaking stress can be written as $f = (\alpha \sqrt{2}\,P)/(t\,l)$ where α has the value of approximately 2/3. This factor has this low value due to the stress concentration at the ends of the weld, causing the break to progress from A^1 and B^1 inward.[9] Despite the lower allowable stress for joint (*b*) (where stress must always be compared with shear stress failure, while in (*a*) the much higher value of tension failure can be approached), it must be noted that the energy of rupture of joint (*b*), is much larger than that for (*a*), and Dustin even goes so far as to favor (*b*) over (*a*) for dynamic loading. A conclusion like this, however, is clearly a case of balance of ductility of break against allowable stress.

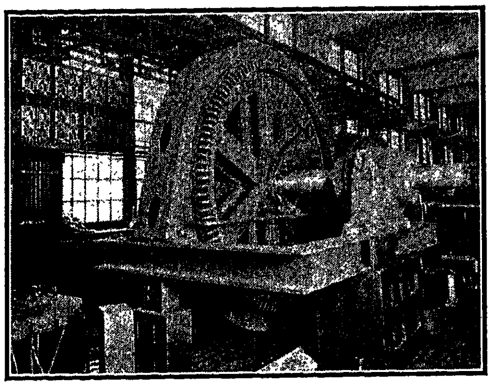

FIG. 4—ARC WELDED A-C. GENERATOR

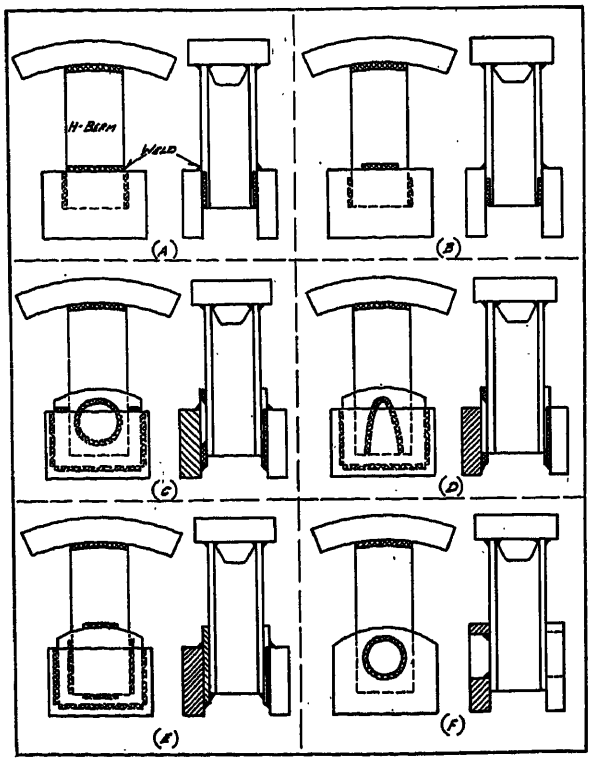

FIG. 5—SPECIMENS TESTED ON VIBRATION MACHINE

The design of machines to be constructed of structural shapes welded together involves a great many factors in addition to the direct stresses in the welds. Fig. 4 shows the type of construction widely used for a-c. generators and motors, the rotors of which are direct connected to geared machines or combustion engines. These machines are subjected to large torque variations of both high and low frequencies and a considerable amount of experimenting has been carried out to determine the best arrangement of the structural members and welded connections.

It was quite easy to make the welds themselves efficient in these structures, but the stress concentration in the structural members was a serious problem. It will be noted that the H-beams of the rotor are placed so that the smallest moment of inertia is utilized for torque loads. This arrangement calls for larger beams but gives the most economical design to keep stress concentration at a minimum.

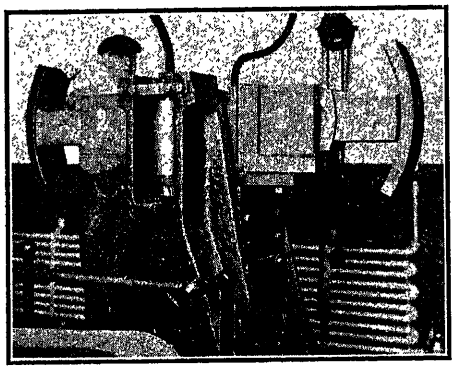

FIG. 6—VIBRATING TEST MACHINE WITH A-C. ROTOR TEST ARMS

Fig. 5 shows several methods of fixing the H-beams into the hub of the rotor which were tested on the machine shown in Fig. 6. This machine subjected the specimens to a very violent oscillating vertical motion corresponding to torque oscillations in the actual rotor. Specimen (*c*) proved to be the best, but specimen (*b*) has proved satisfactory for normal application. Further tests are being conducted on a machine which will give actual as well as relative values of the stresses encountered and will also permit the superimposing of initial stresses in the structure which are independent of the varying stresses.

The initial stresses which are encountered in welded structures proved to be very harmful when the structure is subjected to varying stresses as discussed earlier in the paper. These initial strains are caused by the actual shrinkage of the weld metal itself and by the unequal heating of the various parts of the structure while being welded. Bending and twisting strains are set up while cooling to room temperature. Many complete rotors were built to determine if it was possible to obtain a structure free from initial strains. The method used in determining the residual strains was to drill small gage holes in the pieces before being assembled and recording the variations in the distances between them after being welded. After the rotor was completed, additional strain gages were clamped to the structure at various places and the individual members cut loose from the structure by hack saws. Various welding sequences, peening of welds, artificial cooling of members, and jigs were used, but it was found to be commercially impracticable to obtain a complete rotor free from high residual strains.

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[9] Stress distributions of this type have been qualitatively studied by J. H. Smith, "Stress-Strain Characteristics of Welded Joints," *Journal of Amer. Welding Society*, Sept. 1929.

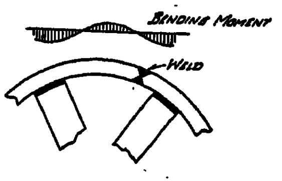

Fig. 7—Weld in Rotor Rim Located at Point where Bending is Absent

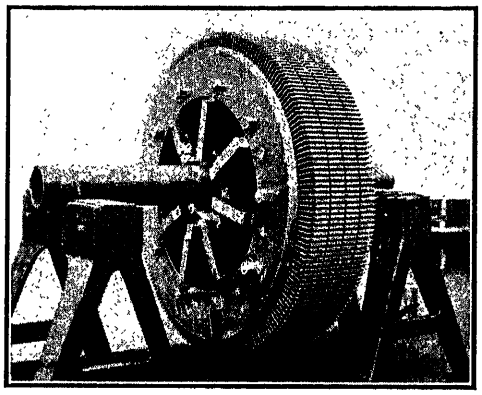

Fig. 8—1000-Kw. Arc Welded D-c. Rotor

Annealing was then adopted in order to remove the strains and tests were made at various temperatures by our Research Dept. It was found that practically all residual strain was removed at 1000 deg. fahr. and this temperature was adopted for all welded apparatus where a strain free condition is necessary. Several complete rotors were annealed at 1000 deg. fahr. and the residual stresses found to be under 1500 lb. per sq. in.

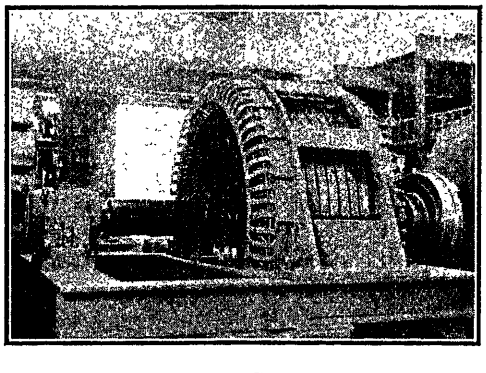

Fig. 9—Arc Welded Single-phase Stator

Annealing at higher temperatures was found to be undesirable because of the greater liability of warping and cracking the welds. All of the physical properties of the weld itself are decreased by high temperatures and because of this it is not considered practicable to obtain refinement of the grain structure of the parent metal adjacent to the weld at the expense of the weld itself. These results concur with previous conclusions of other investigators.[8, 10]

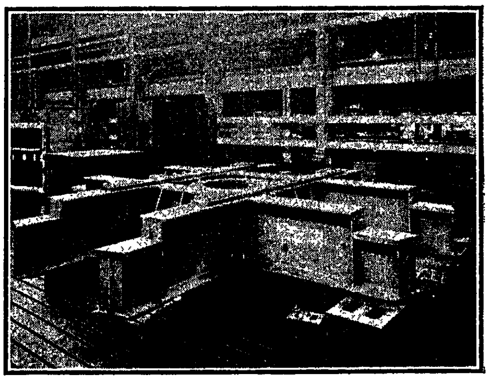

Fig. 10—25000-Kv-a. Lower Waterwheel Bracket

It is of interest to note the design consideration necessary in the use of butt welds for the rims of the above mentioned rotors. These rims are made of bar stock rolled up and butt welded. It was at first thought desirable to locate the welds of one of the rotor arms across the butt weld in order to reenforce it but the bending and tension stresses were above our allowable limit. The location of the weld as shown in Fig. 7 at a point where the bending in the rim is zero and only the centrifugal tension exists, made it possible to use a welded construction.

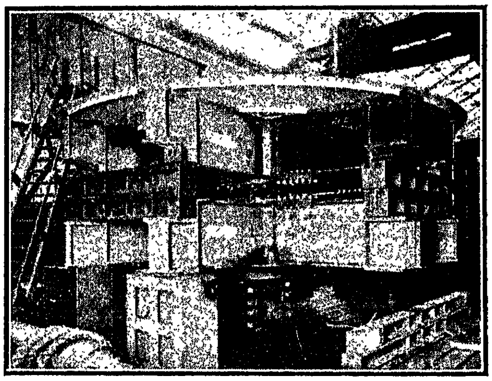

Fig. 11—Two Waterwheel Brackets under Test

In Fig. 8 we have a typical arc welded d-c. rotor, 1000-kw., 300-rev. per min. for steel mill application. The construction here is characteristically different from the a-c. rotors in that there is no welded connection between the spokes and rim, thus eliminating the condition where locked up stresses are serious. This

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[10] J. B. Rose, *Journal of American Welding Society*, Sept. 1929.

permits the use of welding without strain annealing and has been applied with complete success to Diesel driven generators, steel mill drives, ship drives, etc., all of a dynamic stress type. All machines of this type have been tested at repeated short circuit and maximum overspeed, and are designed to withstand a torque load of ten times normal.

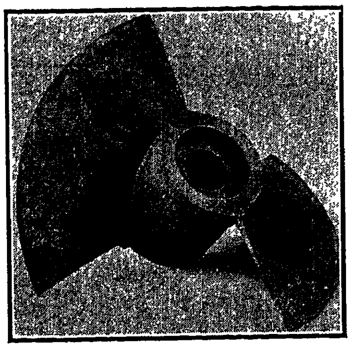

FIG. 12—300-HP. BLOWER PROPELLER

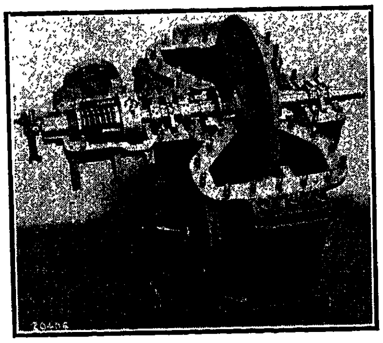

FIG. 13—ARC-WELDED LASHING FOR TURBINE BLADES

Fig. 9 shows a welded single-phase machine in which the varying stresses imposed have as serious an effect on the stator as on the rotor. This type is subjected to 200 per cent torque pulsations at twice the impressed line frequency.

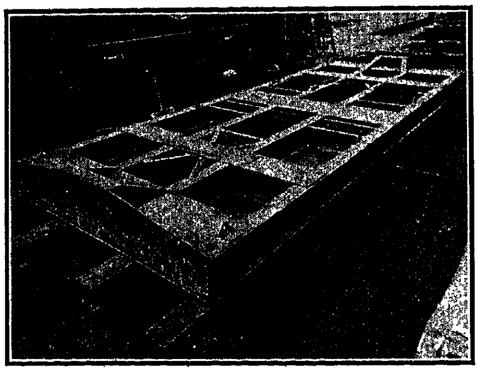

FIG. 14—UNDER FRAME FOR DIESEL ELECTRIC LOCOMOTIVE

Fig. 10 shows an arc welded bracket for supporting the rotor of a 25,000-kv-a. waterwheel generator. This type of bracket must be designed to permit a very small deflection and must withstand varying and shock loads. The waterwheel sets up considerable vertical vibrations and under certain operating conditions water surge impacts can be so severe that the rotor is actually lifted from its thrust bearing. Fig. 11 shows two brackets being tested for deflection by pulling them together with a through bolt. The tests showed that the

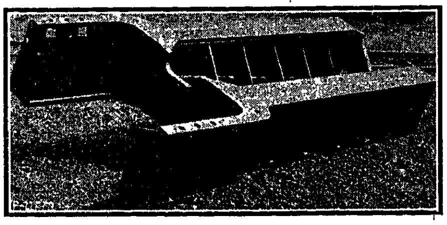

FIG. 15—CRANK CASE FOR HIGH-SPEED DIESEL ENGINE

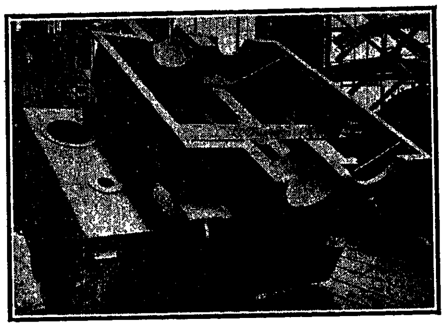

FIG. 16—7000-HP. MARINE REDUCTION GEAR UNIT

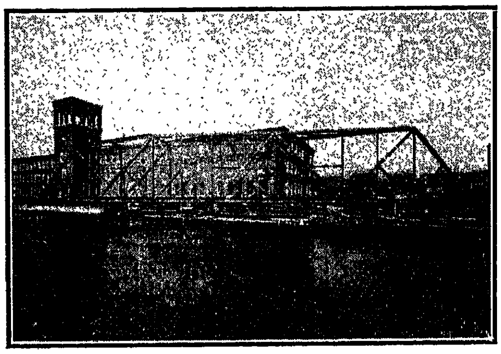

FIG. 17—175-FT. ARC WELDED RAILROAD BRIDGE

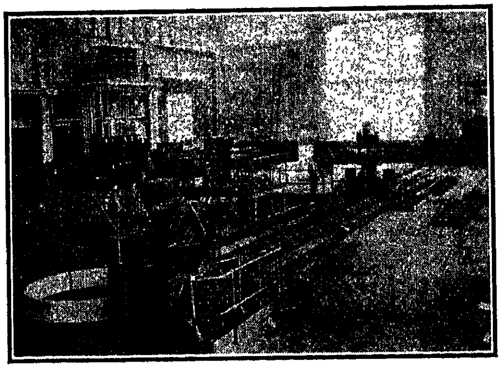

FIG. 18—CORNER OF FABRICATING SHOP
Rolling A-c Rotor Rims

deflection of the welded brackets is comparable with the corresponding cast design and can be predetermined with accuracy. The purpose of the bolted joints shown is to permit the dismantling for shipment. A considerable number of brackets of this general type have been in use for some time.

Fig. 12 shows the propeller[11] of a large volume high-pressure blower which is directly driven by a 300-hp., 3250-rev. per min. induction motor. It is constructed by arc welding the two blades into a slot out in the hub. Various strength and fatigue tests were made on this design and it has withstood severe service conditions. The entire blower housing is fabricated by arc welding and in some applications the propeller is driven by an all welded steam turbine.

Fig. 13 shows the application of arc welding for turbine blade lashing which must withstand unusually severe vibration stresses. This construction has been successfully used for several years.

Fig. 14 shows a welded cab underframe for a Diesel electric locomotive. It has successfully withstood the severe bumping and twisting strains imposed by yard switching service.

Our company has also used arc welded designs successfully in the following applications which are subjected to dynamic stress conditions:

Crane structures.
Marine reduction gear units of several thousand hp.
Trolley pole bases for street cars.
Various ventilating fans.
Lifting lugs on various apparatus. Elevator cages and equipment.
Railway motor housing and gear cases.
Locomotive equipment supports.
Railway bridges.
Crank cases for high speed railway Diesel engines.

We have emphasized, in this paper, the degree to which electric welding may be relied upon in applications where the structure is subjected to dynamic stress. Such applications can be successfully made by giving due consideration to the factors of maximum working stress, residual welding stresses, and stress concentration. The pessimism of the past few years regarding dynamic applications is slowly disappearing and it is only by the careful consideration of all factors that arc welding will be extended to all types of fabrication without restriction.

Discussion

B. L. Barnes: First, it is, I believe, generally agreed that it is possible to deposit welds by hand which will have less initial stress in the deposited metal than would be present in welds that are made by a continuous machine process. Would the authors consider it necessary to anneal all welded machine parts such as rotor spiders shown in the illustrations regardless of whether they are welded by hand or machine?

Second, the paper mentions some limits of allowable working stresses in the deposited metal. I wish to ask whether the allowable stress in the weld is dependent upon the size of the weld. For instance, it is, I believe, considered that metal deposited by a single operation is more uniform and stronger per unit area than that in a larger weld requiring two or more operations. Would the same working unit stress be allowed in a large weld requiring two operations as in a small weld requiring but one operation?

C. H. Jennings: Messrs. Stone and Ritter are to be commended on the very valuable and interesting information presented in their paper *Electrically Welded Structures Under Dynamic Stress.* It has been the aim throughout the paper to show how electric welding has been successfully applied to dynamically stressed structures and to discuss the application of a very useful and convenient method for taking into consideration the dynamic and static stresses when designing welded structures.

This paper is of particular interest to those who are investigating the properties of welds and promoting the use of welding in industry, for it shows how welding has been successfully applied in cases where it was at one time thought unsafe and impossible. This development of welding has only been made possible by the extensive research investigations that have been and are being conducted on the physical properties of welds.

Unfortunately, relatively little work has been done by investigators on the fatigue of welds, a test which is of prime importance for obtaining information on welds subject to dynamic stresses. The company with which the writer is connected has gone deeply into this phase of welded joint investigation and is obtaining some very interesting and valuable information.

It is mentioned in the paper that the use of large specimens has been resorted to in an effort to minimize the effect of stress concentration due to pits and holes, and that in doing so lower endurance values are obtained. This is quite true but the writer feels that this low endurance limit cannot be entirely explained by the injurious effects of residual stresses in large welds. An additional factor that probably aids in producing this low endurance limit is the possibility of some physical change in the structure of large welds as compared with small ones. As yet, however, no definite explanation is available.

The low fatigue results obtained on large specimens are not in disagreement with results published for tensile properties of welded joints,[1] where it was found that the strength of butt welds increased with the thickness of the welded plates, as at first might be expected. The increase in the tensile properties of butt welds as the thickness of the welded plates is increased to at least one inch, is possibly explained to a certain extent, other factors being constant, by the manner in which the plate is welded. In the tests made by Mr. G. Lobo, Jr.,[1] who first published the fact that the tensile properties of butt welds increase with the thickness of the welded plates, up to plate thicknesses of at least one inch, the specimens were welded by weaving the electrode back and forth across the V, when advancing it along the joint. For thin plates and at the bottom of a V, where the crosswise movement of the electrode is small, this gives in effect a bead laid perpendicular to the axis of stress when the plate is tested. When the width of the groove to be welded became larger, as at the top of the V in a large plate, the weld became in effect a number of small beads laid nearly parallel to the direction of stress when the specimen is tested. Also from tests made from pads of weld metal it is known that the tensile properties of weld metal stressed parallel to the direction of deposition are much greater than those obtained by stressing the weld metal perpendicular to the direction of deposition.

This may partly account for the higher tensile values obtained from large plates, where the weld is, in effect, laid nearly parallel to the direction of stress rather than perpendicular to the direction of stress as on thin plates. Also, in large welds the possibilities of better fusion and operating conditions are prevalent. Even though the residual stresses are higher in large specimens, they have little effect on the strength obtained from static tests and it is possible that the advantages obtained from the above items more than overcome them and any other inherent weakness that might be found in large welds.

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[11] Detailed Description, *Power*, July 23, 1929.

[1] G. Lobo, Jr., "Strength of Butt Welds," *Jour. of American Welding Society*, April 1929.

In fatigue tests on large specimens, the directional effect produced by the manner in which the weld metal is deposited and the possibilities of better fusion and operating conditions also have a beneficial effect on the endurance limit, but the high residual stresses and possible structural changes become of great importance and probably more than counteract this beneficial effect.

Another series of fatigue tests which is mentioned and is of importance is that on the different methods of fixing H-beams into the hubs of rotors. Tests on joints similar to those illustrated in the paper are being continued in order to obtain numerical values for comparing the different types. The importance of these tests is quite evident for such joints can be applied to all types of structures and bridges as well as to electrical machinery.

In these tests, a slight digression was made from the conventional fatigue tests (complete reversal of stress) for joints of this type would not be subjected to such conditions when in service. The method being employed in making the tests is to subject the test specimens to varying initial bending loads upon which is superimposed a constant cyclic load. Thus by varying the initial bending loads on the specimens, a critical initial bending load for each type of joint can be obtained. By this is meant an initial static bending load of such a value that greater static bending loads will produce fatigue failure and static bending loads equal to or less than this will not produce fatigue failure when subjected to a superimposed cyclic load of the value used in the test.

With this information, it is possible to obtain actual values of comparison between the various types of joints by comparing the initial bending loads that can be applied without producing fatigue failure.

These few facts are mentioned as they are directly connected with the work mentioned in Mr. Stone's and Mr. Ritter's paper and show the kind of work that has been and is being done in order that welded designs such as they have mentioned are possible.

J. C. Lincoln: (communicated after adjournment) This paper is of considerable interest from an academic point of view, and while the conclusions do not entirely coincide with those of other noted experts, the information given is of undoubted value.

As a matter of commercial application, however, in general the design is determined not from the fatigue limitation but from the amount of metal which must be used in order to make the part machinable rapidly without undue trouble from bending. As a rule, if stiffness and strength sufficient for this are present, they are in excess of the quantities needed for commercial operation.

The authors' recommendation regarding annealing brings up a moot point. Their success with this treatment is not universally conceded. Many other manufacturers with a longer experience state with equal fervor that annealing is of no commercial benefit. As a matter of fact, very few structures which are fabricated by arc welding are now annealed.

The application of any process of this kind is carefully considered by the engineer. It is customary to build the first few structures in the light of this engineering, but after the design has become common commercial necessity or practise, very often the design is changed materially, and in spite of the objections of the engineer it frequently develops that many of the troubles which he predicted failed to materialize.

Arc welded structures in many cases have already gone through this period and the tens of thousands of manufacturers who are using this manufacturing tool, frequently without any engineering, generally with only the faintest conception of the stresses involved, have been able to produce commercially satisfactory results. In doing this they use only the rule of thumb method of making the new design half the section of the previous cast structure.

J. G. Ritter: Mr. Lincoln states that intensive design is unnecessary in making welded structures; in our own plant and in many other large plants the design of welded apparatus has passed the stage of converting from castings. It would be very impracticable in designing a new machine to first make it cast and then make the welded structure half as heavy, as Mr. Lincoln suggests.

It would be ridiculous and unnecessary to anneal all stuctures, but the experience of "most large manufacturers" has proved that annealing is essential in certain rigid structures subjected to dynamic stress conditions.

Mr. Jennings discussion of the relation of fatigue to size of welds is well taken. Most of the data on fatigue of welds have been obtained from built up structures rather than simple welds and much more work should be carried out in testing of the fundamental welds.

Tap Changing Under Load
for Voltage and Phase-Angle Control

BY HOMER B. WEST[1]

Associate, A. I. E. E.

Synopsis.—One of the most important, and sometimes most involved, problems brought about by the many interconnections of electrical systems, is a means of accurate and flexible regulation of power flow. The adjustment of voltage and the control of wattless current by means of tap-changing-under-load equipment has become quite extensive. The regulation of power flow by means of phase-angle control has been put into practise on a limited scale. One means of accomplishing this, which has been used in a few cases and which is being considered on a larger scale, is the adaptation of tap-changing-under-load equipment with properly connected static transformers.

The purpose of this paper is to discuss briefly the connections employed and the equipment used in connection with static transformers for the purpose of voltage and phase-angle control.

INTRODUCTION

THE rapidly increasing number of interconnections between power systems is making possible the shifting of loads from one system to another in either direction in response to the diversity of system loads, and is resulting in improvements in the quality of service and in economies due to savings in plant capacity. The extent of interconnections can be appreciated when one realizes that the aggregate transformer capacity now installed with adjustable voltage ratio under load amounts to more than five million kv-a. A large percentage of this equipment is used for system interconnections.

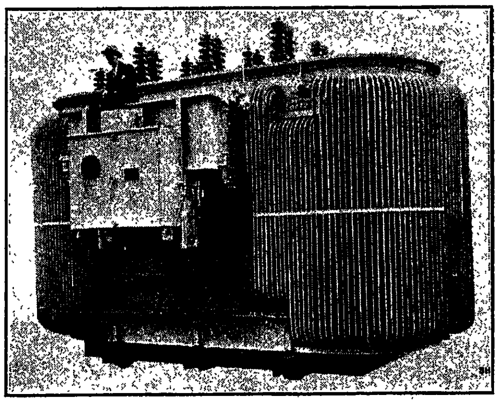

FIG. 1—SEPARATE REGULATING TRANSFORMER COMPLETE WITH TAP-CHANGING-UNDER-LOAD EQUIPMENT FOR 11 PER CENT VOLTAGE REGULATION ON A 50,000-KV-A., THREE-PHASE, 24,000-VOLT CIRCUIT

These interconnections are presenting the problems of interchange of wattless kv-a., which involve voltage control, and the problems of controlling the power flow in definite paths through networks, which involve both voltage and phase-angle control. The purpose of this paper is to show the connections employed and to describe the equipment used for this purpose.

VOLTAGE CONTROL

When two systems are tied together at one point and have no other interconnections, a difference between the two system voltages at the point of interconnection will cause wattless kv-a. to flow from the system with higher voltage to the system with lower voltage. Transformers with equipment for changing the voltage ratio under load are used to compensate for the difference between system voltages and control the interchange of wattless kv-a.

Fig. 1 is an interesting example of tap changing under load equipment for this application. In addition to system interconnections, many other applications for tap-changing-under-load equipment for voltage control have been found. Examples of such applications exist where the equipment is used to compensate for line and transformer drop in transmission circuits, to maintain the voltage at secondary distribution busses, between predetermined limits, and in the supplying of power to synchronous converters and furnace transformers. An example of the latter application is the use of an 8120-kv-a., 25-cycle, single-phase, 12,000-volt furnace transformer with a low voltage range of 60 to 30 volts in 64 steps of less than one-half volt each, obtained by the "UB" equipment later described in this paper.

The needs of many users of voltage ratio adjusting equipment are usually met with a range of 10 per cent adjustment above and below normal voltage. Now and then a greater range is needed, but equipment for more than 10 per cent regulation up and down is becoming more rare. The manufacturer's practise with tap changing equipment seems to be resolving itself into the building of a standardized equipment, giving a total voltage range of 20 per cent, with 10 per cent regulation above normal in four steps, and 10 per cent below normal in four steps. If a greater ratio adjustment than 10 per cent is needed, and it can be used in four steps up and down, the standardized equipment may be used by providing suitable taps in the transformer winding.

There are three fundamental methods of changing the voltage ratio of transformers under load: (1) by induction regulators or step induction regulators; (2) by

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[1] Transformer Engineering Department, Westinghouse Electric & Manufacturing Company, Sharon, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 27-31, 1930.

the parallel-winding method; and (3) by the single-winding method.[2]

1. Induction regulators are generally used on low-voltage distribution circuits of small capacity. Step induction regulators, Fig. 2, are used to obtain small increments of voltage between taps, but since expensive equipment is involved, they are seldom used except for special applications.

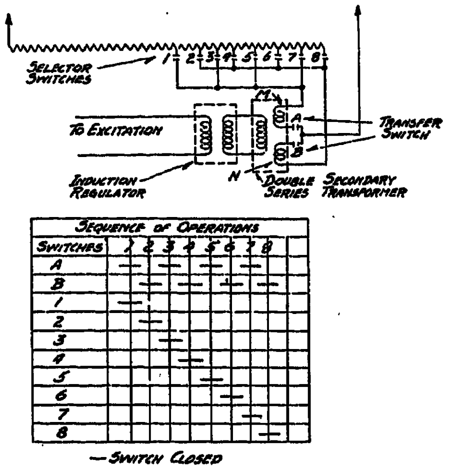

Fig. 2—Development of Transformer Winding and Sequence Chart for Step Induction Regulator Equipment

2. Fig. 3 is a development of the parallel winding method. This method is now obsolete because, (a) it is not economical from the design point of view; (b) the multiplicity of taps usually makes it necessary to place part of the mechanism inside the case; (c) it requires a complicated sequence of operations; and (d) part of the winding is overloaded during transition periods.

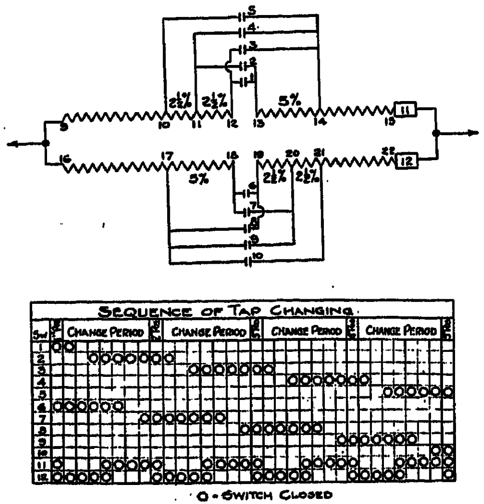

Fig. 3—Development of Transformer Winding and Sequence Chart for the Parallel Winding Method of Changing Transformer Taps under Load.

3. One single-winding method, known as the multiple circuit method, which is an outgrowth of the parallel-winding method, is shown schematically in Fig. 4. The principal difference between this and the parallel-winding method is that, instead of two parallel windings, a multiplicity of double taps on a single winding is used. Also, a separate auto-transformer is used during the transition period to control the circulating current, whereas in the parallel-winding method the inherent impedance between the two windings served this purpose.[3]

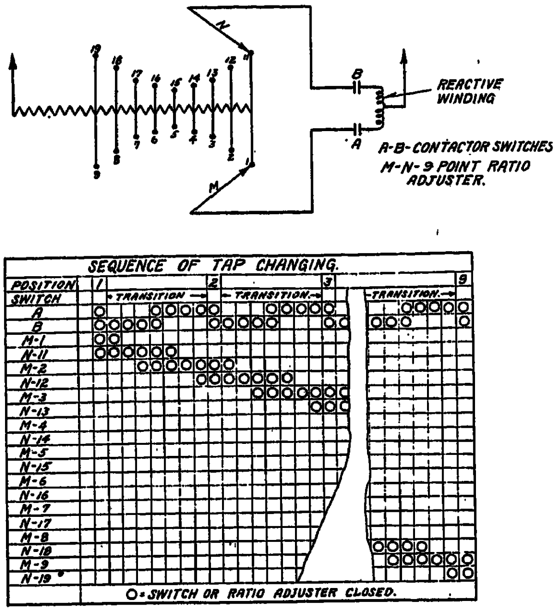

Fig. 4—Development of Transformer Winding and Sequence Chart for the Multiple Circuit Method of Changing Transformer Taps under Load

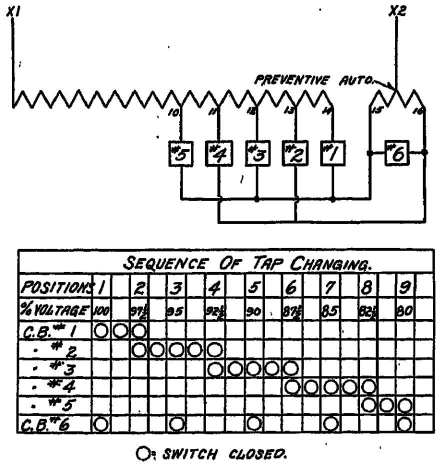

Fig. 5—Development of Transformer Winding Showing Taps, Arrangement of Switches, Bridging Auto-transformer, and Sequence Chart for the Simple Single-Winding Method of Tap-Changing-Under-Load Equipment

A simpler single-winding method which is later described, is shown schematically in Fig. 5.

The single-winding method has outlived the parallel-winding method and is now being used by all leading manufacturers. As a consequence, this paper deals principally with the single-winding method of tap changing and describes the equipment involved, in which every opportunity for simplicity in design has been used.

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[2] L. H. Hill, A. I. E. E. Trans., Vol. XLVI, 1927 p. 582.

[3] L. F. Blume, *General Elec. Rev.*, March, 1928, p. 123.

The simplest method of changing the voltage ratio of a transformer is to provide taps in the transformer winding and a bridging auto-transformer to be switched along the taps as is shown in Fig. 5. A range of 10 per cent voltage above and below normal, each in four steps, is obtained with this simple arrangement.

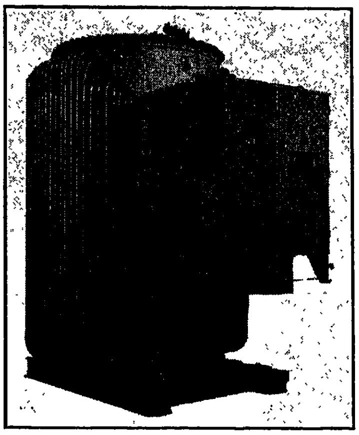


FIG. 6—FRONT ELEVATION VIEW OF TRANSFORMER WITH THE TYPE "UB" TAP-CHANGING EQUIPMENT

To obtain the full winding of the transformer in the circuit, Fig. 5, switches 1 and 6 are closed. The circuit is then through the full transformer winding and divides through the preventive auto-transformer, one-half being through one side of the auto-transformer and one-half being through the other side in the opposite direction. The voltage of the transformer is therefore the voltage induced in the entire winding. To change taps one step, switch 6 is opened and 2 is closed. This connects the auto-transformer across the two taps, and, since the line lead is attached to the center of the preventive auto-transformer, the line voltage comes the same as it would have been had the line lead been attached to a tap midway between the two actually brought out.

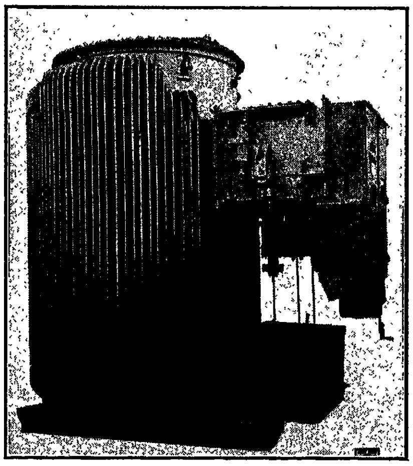
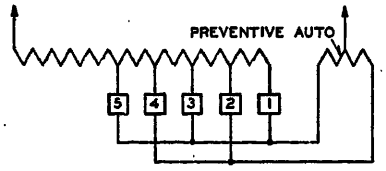


FIG. 7—SIDE ELEVATION VIEW OF TRANSFORMER WITH THE TYPE "U B" TAP CHANGING EQUIPMENT

Similarly, to change taps still further, the process is completed in the same manner as indicated in sequence of tap changing in Fig. 5. This method of switching produces the whole range of nine voltage ratios with only five tap leads and six switches. Each operating position is equivalent to an exact tap position. Figs. 6 and 7 show front and side elevation views of a transformer with this single winding method of tap-changing-under-load equipment.



	SEQUENCE OF TAP CHANGING								
POSITIONS	1	2	3	4	5	6	7	8	9
SW-1	O	O							
2		O	O	O					
3				O	O	O			
4						O	O	O	
5								O	O

O = SWITCH CLOSED

FIG. 8—DEVELOPMENT OF TRANSFORMER WINDING AND SEQUENCE CHART FOR SINGLE-WINDING METHOD OF TAP-CHANGING-UNDER-LOAD EQUIPMENT IN ITS SIMPLEST FORM

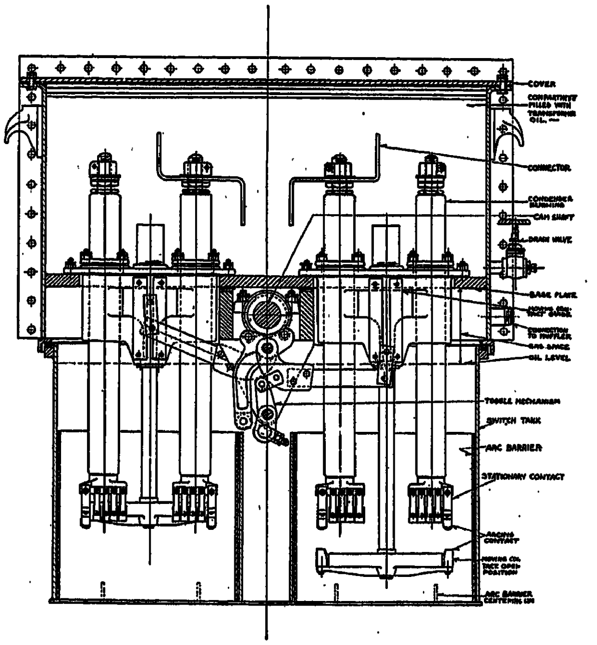


FIG. 9—CROSS-SECTION ELEVATION OF THE TYPE "UB" TAP-CHANGING EQUIPMENT

Fig. 8 is a schematic diagram of a further simplification of the single winding method of tap changing under load with a bridging auto specially designed to meet two different operating conditions. When it bridges across two taps, it operates simply as an auto-transformer. On the alternate tap positions half its winding is connected in series with a tap from the transformer winding and it functions as a reactor. Under the latter condition, it introduces into the circuit a small out-of-phase voltage vector. The special design features of the auto-transformer are devoted to keeping its voltage low when it acts as a reactor. At 100 per cent power factor this vector has no measurable effect on the voltage of the transformer because it is practically in quadrature. At lower power factors it has more effect because it swings around more nearly in phase with the delivered voltage. Its maximum effect would be at practically zero per cent power factor. With power factors that are found in service, the reactor effect of the auto-transformer is of no practical importance.

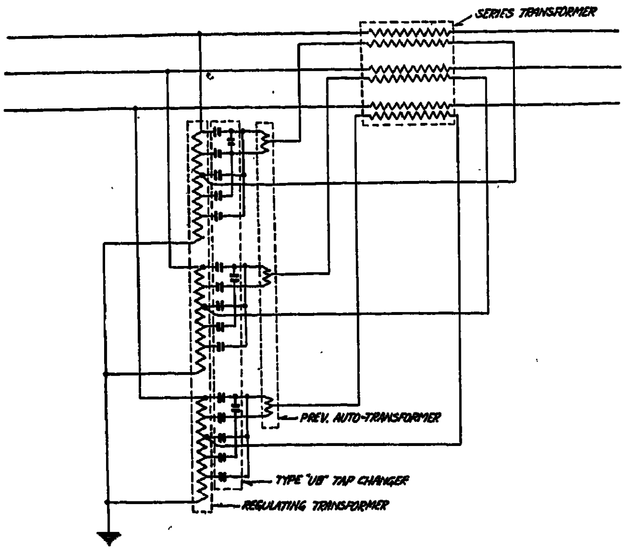


Fig. 10—Separate Regulating Auto-transformer with "UB" Tap Changing Equipment for Changing Taps under Load for Voltage Control

The switches, operating cams, driving motor, and other parts that make up the complete tap changer shown in Figs. 6 and 7, are all placed together in a housing mounted upon the side wall of the transformer case. The tap leads from the transformer winding pass directly through the tank wall into this housing, where they connect to the switches. The compartment enclosing these leads and connections is oil filled, and the switches themselves operate in oil.

In the development of tap-changing-under-load equipment, it was found much better, as a means of simplifying the scheme of connections and operation, to place all parts of it outside the transformer case. Power transformers, as they are built at present, are at best rather complicated affairs, and keeping mechanisms and moving parts out of the main transformer case helps considerably in the effort to keep them as free from complications as possible. With the type shown, only a few leads need be carried through the tank wall. The net result of the combination of transformer and tap changer is one that represents the utmost simplicity and reliability. A cross-section drawing, illustrating the arrangement of the switches and the mechanism that operates them from the cam shaft, is shown in Fig. 9.

Like most tap changing equipments operative under load, the equipment is designed to be operated by motor from a distance, but it can also be operated by hand when desired. The remote control is accomplished at the switchboard with a single switch used either to raise or lower the voltage. As soon as the mechanism starts to move in response to the switch, the control is taken away from the operator and the mechanism automatically completes the change of taps. Indicating lamps and position indicators at the switchboard complete the equipment.

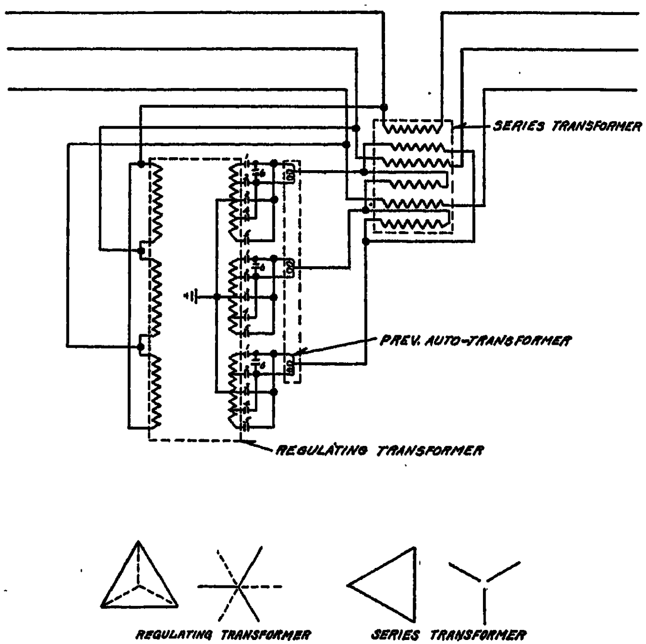


Fig. 11—Separate Two-Winding Regulating Transformer with "UB" Tap-Changing Equipment for Changing Taps under Load for Voltage Control

When the tap changer operates directly in the winding to be regulated, the switches used for changing taps must be insulated for the voltage of the particular part of the winding in which they operate and be capable of carrying the winding current. Sometimes it is cheaper to use a regulating transformer connected as shown in Figs. 10 and 11, with a rating of only the kv-a. of the control required. The voltage and current may be jointly adjusted to give the minimum switchgear cost. For transformers already installed, it is customary to use a tap changing equipment to vary the voltage of a separate regulating transformer in this manner. The regulating transformer may take the form of an auto as shown in Fig. 10, or of a two-winding transformer as shown in Fig. 11. The adjustable voltage is applied to a series transformer connected in the line whose voltage is to be regulated. The principle of changing taps and the equipment are the same as used for new transformers built for the purpose.

Phase-Angle Control

If one of two interconnected power systems is in turn connected to a third power system without forming a closed loop, the problem is still one of voltage control because the division of actual load is controlled through governor speed adjustments.

If two or more interconnected systems or parts of one system form a closed loop, Fig. 12, conditions arise which cannot be corrected simply by voltage control and by governor adjustments. The governors in each generating station can be adjusted to control the power output from that particular station. The closed power loop, however, forms two paths between a generating station and the load at points around the loop. The tap-changing-under-load voltage control equipment can be adjusted to obtain voltages of equal magnitude at a particular location in the power loop, but these voltages may not necessarily be in phase. For example, if the line is opened between stations II and III, Fig. 12, and the voltage E_3 is measured on the station III side and the voltage E_2 is measured on the station II side, there may be a voltage E_z between E_2 and E_3 across the open breakers. The dotted extension of E_2 indicates how the voltages can be adjusted to equal magnitude by means of tap-changing-under-load equipment for voltage control only, but a quadrature voltage E_Q still exists across the breaker, because E_2 and E_3 are not in phase. If the tie line be connected to form a closed ring, current would flow from E_2, the line with leading voltage, to E_3, the line with lagging voltage.

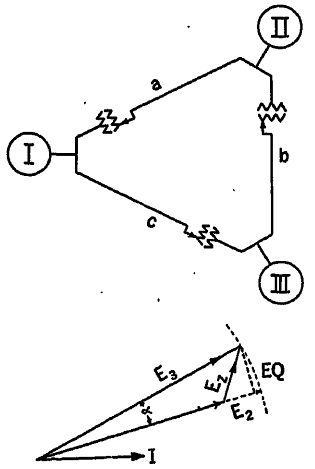


Fig. 12—Three Power Systems Interconnected to Form a Power Loop and Vector Diagram Showing the Conditions which Exist when the Line is Opened between Systems II and III

If a voltage in quadrature with one of these delivered voltages, E_2 for example, is added in the line to compensate for E_Q, the two voltages E_2 and E_3 can be brought into phase with each other. The division of load can be adjusted between the two parallel paths of the loop by adding in-phase components and quadrature components of voltage to compensate for the impedance voltage in one of the parallel paths of the loop.

The voltage relations, of course, would change with a change of load. The added quadrature voltage should, therefore, be available in adjustable form.

This is a simple illustration of the possibilities of phase-angle control. With a ring system involving several generating stations and loads supplied at various points, the problem is more involved. However, if in-phase and quadrature voltages are supplied to the ring to neutralize the resultant impedance voltage of the entire system for a given set of load conditions, the way is cleared for a flexible and an economic distribution of current throughout the ring.

Quadrature corrective currents and voltages may be obtained with synchronous condensers and polyphase induction regulators as well as with static transformers. This paper, however, is devoted only to the use of static transformers for the purpose.

A quadrature voltage may easily be obtained in a three-phase system with transformers. If voltages from phases *B* and *C* of the star-connected regulating winding in Fig. 13 are connected in series, the resultant voltage, *a*, will be displaced 90 deg. from the voltage of phase *A*. With suitable transformer connections it can be superimposed upon the voltage of phase *A*.

In the case of a separate delta-connected regulating transformer feeding a series transformer to add the quadrature voltage to the line, the secondary winding of the regulating transformer is connected in star as in Fig. 14. The voltage from neutral to *C* of this winding, for instance, is displaced 90 deg. from phase *A* to neutral of the main system.

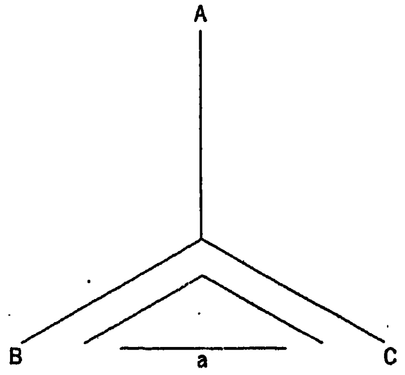


Fig. 13

Equipment which has been developed for adjusting the voltage ratio of power transformers without interrupting the load may be adapted to the control of phase angle without any modifications in the mechanism; it may be used for this purpose also without disturbing the continuous flow of power.

For phase-angle control, a quadrature voltage of

10 per cent will cause a shift of phase angle of approximately 6 deg. Twenty per cent voltage in quadrature will shift the phase about 11 deg., and 36 per cent is required for a phase shift of 20 deg.

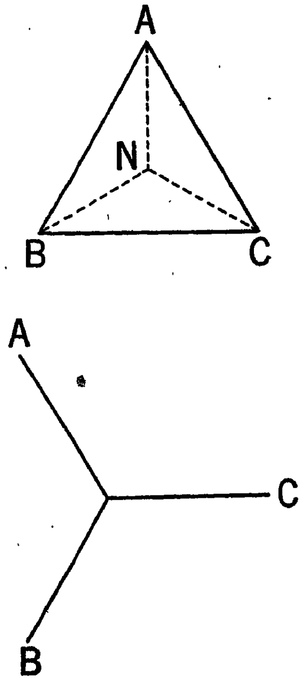


Fig 14.

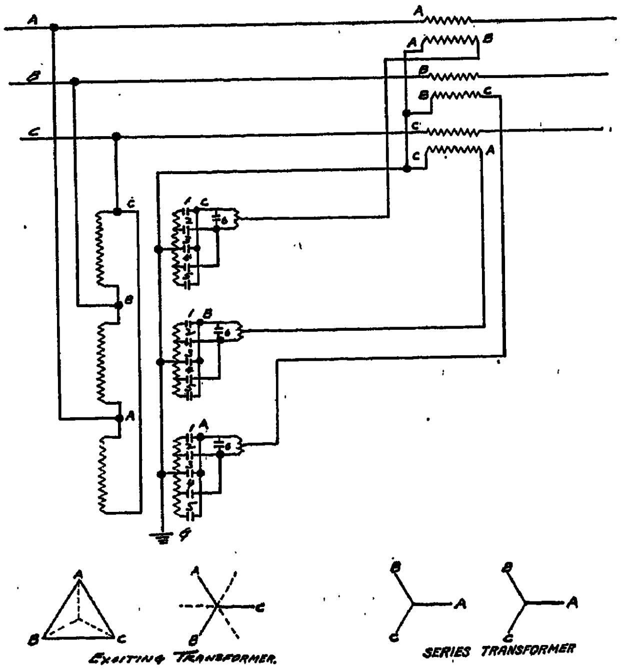


Fig. 15—Separate Two-Winding Regulating Transformer with "UB" Tap Changing Equipment for Phase-Angle Control

Voltage adjustment and phase-angle control may be combined in the same transformer. Separate switching mechanisms and separate series transformers are, of course, required in order to get the separate adjustments. Figs. 15, 16, and 17 show typical schemes of connections for phase-angle control and for combined ratio and phase-angle control.

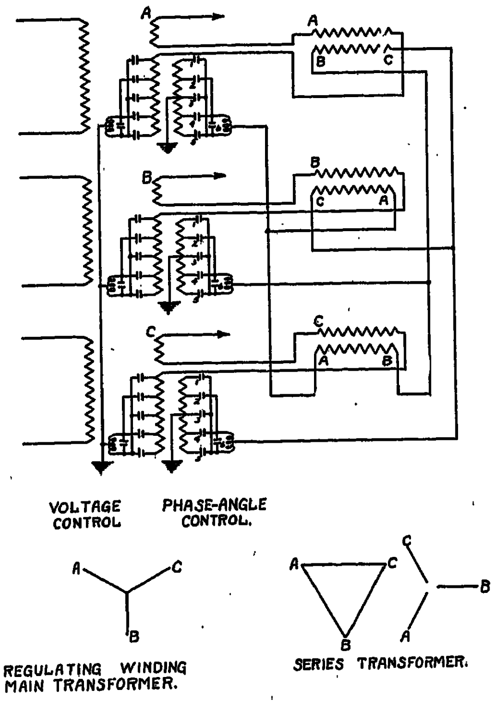


Fig. 16—Transformer Bank with Regulating Winding and Two "UB" Tap Changing Equipments for Combined Voltage and Phase-angle Control

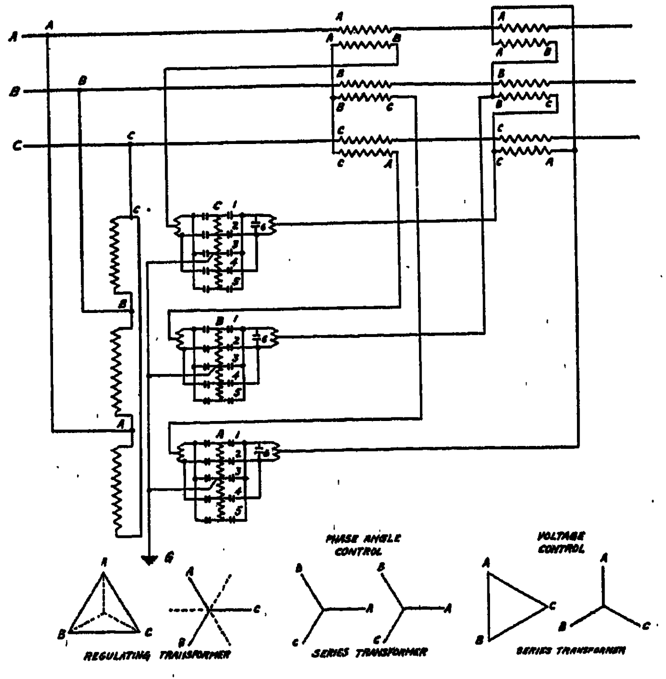


Fig. 17—Separate Two Winding Regulating Transformer with Two "UB" Tap Changing Equipments for Combined Voltage and Phase-angle Control

The greatest problem in connection with phase-angle control with static transformers is not the equipment, for it is available in simple and reliable form. It lies rather in the calculation of the amount of phase shift required to get the operation that is wanted for the varying conditions under which load is delivered. In the case of complicated interconnections with several generating stations and various load centers with variable distributions of load, it is not altogether a simple matter to determine just what shift of phase will produce the best net results.

It would seem that the static transformer with phase-angle control equipment will eventually occupy an important place in the progress of interconnections.

Discussion

J. S. Lennox: Previous to 1922, equipment for controlling the ratio of transformers under load was rather limited in its application and confined largely to the method shown in Fig. 8 of the paper. Since that time the development has been rapid as evidenced by the first part of the paper showing the evolution in methods used by one manufacturing company.

Probably the most outstanding improvements have been in the mechanical design. Modern equipments are mechanically driven throughout, thus eliminating all necessity for electrical interlocks and other similar protective devices.

The mechanical equipments fall naturally into two classes, one in which the tap connections are changed and the current in the reactor circuit broken by the same contacts and the other in which the functions of changing connections and rupturing the current are handled by separate devices developed for each purpose. In the latter case, only two switches are required, one at each end of the reactor.

The first of these types of equipment is only suited for use with electrical circuits which permit bringing all the tap connections out of the transformer, as obviously no switching operations can be permitted in the transformer oil.

The second type is more flexible in its applications but advantage of it is usually taken to place the ratio changing device in the clean transformer oil adjacent to the windings.

In this connection, Mr. West says "in the development of tap changing under load it was found much better to place all parts of it outside the transformer case." I should like to ask Mr. West what operating experiences forced him to this conclusion. So far as I am aware experience covering seven years on a rapidly increasing number of equipments totaling well over two hundred to date has not resulted in a single failure of the parts placed inside the transformer. What few failures have occurred have been confined to bushings or equipment outside of the transformer though damage to the transformer windings may in some cases have resulted.

Most of the electrical circuits shown in the paper can be used with either type of mechanical equipment though some such applications would be quite costly. Fig. 8, for instance, lends itself most readily to the use of the first type of equipment because of the small number of switches required whereas either type can be applied economically to Fig. 5. On the other hand, the second type with ratio changing and current interruption performed by separate devices is best suited to Fig. 4.

I should like to mention one winding arrangement not illustrated in the paper that has come into extensive use in the last three or four years. Electrically it is similar to Fig. 5 of the paper if operation is confined to positions 1, 3, 5, 7, and 9, wherein switch No. 6 is closed, thus short-circuiting the preventive reactor. To obtain nine operating positions it is necessary to have nine taps as the bridging positions are not used.

Considering the sequence chart of Fig. 8, it will be seen that the preventive reactor is either in series or in parallel with the system on every position. In Fig. 5 it is in parallel on alternate positions but never in series. The arrangement referred to is a still further refinement in which the reactor is short-circuited on all positions.

Due to the increased number of taps, this method can be used economically only with the first type of mechanical equipment up to five or possibly seven ratios. For nine or more ratios it is necessary to use the second type of equipment with the ratio changing and the switching functions separated. The additional cost involved in such an application is to a considerable extent overcome by the possibility of quantity production of the various elements and by the fact that the preventive reactor is only one half the size called for in Fig. 5.

PHASE ANGLE CONTROL

As shown by Blume (A. I. E. E. May 1927) if the relation X to R is alike for all portions of the loop the currents will divide to give minimum losses. Where this is not the case it may prove more desirable to readjust the lines by the use of reactors than to employ phase angle control. Certainly it would be difficult, if not impossible, for an operator to control the currents in a loop of any complexity to obtain minimum line losses.

Assuming properly balanced lines, complete flexibility of loop operation, at minimum line losses and including local distribution at fixed voltage from each station bus, can be obtained by installing power factor control on the transformers connected to the loop.

In such a case, however, the power factor or in-phase control would be located between the stations and the loop rather than in series with the loop as shown in Fig. 12.

Operation at minimum losses, however, may result from overloading transformers or cables where such are in series with the loop. Phase-angle control, possibly accompanied by power-factor control, may in such cases offer a more satisfactory solution than adjustments by means of series reactors.

A recent interesting shipment of ratio control covered two large series regulating transformers with the windings so arranged that by a simple change in connections they can be converted from phase angle control to power factor control, thus permitting their use in any manner required by future conditions at the point of application.

F. W. Gay: One would gather from reading Mr. West's paper that the effect of the auto-transformer or reactor is not important. I should like to take exception to this statement. There are three possible types of auto-transformer equipment for tap changing purposes, as follows:

1. The auto-transformer is designed with a saturating magnetic circuit. When operating as an auto-transformer with power supplied to the start and finish of the winding and power taken off the mid tap, the transformer is operated at the knee of the saturation curve. During tap changing there is a brief period during which one-half of the turns of the auto-transformer are in circuit functioning as a reactor. The voltage across the active turns will be approximately 50 per cent greater than the voltage across these same turns when functioning as an auto-transformer. It is therefore evident, if the power factor is low and it is desired to raise the voltage, say 2½ per cent from a tap on the main transformer, that when one-half of the auto-transformer is cut into circuit as a reactor, the voltage will drop 2½ per cent, or more, depending upon whether the power factor is moderately low or very low and the voltage will jump approximately 5 per cent as the winding turns are switched from reactor functioning to auto-transformer functioning. If a purchaser is going to use a tap changing transformer on a low power factor load and is determining on the size of the tap changing steps on the basis of permissible flicker voltage, he should note that the incremental jump in voltage actually obtained on this type of equipment will be approximately twice that of the tap change.

2. In the second type of tap changing auto-transformer an air core auto-transformer is used, having a very high magnetizing current. This auto-transformer when functioning as an auto-transformer is generally designed to take approximately 100 per

cent magnetizing current. If the transformer taps are spaced at 5 per cent intervals, the magnetizing current of the main transformer will be increased approximately 5 per cent when the auto-transformer is functioning as such. The voltage across half the turns of the air core auto-transformer, when it is functioning as a reactor, will generally be less than half the voltage across the saturated iron core reactor just discussed.

3. The third type of tap changing equipment uses auto-transformer with closely interlaced windings having a very low magnetizing current. This auto-transformer is normally shunted by a reactor, having a circuit breaker in series. The circuit breaker is closed at all times except when the auto-transformer is functioning as an auto-transformer across the main transformer taps. In this scheme the magnetizing current drawn by the auto-transformer, when it is functioning as an auto-transformer, is substantially negligible, generally being less than one-tenth the magnetizing current drawn by method one and only two or three per cent of the magnetizing current under method two. The reactance that such an arrangement inserts in the circuit when operating on a main transformer tap should not exceed a quarter of one per cent. However, during the instant of change from full tap to half tap the magnetizing current drawn by the reactor in shunt with the auto-transformer will be approximately five times as great as that drawn by method two.

To sum up, the operating man will do well to remember that no matter what method is used in changing from tap to tap, the effect of the auto-transformer is by no means negligible unless the power factor closely approximates 100 per cent. That method of tap changing should be chosen which best suits the system.

I should like to suggest a possible winding connection for transformers which may be found very convenient for use in conjunction with phase changing equipment.

It is well known that a phase changing equipment to care for much more than fifteen electrical degrees becomes very expensive. I would suggest that each phase of one of the transformer windings be provided with two coils for series multiple connection with 86 per cent taps in each coil. Such a winding may be connected parallel star or series delta. This special winding should be provided on the side not equipped with voltage taps. With such an arrangement on a transformer interconnecting two systems, any phase relationship may be obtained in steps of thirty electrical degrees; in other words, it is possible to meet the requirements of any system with which interconnection is to be made within fifteen electrical degrees.

E. K. Huntington: I should like to discuss Mr. West's paper from the standpoint of its application to systems connected with transformers which are largely for supplying power between these systems; in other words, a superpower connection. Oftentimes these connections normally carry very little power and therefore they may be operating at quite a low power factor. Under these conditions the operation of tap changing equipment at low power factors is quite important.

In considering the purchase of a tap-changing equipment on our system in Rochester, we already had installed a 20,000 kv-a. transformer bank connecting our system with the Niagara 110-kv. system. In applying tap-changing equipment to this transformer, we found that it would be necessary apparatus which would give us a variation of possibly 10 per cent in voltage range. This was determined by shifting the present taps on the high side of the transformer. A shift of 2½ per cent in voltage taps gave us a change of about 3000 reactive kv-a. in the tie between the two systems. This ratio of course varies with different kinds of load on the system but a fair average is 1000 reactive kv-a. for each per cent of voltage tap change.

It is possible, of course, to limit the size of the regulating transformer required by the use of the taps on existing transformers for seasonal changes.

In regard to the dip in voltage which occurs during tap change, it is quite essential that this be determined in advance, so that every time a tap is changed on the transformer a surge will not be created on the system which will be reflected in the system voltage. The amount of this surge is of course entirely dependent upon the design of the preventative auto-transformer connected between transformer taps. The design of this auto-transformer should be one that is adapted to the particular application of the tap changing equipment. This is something that is quite essential if we are going to avoid voltage dips during tap changing, particularly if such changing is made automatic and occurs at frequent intervals.

M. S. Schneider: (communicated after adjournment) The Cincinnati electrical load is supplied partially from Columbia Power Station, which is located approximately twenty miles from the city. The energy from this station is transmitted to the Cincinnati district over four 66-kv. circuits which are connected to the district's 13.2-kv. network by means of suitable transformer substations.

In addition to the four 66-kv. circuits, a 33-kv. circuit connects the Columbia Power Station with the Cincinnati district. This 33-kv. circuit supplies load to consumers located between the station and the city. To provide flexible voltage control on this circuit a three-phase voltage regulating transformer is connected on the 13.2-kv. side of a 15,000-kv-a., 13.2/33-kv. transformer bank at the Columbia station. This transformer bank supplies the 33-kv. circuit.

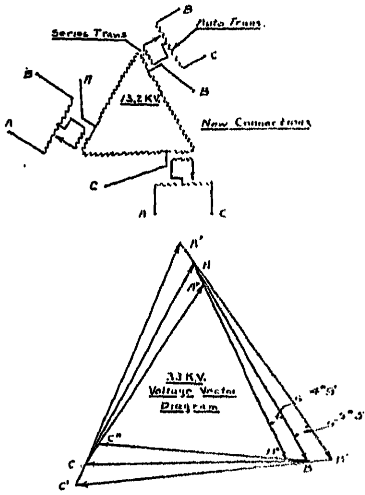


Fig. 1

A, *B*, *C*, input voltages
A′ B, *B′ C*, *C′ A*, full boost voltages
A″ B, *B″ C*, *C″ A*, full buck voltages

During 1928 and 1929, when plans were made for increasing the capacity of the Columbia Power Station from 90,000 to 126,000 kw., it was realized that the division of load between the four 66-kv. circuits and the 33-kv. circuit would be such as to seriously overload the latter. It was necessary, due to load conditions, to keep this circuit energized from the Columbia Power Station and also from the Cincinnati 13.2-kv. network. In order to limit the current flow over the 33-kv. circuit the regulating transformer was installed as previously explained.

This three-phase regulating transformer was designed for insertion in the delta of the 13.2-kv. circuit of a 15,000-kv-a., 13.2/33-kv. transformer bank. It was used to regulate the voltage of the 33-kv. output of this bank under load giving four 2½ per cent steps above and below normal.

The connections between the regulator and main bank were later changed so as to obtain a combined regulation of voltage and phase angle. With the new connections the 33-kv. output of the main bank could be increased 5.44 per cent in voltage and at the same time retarded 5 deg. 54 min. in phase, or the voltage could be reduced 4.61 per cent and the phase advanced 4 deg. 9 min.

Figs. 1 and 2 herewith show original and final general connections and also 33-kv. delta voltages before and after the change. The change in connections was easily made and subsequent tests showed that the kv-a. load was reduced 31 per cent. This was the result desired. Before the transformer was installed the 33-kv. circuit was opened frequently each day at a consumer's plant located at approximately the mid point of the circuit. These oil circuit operations were due to the excessive current being supplied over this circuit from the Columbia Power Station to the Cincinnati district, and they became so frequent that it was necessary to open the circuit at the Cincinnati end at certain times of the day, whenever the load attained certain values, in order to prevent their occurrence. After the regulating transformer was installed, the load was reduced sufficiently so that the oil circuit-breaker operations due to load transfer were eliminated. The customers along the route of this circuit were not affected in any manner.

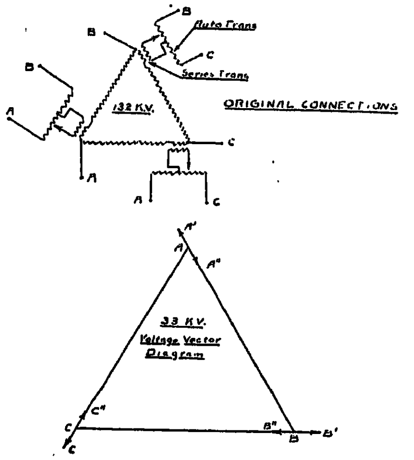


FIG. 2

A, *B*, *C*, input voltages
A B′, *B C′*, *C A′*, full boost voltages
A B″, *B C″*, *C A″*, full buck voltages

After the connections of the regulating transformer had been changed the kv-a. load supplied over this circuit could be regulated within the limits of the transformer. The regulation of voltage causes a variation in the reactive load and at the same time the varying of the phase angle changed the power load. The original connections permitted voltage regulation only, and consequently variation only of reactive load.

H. B. West: I shall refer first to comments by Mr. Lennox. Tap changing under load, as applied to power transformers, is comparatively new, being only six or eight years old. Some of the earlier equipments used parallel winding schemes, as shown in Fig. 3 of my paper. However, experience has shown that the single winding method is much simpler. Therefore, the use of schemes shown in Fig. 5 and Fig. 8 has followed the use of the parallel winding scheme, and they are being used today after the evolution in methods as pointed out by Mr. Lennox. The scheme shown in Fig. 4 is an outgrowth of the parallel-winding method, and while it is a single-winding method, it does not take advantage of all the possible simplicity of the single-winding method.

Mr. Lennox states that only two switches are required for such a scheme as that shown in Fig. 4, but if one refers to the sequence of operation, he will readily see the extra complication over and above that involved in the use of schemes as shown in Figs. 5 and 8.

Power transformers today, even at their best, are somewhat complicated because of high voltages and multiplicity of windings, sometimes three and sometimes four. The designer, therefore, does not care to complicate the design any further, and because of the simplicity in design and operation, all parts of tap changing under load equipment have been kept outside of the transformer tank.

Mr. Lennox referred to two figures in my paper, stating that the number of steps was limited somewhat either to nine or eleven positions, depending upon whether the scheme shown in Fig. 5 or that shown in Fig. 8 was used. This is correct. In making a survey of the range required of the equipments now in service, we have found that a 10 per cent range plus and minus will serve the needs of practically all cases. We have, therefore, standardized on equipment to give this range in voltage in 2½ per cent steps. There have, of course, been some special cases where we have supplied tap changers to give more positions, but in such cases, as well as in those cases where a smaller number of taps is required, we always keep all parts of the tap changing equipment outside of the transformer tank.

In the scheme shown either in Fig. 5 or in Fig. 8, fewer taps are required in the transformer winding than in the scheme discussed by Mr. Lennox, and the operation of the tap-changing equipment is simpler.

It is realized that other methods may be used in phase-angle control, but this paper has been devoted to transformers and tap changing equipment used for that purpose, because it would seem that they will eventually occupy an important place in such applications, because of their simplicity, reliability, and flexibility.

Referring to Mr. Gay's comments, I would estimate that approximately half of the equipments supplied by one manufacturer has been with the scheme shown in Fig. 5, and half has been with the scheme shown in Fig. 8. At 80 per cent power factor, if the voltage between taps in the transformer winding is 5 per cent the steps in voltage using the scheme shown in Fig. 5, would be 2½ per cent, and the steps using the scheme shown in Fig. 8 may be of the order of 1½ per cent and 3½ per cent for alternate steps instead of each step being 2½ per cent.

It may be well to point out that it will take only two seconds to complete the operation in going from one tap position to another, and this means that any dip in voltage that occurs in the transition period would be of a smaller duration. It would seem that voltage dips of the order actually obtained, in a period of such short duration, would not be of much consequence.

Quiet Induction Motors

BY L. E. HILDEBRAND[1]

Member, A. I. E. E.

Synopsis.*—Magnetic noise in induction motors is caused by the vibration of magnetic parts produced by the varying forces associated with the changing flux density in the adjacent air parts of the magnetic circuit. Torsional vibration of the motor is caused by unbalanced windings or applied voltages. Vibration of the rotor simulating a mechanically unbalanced rotor is caused by two fields differing by two poles and not of the same frequency or direction of rotation. Vibration of the stator by distortion from a circular shape results from the presence of other simple combinations of fields. Teeth tend to vibrate axially at double applied frequency.*

The magnetic noise produced by a motor can be predicted from a qualitative analysis of the air-gap field supplemented by an approximate quantitative analysis and consideration of the stiffness of the parts.

INTRODUCTION

DUE probably to the previous lack of engineering importance, there is very little information on the subject of noise in electrical machines readily available in our technical literature. When mentioned at all, it is usually covered by a few general, superficial, and, frequently, inaccurate statements. It has been found that these general statements can be made precise by building up an exact theory from elementary fundamental principles.

A motor is a source of sound because some part is moved periodically by a periodically varying force. This axiomatic statement is made to emphasize the fact that the study of noise is a detailed exact study of the parts which can vibrate and the forces which move them.

Magnetic noise (so called) is that in which the forces are those associated with the varying magnetic flux density. We can limit the study to the vibration of magnetic parts produced by the flux in the adjacent air parts of the magnetic circuit. The conductors themselves can usually be neglected as a noise source as the forces on them, being situated in the relatively weak leakage field, are small. Likewise we can neglect whatever internal stress may exist in the magnetic parts as the rigid parts are not deformed appreciably.

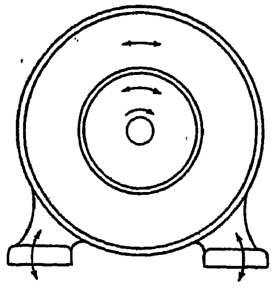


FIG. 1—TORSIONAL VIBRATION PRODUCED BY TWO FIELDS WITH LIKE NUMBER OF POLES BUT DIFFERENT ANGULAR VELOCITIES

The noise producing parts can vibrate in several ways. A torsional vibration of the stator and rotor as a whole results from periodic torque pulsation. Such a vibration may be particularly objectionable as it is transmitted directly through the motor feet to the supporting structure. (See Fig. 1.) Due to the reaction of unsymmetrical supports torque pulsation may also distort the stator from its circular shape. A circular vibration simulating a mechanically unbalanced rotor as shown in Fig. 2 results from an unbalanced magnetic pull. This unbalanced force may revolve in either direction and at other than synchronous or rotor speeds or it may even be stationary. If due to the use of skewed members the phase of the force is not the same for all axial positions it will tend to shake the rotor as well as to bend the shaft, *i. e.*, it simulates two unbalanced weights with different axial and angular positions. The stator may be deflected, as shown in Fig. 3, into a rotating elliptical shape. Similarly, the forces may be balanced on more than two radii resulting in a tendency toward a rotating polygon. The laminations and particularly the teeth may vibrate axially. Experience indicates that teeth seldom if ever vibrate tangentially. The rotor may vibrate axially as a whole, causing noise by striking the bearings.

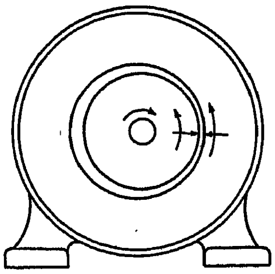


FIG. 2—ROTOR VIBRATION SIMULATING AN UNBALANCED ROTOR

Most vibrations are forced; *i. e.*, their frequency is not near a critical frequency. In this case the vibration is proportional to the force and inversely proportional to the rigidity of the parts. However, since there are several causes for flux density variation and hence several frequencies of applied forces, there is considerable chance of encountering a critical vibration in which a small force produces a large deflection.

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[1] Electrical Engineer, General Electric Company. Lynn, Mass.

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MAGNETIC FORCES

At every point on the air-gap surface there is a directed varying force proportional to the flux density squared. The summation of the tangential components of force is the total torque. The radial components according to their configuration tend to distort the parts.

As is well known and contrary to elementary text book theory, the air-gap field is at no instant distributed according to a simple sinusoid and the distribution changes as it rotates. This is equivalent to saying that in addition to the fundamental field there are other fields with other than fundamental number of poles and some with other than fundamental frequency.

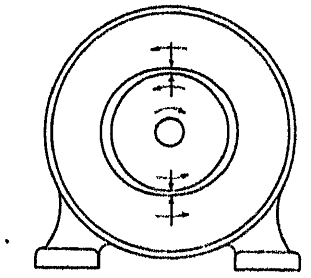

FIG. 3—ELLIPTICAL VIBRATION OF STATOR

It is evident from consideration of energy that torque pulsations will result from the presence of two fields having the same number of poles and rotating with different velocities. The worst case and possibly the only one which is a serious noise factor is that in which there is a backward revolving field produced by single-phase operation, unbalanced applied voltages, or unbalanced windings.

It has been shown by Fritze[2] and also by Chapman[3] that two fields differing by two poles produce an unbalanced magnetic pull as illustrated in Fig. 2. In Fig. 4 instantaneous values of the radial forces are plotted for a field having a two-pole and a lesser four-pole component to show the superposition of this unbalanced radial pull on the high harmonic force components. The shape of this force curve changes as the fields rotate but there is always present the same radial unbalanced component which may be stationary but which usually rotates. The noise produced is like that of a motor with an unbalanced rotor revolving at the speed of rotation of the unbalanced component.

It can also be shown (see Appendix A) that the combination of two fields differing by four poles or a single two-pole field tends to distort the parts into a rotating ellipse. In general the combination of two fields having poles whose sum or difference equals any given number g, or a single $g/2$ pole field tends to distort the stator or rotor into a polygon having $g/2$ sides and usually rotating.

There is always present a double line frequency axial vibration of teeth and punchings best corrected by tight clamping and impregnation. Frequency is double because the forces are proportional to the flux density squared.

The method of determining the noise tendency of a motor is to tabulate qualitatively all of the possibly existing fields and examine for two fields differing by two poles, four poles, etc., or two fields with the same number of poles revolving at different speeds. The disturbing pairs of fields are examined more critically and roughly quantitatively with regard to magnitude, frequency, and rigidity of parts. With these data the quality of the finished design becomes dependent upon the skill of the designer in combining parts to obtain results which can be predicted rather than the chance results of applying less fundamental rules.

It is beyond the scope of this paper to make a thorough study of the air-gap fields except perhaps to enumerate the order of the fields probably existing and the factors upon which their magnitude depends in order to point out how the several design features are related to magnetic noise. The fields existing in an induction motor air-gap may be roughly classified with regard to source as follows.

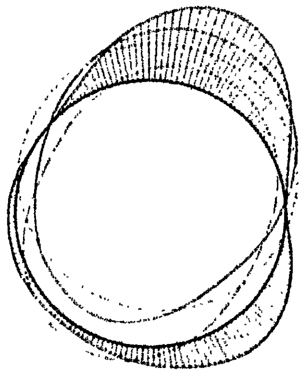

FIG. 4—RADIAL FACE PRODUCED BY THE COMBINATION OF A TWO- OR FOUR-POLE FIELD

Heavy circle is reference zero line
Dotted curve is $\sin x$
Dash curve is $\frac{1}{2} \sin (2x - 15 \text{ deg.})$
Full line curve is flux density, $\sin x + \frac{1}{2} \sin (2x - 15 \text{ deg.})$
Radially shaded area represents radial force, $[\sin x + \frac{1}{2} \sin (2x - 15 \text{ deg})]^2$

1. Fundamental for which motor is designed.
2. Stator magnetomotive force harmonics.
3. Stator magnetomotive force sub-harmonics.
4. Rotor magnetomotive force harmonics.
5. Rotor magnetomotive force sub-harmonics.
6. Gap eccentricity permeance variation.
7. Stator slot permeance variation.
8. Rotor slot permeance variation.
9. Permeance variation due to saturation.
10. Slot pattern permeance variation.

As is well known with a symmetrical winding, the magnetomotive force in addition to the fundamental contains harmonics of order, $N = (\pm)(2 \phi g \pm 1)$

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[2] *Archiv fur Elektrotecnik*, Vol. 10, pp. 73-95, July 23, 1922.

[3] *I. E. E. Jl.*, Vol. 61, No. 313, pp. 39-48, Dec. 1922.

where N = order of harmonic, ϕ = number of phases, and g = any positive integer. The sign indicates the direction of rotation. In a squirrel-cage winding the number of phases equals the number of bars per pair of poles and produces the corresponding harmonic magnetomotive force. A non-symmetrical winding produces other fields having fewer than fundamental number of poles. The magnitude of the magnetomotive force waves is inversely proportional to the order, is proportional to the current in the winding, and is dependent upon the well known pitch and distribution constants. The flux field produced is approximately proportional to the magnetomotive force and inversely proportional to the air-gap unless there is a closed secondary circuit to such a field in either winding. It should be recognized that while the fundamental flux is proportional to the magnetizing current the harmonic flux is proportional to the total current if there is no suitable closed low impedance secondary circuit. In the latter case the flux is greatly reduced.

In addition to these so-called harmonic fields, the presence of slot openings, and other irregularities in the air-gap introduce other disturbances in the flux equivalent to introducing new harmonic fields. These may be called permeance fields. If there are m equally spaced irregularities, the combination of the fundamental magnetomotive force with $2\,N_o$ poles and the m'th order permeance variation introduces new fields, the largest having $2\,(N_o + m)$ and $2\,(N_o - m)$ poles. The important permeance fields are given by m equal to one for eccentricity, m equal to the number of stator slots, m equal to the number of rotor slots, and m equal to the difference in numbers of stator and rotor slots. The frequency may be different from fundamental and the fields may rotate in either direction. The magnitude of the field is approximately equal to the magnitude of the disturbance in the even gap permeance; *i. e.*, the amount of eccentricity or the effective slot opening.

TESTS

Some 1500 motors of about 200 distinctly different designs have been tested with regard to noise. These are assorted over a range of ¼ to 15 hp., 25 to 60 cycles, two- and three-phase, different types of squirrel-cage rotors including single and double windings, slip-ring motors, all overhung slot riveted frame construction. In many cases a motor of a particular rating has been made and tested changing pertinent factors one at a time. As the volume of test results is entirely too large for adequate presentation the following general digest is offered.

No case has yet been found entirely inconsistent with this theory. Only a few cases have been observed noticeably inconsistent. With few exceptions the observed noise has agreed very closely and in detail with that predicted by the methods here presented. The agreement has been particularly good in those cases where a design has been modified to correct an existing noise tendency and in those cases where the design has been changed one factor at a time.

Before presenting any illustrative examples, a description of the method used for measuring noise is in order. It has been found that the human ear and judgment is adequate to measure the disagreeableness of noise if assisted by a standard for comparison. With this standard at one end and the motor being tested at the other end of a room designed with such absorption characteristics as to simulate the absence of walls an observer midway between the two noise sources is impressed equally by them if they are of the same degree of disagreeableness. This method is essentially analogous to the photometer. Contrary to the general impression, the personal error is small and measurements are consistent and adequately precise. In the absence of a commercially recognized unit an arbitrary scale has been used. The scale is an arithmetical progression of disagreeableness such that the noise of the quietest individual commercial motor is recorded as one and that of the noisiest commercial motor about five.

The average of 28 tests on six different sizes of four-pole, three-phase, squirrel-cage motors with 36 stator slots, 7/9 winding pitch and 45 rotor bars, gives a noise reading 1.7 at no-load and 2.4 at full load. The increase in noise with load less rapidly than the primary current is increased would suggest as a cause the combination of primary harmonic fields and permeance fields except that some of the individual motors presumably with even gaps test 1 at no-load and 3 at full load which clearly points to a secondary harmonic field for at least one of the principal noise producing fields. The 23rd stator harmonic with 96 poles and only slightly damped by a 45-bar rotor winding, cooperating with the 94-pole rotor harmonic, the two differing by two poles, constitute the required condition for rotor vibration. The unbalanced pull is proportional to the product of the magnitudes of the two fields. As at no load the rotor harmonic field is zero, the motor is quiet. Under loaded condition both disturbing fields increase and hence the noise rapidly increases.

The use of a 44-bar rotor changes the 94-pole rotor field into one with 92 poles differing by four poles from the 23rd stator harmonic. This constitutes the condition for a rotating elliptical vibration of the stator. However, as the four-pole yoke section is for other reasons quite rigid there should be a distinct improvement. Tests have been made on 17 motors using this 44-bar rotor covering the same range in ratings but averaging 30 per cent larger in horsepower. The average no load noise is 1.3 and the average full load noise is 2. Also, the distinct difference between no load and full load noise in individual machines has disappeared.

A similar case relates to a group of small eight-pole, three-phase, squirrel-cage motors with 48 stator slots, 5/6 winding pitch, and 57 rotor bars. The average of tests on 74 motors gives a noise rating 1.9 at no-load and 2.5 at full load. In this case the noise is caused by the combination of the 104-pole primary harmonic and the 106-pole secondary harmonic. Changing to 68 rotor bars decreases the noise to 1.2 at no-load and 1.5 at full load as the average of 20 somewhat larger motors.

A group of six-pole, three-phase, squirrel-cage motors with 54 stator slots, 7/9 winding pitch, and 68 rotor bars test consistently 2 at no load and 3 to 4 at full load. The noise is caused by the combination of the 138-pole primary harmonic and the 142-pole secondary harmonic, which differing by four poles, constitutes the condition for elliptical vibration of the stator yoke. Changing to 66 rotor bars gives 1½ to 2 at both no-load and full load.

A group of small eight-pole phase-wound secondary motors with 54 stator slots, 8/9 winding pitch, and 48 rotor slots, 5/6 winding pitch, both wound for three phase, was tested with a poor unsymmetrical primary grouping such as to introduce a backward revolving fundamental field and both two- and four-pole sub harmonic fields with their harmonics. The combination of sub harmonic fields differing by two poles caused very bad rotor vibration. The use of a larger shaft reduced the noise. The arrangement of primary groups was next changed to eliminate the backward fundamental and the two-pole sub-harmonic fields but leaving the four-pole sub-harmonic field. Using the original small shaft, the noise was reduced to 4. In this case the disturbing factors are closely associated with the fundamental field and its harmonics differing by four poles from the four-pole sub-harmonic field and its harmonics. The rotor vibration was practically eliminated but the stator vibration was still present. The stator was next made with a symmetrical winding in a 72-slot stator with a reduction in noise to 2.

Several summarized general statements can be made based on this method of noise prediction and confirmed by tests.

An eccentric air-gap is always a serious noise source as it introduces two new fields with two more and two less than the fundamental number of poles and thus tends to cause rotor vibration. In squirrel-cage motors with few poles these fields are partially damped by secondary currents. An increased air-gap usually decreases noise at the expense of power factor as the permeance variations are decreased and also harmonic fields are less since if undamped they are proportional to the total current in the winding and inversely proportional to the gap.

With otherwise equivalent design a motor with a phase wound rotor is usually more noisy than a similar squirrel-cage machine. The slip ring motor generally has fewer rotor slots with wider slot openings and hence the permeance variations are greater and of a lower order. In a slip-ring motor choice of number of rotor slots is more restricted. A squirrel-cage winding damps many more fields than a polar winding. In a slip-ring machine secondary harmonic and sub harmonic fields are frequently more serious.

Noise is reduced by lower flux density if it is caused by fundamental or permeance fields. In this case the forces are proportional to the square of the fundamental flux density. Other fields depend upon primary or secondary ampere turns and may be decreased or increased by changed fundamental flux density.

The use of skewed rotor slots is an effective means for correcting noise due to high order fields. It can be shown that the skew should be equal to an even number of poles as referred to one field of the pair which is the most serious noise producer. A skew of two poles is reasonably effective although four poles is very much better. This desirable skew is usually equal to the pitch of one or preferably two rotor slots.

As until recently more than ordinary quiet operation has been a secondary consideration and as standard parts must be satisfactory for many ratings, there exists in every manufacturer's line a variation in quietness of particular ratings and a lesser variation in individual machines. This will always be the case as the features which make the quietest machine are frequently incompatible with other desirable features.

SUMMARY

As a general rule subject to the usual imperfections of generalized statements, the avoidance of a backward fundamental field and pairs of fields of appreciable magnitude differing by two or four poles is conducive to quiet operation. Gaps must be even and concentric. Parts must be rigid and punchings tightly clamped. As a rule subject to imposing unnecessary design restrictions and possible introduction of undesirable torque characteristics a motor will be quieter if the numbers of poles, stator slots, rotor slots, difference of stator and rotor slots, and winding repetitions bear simple multiple relationship and the motor is quieter the larger the greatest common divisor of these factors.

Appendix A

The radial component of air-gap flux density may be expressed in wave equation form as a series of trigonometric functions of angular position and time.

$$\beta = \Sigma \beta_k \cos (k x - l t + \theta)$$

The summation is made for the several necessary values of k and l. $2 k$ is the number of poles of a given sinusoidal component, $l/2 \pi$ is the equivalent frequency, and θ is a general phase difference angle.

Assuming that the radial force is proportional to the square of the radial component of flux density,

$$F = (1/8 \pi) [\Sigma \beta_k \cos (k x - l t + \theta)]^2$$

This force is a function of angular position and time and can be expanded into a series of sinusoids. Let F_{Mc} be the coefficient of the M'th space order cosine term in the expansion of F.

$$F_{Mc} = (1/8 \pi^2) \int_0^{2\pi} [\Sigma \beta_k \cos(k x - l t + \theta)]^2 \cos M x \, d x$$

Expanding the integrand, the general term is

$$\beta_1 \beta_2 \cos (k_1 x - l_1 t + \theta_1) \cos (k_2 x - l_2 t + \theta_2) \cos M x$$

in which k_1 and k_2 may be the same or different.

$F_{Mc} = 0$ unless $k_1 \pm k_2 \pm M = 0$ in which case

$$F_{Mc} \, \alpha \, \beta_1 \beta_2 \cos [(l_1 \pm l_2) t + \theta]$$

The sign of l_2 is the same as that of k_2 which makes $k_1 \pm k_2 \pm M = 0$

Likewise, if F_{Ms} is the coefficient of the M'th space order sine term in the expansion of F,

$$F_{Ms} \, \alpha \, \beta_1 \beta_2 \sin [(l_1 \pm l_2) t + \theta]$$

Thus $$F_M \, \alpha \, \beta_1 \beta_2 \cos]M x - (l_1 \pm l_2) t + \theta]$$

For $M = 1$, *i. e.*, with two fields differing by two poles there is a force simulating a mechanically unbalanced rotor revolving with speed $l_1 - l_2$ tending to displace the rotor or stator as a whole. For $M = 2$, *i. e.*, with two fields differing by four poles there are forces balanced across any diameter, rotating with speed $\frac{1}{2} (l_1 - l_2)$ tending to distort the rotor and stator into a rotating ellipse. Also a single two-pole field produces the same kind of force rotating at speed l. Carrying out the suggested mathematical deduction in detail shows that in this case the proportionality constant is one-half that in the case of two fields with unlike numbers of poles. Similarly, with two fields differing by any number of poles or a single field of half that number of poles the rotating forces tend to deform the stator and rotor into a rotating polygon of like number of sides. Forces are proportional to the product of the absolute values of the offending fields.

When the vibration is produced by a single field the noise frequency is twice the frequency of the field. When the force is produced by two fields their frequencies or direction of rotation must be different to cause noise producing vibrations. With two fields revolving in the same direction the noise frequency is the difference in frequencies of the two fields. If the fields revolve in opposite directions l_2 is inherently negative and the noise frequency is the sum of that of the two fields. Since most fields are either exactly or nearly fundamental frequency the most common noise frequency is exactly or nearly double applied line frequency.

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Discussion

H. C. Specht: The problem of building induction motors with a minimum of noise has caused the engineer a great deal of worry. In spite of all the theory that is known to us, keeping the undesirable noise out of the motor still remains a difficult art. I agree with Mr. Hildebrand that the unbalanced forces caused by harmonic fields are about the most troublesome factors. Already Dr. Stiel, H. Fritze, and F. T. Chapman have shown this by a very detailed analysis and Mr. Hildebrand by his analysis, arrives at the same results. If this were the only cause of magnetic noise, it would seem to be a rather simple problem to design quiet motors. For ordinary service where the quietness of operation is not of so great importance, this rule alone may be sufficient. However, in recent years there have come up applications, particularly for small motors, where an almost dead silent operation is demanded. For such motors we must take into consideration many other factors that enter into the problem of noise.

One factor, and a very important one, is the skew of either the stationary or rotating member. In order to minimize noise and to obtain uniform starting torque in all rotor positions, most small induction motors are built with a skew. Unfortunately there seems to be no exact theory available as to what degree the rotor should be skewed. Mr. Hildebrand makes a statement in his paper that the skew should be equal to an even number of poles of those harmonics which are the most serious noise producers. I wonder if this can be accurately proved by an analytical theory. So far, all the theory on harmonics is based on straight cores.

I do not quite agree with the author that the teeth vibrate only axially and seldom tangentially. In my opinion this depends very much on the width and length of the teeth. It has been my experience that short, wide teeth give less noise than long, narrow ones. Since the laminations were of the same thickness the axial vibration would not change materially; therefore I assume it can be only the tangential and radial vibration. No doubt the zigzag leakage in induction motors has a great bearing on the noise. The vibration of the teeth as well as the tooth tips may be tangential as well as radial. I have tried motors without dipping them in varnish or bakelite so that the laminations in axial direction could move fairly freely. However I have noticed only a slight difference compared with motors that were dipped and baked having a more rigid body. This seems to me also somewhat of a proof that the axial vibration of the teeth is bothering only a little, if at all. However, axial vibration of the rotor may be of a serious character if the laminations are not built up accurately at right angles to the axis or if the core is skewed considerably. Furthermore, we assume in our theory that the air-gap is always uniform; as a matter of fact, this is seldom true and particularly not when the surface is not ground to size. It should be remembered that the air-gap in induction motors is very small, varying in size from 0.010 in. upwards, depending on the size and speed of the motor. As a rule, manufacturing tolerances of 0.002 of an inch or more have to be allowed; this is certainly a great percentage of the specified air-gap. On motors with a larger number of poles this is not so serious but, for example, on a four-pole motor, which is about the most common case, the unbalanced pull due to this irregularity would be quite considerable.

Another important factor in the problem of noise is the mechanical construction as a whole and the mounting of the motor. If there are periods of natural frequencies that are in resonance with any frequency of the magnetic vibrations, the noise may be amplified to a very objectionable degree. For example, a thin cast iron housing, round in shape, with end brackets of a bell shape, will have periods of natural frequency which very often happen to be in the order of the magnetic noise frequencies. If this is the case, the original noise may be amplified many times. On the other hand, if the motor is to be mounted on a construction which has a natural frequency somewhere near the various frequencies of the vibrations in the motor, the noise may be increased terrifically. As an example, we may take this case of a blower motor for an oil furnace; in this case the motor is usually connected directly to the oil burner structure. It is surprising how noisy a motor, which on the test bench in a noise-proof room was found to be very quiet, would sound running mounted on an oil burner. The same difficulty was experienced with motors for household refrigerators and therefore most of the manufacturers have adopted flexible mounting, using springs or material of a flexible nature. For such quiet operation it will also be necessary to keep the flux density in the gap and iron parts very moderate.

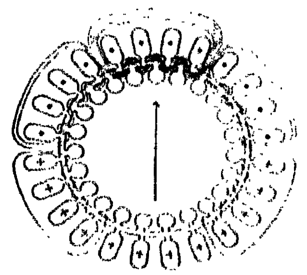
There are many more factors that have to be considered in order to obtain real quiet motors but the few points I have brought out may be sufficient to show that the problem of noise will remain a very difficult one. Nevertheless, this should not discourage us, for by carrying on the study we may succeed some day in predetermining the quietness of a motor. Also, it seems to be very desirable to have more papers on the subject of noise brought before the Institute in the future for discussion.

R. E. Hellmund: A careful study of Mr. Hildebrand's paper reveals that the paper covers in terse statements, either directly or by inference, a very large percentage of all the causes of noise in motors as well as the factors entering into the problem. For this reason, the paper is a very valuable contribution to the art.

Somewhat contrary to Mr. Hildebrand's opinion, I am rather inclined to believe that the scarcity of papers on this subject is not so much due to the fact that the matter has not been held of sufficient importance in the past, but rather to the fact that this problem of noise is a very difficult one as far as correct analytical determinations are concerned. It is true that the requirements for quiet operation of machinery are becoming more rigid right along; nevertheless, noisy motors have been a problem with the designers of motors for a great many years. In addition to the many factors referred to in the paper, others can be mentioned, an example being the great influence of the weight and mass of the parts, a point scarcely touched upon in the paper. The principal difficulty is that many of the factors covered in the paper by one or two sentences would require for their complete analysis calculations and explanations filling several pages.

One fact that makes the analytical study of noise in the induction motor particularly difficult is that the fundamental fluxes, as well as certain of the harmonic fluxes and their influence upon noise, are considerably affected by the damping actions of the secondary, while certain other harmonic fields, especially those caused by zigzag leakage, are not subject to any damping effects. This makes it necessary to study very carefully not only the causes and characteristics of the various fields but also the damping effect to which they are subjected. This may be readily demonstrated by discussing a few simple examples.

Let us consider first the simple case of a two-pole motor and assume that it has a squirrel-cage secondary with very low resistance. In this case, with the motor running, all harmonic fields caused by improper chording of polyphase windings or improper distribution of single-phase windings, that is, all differential leakage fluxes, are damped out. Furthermore, the secondary will damp out any irregularities in the fluxes which might be caused by irregularities of the air-gap. As a consequence, there will be no unbalanced magnetic forces on the rotor from such causes, and therefore no noise from these causes. This is, of course, under the assumption that the main flux is not able to distort the rotor elliptically, an assumption which is usually correct with the very deep and therefore stiff punchings of two-pole motors. The main flux while rotating can, of course, also elliptically distort the stator punchings and thus cause noise. However, here again the relatively deep and stiff punchings of the stator are of considerable assistance in eliminating noise. I do not know of many cases where noise has been traced to the elliptical distortion of two-pole stators in induction motors by the main flux, but I have run across a number of cases where elliptical stator distortion has been responsible for noise in some two-pole induction regulators.



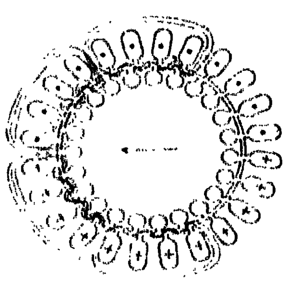
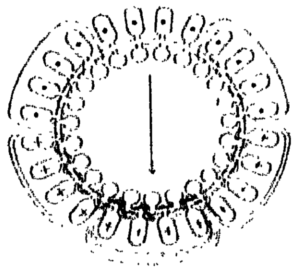


PLATE A

FIG. 1

FIG. 2



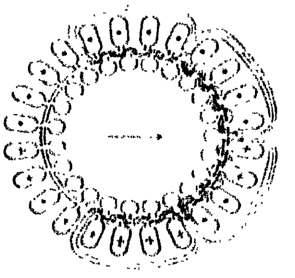


PLATE A

FIG. 3

FIG. 4

As previously mentioned, the damping effect of the rotor does not influence the zigzag leakage fluxes. Figs. 1, 2, 3, and 4, Plate *A*, show these zigzag leakage fluxes for the case of a two-pole motor with one slot less in the rotor than in the stator for four different relative-tooth positions. The four figures show the differences caused by moving the rotor one tooth pitch. It can be readily seen from these figures that the zigzag leakage fluxes are very much unbalanced and that the unbalancing rotates 360 deg. around the circumference and fluctuates in amplitude while the motor moves one tooth pitch. I have found quite a number of cases where this or similar unbalance has caused vibrations and noise in fractional horsepower motors either by a deflection of the shaft or by looseness of the bearing. This condition is especially likely to result in noise with frictionless bearings having slight clearances. A difference of one slot between the stator and rotor has been chosen here as an extreme case. Such a condition is not likely to occur in practise, but similar conditions, although less extreme, may exist with any odd number of slots in the rotor, a construction which has been frequently employed in the past in order to avoid dead points under starting conditions. With the greater masses in large motors, this condition is of course less likely to cause noise. Nevertheless, it is nearly always safer, especially in small motors, to use an even number of slots in the rotor. Figs. 1 and 2, Plate *B*, indicate that in this case the zigzag leakage is always balanced. With uniform air-gap it is of course possible that in both Plates *A* and *B* the zigzag fluxes will bring about distortion in the stator and, especially in Plate *B*, an elliptical distortion would seem possible, assuming that the stiffness and mass of the stator are not sufficient to counteract such tendencies.

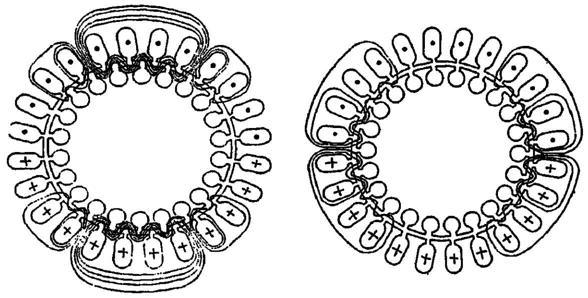


Plate B

Fig. 1 Fig. 2

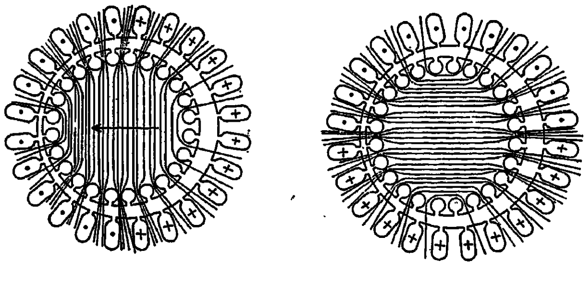


Plate C

Fig. 1 Fig. 2

So far, low secondary resistance has been assumed. As soon as rotors with high secondary resistance are used, certain main flux unbalanced conditions are possible, as well as the existence of winding harmonics or differential leakage fields. Not only will such conditions be influenced by the total secondary resistance, but a certain influence upon the various fields will even be caused by the distribution of the total squirrel-cage resistance between the bars and the resistance rings. It is therefore evident that with partially damped main flux irregularities, partially damped differential leakage fields, and undamped zigzag leakage fields, the final outcome is rather difficult of correct analysis.

As indicated in the paper, with wound secondary the damping effects are again different from and generally not as good as in squirrel-cage motors. On the other hand, certain favorable balancing effects, especially with regard to the main flux, can be obtained by connecting oppositely located stator or rotor winding groups in parallel, and the existence of differential leakage fields can of course be greatly minimized by proper chording and winding distribution.

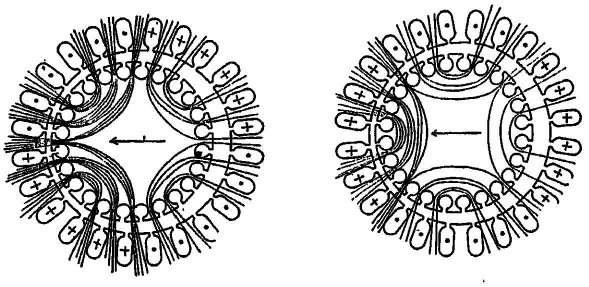


Plate D

Fig. 1 Fig. 2

So far, we have essentially considered a two-pole motor, but several of the statements apply to motors of four poles as well. On the other hand, there are certain differences which must be taken into account. Plate *C*, for instance, shows a two-pole motor with eccentric rotor and no damping effect upon the main flux. It is evident that, no matter how unbalanced the air-gap may be in a two-pole motor, the total flux leaving and entering the rotor on two opposite sides thereof must be the same if the center of the flux is as shown in Plate *C*, Fig. 2. However, with the center of the flux as shown in Fig. 1, certain unbalances are possible. This means that with the eccentric and varying pull on the rotor bearings, there is again the possibility of rotor vibration and noise. In Plate *D*, an equivalent four-pole motor is shown, and in this case the eccentric pull on the rotor is always present and reasonably uniform in the same direction, which therefore means that with a four-pole motor less noise might be expected from this cause.

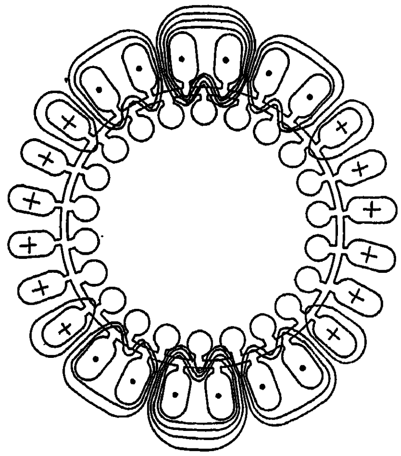


Plate E

The main flux distorting effect on the stator of motors having four or more poles should, of course, be less if we assume the same stiffness, because such distortion is no longer elliptical. On the other hand, the distortion caused by zigzag leakage fluxes may be elliptical, as shown in Plate *E*, for an even number of rotor slots

As in the case of the two-pole motor, the zigzag fluxes, however, are balanced with regard to the rotor and therefore should not cause any rotor vibrations. Plate *F* shows a four-pole motor with one less slot in the rotor than in the stator, and here the conditions are much the same as with the two-pole motor having the same number of slots, indicating the possibility of rotor vibration in small motors.

Examples of the different possibilities can be multiplied indefinitely, but it is believed that the few examples given will indicate the great many possible varieties and give a somewhat clearer picture of some of the conditions to those not familiar with the superimposition of various harmonic fields of different types.

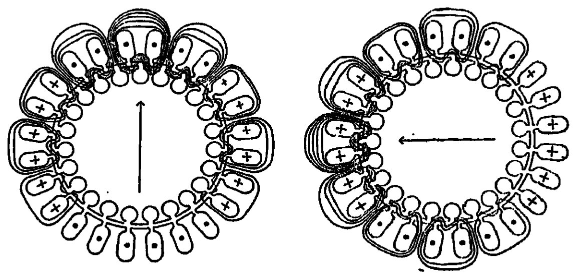


PLATE F

FIG. 1 FIG. 2

While, as pointed out, there are distinct difficulties in completely analyzing noise tendencies, it nevertheless seems desirable to devote a good deal of further attention to this subject and to work up various additional papers and calculating methods. Even data collected relating to the predominating causes of noises in different types and sizes of motors should prove of value as they would reduce the analytical work necessary for the various ranges of size in individual types of motors.

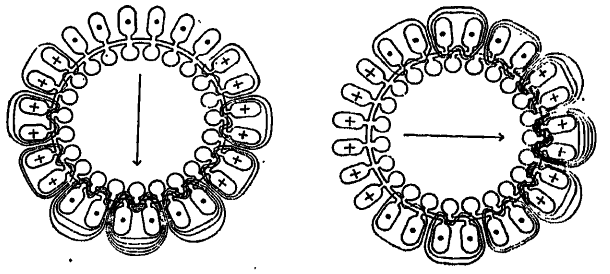


PLATE F

FIG. 3 FIG. 4

R. L. Young: We have been following motor noise characteristics closely for the past 20 years or more. It seems obvious that when a number of machines ranging perhaps up to 75 hp. are to be put in an office near where a wire chief's staff must carry on telephone testing, they must be reasonably quiet. Further, it is often desirable to run these machines all night and some central offices are located in residential districts where a very small noise is noticeable while people are trying to sleep. A great deal of work, therefore, was done in cooperation with the manufacturer during 1911 and 1912 involving trials of skewed slots, reduction of windage, low magnetic saturation, prime rotor slots, and other features. Finally, a very satisfactory line of induction motors was evolved, although there were as many as a dozen different designs of a single size, for example 20 hp., built up before the desired results were obtained. It might be mentioned that these highly satisfactory results were obtained at that time largely through the work of Mr. F. J. Rudd and others in the same engineering organization to which the author belongs.

The difficulty mentioned by the author in securing a suitable method of noise measurement was experienced and finally two or three representative motors were picked out, which were run under specified conditions, as standards of maximum noise. They were polyphase motors run single-phase to make a fair amount of noise, and test inspectors were told that all the motors accepted must sound at least as quiet as these. If the work were being done now, however, I believe we should consider the audiometer recently developed by the Bell Telephone Laboratories which has been used with success in measuring certain mechanical noises. If desired it would appear practicable to make a phonograph record from a typical motor and use this as a standard of comparison in terms of which other motors could be rated, without the necessity of operating the standard motor during tests.

At present the noise situation isn't so acute. With the coming of the dial central offices, which require considerably more power, it is usually desirable to have a separate machine room which is now seldom located in the part of the building where the wire chief has to test or where the commercial department may have offices. Another complication, however, has arisen, resulting from the practise of power companies in gradually putting into effect requirements for higher power factor. Induction motors are likely to draw penalties, since they don't have very high power factors, particularly if run at light load for parts of the day. It is then necessary, in general, to use some synchronous motors to bring the average up to perhaps 85 or 90 per cent.

The synchronous motors have been found to be very much noisier than the specially quiet induction motors formerly used. This is to be expected, since, to mention a single feature, they have salient poles which make a considerable whistle as they pass the ventilating openings, and the design has apparently not received much attention to secure quiet operation. The large increase in noise has led to some unfavorable comment.

This brings me to the question that I should like to ask the author, assuming that the synchronous motor is nearly enough like the induction motor to benefit in some degree by the studies of quiet operation made for the latter. I should like to know what the possibilities are of obtaining quiet synchronous motors in sizes from 25 to 100 hp.

Q. Graham: The methods employed by Mr. Hildebrand in the study of induction motor noise are applicable to other types of machines. I have been interested in similar noise problems in synchronous motors and generators and have found the study of various harmonic fields to be fruitful. In large synchronous machines using fractional slot windings the sub harmonics are particularly troublesome unless care is taken in the selection of the number of stator slots.

As pointed out in the paper a table of the orders of the existing harmonics will show very quickly whether or not the possibility of trouble exists, although a quantitative examination of these harmonic fields is necessary before one can be sure that trouble will occur. It sometimes happens that two sets of forces exist, each of which alone is capable of producing elliptical vibration, although their phase positions may be such that they nearly cancel one another, and a quiet machine results. I have in mind a case in which a two pole sub-harmonic field was present, but the machine was noticeably quiet. Further analysis showed that there was also a pair of fields differing by four poles, whose sum almost completely cancelled the effect of the two pole field. The outcome is less fortunate if the two kinds of fields happen to be in phase instead of being out of phase.

It has been found that the question of *resonance* is important in large machines. Calculations and measurements of the vibrating periods of stator structures have shown that it is quite possible to get natural frequencies in the range of 120 cycles per second, which is the frequency of the major flux pulsations in 60-cycle machines.

Another point of difference between the large diameter multipole class of machines and the smaller motors with few poles and relatively rigid stators is that in the former a force distributed in four poles or even more may give trouble. Mr. Hildebrand points out the source of such forces in his statement covering the force polygon with $g/2$ sides.

One further difference which I believe exists between the synchronous machine and the induction motor noise problem is the absence, in synchronous machines, of the case in which the offending fields differ by two poles. It seems to me that this type of unbalance requires primary and secondary members with different numbers of phases. The synchronous machine, in addition, has another type of magnetic noise not dependent upon stator current which produces sounds in the range of slot frequency.

C. A. M. Weber: The noise problem which is covered in part by Mr. Hildebrand is not a new problem but rather a problem which is receiving more consideration today than formerly because the public is rapidly becoming more noise-minded. Designers have always considered noise problems in connection with their designs but in view of the increasing demand for quieter motors, designers have had, during the past few years, a greater incentive to investigate the question of noise in rotating electrical machinery.

As an illustration of the importance of this matter I wish to mention that the National Electrical Manufacturers Association, at its last annual meeting, appointed a committee to investigate the question of noise in rotating electrical machinery particularly from the point of view of the various instruments and devices available for the purpose of measuring such complex noises as occur in rotating electrical machinery.

Such contributions as the author's paper and the ensuing discussion and future papers on this subject will be of considerable assistance to this committee of the National Electrical Manufacturers Association. As a member of this committee I feel that designers familiar with the noise problem in rotating electrical machinery should be encouraged by the Institute and others to write papers on this subject and to enter freely into discussions of the various phases of noise so that this committee and others may benefit by the many years of experience of designers in reducing the noise in rotating electrical machinery.

C. J. Fechheimer: In the analysis of noise problems of electrical machinery, we must consider three sources:

1. The noise due to windage;
2. The noise due to the main magnetic flux;
3. The noise due to the leakage fluxes.

These can be readily separated in machine. To determine whether it is windage noise, the machine can be run at normal speed without any excitation. For the induction motor it means that the machine is run up to normal speed without any load and then disconnected from the circuit. For the synchronous and other machines, of course, the method is obvious.

The noise due to the main magnetic flux can be determined by exciting the machine without any load. Then, as the load comes on, we can determine whether the noise is increased and if so it would be due to the leakage fluxes.

L. E. Hildebrand: The discussion has indicated that some parts of this paper are too brief. As Mr. Hellmund has pointed out the last three paragraphs of that part of my paper under the general subject of magnetic forces contains either by terse statement or inference a skeleton for the analysis of the air-gap flux which leads to an understanding of the magnetic forces. My method is to use the approximate so-called magnetic Ohm's law to express the magnetomotive force and the permeance each as trigonometric series in wave equation form. The product of these two series gives an indication of the number of poles and equivalent frequency of the principle harmonic fields together with an indication of the magnitude. Of course as has been pointed out both in my paper and by Mr. Hellmund, damping must be considered. Mr. Hellmund's diagrams show pictorially and physically rather than mathematically what these harmonic fields are like.

With regard to Mr. Specht's discussion, his observation of tangential vibration of teeth will lead me to watch more closely for such phenomena. His question with regard to skewed slots may be answered generally by a reference to the force equation in the appendix. If we consider that this equation expresses the force on one lamination, add a phase angle to take care of the other laminations and then integrate the expression axially, the result indicates that the radial force has been broken up into a main force and a couple. The case is exactly analogous to two unbalanced weights instead of one. If the rotor slots are skewed exactly two poles as referred to one of the offending fields, the main force is eliminated but there still exists a couple tending to shake the rotor. If the rotor is skewed four poles both the main force and the couple are eliminated, *i. e.*, the case is analogous to three rotating weights.

A statement has been made with regard to the avoidance of an odd number of slots with which I cannot agree. We may consider a case in which the poles, stator slots, and rotor slots are all divisible by three. Whatever is happening on one radius at one instant is also happening on two other radii spaced exactly 120 degrees at the same instant and the forces are balanced as a "Y" connected system. In this case the number of slots can be odd.

Mr. Young's question with regard to synchronous machines has already been answered in general by Mr. Graham. The same general methods of analysis and detailed study will make a quiet synchronous machine or d-c. motor. The main thing is to do it by detailed analysis rather than by rule of thumb methods.

In reply to Mr. Bela Gati's question regarding the generation of a second harmonic voltage no such phenomenon has been observed. Of course any vibration must produce a voltage distortion but of what magnitude and frequency I do not know.

I am indebted to Mr. Graham and others for pointing out an error in the first example given under the subject of test. The 23rd harmonic is 92, not 96 poles. But the general reasoning is not materially changed. The 23rd harmonic with 92 poles differs by two poles from the 94 pole rotor harmonic field and constitutes a cause for rotor vibration. Changing to 44 rotor slots changes the 94 pole rotor harmonic to a 92 pole field which by cooperation with the 23rd harmonic constitutes a cause for torque pulsation which according to test results certainly is not very serious.

SYMPOSIUM ON LIGHTNING

Lightning on Transmission Lines

Cathode Ray Oscillograph Studies

BY J. H. COX*
Associate, A. I. E. E.
and
EDWARD BECK*
Member, A. I. E. E.

Synopsis.—This paper is a review of the work done and results obtained in Norinder-type cathode ray oscillograph studies of lightning on transmission lines during 1929. The various installations and methods of test are described. The wave shapes recorded are illustrated and their meaning discussed.

The paper also contains a preliminary description of an artificial lightning investigation, using a portable 1,000,000-volt surge generator and a portable cathode ray oscillograph. The methods used and some of the preliminary results are discussed.

* * * * *

THE summer of 1928 saw the first concerted effort, likely to bring results, to determine the exact characteristics of lightning disturbances on high-voltage transmission lines.[1,2] This was made possible by the development of the cathode ray oscillograph, the only available device for making complete records of voltage transients of the speed of lightning surges, to the automatic stage so that it will respond to and record transients whose time of occurrence cannot be foretold. An outstanding contribution in this application of the cathode ray oscillograph was made by Dr. Norinder, whose development permits the oscillograph to be energized prior to the time of an expected transient and which acts as its own relay to record any chance lightning surge in its entirety. This instrument was briefly described at the Winter Convention of 1928.[1] During the lightning season of 1928 the Westinghouse Company, in cooperation with the Aluminum Company of America, set up and operated two stations with such oscillographs on the 154-kv. Cheoah-Alcoa transmission line in Tennessee. One oscillographic record of a natural lightning transient was secured in this preliminary investigation. This first year's work proved the adequacy of this type of recording apparatus and laid the foundation for further effort in 1929.

In 1927 preliminary work had also been done in the investigation of the behavior of transients on a transmission line using a surge generator, cathode ray oscillograph and a five-mile, 22-kv. idle line loaned by the Duquesne Light Company.[3] This work also has been continued on a much greater scale on a 220-kv. idle line with a one million-volt portable surge generator and a portable cathode ray oscillograph of the Norinder type.

The complete lightning problem on transmission lines may be divided into three phases. The first is the nature of natural lightning surges as they occur on lines, and this must be determined by collecting data on the characteristics of this phenomenon by means of automatic cathode ray oscillographs connected to lines during storms. The second is the behavior of such transients on transmission lines and is to be determined by applying controlled surges to a line from a surge generator and measuring them under various circuit conditions with cathode ray oscillographs. This method of studying the second phase of the problem is made necessary by the infrequent occurrence of natural lightning transients. The third phase of the problem is the impulse characteristics of insulation, apparatus, and protective devices used on transmission systems. This, of course, is best determined by laboratory studies with a high-voltage surge generator and a cathode ray oscillograph.

1929 LIGHTNING PROGRAM

Laboratory investigations with surge voltages have been in progress for several years, as has been the study of lightning on transmission lines by means of the klydonograph. In 1927 and 1928 the field studies of lightning and controlled surges was given greater impetus by the use of the cathode ray oscillograph. To complement the laboratory work with adequate field activity on the problem, the following program of field lightning investigations during the season of 1929 was entered into with the cooperation of the operating utilities mentioned below:

1. Aluminum Company of America. The two stations on the 154-kv. Cheoah-Alcoa transmission line in Tennessee to study natural lightning, and operated in 1928, were continued.
2. Public Service Company of Northern Illinois. A third field laboratory similar to those in Tennessee and to study natural lightning, was installed on the 132-kv. Joliet-Chicago Heights line in Illinois.
3. American Gas & Electric Company. A fourth field laboratory similar to those in Tennessee, and to study natural lightning, was installed on the Turner-Logan section of the 132-kv. system of the Appalachian Electric Power Company in West Virginia.
4. Public Service Electric & Gas Company. A fifth laboratory was installed on the 220-kv. Bushkill-Roseland line in New Jersey, for the purpose of studying both natural lightning and controlled surges.

Experience in Tennessee during 1928 with the first design of Norinder type oscillograph suggested certain modifications in design which would further improve

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*Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.

[1] For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 27-31, 1930.

the quality of the records, make the instrument more convenient and flexible in operation and simplify the construction. The new oscillograph was designed by Mr. O. Ackermann and is described by him in a companion paper presented at this Convention.[4] In the 1929 program the three original instruments, with certain refinements, were used in the two Tennessee stations and the Chicago station. Instruments of the new design were used in the West Virginia and New Jersey stations.

The stations, with the exception of that in New Jersey, were essentially the same as described in the Fortescue, Atherton, and Cox paper in that they consisted of a cathode ray oscillograph connected through a potentiometer to the most exposed line conductor, supported by klydonographs connected to all three conductors at the cathode ray oscillograph station and at a short distance on either side, and other accessories. The line on which the test in New Jersey is being conducted, being in the state of construction and therefore without load, presented a rare opportunity for the pursuit of the second phase of the lightning problem, that of the study of controlled surges, in addition to permitting the recording of natural lightning transients on the highest insulated line in existence in this country. For this reason the oscillograph with all accessories at this location was made entirely mobile by mounting in an automobile trailer and in addition a mobile surge generator good for one million volts and built on a second automobile trailer was provided. Both of these equipments were made entirely self-sustaining by the provision of independent power supply units consisting of mobile gasoline engine generator sets. Either equipment could be moved to various points along the line for the generating or recording of controlled surges with the oscillograph always available for the recording of natural lightning.

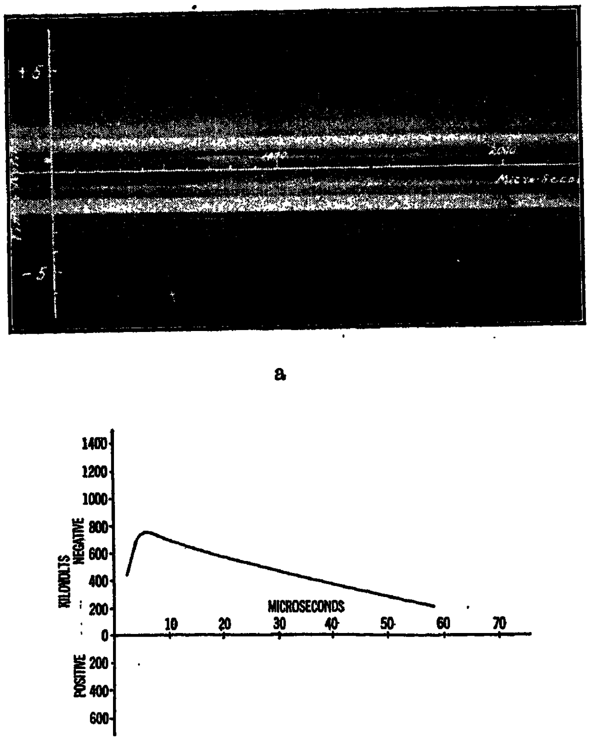


FIG. 1—(a) 750-KV. LIGHTNING SURGES RECORDED AT CHILHOWEE, TENNESSEE, IN 1928

(b) SURGES SHOWN IN (A) REPLOTTED TO AN EXTENDED TIME SCALE

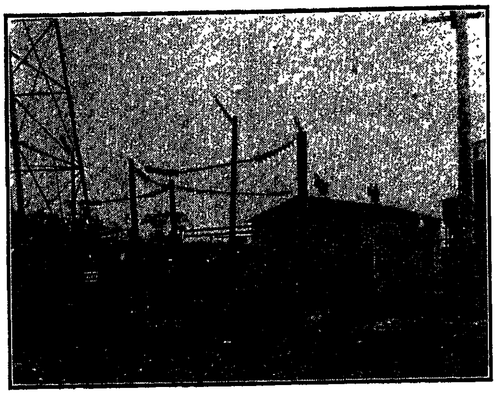


FIG. 2—STATIONARY LIGHTNING LABORATORY

Tower 89, Joliet-Chicago Heights Line, Illinois

POTENTIOMETERS

In the stations erected in 1928 a parallel pipe type of potentiometer was used consisting of three 3-in. iron pipes 10 ft. long mounted with suspension and apparatus type insulators as in Fig. 2. In those stations erected in 1929 a potentiometer consisting of iron rings suspended entirely with suspension insulators was found to be more convenient. These potentiometers are illustrated in Figs. 4a and 5. It has been found by experience both in the field and in the laboratory, that a potentiometer suitable for this sort of work should have as high a capacity as possible without exerting appreciable influence on the transients being studied. At best leakage currents under rain conditions, corona under high-voltage conditions, and changes in surrounding

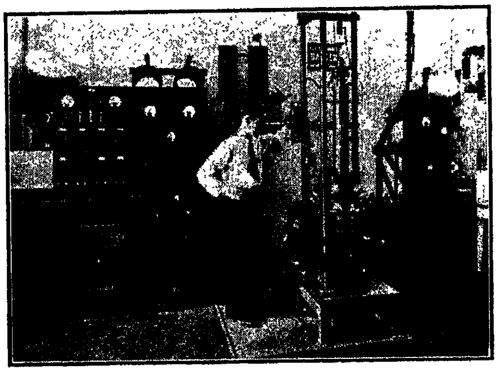


FIG. 3—CATHODE RAY OSCILLOGRAPHIC EQUIPMENT OF LIGHTNING LABORATORY

Interior of station shown in Fig. 2

objects, somewhat modify the potentiometer multiplying ratio. The error involved is not considered to be sufficient to justify elaborate precautions for use in klydonograph tests, but in cathode ray oscillograph tests where the greatest accuracy possible is justified, the rings were made quite large and the upper ring on the potentiometer was made out of 4-in. pipe to keep down the influence of corona at high voltages. The most accurate type of potentiometer has been found to be a non-inductive resistance; therefore, this type has been used in the tests on the idle 220-kv. line. Of course this type of potentiometer is not feasible on energized lines.

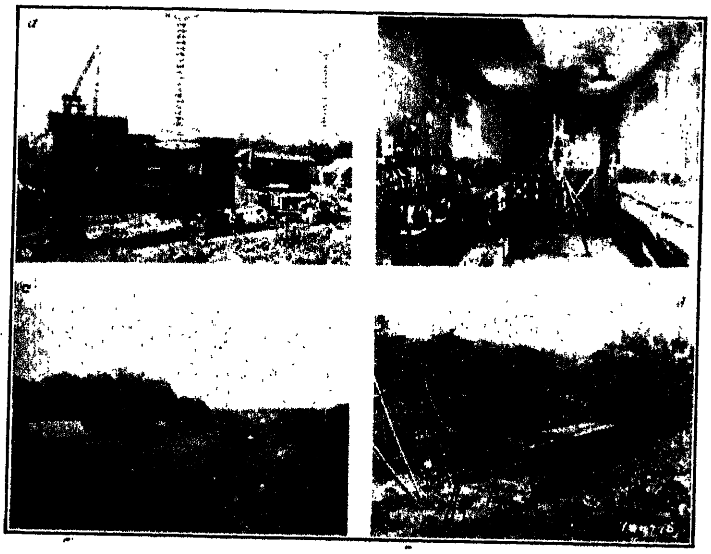


FIG. 4—LIGHTNING LABORATORY AT STILLWATER, NEW JERSEY

(a) Mobile cathode ray oscillograph laboratory
(b) Interior of oscillograph laboratory
(c) Lightning station showing oscillograph laboratory, potentiometers, and living quarters
(d) Mobile surge generator extended for use

TIME SCALES

The first Norinder oscillograph used in these field investigations was equipped with a moving film, a high speed being secured by revolving the film on a drum spinning at between 6000 and 8000 rev. per min. This permitted a time scale of about 400 microseconds to the inch and provided a simple method of timing. In the earlier work this had some advantages as the length of the lightning transients expected was not known and the scheme had been used successfully by Norinder. The revolving film drum has proved useful in the measurement of switching surges of long duration. However, the moving film has proved to be too slow for detailed study of lightning or artificially generated transients. Consequently, in the 1929 investigation the film was left stationary and the timing axis was provided by a high-frequency oscillator according to the method of Harrington and Opsahl.[5] This permits great flexibility in timing with possible speeds much faster than have been found necessary for any type of work, thereby securing detail; it also has the advantage of combining this fast timing with long duration due to the fact that the time scale can double back upon itself repeatedly, thus recording the entire transient and any subsequent reflections that may occur. It has one disadvantage in that the records, if they are long, may become complicated because of the repeated crossing of the film but this is overshadowed by its advantages, particularly as it is a simple matter to replot the records to Cartesian coordinates, as has been done in Fig. 7, assuming the frequency of the timing oscillator is properly chosen. It has been found that for the recording of natural lightning disturbances a sweep across the film in 10 microseconds is satisfactory in that it gives detail and records the entire transient without an objectionable number of crossings of the film.

Another method of timing is by the use of a circuit which will sweep the beam across the film only once, thereby providing a linear time scale. This method has a disadvantage in that the record embraces only such phenomena as occur during one passage of the beam across the film. Moreover, to secure a complete record it is necessary to initiate the timing sweep without delay when the transient occurs, otherwise the beginning of the disturbance may be lost. However, where the order of duration of the expected transient is known, and provided a system can be secured which produces the sweeping movement with negligible time delay, the inconvenience of replotting each record may be avoided by the use of this time scale. A relay of the Norinder type because it acts with zero time delay lends itself very well to such a timing method as is described in the contemporary Ackermann paper. This has been found very useful in studying controlled transients where the test can be repeated at will, and the time scale adjusted

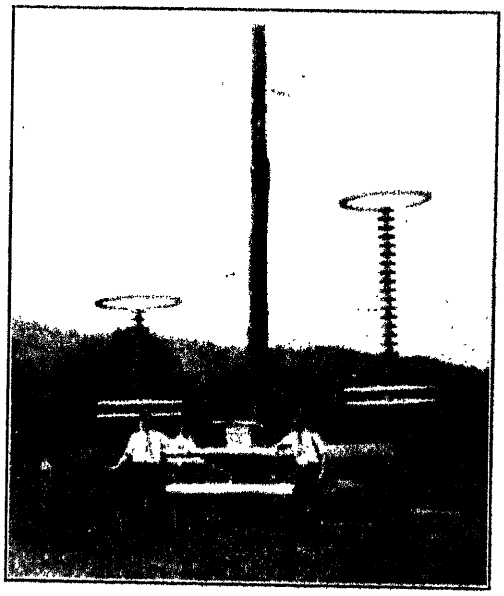


FIG. 5—GRADIENT STATION NEAR CHILHOWEE, TENNESSEE, SHOWING KLYDONOGRAPH AND POTENTIOMETERS

to the best advantage. Furthermore, now that the approximate duration of lightning transients is known, it is probable that our future investigations of natural lightning will be conducted with this method.

GRADIENT STATIONS

In addition to the equipment connected to the line in Tennessee, four antennas were erected and connected to klydonographs through potentiometers for the purpose of gaining additional information on cloud field gradients. Two of these were 10 ft. above the earth and the other two were 20 ft. above the earth. One of each height was installed on the mountain top and one of each in the valley. Because of the relatively less importance of gradient data the installation of cathode ray oscillographs at these stations was not considered justified and only klydonographs were installed. Although there was a large number of storms there were not many records. The maximum gradient recorded was 20 kv. per ft. and this appeared uniform as the gradient indicated by the high and the low antennas was the same.

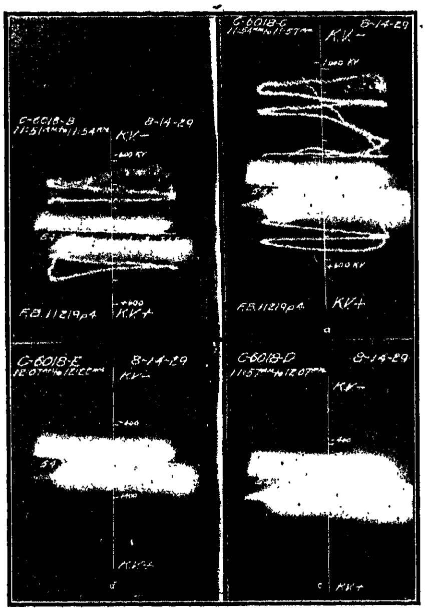
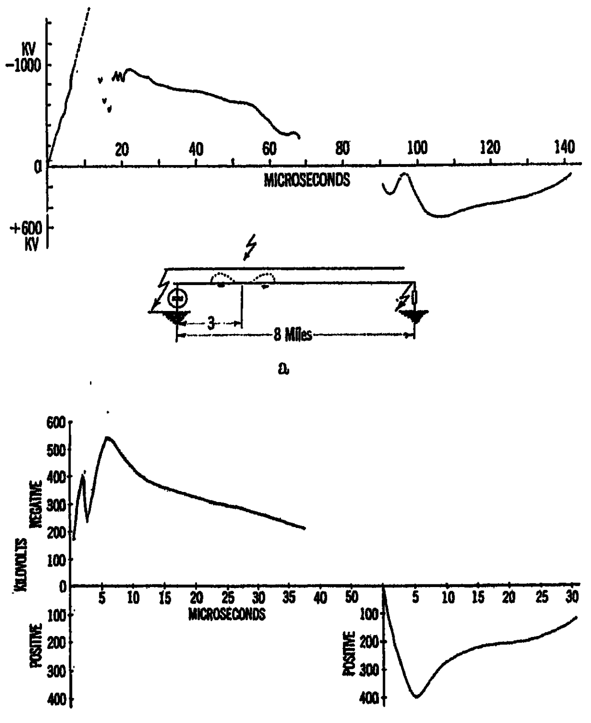


FIG. 6—OSCILLOGRAMS OF LIGHTNING SURGES TAKEN ON 220-KV. LINE

LIGHTNING SURGE RECORDS

Considering the fact that in 1928 only one surge was obtained with the two stations in Tennessee, remarkable progress was made in 1929. A large number of records of transients of various magnitudes has been obtained. A striking feature of the season's work was the absence of lightning surges during the spring and early summer at the locations of the stations. All the records obtained were recorded after July 15. Typical examples of the surges recorded are shown in Figs. 6 and 7. Of particular interest is the surge shown in Fig. 6a and replotted in Fig. 7a. In this case the wave front increased at the rate of 143 kv. per microsecond. The voltage rose considerably beyond the range of the film at the particular setting used and at approximately 12 microseconds a flashover occurred on an adjacent conductor reducing the voltage on the conductor to which the oscillograph was connected to one million volts, after which it decayed to 600 kv. at 55 microseconds. At this point the reflected wave arrived from the far end of the line where it had flashed over a non-inductive resistance located at that point, thereby modifying the original tail of the wave. It appears from this record that the voltage on the conductor studied rose to the order of 2,000,000 volts before the flashover on the adjacent conductor took place and did so in the order of 12 microseconds. After this, had not the reflection modified the surge, it would have remained above half value for the order of 75 microseconds. This is one of the longer waves recorded but is typical of most of the surges measured as is apparent from Fig. 9, which shows a group of typical records replotted to the same linear scale. The principal characteristics of these surges are presented in tabular form below the figure. It should be noted that the slopes of both fronts and tails are very much alike for the surges shown in Fig. 9, and this is also true for those which have been recorded but which are not included in this figure. Furthermore, the surge recorded at Wallenpaupack in 1928 and presented at the 1929 Winter Convention[2] had the same general shape. The surges recorded indicate that the lower valued surges are shorter in duration than those



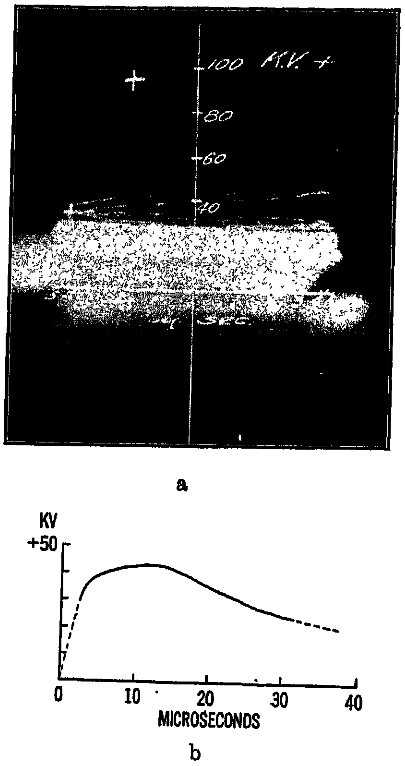
a

b

FIG. 7—(a) SURGE SHOWN IN FIG. 6a REDRAWN
(b) SURGE SHOWN IN FIG. 6b REDRAWN

of the higher voltages. Of the waves recorded the fronts have varied from two to ten microseconds, the durations above half value from 10 to 75 microseconds, and voltages from just above line voltage to beyond the range of the instrument at the calibration used.

The field experience of 1929 further substantiates the statement made at the last Winter Convention[1] that those surges which have voltages and durations high enough to be dangerous on high-voltage lines are caused by direct strokes, and that induced strokes are innocuous, except perhaps on the lower voltage circuits such as distribution lines. This is indicated by visual observation of many severe strokes close to the line and station which did not cause surges and by the small number of surges per storm. It is also indicated by instances of surges recorded which were known to be caused by direct strokes. These particular records conform in characteristics to the general type of surge recorded and from theoretical considerations it is not logical that direct stroke and induced stroke surges should be similar. It is unlikely that induced transients would have the steepness of front, magnitude, and duration of the higher voltage records. The authors believe that some of the smaller surges, which were shorter in duration, may have been induced by strokes near the line.



a

b

FIG. 8—(a) 45-KV. LIGHTNING SURGE ON 220-KV. LINE
(b) SURGE REPLOTTED TO LINEAR TIME SCALE

The conclusions that all important surges on high-voltage lines are caused by direct strokes somewhat modifies the point of view formerly held regarding lightning on transmission lines. Instead of there being two entirely different types of important surges, the induced stroke amenable to control and the direct stroke entirely beyond control, it now appears that there is no pronounced demarcation and that protection is a matter of degree. This being the case it seems hopeful that protection methods may soon be extended to include those occasional severe strokes which now go beyond control.

SWITCHING SURGE RECORDS

Although previous klydonograph tests have shown that switching surges are not of sufficient magnitude to be important on properly insulated transmission systems, tests were also made to determine the characteristics of such surges. The first records showed that the time scale frequency used for lightning was entirely too fast and the records obtained crossed the film so many times that they were difficult to analyze. Examples of such records are shown in Fig. 10. Subsequent tests were made using the rotating film drum as discussed above and oscillograms taken in this manner are shown in Fig. 11. The striking feature about these records is their duration, which is extremely long compared to that of lightning surges. It is interesting to note the step-like form of these surges, due to successive reflections from the line ends, showing the typical surge charging or discharging of a line. Further, the low value of these surges confirms the conclusion based on klydonograph tests that switching surges are not of dangerous magnitude, the normal crest voltage to neutral on this line being 108 kv.

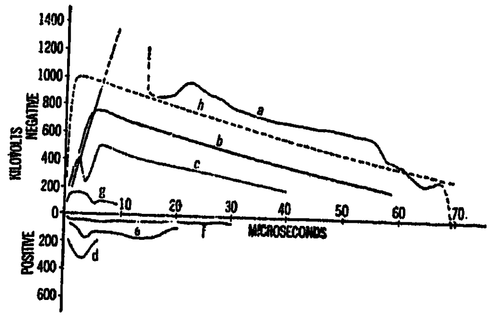


FIG. 9—TYPICAL LIGHTNING SURGES RECORDED WITH CATHODE RAY OSCILLOGRAPHS

Surge No.	Oscillo-gram No.	Polarity	Rate of rise kv. per μ-sec.	Crest kv.	Time To crest μ-sec.	 To ½ max. E μ-sec.	Where taken
a	O-6018-O	Neg.	143	Unknown 1000 recorded	7 m-sec. to 1000 kv.		New Jersey
b		Neg.	183	750	5	33	Tenn.
c	O-6018-B	Neg.	270	550	6	26	N. J.
d	O-6018-D	Pos.	137	325	3	4	N. J.
e	O-5017-I	Pos.	30	180	5	15	W. Va.
f	6039-E	Pos.		45		30	N. J.
g	5015-B	Neg.	167	160	2.5	8	W. Va.
h		Pos.	500	1000	2	43	Trafford Test

APPLICATION OF LIGHTNING DATA

The particular significance of the lightning data thus far obtained is their relationship to transmission line insulation and protective apparatus. The impulse flashover characteristics of line insulation, as well as the operating characteristics of lightning arresters, have been quite accurately determined by laboratory studies. The work on insulation characteristics is described in a companion paper being presented before this convention by J. J. Torok[6] and the lightning arrester characteristics are accurately known. With this information the performance of transmission lines when subjected to lightning surges of the types recorded can now be predicted. The impulse flashover time lag characteristic of insulation as determined at the Trafford High-Voltage Laboratory has been studied with the use of the wave as shown by the dotted curve in Fig. 9h. This wave has a front of the order of one microsecond, which is considerably shorter and therefore more severe than any of the surges thus far recorded, and is steeper than the steepest lightning surge which is likely to occur. It also conforms in shape to the severe observed transients.

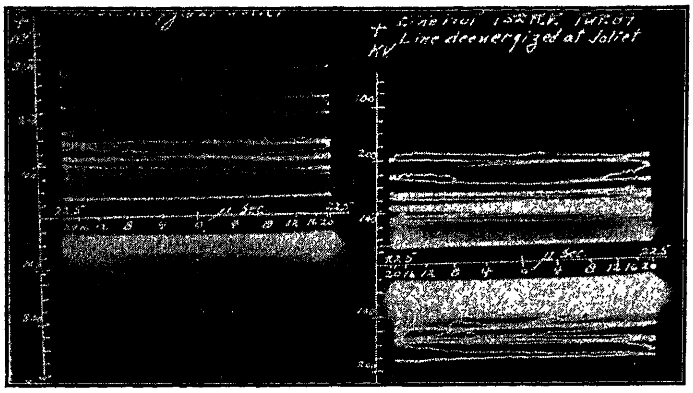


FIG 10—SWITCHING SURGE OSCILLOGRAMS—OSCILLATING TIME SCALE

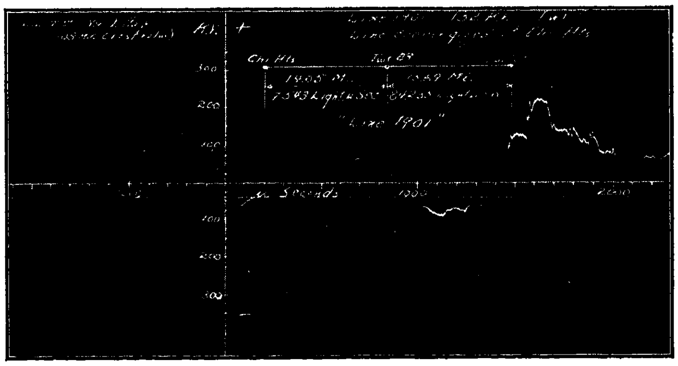


FIG. 11—SWITCHING SURGE OSCILLOGRAM, LINEAR TIME SCALE

An example of the flashover—time lag characteristics of insulation is shown in Fig. 12, reproduced from Torok's paper[6] for purpose of illustration, and shows the impulse ratio curves for standard suspension insulators in strings from 2 to 16, and needle-gaps in air. The significant feature of these curves is that they flatten out at a low impulse ratio in the order of six microseconds. This means that when this type of structure is subjected to lightning surges having the durations secured in the tests thus far, the maximum surge which can be impressed without flashover is very little above the 60-cycle crest flashover of the structure in question. They also show that for a very high, steep surge which reaches the flashover of the insulation in the order of one microsecond, the actual flashover may occur at a voltage of the order of 1.8 times the 60-cycle crest flashover. This latter, of course, determines the magnitude of the chopped wave which passes on and may be impressed upon a piece of apparatus unless it is reduced by lightning arresters, but it should be emphasized that it is impossible for a wave of this magnitude to be impressed upon the line insulation without causing a flashover if the surge has a duration of the order of lightning surges. These impulse flashover curves, together with the knowledge of the types of surges experienced, permit the intelligent coordination of the various types of insulation in a transmission system. The records secured of natural lightning also furnish data on the amounts of energy that lightning arresters have been called upon to handle and provide valuable information for use in the development of new types of arresters.

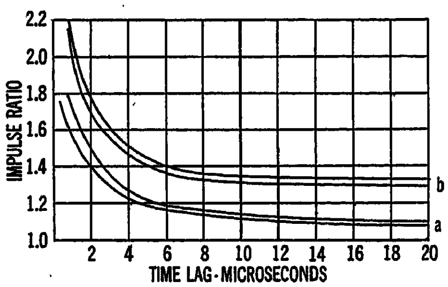


FIG. 12—IMPULSE RATIO-TIME LAG CURVES OF INSULATION

(a) Suspension insulators
(b) Needle-gaps in air

CONTROLLED SURGE STUDY

In the second phase of the work, that of the study of controlled transients, a great amount of valuable information has been and is being obtained. However, these data are not sufficiently analyzed or complete at this time for adequate presentation but will be reported in a subsequent paper. As stated above, the idle 220-kv. line with open ends and sectionalized at one point presented a rare opportunity for the pursuit of this program. Such factors as surge impedance, mutual surge impedance, reflections, and so forth, have been rather completely defined on a theoretical basis but certain quantities in the calculations such as distance of the ground return, size of wire under corona conditions, and so forth, need further investigation. These are being studied. In addition such factors as attenuation due to corona at high voltages and the influence of ground resistance in various connections are relatively unknown. These also are being studied and determined completely with considerable accuracy.

The line on which this work was performed proved particularly convenient for this type of study. With the wide spacing, the mutual coupling was great enough to give induced waves amply large for measurement, and yet not so great that the reverse reaction appreciably modified the original wave. In this way by making various cross connections, such as connecting the outside conductors at the far end and impressing a surge on one conductor with oscillographic measurement on the return of the wave on the other conductor, a great variety of set-ups and tests could be made with all the equipment and the entire force working at one location. However, although the tests require somewhat more time to perform when the surge generator and oscillograph are located at different points, this does not involve great difficulty and operation under such conditions is quite satisfactory.

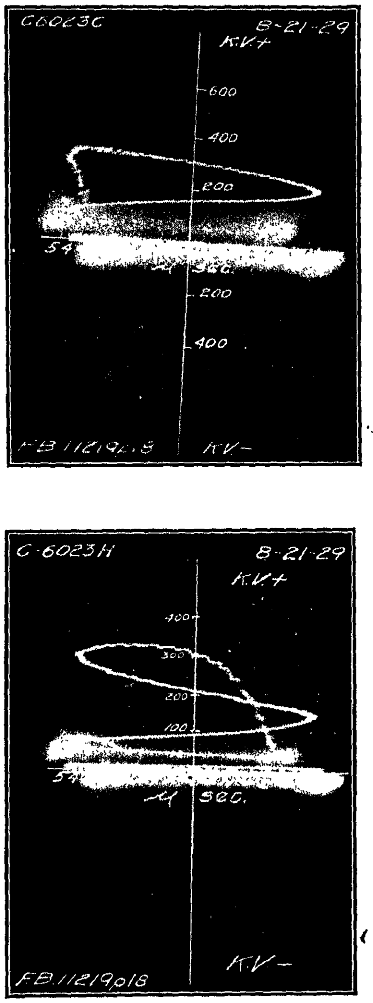


FIG. 13—OSCILLOGRAMS OF MOBILE SURGE GENERATOR WAVE USING OSCILLATING TIME SCALE

Furthermore, as mentioned above, the necessity of replotting surges when taken with an oscillating time scale is somewhat inconvenient and this inconvenience is particularly prominent in controlled surge tests where a great number of records are taken in a short time. Therefore, in these tests the circuits were modified to provide a linear time scale. In controlled surge tests this scale has no disadvantage since the length of the surge expected is definitely known and its time of arrival can be chosen at will within the desired limits. The circuit used was a modification of the timing circuit used at the Trafford High-Voltage Laboratory, and described by Torok and Fielder[7] in the *Electric Journal* of July 1929, adapted to use with the Norinder type of relay.

Fig. 13 shows oscillograms generated by the one million-volt mobile surge generator taken with the use of the oscillating time scale. It is noted that this gives a sharp record with a long time scale but requires replotting to be easily readable. Only a small amount of the surge within the target band is missing. Figs. 14, 15, and 16 show oscillograms taken with the linear time scale. These are directly readable and also with this system the entire wave down to zero voltage is obtained.

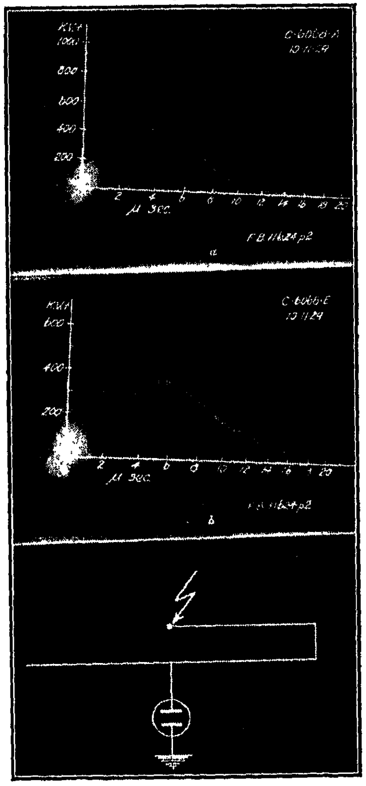


FIG. 14—OSCILLOGRAMS OF MOBILE SURGE GENERATOR WAVE SHOWING ATTENUATION

(a) Original surge, ½ microsecond front, 940 kv.

(b) Attenuated surge after 2.65 miles of line, 3.5 microsecond front, 350 kv.

The waves shown in Figs. 14, 15, and 16 are typical of the work being done. Fig. 14 shows a typical example of records showing attenuation. The upper oscillogram (*a*) shows an original surge having a front of about ½ microsecond and a magnitude of 940 kv. The lower oscillogram (*b*) shows the same surge after 2.65 miles of line, when it has a front of 3.5 microseconds and a magnitude of 350 kv. Fig. 15 shows similar records except that the original surge had a front of 2.5 microseconds and a magnitude of 875 kv., and the attenuated surge has a front of five microseconds and a magnitude of 425 kv. This indicates that the short steep front surge attenuated at a greater rate than the larger, slower front surge. These values of attenuation check quite closely with previous klydonograph data. Complete attenuation curves for various voltages and for various conditions are being obtained. Fig. 16 shows records on coupling. Of particular interest is the break in the curves of Figs. 14b and 15 b at about 200 kv. This marks the point where corona begins and shows its effect on attenuation. In Fig. 16 the upper oscillogram (*a*) shows the wave as generated and impressed upon Conductor *C*, the middle oscillogram (*b*) shows the induced wave on the middle or nearest conductor, phase *B*, and the bottom oscillogram (*c*) shows the induced wave on the other outside conductor, phase *A*, 56 ft. distant. It is noted that the induced wave in the conductor 28 ft. distant from the conductor on which the surge is impressed is 22 per cent and on that 56 ft. distant is 11 per cent of the original surge. These values are only slightly above the calculated values, which indicates that the actual image of the conductor is very little below the true image. The self-surge impedance of the line conductors has been determined quite accurately and checks very closely with calculated values when various factors of influence such as distance of the ground return, and increase in size of conductor due to corona, are taken into account. The influence of these factors has been evaluated. In addition, the other problems mentioned, such as the amount and nature of ground wire protection, the influence of ground resistance, the effect of buried conductors, the performance of lightning arresters on lines, and so forth, are being accurately determined and when completed the uncertainty regarding all these factors will have been removed.

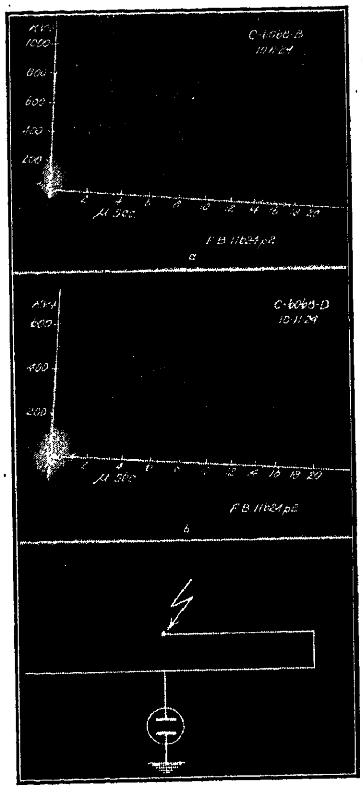


FIG. 15—OSCILLOGRAMS OF MOBILE SURGE GENERATOR WAVES SHOWING ATTENUATION

(a) Original surge 2.5 microsecond front, 875 kv.
(b) Attenuated surge after 2.5 miles of line, 5-microsecond front, 425 kv.

STATUS OF THE PROBLEM

Much progress has been made toward the solution of the lightning problem by the work of all the groups engaged in lightning investigations. Yet it is apparent that caution must be exerted in the interpretation of the results thus far achieved to prevent the hasty drawing of conclusions. Although accurate measurements have been made of many wave shapes and voltages, much more work is necessary to determine the variations in these values and to make our knowledge more definite. It must be realized that it is impossible to draw definite conclusions from what must still be regarded as a relatively small number of records. Considering the infrequent occurrence of lightning surges it is quite probable that a measurement has not been made of a lightning stroke at the immediate point of generation and such a stroke might have a considerably steeper wave front and higher magnitudes than those measured thus far. At best all that has been measured thus far has been the maximum voltage as

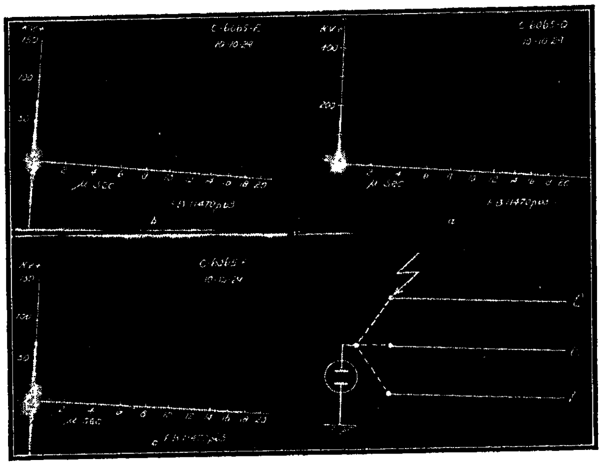


FIG. 16—OSCILLOGRAMS OF MOBILE SURGE GENERATOR WAVES SHOWING MUTUAL COUPLING

(a) Original surge, 515 kv.
(b) Induced surge in conductor 28 ft. distant, 115 kv., 22 per cent
(c) Induced surge on conductor, 56 ft. distant, 70 kv., 11 per cent

limited by line insulation and although the crest can be estimated from the shape of the front and tail it is desirable to obtain records of disturbance occurring on lines having much greater insulation than is now available. The work by all groups is being continued on an expanded scale and additional groups are becoming active which means that the remaining questions will all be determined in time.

The authors wish to stress again the importance of this work to the industry as a whole. Not only does the volume of data available require great expansion, but different transmission systems have different problems. The satisfactory answer to the lightning question will be hastened by the active participation of many in a coordinated program to increase the number of observation stations and to avoid inefficient repetition of tests.

ACKNOWLEDGMENT

The authors of this paper have assumed the writing of it only because the number of men working on the problem was too large to permit the co-authorship of all who made major contributions. Particular acknowledgment is due the utilities whose hearty co-operation made the work possible and to the following men whose enthusiastic efforts produced the results discussed in this paper: O. Ackermann, A. M. Opsahl, J. J. Torok, C. G. Archibald, R. H. Southgate, A. S. Brooks, E. R. Whitehead, W. G. Roman, P. E. Stephens, J. E. Garrett, Robert Sparks, W. J. Kroeger, M. E. Gainder, and L. R. Smith.

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Discussion

For discussion of this paper see page 928.

Surge Characteristics of Insulators and Gaps

BY J. J. TOROK*

Associate, A. I. E. E.

Synopsis.—This paper presents the surge characteristic, in the form of time lag curves, of transmission line and substation insulation. The forms of insulation discussed are:

I. Line insulation: suspension type with and without arcing rings, pin type, and wood alone and in combination with porcelain.

II. Post type insulators with and without arcing rings.

III. Safety gaps with either needle or spherical electrodes.

IV. Oil gaps.

* * * * *

INTRODUCTION

It has long been recognized that the duration as well as the magnitude of an impulse voltage will determine the nature of the flashover of an insulator or gap. Accordingly sporadic efforts to determine these characteristics have been made with varying successes. The methods employed can be classified into two groups, the "comparative" and the "absolute." The comparative method is based upon a particular wave and an assumed standard of insulation, such as the suspension insulator. Other forms of insulation are then tested in parallel with this standard, and the results are computed in terms of the insulating strength of the standard. With the "absolute" method the characteristics of the apparatus are determined directly from cathode ray oscillograms, and the results are plotted in the form of volt-time curves. The advantage of the latter method is quite apparent. If the volt-time curves of different forms of insulation are known, the relative strength of each under all types of waves can be seen by merely comparing the curves.

An intelligent application of such volt-time curves in the design of transmission lines and sub-station equipment will add greatly to the reliability and safety of the system. Furthermore, they make possible a more economical construction in terms of both initial cost and maintenance. With an exact knowledge of the performance of each piece of apparatus, the designer can proportion the insulation of the line and substation to obtain proper grading of line insulation, prevention of flashover at one or more points on the line, or in the substation, and correct placement of arresters. In addition, safety gaps for the protection of adjacent equipment can be properly designed and applied.

METHOD OF OBTAINING VOLT-TIME CURVES

A thorough description of the equipment and system used in these surge studies has been given elsewhere,[1] but briefly it is as follows: The high-voltage impulses are generated by a bank of condensers charged in parallel and discharged in series. For voltage and time determinations, the apparatus under test is parallel by a wire-wound, non-inductive potentiometer terminating in a cable which leads to the cathode ray oscillograph. The synchronization of the oscillograph and surge generator is obtained by means of three-electrode gaps. The constants of the generator are adjusted to give a wave having an abrupt front and a slowly attenuating tail, which, in general, simulates a rectangular wave.

The rectangular wave is the most desirable in the "absolute" method.[2,3] The procedure is as follows: The voltage of the surge generator is raised to a value just sufficient to cause flashover. A record of the breakdown is then made with the cathode ray oscillograph. Such an oscillogram is shown in Fig. 1. The voltage rises abruptly to a given value at which it remains until breakdown occurs. The time lag of the insulator for this voltage is the time from the point at which the voltage first exceeds the 60-cycle crest breakdown value, to the time when the voltage is suddenly reduced by breakdown. The voltage of the generator is then raised to a slightly higher value and another record is taken. This procedure is followed until the time lag of breakdown becomes very short. With the data so obtained a breakdown volt-time curve can be easily constructed.

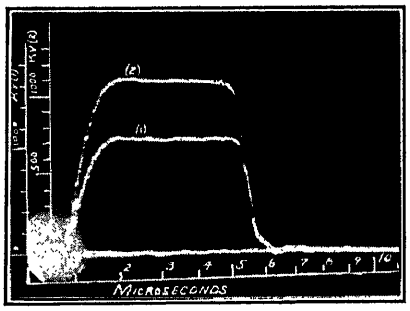

FIG. 1—CATHODE RAY OSCILLOGRAM OF THE SURGE FLASHOVER OF AN INSULATOR

The conventional types of surge generator cannot produce a strictly rectangular wave. It is physically impossible to produce a perpendicular front as the charging current would become infinite. As a result, the fronts are sloped, but they may be made so steep that, for practical purposes, the time of rise can be neglected

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*Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.

1. For references see Bibliography.

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except for very short time lags. Also the potentials of generators attenuate after reaching the crest. This reduction in voltage is very often accentuated by the formation of space charges and streamer currents.[4] Thus it cannot be strictly said that the voltage remains at a fixed value until flashover occurs. But here again, the deviation can be neglected except for extremely long time lags and for those of short duration, where the streamer currents become very large. At such points some minor approximations must be made to avoid discrepancies caused by changing wave shapes.

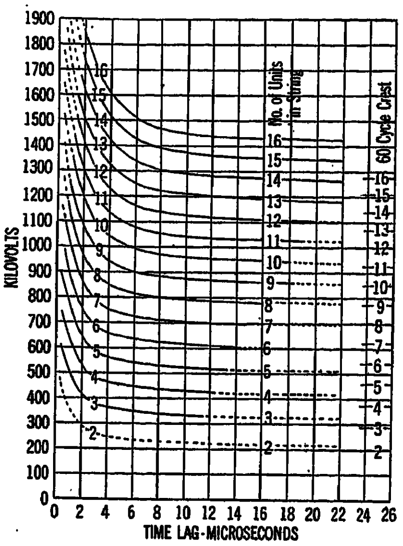

FIG. 2—SURGE FLASHOVER CURVES OF SUSPENSION INSULATORS, 10-IN. DISKS, 5¾-IN. SPACING

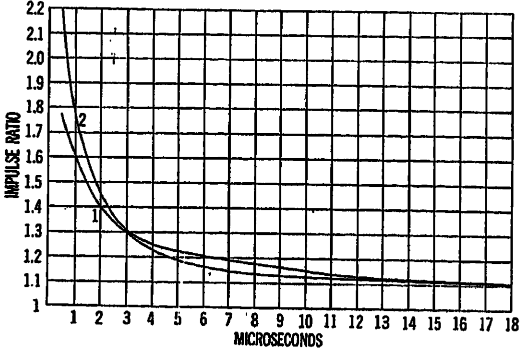

FIG. 3—IMPULSE RATIO BOUNDARIES FOR SUSPENSION INSULATOR STRINGS

(1) 11-unit string
(2) 2-unit string

All tests discussed below were made with the high-voltage end of the surge generator positive in polarity.

INSULATOR TESTS

A group of tests was made on suspension insulator strings comprising 3 to 16 units of 10-in. disks having 5¾-in. spacing. A complete time lag curve was obtained for 3, 5, 7, 9, 10, 11, 12, 13, 14, and 16-unit strings. The other string lengths shown in Fig. 2 were extrapolated from these curves.[3] The relationship between the duration of the applied wave and the potential is well exemplified in the above curves.

The striking similarity of the long and short string characteristics suggests that they might all be represented by a single characteristic or by two boundary curves. Such a curve can be drawn by using the 60-cycle crest breakdown voltage as a basis, plotting against time the impulse voltage in terms of the 60-cycle value. The two curves shown in Fig. 3 are obtained in this manner. The upper curve corresponds to a two-unit string, while the lower curve is for eleven units. The curves for all other string lengths lie between those of the two- and eleven-unit strings. The term "Impulse Ratio Curves" is commonly applied to this form of characterization.

Quite often a lightning storm precedes or occurs at the beginning of the heavy precipitation, when the insulators have not yet become wet. For this condition the surge flashover characteristic can be assumed to be that of a dry insulator. Since such an assumption is not always correct, it is necessary to determine the performance of insulators when dripping wet. The result of tests made on suspension insulators under heavy rain fall (0.2 in. per min.) are given in Fig. 4 with dry insulators for comparison.

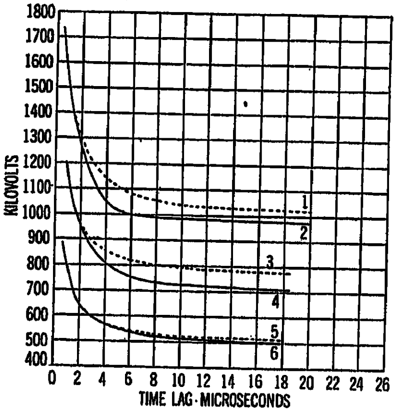

FIG. 4—COMPARISON OF WET AND DRY SURGE FLASHOVER OF SUSPENSION INSULATORS

(1) 11 units dry
(2) 11 units wet
(3) 8 units dry
(4) 8 units wet
(5) 5 units dry
(6) 5 units wet

The pin type insulators most commonly used on low-voltage lines, must also be considered in this analysis. There are many types of pin insulators in use, but fortunately their impulse characteristics are very much alike, and therefore can well be represented by a single example. Time lag and impulse ratio curves of a typical 66-kv. pin type insulator are given in Fig. 5.

WOOD INSULATION

A crude survey of pin insulator installations indicated that only in exceptional cases were the pins grounded; all others were used in conjunction with wood construction. It happens also that ungrounded suspension insulators are very often used on wood pole lines. It is, therefore, essential to include in this study the effects of wood on line insulation.

It was believed that a more comprehensive idea of the performance of mixed insulation could be had if the characteristics of each were first determined, and then ompared with various combinations as units. Acordingly tests were made on oak crossarms with spacngs of 1, 2, 4, and 8 ft. between electrodes. A similar est was made on a spruce pole with 1, 2, 4, 8, and 13.5-t. spacings. The results of these tests are given in Figs. 6 and 7. The inconsistencies of flashover at any given condition and the complete lack of breakdown at long time lags is very marked. The scattering of the points can be attributed to the electrical instability of the material. But the strong tendency to breakdown during the first few microseconds of voltage application, or not at all, was unexpected.

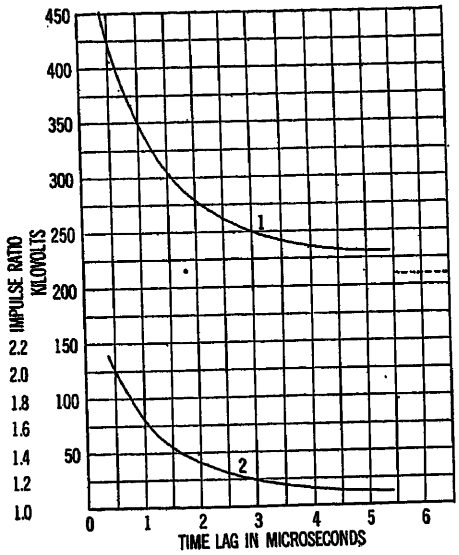

Fig. 5—Surge Flashover Curve of 66-Kv. Pin Type Insulator

(1) Volt-time curve
(2) Impulse ratio curve

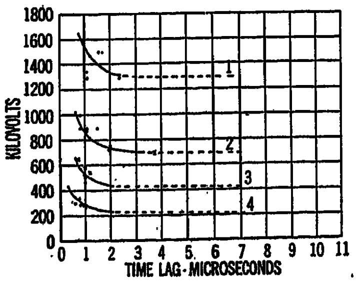

Fig. 6—Volt-Time Curves of the Surge Breakdown of Oak Crossarm, 5 In. by 5 In.

(1) 8-ft. length
(2) 4-ft. length
(3) 2-ft. length
(4) 1-ft. length

This phenomenon may perhaps be explained in this way: When the voltage is first applied, the potential gradients along the wood will be determined by electrostatic relationships and space charge formations. When the resistivity of the wood is sufficiently low enough current will flow to cause a redistribution of potentials and space charges within and along the surface so that the breakdown gradient is maintained at no point long enough for flashover to occur. The curves have been extended by a dotted horizontal line, below which breakdown did not take place.

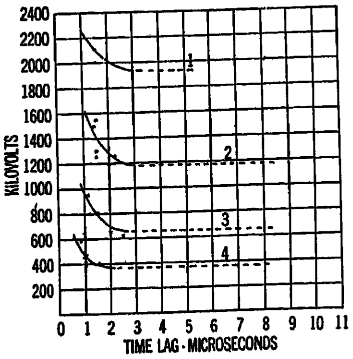

Fig. 7—Surge Breakdown Characteristics of Wood Poles

(1) 13½-ft. length
(2) 8-ft. length
(3) 4-ft. length
(4) 2-ft. length

Many combinations of wood and porcelain insulators may be obtained by merely varying the length of cross section of the wood and the type or number of insulators. But for the sake of simplicity and clarity, a single group of combinations was chosen, wherein the porcelain insulation was fixed and consisted of a string of 5 suspension units, only the length of the wooden crossarm being varied. The data obtained from this group of combinations are shown graphically in Fig. 8.

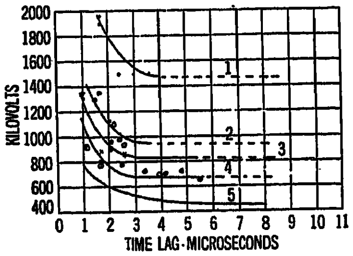

Fig. 8—Surge Breakdown Characteristics of Wood and Porcelain Insulation

(1) 5 units and 8 ft. of crossarm
(2) 5 units and 4 ft. of crossarm
(3) 5 units and 2 ft. of crossarm
(4) 5 units and 1 ft. of crossarm
(5) 5 units only

ARCING RINGS

Line insulators, especially the suspension types, are often equipped with arcing rings or horns. These devices change the electrostatic field about the insulator as well as the path of the arc. It is, therefore, essential to obtain time lag curves on strings of insulators on which are mounted arcing rings or horns. However, the large variety of rings now in use would require hundreds of tests if each type were considered separately. Fortunately, the characteristics are nearly alike, and therefore they can be well represented here by a few typical samples.

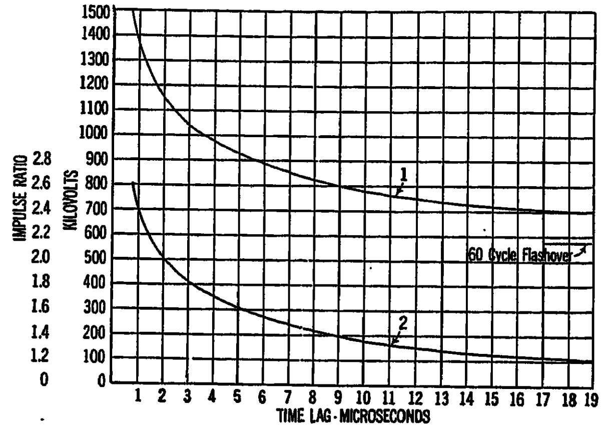

FIG. 9—SURGE CHARACTERISTIC OF SUSPENSION INSULATOR STRING OF NINE UNITS WITH ARCING RINGS (24 IN. BY 30 IN.)

1—Volt—time curve
2—Impulse ratio curve

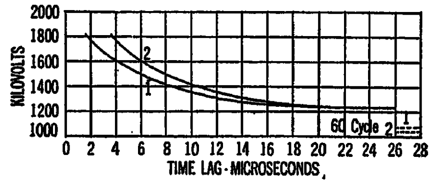

FIG. 10—SURGE CHARACTERISTICS OF STRAP AND TUBE ARCING RINGS FOR A 16-UNIT STRING

(1)—4-inch tube ring
(2)—¼-in. by 2-in. strap ring

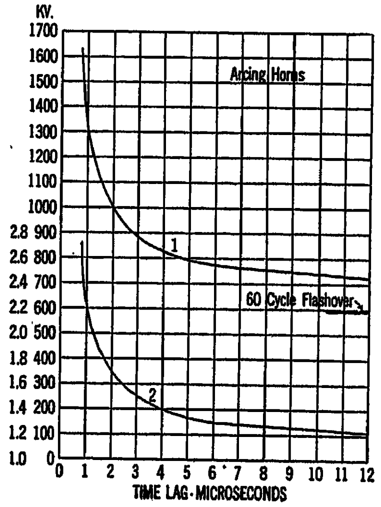

FIG. 11—SURGE CHARACTERISTICS OF SUSPENSION INSULATOR STRING OF NINE UNITS WITH ARCING HORNS

1. Volt-time curve
2. Impulse ratio curve

The time lag curve of a typical ring, used quite extensively, is shown in Fig. 9. The ring is of the flat strap type, oval in shape, its axial dimensions being 24 by 30 in., while the spacing between rings is 43.5 in. when mounted on a string of 8 units. The effect of increasing the projected area of the material of which the ring is made can be seen from Fig. 10.

The status of the arcing horn is much like that of the arcing rings, *i. e.*, many different types and shapes are in use, but their surge performances are all so similar that they too may be represented by a single example. Fig. 11 is the volt-time curve of a string of 9 suspension units equipped with arcing horns. The length of the string was 51¾ in., whereas the spacing between horns was 46½ in.

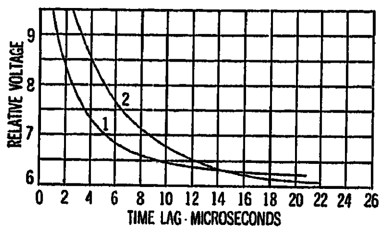

FIG. 12—RELATIVE IMPULSE FLASHOVER CURVES OF AN INSULATOR POST OF SIX PILLAR UNITS

1—Without arcing rings
2—With arcing rings

The above data will yield a reasonably good working knowledge of the behavior of transmission lines when subjected to surges. The nature of the impulses coming up to the substation can be established by examination of the curve or curves of insulation which most nearly agree with the specifications of the line.

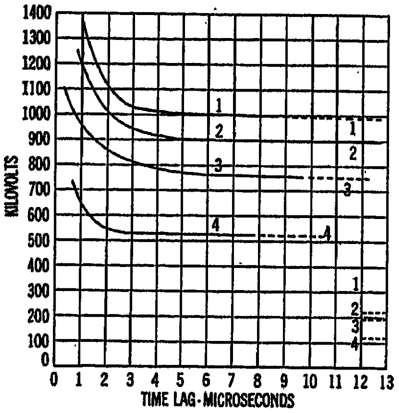

FIG. 13—SURGE BREAKDOWN OF 6.25-CM. SPHERE-GAPS UNDER OIL

(1) 3-in. separation
(2) 2-in. separation
(3) 1-in. separation
(4) ¼-in. separation

PILLAR TYPE INSULATORS

Similar flashover time lag tests were made and curves obtained with pillar insulators comprised of six units with and without arcing rings and the relative values are shown in Fig. 12. It will be noted that these curves correspond very closely to those obtained for suspension insulators so that by choosing the proper number of suspension insulators the impulse strength may be made to correspond at practically all points with that of the pillars. It will be further noted that the addition of the arcing rings has caused an appreciable change in characteristics just as on suspension insulators, increasing the time lag at the higher voltages and reducing the flashover of the lower surge voltages. It is questionable, however, whether any advantage is obtained by the addition of rings to pillars, for the reason that when properly rationalized with respect to the line there is no occasion for them to flash over and when lightning arresters are installed, the rings will have no function whatever.

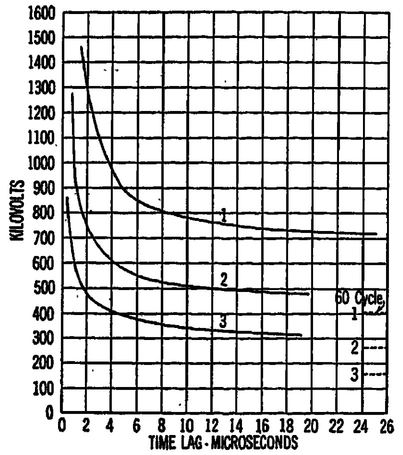

Fig. 14—Surge Breakdown of Needle-Gaps Under Oil

(1) 10-in. separation
(2) 5-in. separation
(3) 2½-in. separation

OIL-GAPS

Two types of gaps were used in the oil tests, one formed with needle electrodes to simulate sharp corner conditions, the other with 6.25-cm. spheres to represent rounded electrode conditions. A complete volt-time curve was made for a given gap setting. Then the spacing between electrode was changed and the test repeated. By this means some concept of the surge dielectric strength of oil is obtained for various shapes of gaps set at different gap lengths. The results of this group of tests are given in Figs. 13 and 14. It must be remembered, however, that these curves are for the breakdown of oil alone and not for creepage over insulating surfaces. The 60-cycle breakdown curve for these gaps is given in Figs. 13 and 14. Unfortunately the surge breakdown curves cannot be converted into impulse ratio time curves as a slight change in the quality of the oil will affect the surge characteristics but little, while the change in the 60-cycle breakdown value will be quite marked. Hence any impulse ratio curve of oil is meaningless unless all the electrical properties of the oil are specified.

AIR-GAPS

Occasionally air-gaps are connected to the lines, the object being to limit the surge voltage entering the substation and thereby afford some measure of protection for the transformers. If such gaps are to be used it is essential to know the surge characteristics of a group of gaps, so that a correct application can be made.

Again, as in the oil-gap tests, two common types of electrodes were chosen, the needle and the sphere. In the tests employing needle electrodes, it was only necessary to find the volt-time characteristic of the gap at several settings. The results of these tests, together with 60-cycle values, are given in Fig. 15. However, the sphere-gap tests were not so simple, for at spacings beyond the diameter of the sphere the breakdown characteristics change. It was, therefore, necessary to make tests on at least two sizes of spheres, wherewith some relationship may be set up between diameters and spacings of the spheres.

The sphere-gap tests were made with 6.25-cm. sphere electrodes in one case and 20-cm. electrodes in another. The 6.25-cm. spheres were tested at 10, 20¼, 40, and 58½-in. separations. The volt time curves at these separations are given in Fig. 16. Fig. 17 represents the characteristics of the 20-cm. sphere-gaps at 10, 20, 40, and 59¾-in. separations.

In many installations it is very desirable for both economic and space considerations to use small electrodes. From the above curves it can be seen that if the spheres are separated by a distance no greater than their diameter, the impulse ratio is nearly unity under all types of waves. Increasing the separation at first causes an increase in impulse ratio only for waves of short duration, and then with further separation, the impulse ratio rises for even the longer waves. Although a constant voltage characteristic is the most desirable,

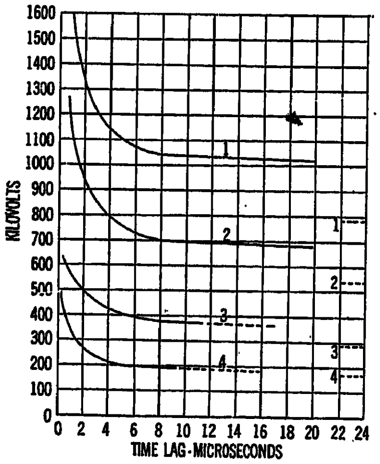

Fig. 15—Surge Breakdown Curves for Needle-Gaps in Air

(1) 60-in. separation
(2) 40½-in. separation
(3) 20-in. separation
(4) 10-in. separation

a gap setting allowing moderate overvoltages may be used as long as the time lag curve of the gap at that setting lies well below that of the apparatus protected and yet is high enough to maintain the insulation of the system at a reasonable value.

The curves in Fig. 16 cannot be placed upon the same basis as those of Fig. 17 by merely conversion of voltage into impulse ratio, because, as pointed out above, the diameter and spacing must also be considered. It was believed that these curves could be used in establishing a family of curves if one of the axes represented impulse ratio, and the other, spacing in terms of the diameters. However, either the basis of comparison was faulty or the nature of breakdown alters at large separations, because these curves when plotted were so different that no set of universal curves could be deduced from them.

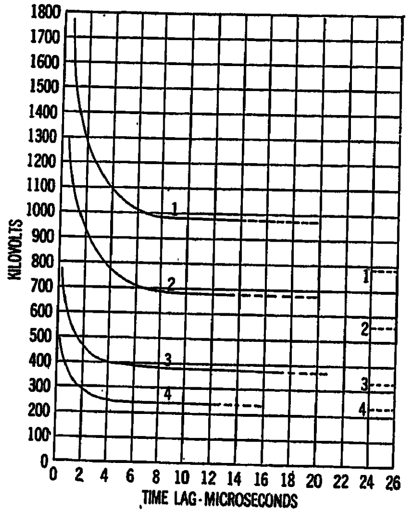

FIG. 16—SURGE BREAKDOWN CURVE FOR 6.25-CM. SPHERE-GAPS IN AIR

1-58½ inch separation
2—40 inch separation
3—20¼ inch separation
4—10 inch separation

CONCLUSIONS

I. The surge flashover of an insulator is a phenomenon dependent upon the duration of the applied potential as well as its magnitude, and curves showing the volt-time relationship can be obtained.

II. The surge characteristics of an insulator can be plotted if its 60-cycle breakdown value and the impulse ratio curve for typical equipment are known.

III. By using volt-time surge breakdown curves the relative performance of widely different forms of insulation can be directly compared.

IV. In the event of an insulator flashover on a transmission line, the entire surge does not go to ground. The section of the wave preceding breakdown continues its course on the line, thereby making it possible to flash over other insulators on the line, should they be weaker than the one which chopped the wave.

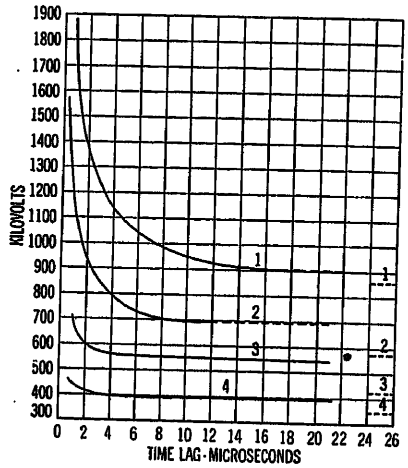

FIG. 17—SURGE BREAKDOWN CURVE FOR 20-CM. SPHERE-GAPS IN AIR

(1) 59¾-in. separation
(2) 40¼-in. separation
(3) 20-in. separation
(4) 10-in. separation

V. The added insulation offered by wood is roughly proportional to the length of wood used.

VI. Rain affects the surge flashover of insulators only at long time lag values.

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Discussion

For discussion of this paper see page 928.

Lightning Laboratory at Stillwater, New Jersey

BY R. N. CONWELL[1]
Associate, A. I. E. E.

and

C. L. FORTESCUE[1]
Fellow, A. I. E. E.

Synopsis.—This paper outlines the purpose of the cooperative installation of a lightning laboratory, the organization employed, and the various tests which have been made during the summer of 1929. In general, the paper presents the preliminary analysis of some of the most outstanding of a rather long series of tests and forms the background for a more complete paper giving a more detailed analysis which will be presented in the near future.

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THE work done in investigating lightning in the field by means of cathode ray oscillographs during the year 1928 demonstrated that important data can be obtained in this manner. By use of the instruments now available information can be acquired which will place expenditures for lightning protection on a sound basis.

The Public Service Electric & Gas Company of New Jersey, having a large amount of 220-kv. lines under construction, realized that these lines afforded an exceptional opportunity for investigating lightning conditions free from the exigencies of operating conditions and for obtaining information on the characteristics of such lines under lightning surge conditions, the characteristics of ground wires under grounding conditions that obtain on certain parts of the right-of-way, etc.

Such considerations led the Public Service Electric & Gas Company to cooperate with the Westinghouse Electric & Manufacturing Company in a joint investigation. The program consisted of an investigation of the characteristics of actual lightning strokes and of those physical characteristics of a transmission line which might have an effect on the performance of such lines under lightning conditions. Some of the features considered were the effects of attenuation, coupling between ground wire and phase wires, coupling between phase wires, the effect of the ground wire in reducing voltages on phase wires, and the surge impedances of towers and of lines.

The Site. The site selected for the conduct of these investigations was admirably suited for the purpose. Located in the hills of New Jersey approximately six miles from Newton in Sussex County, at an elevation of 500 feet, the power company had available a section of new 220-kv. transmission line extending 7.75 miles west to the Delaware River and approximately 1.3 miles east. This locality according to previous records was subject to from 30 to 40 lightning storms per year. In addition, the typography was rough and a great variety of ground conditions was available for test.

The line is a part of the Pennsylvania-New Jersey interconnection of the Public Service Electric & Gas Company with the Pennsylvania Power & Light Company and the Philadelphia Electric Company. The type A suspension towers have an over-all height of 77 ft., 9½ in. with the conductors arranged for flat horizontal configuration. The crossarm has an over-all length of 57 ft., the spacing between conductors being 28 ft., 6 in. Two ground wires are located midway and above the conductors at a height of 13 ft., $10\,^9/_{16}$ in. The line is designed for 18 insulator units and is equipped with 40-in. arcing rings. The conductor is 795,000 cir. mils A. S. C. R., 40 per cent steel by weight. The ground wires are 203,200 cir. mils A. S. C. R., 72 per cent steel by weight. In order not to influence the results from lightning strokes, the ground wires were not installed on the section of line under test. The availability of such a line as this, not yet in service, was particularly fortunate since during those periods when there was no possibility of measuring lightning strokes, the test crew could utilize their time, with the aid of a lightning generator, to investigating the surge characteristics of the line.

Equipment. The equipment was made mobile so that investigations might be carried on at other sections of the line with very different ground conditions or where the ground wire had already been installed, but the volume of work, together with the delay in getting the laboratory into operation this summer, has resulted in its being located on the one site.

The test equipment consisted of two trailers and two Fordson tractors. On each of the tractors there was installed a 5-kw. generator with an exciter to be utilized in supplying power for the Norinder oscillograph and the 1,000,000-volt lightning generator. In one of the trailers the Norinder oscillograph was located together with its auxiliary equipment, consisting of an oscillator, linear timing device, high voltage rectifier, vacuum pumps, switchboard, and work bench. At the other end of this trailer there were installed facilities for the development and printing of films and an electric refrigerator so that an ample supply of cold water will be available for the photographic processes. The second trailer contained the 1,000,000-volt lightning generator patterned after that at the Westinghouse High-Voltage Laboratory, at Trafford, Pa. This generator consisted of a 100,000-volt transformer arranged to charge 14 banks of $5 - 0.5\ \sigma f.$ condensers in parallel through a full-wave kenetron rectifier. Through an arrangement of gaps and resistors, these banks were then discharged in series. The auxiliary equipment, in addition to the Fordson tractors with their 5-kw. generators, consisted of numerous surge impedances, a 50-cm. sphere-gap, klydonographs, and a number of capacity and resistance potentiometers. A portable signal system was utilized between the lightning generator and the oscillograph laboratory.

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[1] Of the Public Service Electric and Gas Company and the Westinghouse Electric and Manufacturing Company, respectively.

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Organization. The laboratory was constantly attended by a competent staff consisting of not less than six engineers and at times during the summer there have been as many as nine engineers at work on this project. Although the personnel was derived from two organizations, it operated completely as a single unit during the entire period of tests.

We desire to pay tribute to those men who have actively participated in the work and whose technical skill and knowledge, together with their enthusiasm and tenacity of purpose, have produced such excellent results in the limited time available.

Lightning. Lightning generator and Norinder oscillograph arrived at Stillwater on June 17 and were put into operation on July 1, 1929. The spring and summer preceding the placing of the laboratory into operation had been remarkable for the severity and frequency of lightning storms in this vicinity and although many storms were observed on either side of the line, it was not until August 14 that a storm traveled over the line. Between 11:51 a. m. and 12:07 p. m. four oscillograms were obtained which showed a range of fronts varying from 250 or 300 kv. per microsecond to 90 kv. per microsecond and a range of crests from approximately − 1600 kv. to + 350 kv. (Fig. 1).

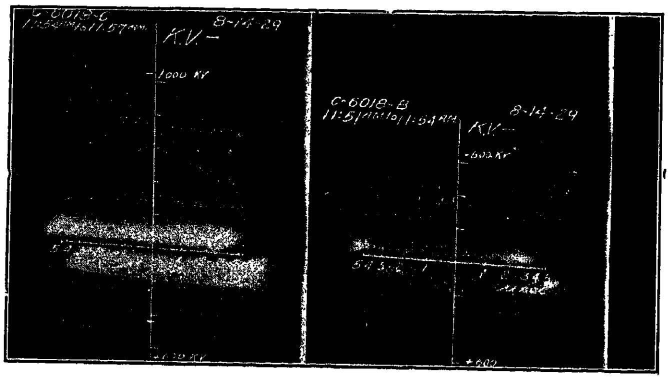


FIG. 1—TWO OF THE LIGHTNING SURGES RECORDED AT STILLWATER, NEW JERSEY

a. 1600-kv. lightning surge
b. 350-kv. lightning surge

It is to be regretted that more data on natural lightning were not obtainable, for a large volume is necessary under conditions where the line characteristics are known before the results can be derived and a practical application to transmission line design made.

TESTS WITH LIGHTNING GENERATOR

Attenuation. In order to determine the duration, as measured by the length of line, of surge voltages which might prove dangerous to the operation of a transmission line, surges were impressed with different wave shapes, the voltages varying from 200 to 1000 kv. over a length of line ranging from zero to 18 miles. Both positive and negative polarities were employed during these tests. Figs. 2, 3, and 4 show attenuation data in the form of plotted curves for the several types of waves used.

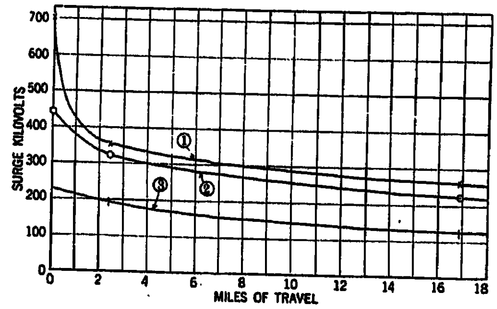


FIG. 2—ATTENUATION OF SURGES OF VARIOUS ORIGINAL MAGNITUDES (STEEP WAVES)

1. Attenuation of negative 700-kv. surge (½ μ sec. wave-front)
2. Attenuation of negative 436-kv. surge (½ μ sec. wave-front)
3. Attenuation of negative 218-kv. surge (½ μ sec. wave-front)

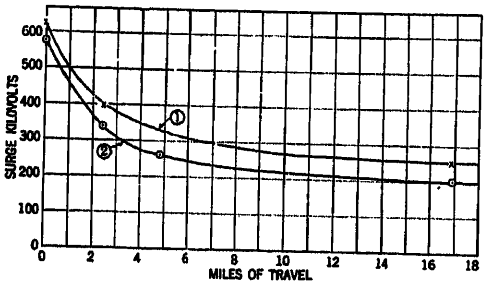


FIG. 3—ATTENUATION OF SURGES OF DIFFERENT POLARITY

1. Attenuation of negative 621-kv. surge (wave-front of 3 μ seconds)
2. Attenuation of positive 574-kv. surge (wave-front of 3 μ seconds)

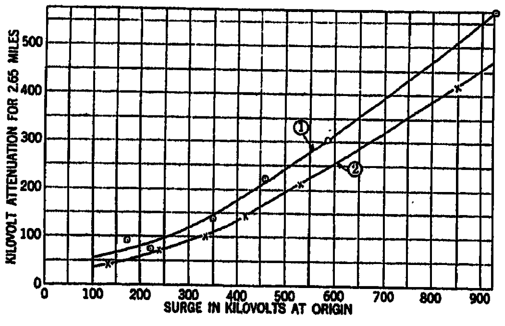


FIG. 4—ATTENUATION OF SHORT STEEP WAVES AND SLOWER SLOPING WAVES

1. Attenuation of positive surge (wave-front of ½ μ sec.)
2. Attenuation of positive surge (wave-front of 2 μ sec.)

The trend of the data obtained indicates the following relationships:

1. Considering crest voltage of the original surge and its attenuation, the attenuation is increased with the crest voltage over the length of line used. The rate of increase in attenuation is much more rapid

when the surge voltage exceeds the corona point of the line. The influence is shown by breaks in the line in Fig. 5. (Oscillogram C-6070-B.)

2. Considering the relation between the length of wave and its attenuation, the short wave attenuates more rapidly than a long wave of equal crest voltage.

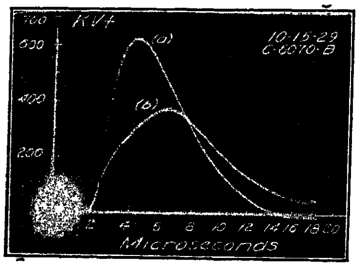


Fig. 5—Oscillogram Showing Effect of Corona on Attenuation

(Oscillogram C-6070-B)

3. Considering the relation between the polarity of the surge and attenuation above the corona point, positive surges attenuate more rapidly than negative surges of equivalent crest voltage and shape.

4. Considering the effect of distance traveled on wave shape, with increased distance there seems to be an increased tendency to slope off the front of the wave and to increase the length of the tail. This change in form is more accentuated above the corona point, and is shown both in Figs. 6 and 7. (See oscillogram No. 6068-A and 6068-E.)

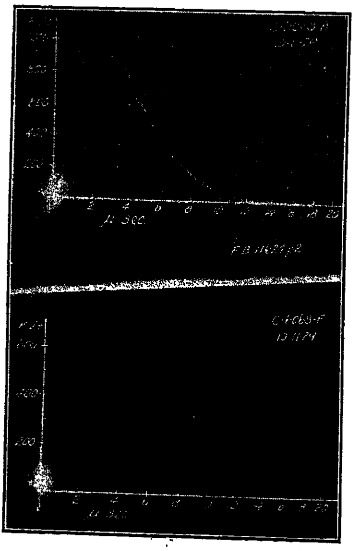
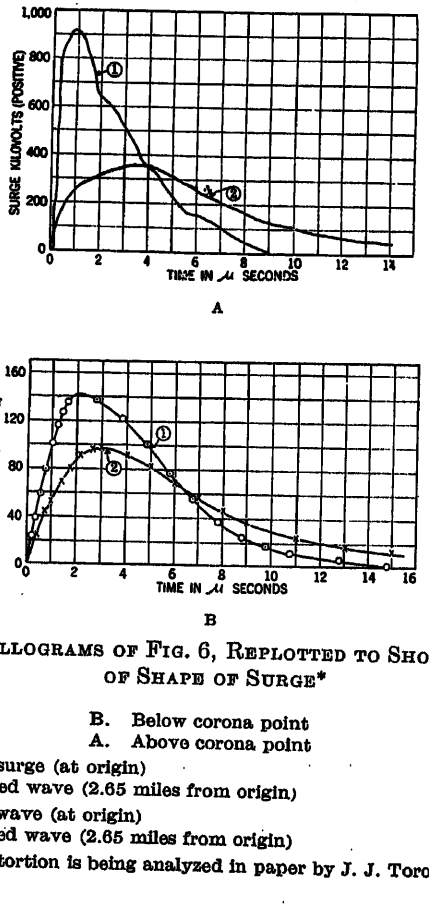


Fig. 6—Oscillograms Showing Attenuation

A. Original wave, (oscillogram C-6068-A)
B. Wave after 2.65 miles, (oscillogram C-6068-E)

Fig. 7A shows the surges of Fig. 6 replotted to show the change in wave shape more clearly and is for a wave above the corona voltage. Fig. 7B shows similar waves replotted but below the corona point. Since oscillograms of natural lightning surges generally show a long tail, they will attenuate less rapidly because they are of the type indicated above as being the least susceptible to attenuation.

Coupling. In order to determine the effect on a phase wire of a surge on another phase wire or a ground wire, the coupling between the phase wires was measured varying both the crest voltage and polarity. The coupling factor may be defined as the ratio between the voltage of an impressed surge and the resulting induced surge on an adjacent wire. These data are important in the more advanced theory of lightning protection by means of ground wires. It is important to find out how close to actual values are the results obtained by the usual methods of computation using the method of images. In the tests, since no ground wires were available on the structures, the middle phase wire was utilized as a ground wire.



A

B

Fig. 7—Oscillograms of Fig. 6, Replotted to Show Change of Shape of Surge*

B. Below corona point
A. Above corona point
1. Original surge (at origin)
2. Attenuated wave (2.65 miles from origin)
1. Original wave (at origin)
2. Attenuated wave (2.65 miles from origin)

*Effect of distortion is being analyzed in paper by J. J. Torok

Fig. 8 shows curves of the coupling between adjacent conductors and outside conductors. The values as calculated by the usual formula and assuming the path in the earth to follow the image of the conductors are 11 per cent and 20 per cent between adjacent conductors and outside conductors, respectively. The curves show that these values increase with increase in voltage, which is possibly due to the effective increase in the size of conductors due to corona. The increase in the coupling factor is less for negative surges than for positive surges and this seems in agreement with the fact that the attenuation is also less for negative surges than for positive surges.

In general, the results are summed up as follows:

1. Considering the effect of the surge crest voltage on coupling, the higher the crest voltage, above the corona point, the greater the coupling factor. (Oscillogram C-6070-A.) The oscillograms of Fig. 9 show the nature of this coupling.

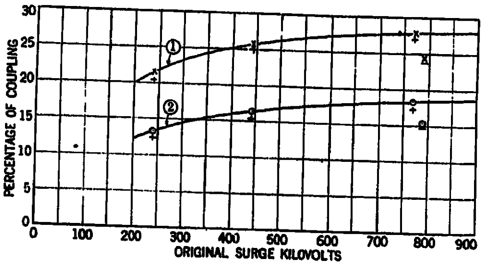


FIG. 8—CURVES SHOWING COUPLING BETWEEN PARALLEL WIRES

1. Horizontal configuration 28 ft. spacing. Approximate 50 ft. above ground
2. Horizontal configuration 56 ft. spacing. Approximate 50 ft. above ground

2. Considering the effect of polarity on coupling at the higher voltages, the coupling factor is greater for a positive surge than for a negative surge of equal crest value.

3. Considering the effect of grounding the middle phase wire, the voltage induced between the two outside phase wires is decreased.

The tests are valuable for the determination of the correction to be made in line characteristics under transient conditions as calculated and are of use in conjunction with other tests in estimating the behavior of natural lightning strokes to ground wires and lines and their probable effect in producing outages.

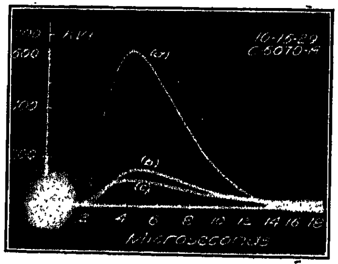


FIG. 9—OSCILLOGRAMS SHOWING COUPLING BETWEEN CONDUCTORS, (OSCILLOGRAM C-6070-A)

a. Original surge
b. Induced surge in conductor 28 ft. distant
c. Induced surge in conductor 56 ft. distant

Surge Impedance. In order to determine surge impedances to be used in connection with computed values of natural lightning strokes, 1, 2, and 3 phases of the line were surged. Additional tests were made with one phase wire surged and the adjacent phase wire grounded to determine the effect of this condition simulating a line protected by ground wires. Tests were made with both polarities, various crest voltages, and wave fronts. Fig. 9 shows the points obtained in these tests together with a median curve indicated. Although the points are rather scattered they indicate that at low voltages the surge impedance is higher than the calculated value, which was 450 ohms, and that as the voltage is raised this value becomes less. This is probably due to the fact that the calculated value of 450 ohms is based on a return current coinciding with the image of the conductor, whereas it actually is probably somewhat lower and the drooping of the curve at higher voltages is undoubtedly due to the increase in the size of the conductor brought about by corona.

The results may be summed up briefly as follows:

1. The effective surge impedance of the line is reduced as the surge potential increases above the corona point.

2. The distortion factor of the line with corona is appreciable, and will be discussed fully in a subsequent paper.

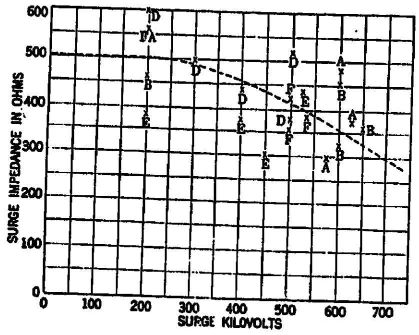


FIG. 10—MEASURED SURGE IMPEDANCE OF ONE CONDUCTOR

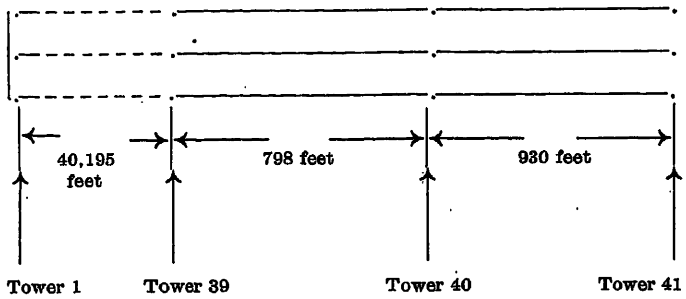
Tower and Tower Ground Impedances. Since the impedance to a surge between ground wire and ground is the dominant factor in determining the effectiveness of a ground wire in protecting against direct strokes, it is extremely important that this impedance be evaluated and its relationship to the tower footing resistance, measured in the usual manner, determined. Surge measurements were made on towers having meggered resistance of the order of 15 to 50 ohms. The results of these tests are shown in Table I. These indicate that the impedance to surges which of course includes the impedance of the tower itself may be from 10 per cent to 80 per cent greater than the meggered resistance.

Buried Cable Tie Between Tower Footings. Theoretical analysis has indicated that a buried cable tie between tower footings will effectively lessen the surge impedance of high resistance tower footings. The tower footings of several adjacent towers were tied together by ⅝-in. stranded cable buried from 6 in. to 8 in. below the surface of the ground. The values are included in Table I and it will be noted that while the cable results in a reduction of resistance of the order of 20 to one, the corresponding reduction in surge impedance is only of the order of two to one. This of course is to be expected

TABLE I

SURGE IMPEDANCE OF TOWER GROUNDS

Location ground	Cable connection	Megger resistance	Ground surge* impedance
T –39	Off	44 ohms	76
T –40	Off	60 ohms	100.
T –39 +40	Off	25.3 ohms	40 ohms
T –41	Off	16 ohms	18.3 ohms
T –39	On T –39 – 40 –41	2.2 ohms	48.5
T –40	On T –39 – 40 –41	3.7 ohms	34.2
T –41	T –41	3.5 ohms	15.6 ohms



*Based on a surge impedance of 450 ohms for the line.

since the surge impedance is a transient value and will depend upon the distributed constants of the buried cable, whereas the ohmic value will depend upon the integrated effect for the leakage of the cable to earth. The amount of reduction in surge impedance, however, is not to be regarded as insignificant. In fact, considering the fact that the occurrence of lightning surges follows the probability curve, this gain might easily be sufficient to change an entirely unsatisfactory line's performance to one practically free from trouble.

Ground Wire Protection. By using one of the phase conductors as a ground wire some measurements were made to evaluate the protection afforded by ground wires against induced strokes. The measurements made checked fairly well the values of this protection as calculated by the advanced methods including reflections at grounded points. However, since our own work on lightning in recent years as well as the independent work headed up by Professor Matthias in Germany, an account of which is given in *E. T. Z.*, of October 10, 1929, indicate that induced strokes are not an important factor on high-voltage lines, no great amount of time was spent in these tests and they are not considered of sufficient importance to justify the space of including them here. As previously brought out, ground wires with low surge impedance grounds are extremely effective in protecting against direct strokes. The results of these tests, discussed under surge impedance, coupling, and tower ground impedance, is directly applicable in evaluating the amount of this protection. The remaining unknown factors for a complete analysis are the voltage of the natural lightning where it strikes the ground wire and the characteristics of the lightning path. These can only be determined by a further, more detailed study of natural lightning.

CONCLUSION

While the time available during the past summer has been limited to practically four months, a considerable amount of valuable information has been obtained which, although in many respects incomplete, has provided a better understanding of the effects of natural lightning surges on transmission lines. Since the work was carried on out of doors on a section of 220-kv. transmission line, weather conditions played an important part both for the recording of natural surges and for the investigation of line characteristics with the surge generator. On account of this fact, comparatively little time was spent in the analysis of tests, every effort being made to obtain data which could be thoroughly analyzed during the winter.

The present paper gives only a cursory analysis of some of the data particularly pertinent to the immediate requirements for line and ground wire construction. The total data obtained during the summer comprised something over one thousand films and in the short time available it was not possible to even examine all these data, but the contents will be found, it is believed, of considerable scientific as well as practical importance. A paper giving complete analysis of the results of these tests will be submitted to the Institute at a later date by Mr. Torok.

A large volume of additional data is needed in order to adequately classify and determine the effects of natural lightning strokes, and it is expected that the work started at Stillwater in the summer of 1929 will be continued next summer, although the location may be changed. The work with the surge generator, which is necessary for the determination of the individual factors involved in the design of transmission lines and their effect upon lightning strokes, will also be continued.

It is to be hoped that the continuation of this work will result in the final determination of the mathematical relationships existing between those factors affecting lightning surges on transmission lines, thus making possible the design of steel tower transmission lines which will render better service when constructed in situations exposed to severe lightning storms.

We wish to acknowledge gratefully the assistance of our colleagues in carrying out computations in connection with the data submitted and the general assistance in connection with the writing of the paper.

Discussion

For discussion of this paper see page 928.

Lightning Voltages on Transmission Lines

BY R. H. GEORGE* and J. R. EATON†

Synopsis.—During the summer of 1929 studies were made to determine the characteristics of voltage transients set up on transmission lines by lightning. A cathode ray oscillograph and a number of surge voltage recorders were connected to a 140-kv. transmission line in such a manner that lightning disturbances were recorded by the instruments. This paper describes the equipment used, and presents the results obtained from the season's investigations.

* * * * *

INTRODUCTION

During the summer of 1929 extensive field studies were made to determine the characteristics of transient voltages set up on transmission lines due to lightning. Tests of this kind have been found necessary because in the present stage of the art, transmission line design and operation have advanced to such a point that lightning probably presents the only serious remaining problem pertaining to continuity of service. To attack this problem it has been found necessary to determine first the nature of the disturbances set up by lightning, and therefore fundamental studies have been in progress to study the electrical effects of lightning discharges on actual high-voltage transmission lines.

For some years studies of this general nature have been in progress in this and other countries, and many papers have been published describing the results. In 1926 the General Electric Company and the Consumers Power Company began a cooperative investigation, which has been continued in succeeding years, using the former's surge voltage recorders on the power company's transmission lines.[1] In 1928 the Consumers Power Company, in conjunction with Purdue University, installed on its transmission system a new type cathode ray oscillograph which had been developed in the University's laboratories. In the spring of 1929 the surge voltage recorder and cathode ray oscillograph investigations were combined into one extensive study. The cathode ray oscillograph was used to record the wave shape of transient voltages appearing on a transmission line during lightning storms, while the surge voltage recorders registered the magnitude and time of occurrence of these transient voltages at various points along the line.

DESCRIPTION OF EQUIPMENT

Transmission Line. This investigation was conducted about 40 mi. north of Grand Rapids, Michigan on a 140-kv. transmission line (known as T-20) which extends north and south a distance of 101 mi. from Junction Dam to Grand Rapids, Fig. 1. This is a single circuit line erected on one side of double circuit towers, as shown in Fig. 5. The conductors on the line are 110,000-circular mil copper, insulated with 10 Ohio Brass No. 25,622 disk insulators which are equipped with arcing horns on the lower end of the strings. The north half of the line and a section of about 10 mi. on the south end of the line are equipped with a No. 4 copperweld ground wire suspended from the peak of the towers.

The line traverses a relatively flat country, the soil of which varies from light sand on the northern half of the line, to heavy clay on the southern end. Tower footing resistances vary over extreme limits of 1000 to 1500 ohms in the sandy section down to 3 to 5 ohms in the clay and swampy sections. Operation over a period of 10 to 12 years has shown that this line is subjected annually to many lightning storms which are swept eastward from Lake Michigan.

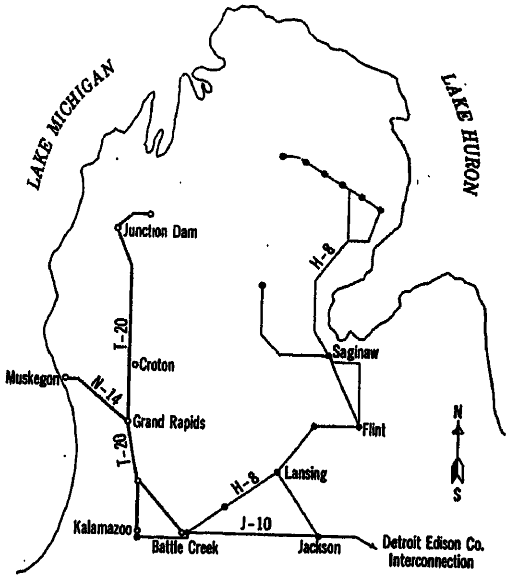


FIG. 1—CONSUMERS POWER COMPANY 140-KV. SYSTEM

Heavy lines indicate 30-cycle. Light lines indicate 60-cycle

Cathode Ray Oscillograph. The cathode ray oscillograph used in this investigation is of the hot cathode type and with the exception of some improvements, has been described in a previous A. I. E. E. paper.[2] The principal improvements have been the change to a single sweep time axis and the addition of a signal for indicating the occurrence of a surge.

The sweep circuit which is set in operation by the lightning surge, sweeps the beam once across the photographic film at a known rate of speed, producing a logarithmic time axis. On the arrival of a surge the grid-glow tubes, Fig. 2, close the circuit between points *A* and *B*, permitting current from the source 3 to flow through the resistance 4 and condenser 5, charging condenser 6, across which the deflecting plates are connected. The resistance 7 maintains the deflecting plates at ground potential previous to the arrival of a surge.

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*Research Associate in High Voltage, Engg. Experiment Station, Purdue University, West Lafayette, Ind.

†Transmission Line Engineer, Consumers Power Company, Jackson, Mich.

1. For references see Bibliography.

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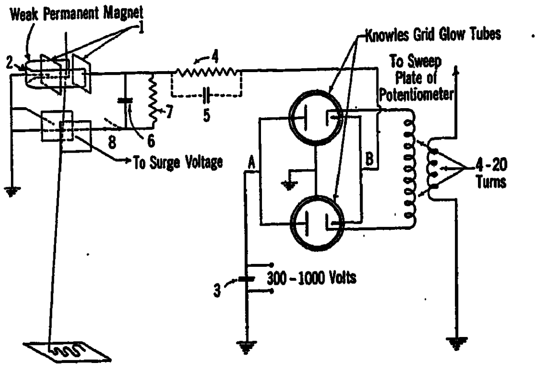


FIG. 2—CIRCUIT FOR AUTOMATICALLY STARTING TIMING SWEEP WITH SUPGE

The occurrence of a surge is indicated by the flash of the grid glow tubes, which providing the resistance 7 is set below a critical value, will continue to glow until the switch 8 is opened momentarily.

The oscillograph was installed in a sheet metal building, Fig. 3, all parts of which were well grounded to shield the instrument from extraneous electrostatic fields. Connections from the transmission line to the oscillograph were made through an air insulated capacitor potentiometer shown in Fig. 4. The large plate *A* supplied voltage to initiate the sweep circuit which also denoted the arrival of a surge. The antenna wire *B* supplied voltage to the vacuum tubes which initiate the cathode beam. Voltage to the deflecting plates of the oscillograph was picked up by the small plate *C*, connections to which were carefully shielded from electrostatic fields which might be present due to other equipment in the vicinity. The plate *C* consisted of two parts, one below and one above the grounded wire screen. With the lower section only connected to the deflecting plates the beam sensitivity was one inch of deflection per million volts on the line, whereas with both upper and lower sections connected, the sensitivity was one inch deflection per 400 kv. on the line. These sensitivities were carefully checked using a million-volt lightning generator which was set up for studies of artificial lightning on another transmission line.[3] Transient voltages of various values were applied to the upper grid of the potentiometer and oscillograms were recorded in exact fashion as when the station was being used for recording lightning voltages. The applied voltage from the lightning generator was measured at the potentiometer by means of sphere-gaps, and these measured values of voltage were compared with the deflections produced in the oscillograph.

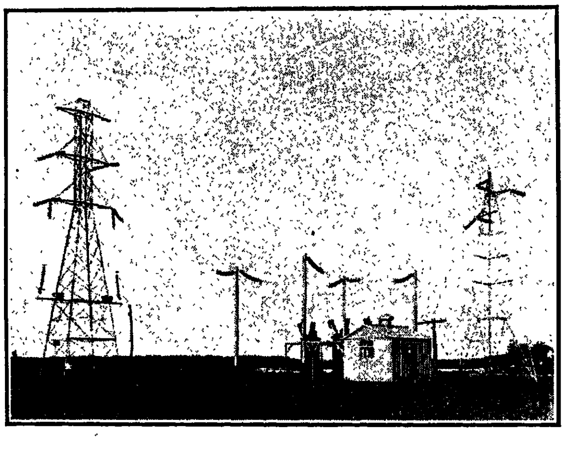


FIG. 3—CATHODE RAY OSCILLOGRAPH STATION, SHOWING POTENTIOMETER AND CONNECTION TO LINE

Surge recorders on tower at left

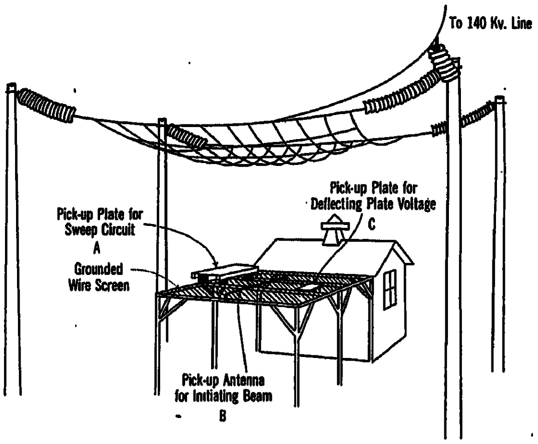

To 140 Kv. Line
Pick-up Plate for Deflecting Plate Voltage
C
Pick-up Plate for Sweep Circuit
A
Grounded Wire Screen
Pick-up Antenna for Initiating Beam
B

FIG. 4—CATHODE RAY OSCILLOGRAPH POTENTIOMETER ARRANGEMENT

The oscillograph was first put into operation for recording lightning voltage on May 11, and was operated during all lightning storms until the end of the season. The instrument was maintained in operating condition at all times but was not evacuated to operating pressures when no storms were present on the system. Upon warning from the system operator of storms in the vicinity, the oscillograph bell was pumped down to operating pressure and the set was put in readiness for registering voltage surges. As a rule about twenty minutes time was required for the set to be put in operation after a storm warning was received. The potentiometer was set for a ratio of one inch deflection per 400 kv. when storms were distant, and one inch deflection per 1000 kv. when storms were in progress in the immediate vicinity of the station.

Surge Voltage Recorder. The surge voltage recorder used was the General Electric two-electrode instrument which has been described in previous papers.[4] The potentiometer used to connect it with the 140-kv. line, Fig. 5, was the conventional type consisting of an insulator string with grading shields and a ground connection to by-pass leakage currents around the shield supplying voltage to the instrument.[5] This type of potentiometer has been used during the past two years because it eliminates the formation of slightly damped Lichtenberg figures which are known to give an erratic indication of line behavior. Voltage ratios of this type of potentiometer have been carefully checked in the General Electric laboratories and found to be 68:1, 75:1, and 77:1 for the various arrangements used in these studies. In all, 13 surge voltage recorders were kept in operation, the first one being installed and started May 8; all recorders were in operation by May 27, and were kept running continually until October 4.

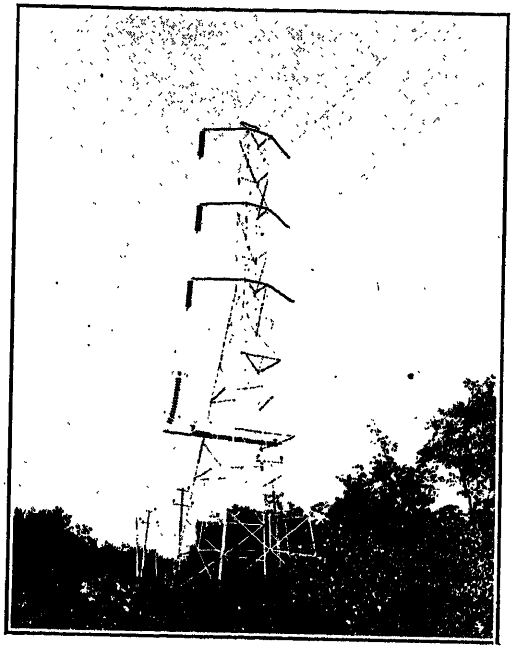


Fig. 5—Surge Voltage Recorder Installation, Showing Standard Tower

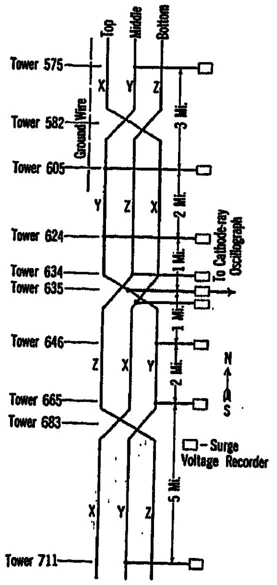


Fig. 6—Location of Surge Voltage Recorders and Cathode Ray Oscillograph on T-20 Transmission Line

Location of Equipment along Line. It was thought desirable to conduct these studies at a point considerably distant from the nearest substation in order that wave forms of lightning voltages would not be influenced by lightning arresters and other terminal equipment. Croton was chosen as the center of the investigation as it afforded a low-voltage power supply near the T-20 140-kv. transmission line, 40 miles from Grand Rapids, the nearest 140-kv. terminal. The oscillograph was located at a convenient point on the T-20 line at Croton, and surge recorders were connected along the line at points as indicated in Fig. 6. It may be noted that at the oscillograph station, recorders were connected to X, Y, and Z phases. At points one and three miles both north and south from the oscillograph station, recorders were connected to the Y phase, in order that some record might be obtained pertaining to the point of origin of lightning disturbances and to the voltage magnitude at various points along the line both directions from the oscillograph. With these instruments it was thought that data could be obtained which would correlate with that of the cathode ray oscillograph and which would also give information pertaining to attenuation of surge voltages as they travel along the line.

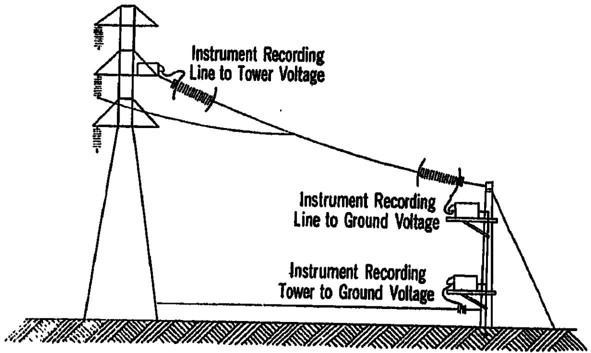

Instrument Recording Line to Tower Voltage
Instrument Recording Line to Ground Voltage
Instrument Recording Tower to Ground Voltage

Fig. 7—Surge Voltage Recorder Installation at Towers 575 and 711

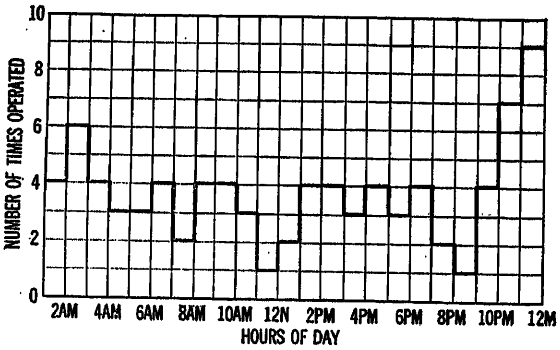
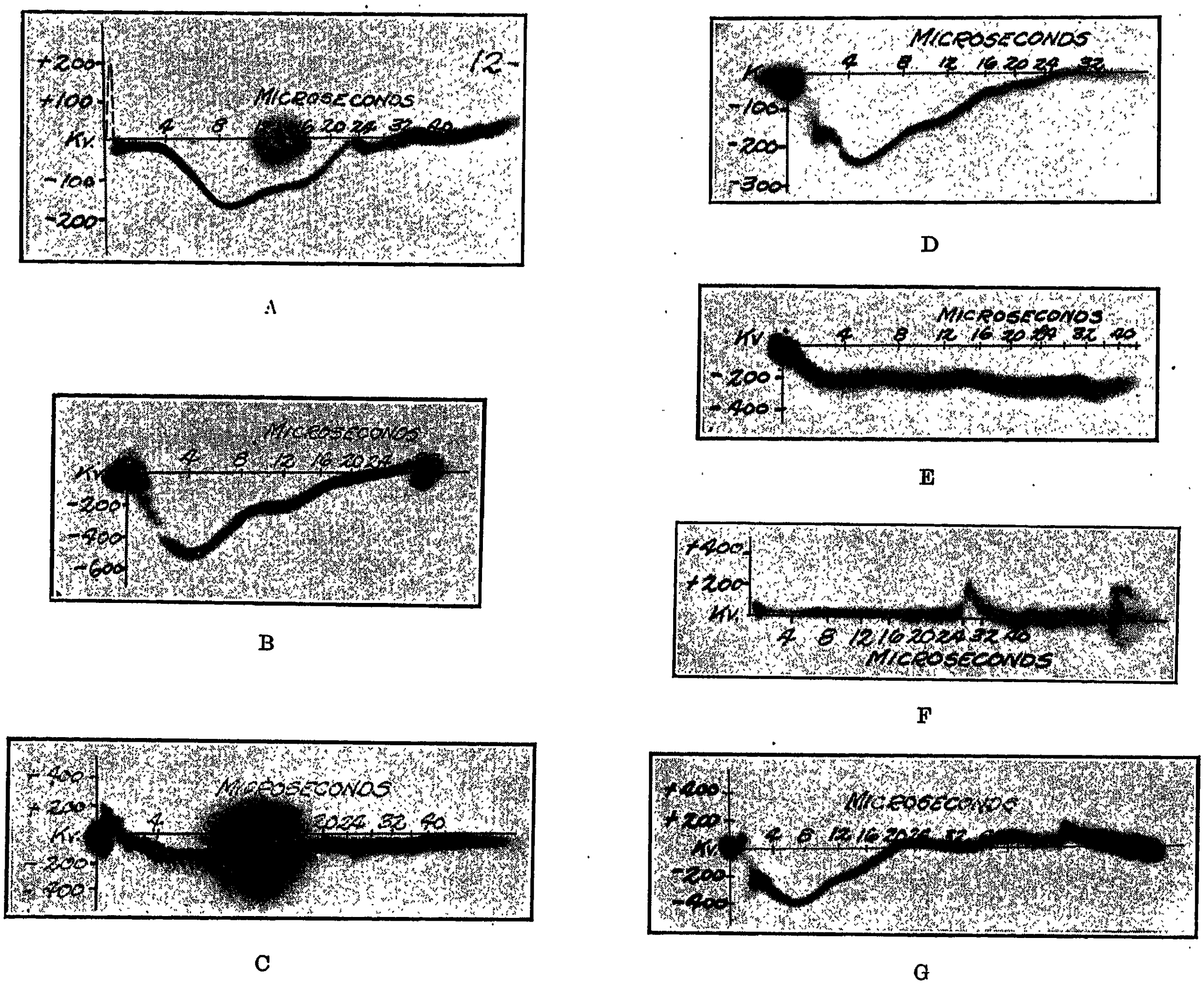


Fig. 8—Hours of Operation of Oscillograph

At tower 575 about 6 mi. north, and at tower 711 about 7½ mi. south of the oscillograph station, set-ups were made of three surge voltage recorders as shown in the diagram Fig. 7. These instruments, as may be seen from the diagram, were so arranged that one recorded voltage between line and tower, another voltage between line and ground, and a third between tower and ground. This set-up was made in an effort to study the distribution of voltage between the above mentioned parts when the line was subjected to lightning voltages. Theoretically, it appears that the voltage between line and ground should equal the sum of the voltages from line to tower, and from tower to ground. On lines equipped with ground wires there is a tendency for the towers to be raised to high voltage due to the charges induced on the ground wire. On towers whose footings rest in sandy or rocky soil of high resistance, the tower-to-ground voltage may be very high and may possibly be the same order of magnitude as the line-to-tower voltage to which the insulators are subjected. As tower 575 is in a section equipped with ground wire and as it is in sandy soil with a footing resistance of 1150 ohms, it offered an excellent opportunity for checking the above mentioned theory. Tower 711 presented the other extreme conditions in that it is in a section not equipped with a ground wire and stands in marshy



A B C D E F G

Fig. 9—Oscillograms of Lightning Voltages

A. 12-6 6-26-29 12:22 P. M. Low potentiometer ratio. Surge on T-20 line. No lightning visible at Croton but reported at station 60 mi. north. Corresponds to Surge No. 14. See voltage profile, Fig. 11A

B. 15-8 6-30-29 2:08 A. M. High potentiometer ratio. Surge on T-20 line. Lightning seen to strike near line south of Croton. Flashover found on X and Y phases at tower 682 (4.7 mi. from cathode ray oscillograph) thought to have resulted from this flash. Corresponds to Surge No. 21. See voltage profile, Fig. 11B

C. 19-7 7-13-29 5:43 A. M. High potentiometer ratio. T-20 line arced over and burned down at Tower 608 (2.7 mi. from cathode ray oscillograph). Corresponds to Surge No. 31. See voltage profile Fig. 11C

D. 19-19 7-13-29 9:41 A. M. Low potentiometer ratio. T.20 transmission line was deenergized and out of service at the time of this registration. Corresponds to Surge No. 33. See voltage profile Fig. 11D

E. 20-8 7-24-29 11:10 P. M. High potentiometer ratio. Lightning near cathode ray oscillograph station. Corresponds to Surge No. 40. See voltage profile Fig. 11E

F. 21-4 8-13-29 8:48 A. M. High potentiometer ratio. Storming at Croton. Corresponds to Surge No. 45. See voltage profile, Fig. 11F

G. 21-9 8-13-29 1:33 P. M. High potentiometer ratio. Storming at Croton. Corresponds to Surge No. 48. See voltage profile Fig. 11G soil with a footing resistance of three ohms. In order that data obtained on both towers 575 and 711 might be correlated with that obtained on the cathode ray oscillograph, the connection to the line was made to *Y* phase.

RESULTS

Cathode Ray Oscillograph. The cathode ray oscillograph was operated during 28 storms through the course of the 1929 lightning season. Fig. 8 shows the number of times the oscillograph was operated during the various hours of the day.

Transient voltages on the line caused the beam of the oscillograph to be initiated on 26 occasions, resulting in time-voltage lines on the photographic film. The characteristics of these oscillograms are listed in Table I. Fig. 9 shows reproductions of the seven registrations which represented voltages above 200 kv. Some of the registrations showed almost no deflection and were practically straight lines on the zero axis. The oscillograph was so adjusted that it was actuated by a relatively low value voltage transient, in order that it might be fully turned on by any surge of appreciable magnitude. As the potentiometer ratio was necessarily adjusted to give the desired deflection for the highest value of voltage expected, many low magnitude transients turned on the beam but did not produce enough voltage on the oscillograph plates to give an appreciable deflection.

Surge Voltage Recorders. Throughout the lightning season the surge voltage recorders registered a total of 53 surges. Of these, 48 were due to lightning, 1 was due to switching, and 4 resulted from unknown causes. The number and magnitude of surge voltages recorded are shown by the curve in Fig. 10. The characteristics of the lightning surge voltages are shown in Table II.

Lightning transients were simultaneously recorded on a number of the surge voltage recorders in operation, and therefore are indicative of the attenuation of voltage surges as they travel along the line. Voltages of high values (1000 to 1500 kv.) attenuated very rapidly as is illustrated by the profile of surge No. 21, Fig. 11B. This particular transient reduced in value from 1650 kv. to 540 kv. in a distance of three miles, or an average rate of almost 400 kv. per mile. Voltages of low values, of which Surge No. 37, Fig. 11H is an

TABLE I

CHARACTERISTICS OF CATHODE RAY OSCILLOGRAMS OF LIGHTNING VOLTAGES

Cathode ray oscillogram No.	Date	Time	Deflection		Wave			Surge voltage recorder registration on 635 *Y*		Remarks
			Pos. kv.	Neg. kv.	Front msc.	Time to half voltage msc.	Duration msc.	Pos. kv.	Neg. kv.	
11- 3	6-19-29	12:11 a. m.	..	..	..	..	..	Trace	Trace	Deflection not sufficient for analysis
11- 7	6-19-29	4:00 a. m.	..	..	..	..	..	None	None	Deflection not sufficient for analysis
11- 8	6-19-29	4:05 a. m.	0	150	3	20	40	250	Trace	Arc-over on line. Location not known
12- 6	6-26-29	12:22 p. m.	200(?)	170	8	18	36	Trace	Trace	May have been positive voltage of very short duration preceding main surge
15- 2	6-29-29	11:27 p. m.	..	..	..	..	..	260	260	Deflection not sufficient for analysis. Line apparently grounded. Point unknown. Surge recorded by M. D. figures
15- 3	6-30-29	12:05 a. m.	..	..	..	..	..	None	None	Deflection not sufficient for analysis.
15- 4	6-30-29	12:19 a. m.	..	..	..	..	..	None	None	Deflection not sufficient for analysis
15- 6	6-30-29	1:01 a. m.	150	100	3	6	40	None	None	Deflection not sufficient for analysis
15- 8	6-30-29	2:08 a. m.	0	530	4	8	30	Trace	540	Line may have arced over on *X* and *Y* phases at tower 682
18- 7	7- 8-29	10:49 p. m.	140	80	1 –	3	50	None	None	Front not registered. High frequency following crest for first 6 msc.
18- 8	7- 8-29	11:00 p. m.	..	..	..	..	..	None	None	Deflection not sufficient for analysis
19- 7	7-13-29	5:43 a. m.	250	150	0.5	2.0	50	Trace	300	Oscillations during first 8 msc. Arc-over on line 2.7 mi. north of Croton. Beam poorly focused
19- 8	7-13-29	5:47 a. m.	..	..	..	..	..	None	None	May have been high frequency for first 2 msc. Probably result of switching surge
19 -9	7-13-29	5:51 a. m.	..	..	..	..	..	None	None	Deflection not sufficient for analysis
19-10	7-13-29	5:54 a. m.	..	..	..	..	..	None	None	Deflection not sufficient for analysis
19-19	7-13-29	9:41 a. m.	0	225	4	11	30	None	275	High amplitude oscillations for first 3 msc. Line de-energized at time of surge
19-20	7-13-29	9:44 a. m.	80	120	..	..	..	None	None	Main wave very low amplitude. Damped oscillations beginning at 25 msc. Line de-energized at time of surge
20- 3	7-24-29	5:14 a. m.	..	..	..	..	..	Trace	Trace	Deflection not sufficient for analysis
20- 7	7-24-29	11:07 p. m.	..	..	..	..	..	None	None	Deflection not sufficient for analysis
20- 8	7-24-29	11:10 p. m.	0	220	3	Flat top	40*	None	None	Wave flat top for 40 msc. after which no record was obtained
20- 9	7-24-29	11:11 p. m.	..	..	..	..	..	None	None	Deflections not sufficient for analysis
20-11	7-24-29	11:39 p. m.	..	..	..	..	..	Trace	Trace	Oscillogram indefinite with very little deflection. Arc-over in Grand Rapids sub. Arcing ground
21- 4	8-13-29	8:48 a. m.	200	0	..	..	..	None	None	No deflection of beam except sharp peak at 30 msc.
21- 8	8-13-29	1:29 p. m.	100	0	..	..	..	None	None	No deflections of beam except sharp peak at 12 msc.
21- 9	8-13-29	1:33 p. m.	100	420	7	12	50	Trace	480	High amplitude oscillation throughout first 3 msc.
21-10	8-13-29	2:30 p. m.	100	..	..	..	..	None	None	Sweep circuit failed to operate. Only vertical line recorded

example, apparently travel along the line with almost no decrease in voltage value.

The information gained at towers 575 and 711, Fig. 7, was not so conclusive as might have been expected.

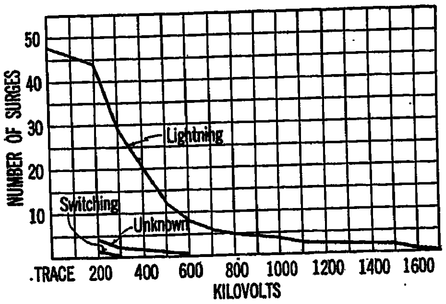


FIG. 10—NUMBER OF SURGES EXCEEDING VARIOUS VALUES OF VOLTAGE

Surge voltage recorder data

Evidently the difference between the line-to-tower and line-to-ground voltage is not ordinarily of sufficient magnitude to be definitely indicated by the surge voltage recorder, which due to its inherent characteristics, records only approximate values of voltage. The line-to-tower voltage sometimes registered higher than the line-to-ground voltage, and vice versa. However, a number of registrations of tower-to-ground voltages was obtained which indicated that transmission line towers may be frequently raised to a rather high voltage with respect to ground. Registrations of tower-to-ground voltages are listed in Table III.

TABLE II

CHARACTERISTICS OF LIGHTNING VOLTAGES

Surge Voltage Recorder Data

Type of surge	Number of surges	Highest recorded voltage Kv.
Oscillatory	39	1650
Unidirectional	9	280
Oscillatory having positive value exceeding the negative value	10	620
Oscillatory having negative value exceeding the positive value	21	1650
Oscillatory having positive and negative values equal	8	570
Unidirectional of positive polarity	1	280
Unidirectional of negative polarity	8	280
Recorded by highly damped Lichtenberg figures	35	1650
Recorded by highly damped and medium damped figures mixed	12	
Recorded by medium damped Lichtenberg figures	1	400

TABLE III

VOLTAGES RECORDED BETWEEN TOWER AND GROUND

Surge Voltage Recorder Data

Tower 575. Supports ground wire. Footing resistance 1150 ohms.

Surge No.	Voltage in kv.					
	Line-to-tower		Line-to-ground		Tower-to-ground	
	+	−	+	−	+	−
9	275	530	280	1100	10	34
22	270	270	None	None	None	Trace
31	400	290	285	625	28	Trace
32	450	450	285	280	54	12
48	675	600	350	1070	None	140 +

Tower 711. No ground wire. Footing resistance 3 ohms.

Surge No.	Voltage in kv.					
	Line-to-tower		Line-to-ground		Tower-to-ground	
	+	−	+	−	+	−
18	None	None	None	None	None	Trace
31	285	390	No record		6.8	5.5
33	265	300	No record		6.8	3.5
42	265	275	No record		3	3
47	330	290	265	330	3	3
49	650	1500	520	650	6.8	3

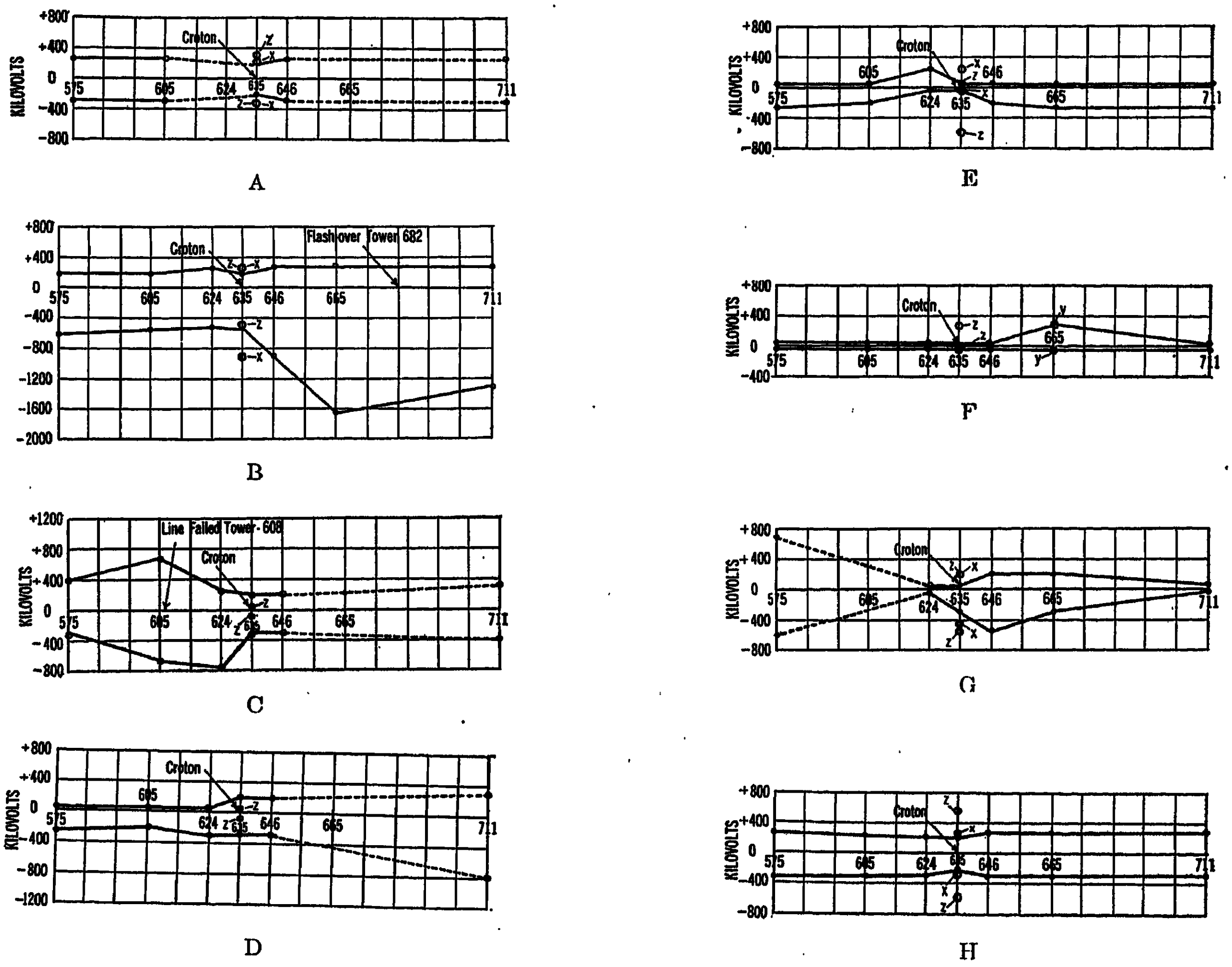
The one switching surge recorded reached a maximum value of 260 kv. and was recorded by Lichtenberg figures of highly damped characteristics. Surge voltages resulting from unknown causes were four in number with a maximum value of only 500 kv.

Correlation of Cathode Ray Oscillograph and Surge Voltage Recorder Data. Profiles of voltages measured on the surge recorders simultaneously with registrations on the cathode ray oscillograph are illustrated by the curves of Fig. 11. Some of these profiles are of individual interest.

Surge No. 21, Fig. 11B, and its corresponding oscillogram 15-8, Fig. 9B, gives the most complete record of any obtained throughout the season. The stroke of lightning which produced it was observed by the oscillograph operators, and later inspection of the line showed flashovers on phases *X* and *Y* at tower 682, approximately 4.7 miles from the oscillograph station. Scrub oak in contact with the base of the tower were killed by the electrical discharge although very little burning was noted. The wave recorded, Fig. 9B, is smooth and shows no indication of having been chopped, although flashover took place on the phase to which the oscillograph was connected. The record seems quite consistent in view of the fact that the footing resistance of tower 682, on which flashover occurred, is 600 ohms. With this value of resistance in series with the discharge path, it might be expected that the wave would have a point of discontinuity representing the time of flashover; but thereafter the shape of the wave would not be materially affected by the breakdown, although the magnitude would be reduced. The values of voltage recorded by the surge voltage recorders are very high as compared with the commonly accepted value of insulator flashover for the type of wave recorded. Cloud fields terminating on the rings of the potentiometers, or other factors, may possibly have caused the recorders to give erratic readings.

Surge No. 31, Fig. 11C and its corresponding oscillogram 19-7, Fig. 9C, represent the records obtained at the time of a line failure at tower 608, 2.7 mi. north of the oscillograph station. A lightning discharge apparently established an arc between the top and middle conductors (*Y* and *Z* phases) and flashed over the bottom insulator string (*X* phase). The middle conductor was burned off at a point about four feet from the insulator string. Tower 608 is only 1600 ft. from tower 605 on which a surge voltage recorder was connected to Y phase. The recorded value of voltage, 675 kv. from line to tower, seems somewhat low. This section of line is equipped with a ground wire and is characterized by tower footing resistances ranging from 100 to 250 ohms. It is possible that the charge on the ground wire raised the potential of the structure of tower 605, thereby reducing the line-to-tower voltage. The oscillogram recorded is somewhat lacking in detail as the beam was poorly focused.

Oscillogram 19-19, Fig. 9D, and its voltage profile, Surge 33, Fig. 11D, are of interest in that they were recorded at a time when the line was out of service, and therefore are in no way influenced by the power frequency voltage. The wave shape is, however, quite similar to others recorded at times when the line was in operation.



A

B

C

D

E

F

G

H

FIG. 11—PROFILES OF VOLTAGES ALONG LINE

Surge voltage recorder data

Horizontal divisions represent miles

A.	Surge No. 14.	See oscillogram	12-6.	Fig. 9A
B.	" " 21.	" "	15-8.	" 9B
C.	" " 31.	" "	19-7.	" 9C
D.	" " 33.	" "	19-19.	" 9D
E.	" " 40.	" "	20-8.	" 9E
F.	" " 45.	" "	21-4.	" 9F
G.	" " 48.	" "	21-9	" 9G
H.	" " 37.			

From Fig. 6 it may be seen that both the oscillograph and a surge voltage recorder were connected to the Y phase at tower 635. Comparison of the data obtained from these two instruments, Table IV, shows their relative performance.

From the above table it may be seen that the oscillograph recorded many voltages which produced no registrations on the surge voltage recorder. Of the four surges which registered on the recorder but not on the oscillograph, only one was sufficient to produce a measurable Lichtenberg figure on the film. This figure represented a positive voltage of very low value and a negative voltage of 300 kv. A momentary interruption to the oscillograph power supply experienced at about the time of this registration may account for the failure of the oscillograph to operate.

DISCUSSION OF RESULTS

Wave Shapes. As may be seen from Fig. 9 and

TABLE IV

RELATION BETWEEN OPERATION OF CATHODE RAY OSCILLOGRAPH AND SURGE VOLTAGE RECORDER ON *Y* PHASE AT TOWER 635

Total number of cathode ray oscillograms	26
Total number of surge voltage recorder registrations	16
Number of times surge voltage recorder registered when cathode ray oscillograph was not in operation	2
Nimber of times both cathode ray oscillograph and surge voltage recorder registered	10
Number of times only cathode ray oscillograph registered	16
Number of times only surge voltage recorder registered	4

Table I, the oscillograms showed certain definite characteristics of lightning voltages. The wave fronts ranged from approximately one-half to eight microseconds, the majority being less than five microseconds. On oscillograms 12-6, 18-7, 19-7, 19-19, and 21-9, high-frequency oscillations of considerable value are shown superimposed on the front of the main wave. The fronts of waves shown on oscillograms 15-8 and 20-8 are very smooth and free of oscillations. With one exception the total duration of the transients ranged from 30 to 50 microseconds. Oscillogram 20-8 showed a flat top wave which was sustained at practically constant value for a period of 40 microseconds, at which time the record was ended by the beam being cut off automatically. The total duration of the transient may have been far in excess of 40 microseconds.

In considering the oscillograms recorded, it must be borne in mind that they represent the time-voltage characteristics of traveling waves, and therefore may not be indicative of the voltage impressed on insulator strings at the centers of the disturbances. In no case was an oscillogram recorded of a stroke of lightning in the immediate vicinity of the station. The wave shapes recorded are probably dependent on at least three factors: the time characteristics of the lightning discharge, the geometry of the cloud field, and alterations from various influences as the waves traveled along the line to the oscillograph station. The records, however, are probably representative of the waves to which station equipment is subjected due to storms over connecting transmission lines.

The records of both the cathode ray oscillograph and the surge voltage recorder throughout the past summer showed a predominance of negative surges. Since both instruments recorded a number of positive surges, the above tendency cannot be due to inherent characteristics of the equipment.

The rates of attenuation of voltage surges recorded in these studies have checked in general those obtained in other tests with surge voltage recorders and with artificial lightning. No data are available from these tests which would indicate the factors affecting reduction of voltage with travel along the line. Studies with artificial lightning will probably be necessary to determine if loss of energy, change of wave shape, or other causes produce this decrease in voltage.

The voltages measured between tower footing and ground were of surprisingly high magnitude and demonstrate the necessity of auxiliary grounds on towers whose footing are of high resistance. From both the standpoint of electrical operation and the standpoint of safety, tower resistances should be carefully checked, particularly on lines provided with ground wires.

Studies of lightning made during the past few years have assisted in solving many of the problems encountered in transmission line operation. Continuation of this work in the future should add materially to the information now at hand, and some day it is hoped, make possible the design of a lightning-proof line of reasonable cost.

ACKNOWLEDGMENTS

The writers wish to acknowledge the assistance of the General Engineering Laboratory of the General Electric Company who furnished the surge voltage recorders for the investigation and of the Lightning Arrester Engineering Department whose members assisted with an artificial lightning generator in checking the operation and calibration of the oscillograph.

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Discussion

For discussion of this paper see page 928.

Traveling Waves on Transmission Lines
with Artificial Lightning Surges

BY K. B. McEACHRON,*
Member, A. I. E. E.
J. G. HEMSTREET,†
Associate, A. I. E. E.
and
W. J. RUDGE*
Associate: A. I. E. E.

Synopsis.—*The paper describes tests on a transmission line of the Consumers Power Company using a portable million-volt impulse generator.*

Data were secured on attenuation with one, two, and three conductors in parallel, with and without ground wires. A study was made of the influence of ground wires in reducing voltages induced by overhead discharging clouds and also the effect on surges of entering or leaving a section of line having ground wires. A brief study was made comparing steel and copper for ground wires. The effect of ground resistance in tower footings was considered and a method given for measuring surge impedance of transmission lines.

IN May 1929 the General Electric Company and the Consumers Power Company began a joint investigation, having for its object the securing of information which will serve to give definite figures for the various effects which take place when overhead circuits are subjected to traveling waves such as those caused by lightning. During the previous summer a certain amount of preliminary work was done using a portable impulse generator having an open circuit potential of 400 kv. on a 66-kv. circuit of the Turners Falls Power and Electric Company.[1] This work has been continued this summer and some of the data obtained, particularly on attenuation, will be found in this paper. Other data dealing with choke coils have been published ecently.[2]

The historic S-19 transmission line of the Consumers Power Company between Croton Dam and Grand Rapids, which was the first line to transmit electrical energy at 110,000 volts, was set aside for the study of transients produced artificially. At Croton Dam an outdoor laboratory, housing a cathode ray oscillograph, was erected for the purpose of securing records of natural lightning on the T-20 line.[3] Results of an investigation with surge voltage recorders on the Consumers system have already been published.[4] In making the study on the S-19 line the cathode ray oscillograph was available for determining the wave form of impulses at the laboratory end of the line.

The impulse generator used in these tests was a portable million-volt equipment[5] consisting of 40 banks of capacitors, each bank having a capacity of 0.50 μ f. The banks are charged in parallel to 25 kv. and discharged in series as is done in the Marx circuit.[6] The connections are so arranged that the polarity of the generator may be changed at will, which is very necessary for the type of tests to be described in the paper. The power requirements for the impulse generator are such that it may be operated from a 110-volt lighting circuit. Fig. 1 shows the million-volt generator set for test at Croton Dam.

The S-19 line is approximately 45 mi. in length and is not paralleled by any other lines except for a short distance near Grand Rapids. It is of three legged steel tower construction, the steel being set in the earth with concrete footings somewhat similar to the present Malone anchors. The conductors are 72,670 circular mil, seven strand copper, insulated with seven insulators spaced 4¾ in. The line does not have a ground wire nor are the conductors transposed. Fig. 2 shows a typical tower construction.

The cathode ray oscillograph for recording lightning surges is installed on the T-20 line about 1000 ft. from the Croton Dam generating station and the S-19 line was extended to this point, four additional towers being required. The impulse generator was set up one span away from the oscillograph and the line provided with jumpers at this point, so that the impulse could be sent either to the oscillograph for checking the open circuit wave, or out on the line. It is also possible to connect with the T-20 line and this was done for the purpose of checking the calibration of the cathode ray oscillograph on this line.[3]

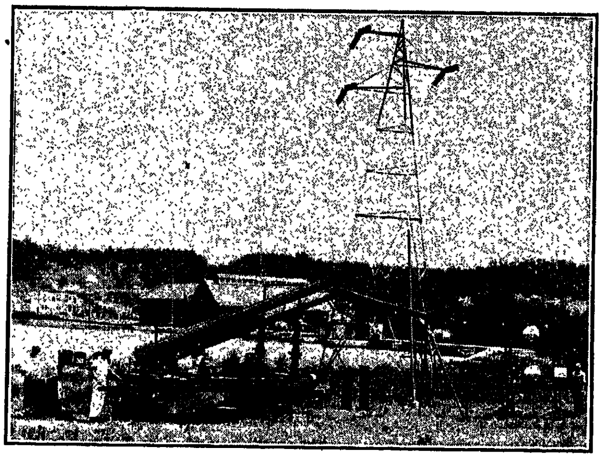

FIG. 1—ONE MILLION-VOLT IMPULSE GENERATOR SET UP FOR TESTS

Most of the measurements given in this paper were made with sphere-gaps which were moved along the line by truck, communication being maintained with the impulse generator and oscillograph stations by tapping local telephone lines.

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*General Electric Company, Pittsfield, Mass.

†Consumers Power Company, Jackson, Mich.

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 27-31, 1930.

ATTENUATION

The problem of attenuation of surges on transmission lines is important from the operating standpoint as well as being of great importance from a purely scientific point of view. Knowledge of attenuation makes it possible to determine the distance that surges can travel and still be dangerous to connected apparatus. In this paper attenuation will be understood to refer to the decrease in crest surge voltage without regard to change of wave shape.

Corona loss[7] is now generally regarded as one of the most important factors affecting attenuation. Some attempts have been made to deduce the rate of attenuation of surges from the constants in the known empirical formulas for corona loss on lines at power frequency voltages. It is extremely improbable, however, that the final state of breakdown of the air occurring at these frequencies is reached in the duration of the lightning surge especially if its time above the corona voltage is short as in the case of an insulator flashover on or near the front of the wave. Without entering into any extended theory of attenuation, a brief discussion of the more important factors will be given.

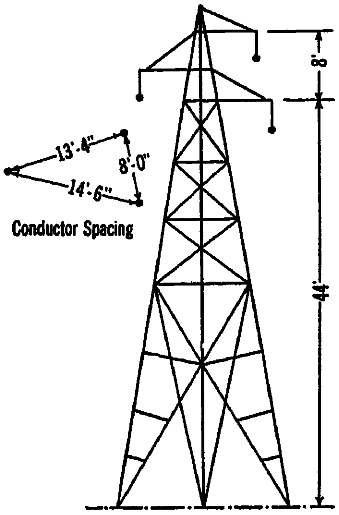

Fig. 2—Typical Three-Post Tower Showing the Arrangement of Conductors

EFFECTS OF PARALLEL CONDUCTORS

The formation of corona around a conductor occurs when the potential gradient in this space exceeds a value known as the critical corona voltage. This gradient is very nearly proportional to the charge on the conductor. At a given potential the charge is proportional to the electrostatic capacity of the conductor to the earth. The electrostatic capacity of the conductor to earth is affected by the presence of other charged conductors nearby, being increased when the other conductors are charged to the opposite sign and decreased when they are changed to the same sign.

When a surge is traveling on a number of conductors in parallel, as is always the case on an actual transmission line, the capacity to earth is of one conductor reduced by the presence of surges of the same sign on the other conductors. Thus the charge on each conductor at a given surge voltage is less and so the potential gradient and consequently the corona is less on each conductor than would be the case with a single conductor. Following out this line of reasoning one would expect to find the attenuation less when several conductors in parallel were carrying the same surge. The tests made show conclusively that increasing the number of conductors in parallel decreases the attenuation.

EFFECT OF POLARITY

It is a well-known fact that there is a considerable difference in the appearance of positive and negative corona.[8] With dissimilar electrodes, as in a pin-type insulator, flashover will take place at a lower potential when the cap is positive than when negative, although as the rate of potential application is increased the positive and negative spark potentials approach a common value.[9] Thus one would expect that a difference would exist in the corona loss for positive and negative impulses and that in general the loss for the positive impulse would be greater. On this basis, the attenuation with negative impulse should be less than for the positive impulse, which was in general found to be true. Since, however, the insulator tests referred to showed that as the time to breakdown became less the difference between positive and negative impulses disappeared, it is reasonable to expect that with extremely short waves positive and negative attenuation would be equal. The only tests made with short wave were made with a chopped wave having a seven-microsecond front which was probably much too long to give equal values of attenuation for positive and negative surges.

EFFECT OF LENGTH OF WAVES

The state of breakdown of the air in the corona area around a conductor is changing very rapidly during the passage of a surge, and consequently the attenuation varies on different parts of the surge. This would affect the attenuation of surges, the amount depending upon the wave shape. Thus if the corona loss[10] was greatest during the initial stages of breakdown, a short wave would suffer more attenuation than a long wave. The tests made have shown that attenuation is dependent on wave shape.

EFFECT OF GROUND WIRE

With a traveling wave the ground wire takes up a charge of opposite sign to that of the wave in the line conductor and as pointed out previously this results in an increased charge on the line conductors which results in an increased corona loss at a given potential. This effect tends to increase the attenuation. On the other hand the increase in charge results in an increase of energy of the surge which tends to decrease the attenuation. It has been assumed[11] that the resultant effect of the ground wire is to increase the attenuation, but the attenuation was found to be less with the ground wire, especially in the case of the negative impulse on the line conductor. Apparently the increases in surge energy and corona loss are more or less balanced in the case of the positive impulse but with the negative surge the increase in surge energy seems to be greater than the increase in corona loss resulting in a decreased attenuation.

TEST RESULTS

The attenuation tests were made using three different wave forms, a fast wave reaching its crest in less than one microsecond, a slow wave reaching its crest in seven microseconds, and a chopped wave produced by cutting off the slow wave at its crest by the use of a parallel sphere-gap. These waves of both positive and negative polarity were applied to a single conductor and also to two and three conductors in parallel. As the S-19 line was not provided with a ground wire the influence of the ground wire on attenuation was studied by grounding the conductors at approximately one-half mile intervals.

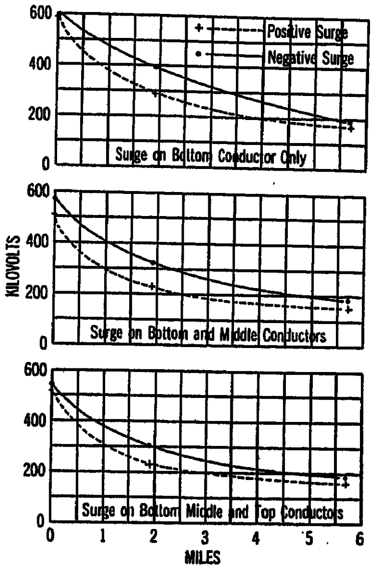

FIG. 3—ATTENUATION WITH STEEP WAVES ON ONE, TWO, AND THREE CONDUCTORS

In Fig. 3 are given curves showing the variation in measured crest voltage with distance along the line for both positive and negative steep waves applied to a single conductor and to two and three conductors connected in parallel at the impulse generator. The applied voltage decreases with the number of conductors in parallel because of the increased load on the surge generator due to the decreased surge impedance when several conductors are connected in parallel. The positive wave in each case shows more attenuation for potentials in excess of 200 kv. than does the negative.

The attenuation tests with the slow wave extended over a distance of 30 mi. from the impulse generator. The results are given in Fig. 4. These data cover both positive and negative impulses on one, two, and three conductors. The results with a single conductor show that the negative impulse has decreased from 475 kv. to 90 kv. and the positive from 415 kv. to 60 kv. as a result of traveling 30 miles. It is interesting to compare these results with those given by Lewis[7] which were obtained from studies with the surge records from lightning storms. Lewis gives a formula derived by Foust and Menger in which the voltage e at a point s miles from the point of inception is as follows:

$$e = \frac{e_o}{k\,s\,e_o + 1}$$

where e_o is the voltage at the point of origin, and k a factor found empirically.

This equation fits the data given in Fig. 4 fairly well for the first ten miles beyond which the loss due to corona becomes small. Mr. E. W. Boehne has suggested that s in the equation be replaced by s^n which with the proper values for n checks the curves closely. In Fig. 5 are given the values for n and k for the various conditions of the test. The values of both n and k are dependent on many factors which the present investigation did not diclose. The values given apply only to the slow wave used and the particular line tested.

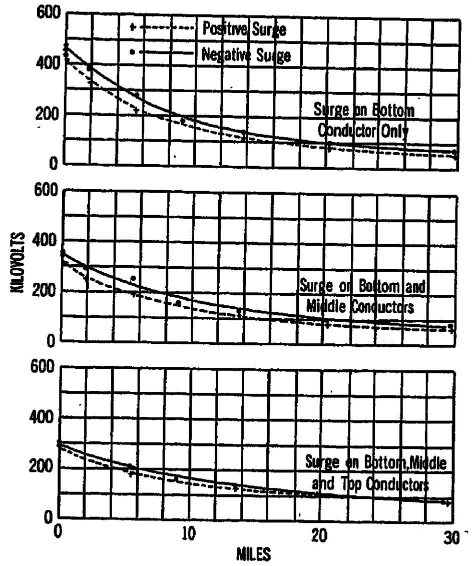

FIG. 4—ATTENUATION WITH SLOW WAVES ON ONE, TWO, AND THREE CONDUCTORS

It is of interest to note that on the Turners Falls Line[1] a brief study of attenuation on a single conductor gave for n a value of 0.75 and k 0.00069 with an e_o of 230 kv. These values are quite different from those obtained on the S-19 line and indicate that the values of n and k must be determined for each individual condition considered.

The results of the tests made with the chopped slow wave are given in Fig. 6, which shows not only the decrease of the positive and negative surge with distance but also show the potential on the free conductor where such existed. It is interesting to note that the change of potential of the free wire with distance is very small and approaches the potential of the conductors carrying the original surge. When the surge was applied to the bottom conductor the top conductor assumed an initial potential of about 30 per cent of the applied. This increased to 50 per cent when the impulse was applied to the middle and bottom conductor in parallel. It is to be noted that the potential of the free wire and of conductors carrying the surge have all come to the same value in a distance of 5.7 mi. Comparing the results of the chopped wave with those given in Figs. 3 and 4 for the fast and slow wave it is evident that the attenuation is greater with a single conductor than with simultaneous surges on multiple conductors, and that this effect is much more pronounced with the chopped wave than with the others. This may be due to the transference of a larger percentage of the wave energy to the free conductors in the case of the chopped wave. Some indication of this energy transfer is found in the voltages measured on the free conductor as seen in Fig. 6.

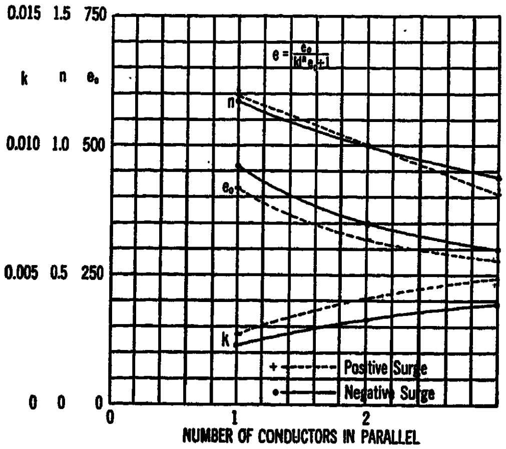

FIG. 5—ATTENUATION CONSTANTS FOR S-19 LINE

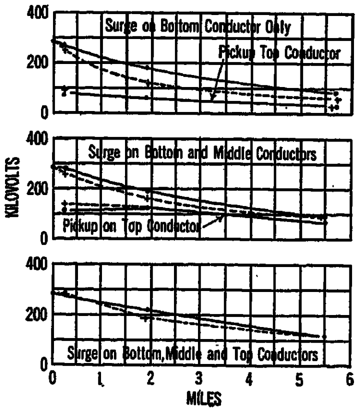

FIG. 6—ATTENUATION WITH CHOPPED SLOW WAVES ON ONE, TWO, AND THREE CONDUCTORS

Considerable work was done in the study of attenuation with ground wires, the results shown in Fig. 7 being representative of the results obtained. Two sets of curves are given, one with the top conductor grounded and one set with the top and middle conductors acting as ground wires. Plotted with these curves are the results for the same conditions with no grounded wires. The impulse was always applied to the bottom conductor at tower *C*. Tests were made with both positive and negative impulses. In comparing these tests it should be remembered that the top and middle conductors are 8 ft. and 14 ft. respectively, from the bottom conductor, which means that the top conductor acting as a ground wire will influence the bottom conductor more than the middle wire will when it is grounded.

Referring to Fig. 7 the attenuation with the positive impulse seems to be only slightly affected by the presence of one or two ground wires, although the tendency seems to be definite at potentials above 200 kv. for a decreased attenuation with ground wires. Expressed in distance of travel to suffer the same reduction in potential, the positive impulse without ground wires traveled 5.8 mi. to reduce from 360 kv. to 190 kv. while with one ground wire the corresponding distance was 6.8 mi. and with two ground wires the distance was 7 mi.

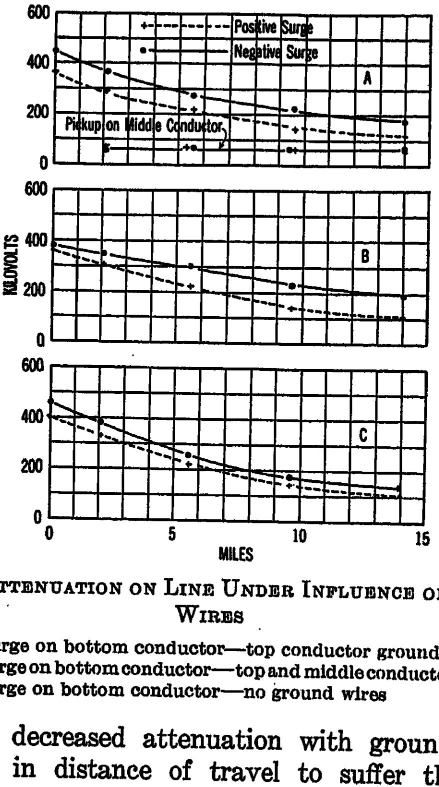

FIG. 7—ATTENUATION ON LINE UNDER INFLUENCE OF GROUND WIRES

A surge on bottom conductor—top conductor grounded
B surge on bottom conductor—top and middle conductor grounded
C surge on bottom conductor—no ground wires

With negative impulses of a potential of the order used in these tests the attenuation was considerably decreased by the presence of ground wires. To illustrate, from the curves without the ground wires it may be seen that the potential decreased from 380 kv. to 180 kv. in a distance of 7.4 mi. while with one ground wire the distance required for the same decrease was 14 mi. and with two ground wires, extrapolating the results slightly, this distance is of the order of 15 mi.

These results indicate that for positive impulses the ground wires have but little effect on the attenuation. In the case of negative impulses it appears that the presence of one ground wire so decreases the attenuation that the impulse may travel twice as far with the ground wire as without. Two ground wires increase this effect slightly. The idea that ground wires should damp out traveling waves quickly does not seem to hold; in fact the tests show that the reverse is true. This does not mean that ground wires are not useful in reducing the potential which would otherwise have been present when the line is under a discharging cloud, nor does it mean that the ground wire would not be helpful in case of a direct stroke of lightning to the ground wire or line wires. As shown in the section dealing with the "Study of Ground Wires" the reduction of potential during the release of the bound charge on the S-19 line is of the order of 20 per cent, while the calculated value was in the neighborhood of 23 per cent, considering the middle wire acting as the ground wire. When the top wire acts as the ground wire the calculated reduction is of the order of 30 per cent and it is assumed that by test this reduction might also be of the order of 30 per cent. Assume by way of illustration that a potential of 460 kv. be taken as the induced potential without ground wire and 300 kv. as the potential on the bottom wire with the top wire acting as a ground wire. From Fig. 7 the respective attenuations for the bottom conductor bring the curves together after a travel of 7 mi., which means that as far as terminal apparatus is concerned the effect of the ground wire is lost after traveling over the distance of 7 mi. With the positive impulse the case for the ground wire is better for in traveling 7 mi. impulses which began as a traveling wave of 400 kv. without the ground wire and 260 kv. with it would be 170 kv. and 135 kv. respectively, which shows that the original 30 per cent difference had been reduced to 20 per cent by traveling 7 mi. on the S-19 line. In 11.5 mi. this difference has been reduced to about 10 per cent.

These tests indicate that the ground wire acts most efficiently when the storm is close to the point considered as will always be the case with line insulators, or when storms are close to station apparatus. It is fortunate that this condition exists for the most damaging impulses undoubtedly will be found nearest the point of inception.

Study of Ground Wires

A brief review of the theory of the ground wire will first be given in order that the significance of the tests to be described may be more fully appreciated.

When a cloud over a transmission line becomes charged all of the conductors of the transmission line including the ground wires take up charges having a polarity opposite to that of the cloud. This action is easily understood when it is remembered that charges of unlike sign attract with the result that charges of a polarity opposite to that of the cloud are drawn up from the ground onto the ground wires while similar charges on the line conductors are not only drawn from the ground by leakage over insulators, but from other sections of line not under the cloud.

When the lightning discharge takes place the charges on the ground wires are free to pass directly to the earth while the charges on the line conductors travel into other portions of the system as traveling waves. It is the function of the ground wires to reduce the potential appearing on the line conductors just after the lightning discharge takes place but before the charges begin to travel along the line wires. The mechanism of this reduction is frequently divided into two parts which will be referred to as the first and second actions of the ground wire.

It has already been pointed out previously in this paper that when parallel conductors have charges of the same sign, the capacity to ground and the charges at a given potential are both less than they would have been without the charges on parallel conductors. This is exactly the case with the ground wire which initially has a charge of the same sign as that on the line conductors so that the line conductors have a smaller charge and a smaller capacity to ground because of the presence of a similar charge on the ground wire. It is assumed that the charge on the ground wire runs off to earth in zero time, so that the line conductors find themselves with an increased capacity, because of the elimination of the charge on the ground wire, but with the same charge that they had with the original capacity. As the product of the potential and the capacity is equal to the charge which is constant, an increase in capacity results in a decrease in potential which constitutes the *first effect* of the ground wire.

The *second effect*, which is smaller in magnitude than the first, results from a further increase in capacity of the line conductors as the ground wires take up charges of the opposite sign. In this case the grounded conductor acts in a manner similar to the manner in which all of the conductors acted when they were in the original cloud field, except that the cloud field is replaced by the field of the line conductors to ground. The total effect of ground wires in reducing the potential on the line conductors during the release of a bound charge is the sum of the first and second effects. It is interesting to note that the sum of the first and second charges on one or more ground wires is just equal to the charges they would have had if no other conductors were present. For purposes of calculation, the first and second effects of the ground wire are conveniently considered together, and in the tests to be described the effects were studied together, but the second effect was also studied separately.

Study of the Total Effect

It is not possible to reproduce exactly the conditions existing in a cloud field with the apparatus at present available. However, a fair measure of the effect on the potential of the line conductors, of changing the charge on the ground wire may be obtained by the use of traveling waves, although the ground wire theory is predicated on static conditions. As pointed out previously, the potential of any one conductor is affected by charges on the parallel conductors, and this is true whether the charges are moving or stationary.

In making the tests to show the total effect, the three conductors were connected together at Croton Dam and an impulse applied to all three in parallel, as shown in Fig. 8. The middle conductor was grounded successively at towers 50, 52, and 55 which had resistances of 76.4 ohms, 28.4 ohms, and 7.5 ohms respectively. Voltages were all measured at tower 56.

Although in practise the different conductors will assume different potentials because of their different heights, this difference was neglected as being small, the same surge being applied to all three conductors. The surge applied had a front of about 10 microseconds and traveled a distance of 5.5 miles to reach tower 56, where the measurements were made. The results obtained are given in Table I and show the protective value of the ground wire which is the same for both positive and negative impulses.

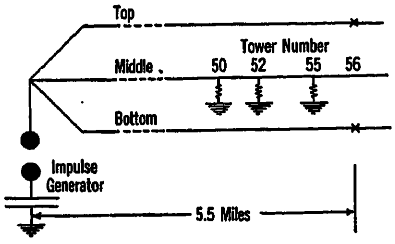

Fig. 8—Set-Up for Study of Total Ground Wire Effect

TABLE I
TOTAL GROUND WIRE EFFECT

Tower at which middle conductor was grounded*	Voltages measured kv. at tower 56			Protective factor	
	Top	Middle	Bottom	Top	Bottom
No grounds	178	168	177	100%	100%
50	..	40	146	..	83.5
50, 52	..	34	145	..	83.0
50, 52, 55	143	4.3	142	80.6	80.0

*The ground resistances were—Tower 50, 76.4 ohms; Tower 52, 28.4 ohms; Tower 55, 7.5 ohms.

The calculated protective factors for the total effect assuming the middle conductor at earth potential are 79 per cent and 77 per cent for the top and bottom conductor respectively.

The data obtained for the bottom conductor are plotted in Fig. 9 and show a protective factor of slightly less than 80 per cent where the curve crosses the zero ground wire voltage axis.

It is interesting to note that the greatest reduction of potential occurred at the first tower although the tower resistance was 76.4 ohms, the ground wire potential being reduced to 24 per cent and the line conductor to 83.5 per cent of the original values. This indicates that a large part of the benefit of the ground wire for reduction of induced discharges will be retained with tower footing resistance of the order of 75 ohms. Of course still better results will come from still lower resistances, particularly from the standpoint of direct strokes of lightning near the towers.

Numerous other tests have been made at different locations on the S-19 line, some with steel and some with copper ground wire, which also check the calculated values closely.

Study of the Second Effect

The second ground wire effect involves a consideration of the change in potential of a conductor when the ground wire assumes a charge of opposite sign as a result of being placed in the field of the charged conductor.

In order to get experimental data on this effect a transition point was created on the S-19 line at a distance of 5.5 mi. from the impulse generator which was far enough for measurements to be unaffected by reflections from the impulse generator. At this point the top and middle conductors were cut. In one case grounds were put on the top and middle conductor from tower 58 to 160, a distance of 11.5 mi. while the same conductors were left floating from tower 57 to the impulse generator. Both positive and negative waves having fronts of 7 microseconds were applied to the bottom conductor and voltage measurements made with spheres at suitable points along the line both in the grounded and ungrounded sections of the line.

This test was repeated with the reverse conditions, the top and middle conductors being grounded from tower 57 to the impulse generator location. The impulses were applied as before to the lower conductor and emerging from under the grounded conductors at tower 57. These data are shown plotted in Fig. 10 and bring out some important conclusions. Considering case I, in which the impulse enters the grounded section at tower 57, a point of inflection occurs in the attenuation curves which shows a reduction in potential due to the presence of the ground wires. The change in the curves occurs at a point about one mile back from the beginning of the ground wire section. The effect of the ground wires may be seen by extrapolating the curves

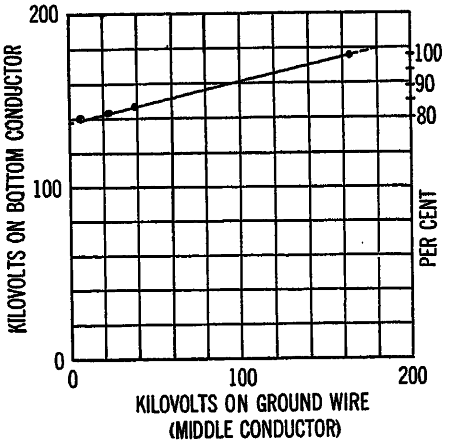

Fig. 9—Protective Factor using Ground Wire

without the ground wires which indicates a reduction of about 8 per cent due to the ground wires. It should be possible to apply the attenuation curve *C* of Fig. 7 for the first three or four miles of this curve, but for some unknown reason curve *C* shows considerably increased attenuation for the same initial value of potential.

In Fig. 10, case II, the attenuation curves are shown for an impulse passing from the ground wire section into a section without ground wires. In order to show the change in potential clearly, the attenuation curve *B* from Fig. 7 has been superimposed and indicates an increase of potential of 8 per cent for the negative impulse and slightly more for the positive impulse. As with case I, the effect of the ground wire is felt a distance of about one mile back from the transition point. Measuring only crest potentials with a sphere-gap the effect of the partial reflection would extend back from the point of change in impedance and a distance slightly greater than half the length of the wave front. If the wave had been perpendicular, full crest potential would have measured up to the reflection point, assuming, of course, no time lag for the measuring sphere-gaps.

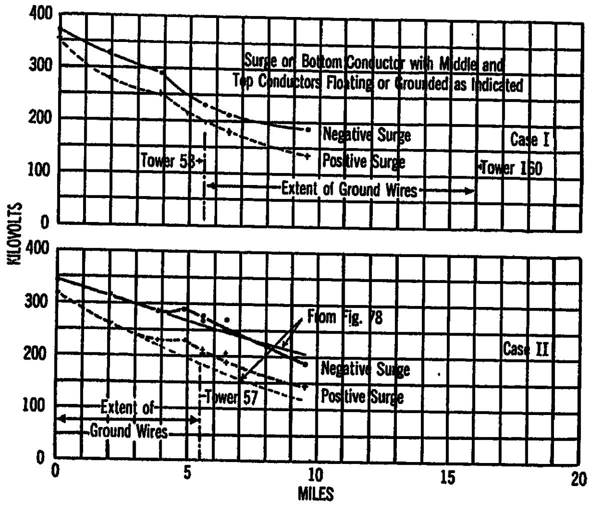

Fig. 10—Effect on a Surge of Entering or Leaving a Section Equipped with Ground Wires

It should be noted in connection with these curves that the attenuation is much the same for positive waves, whether under the ground wire or not, while with the negative waves there seems to be a definite tendency for less attenuation under the ground wires. This situation is clearer in connection with case II than it is for case I. These results agree with those discussed in connection with Fig. 7 under the general subject of attenuation.

Considering all of the values obtained a value of 8 per cent may be taken as the second ground wire effect, which is to be compared with the 6 per cent calculated reduction when passing from a conductor having a surge impedance of 530 ohms to a surge impedance of 470 ohms which was calculated for the S-19 line with the top and middle conductors grounded.

Thus the sum of the first and second ground wire effects gives a total reduction of 20 per cent for one ground wire. With two ground wires the first effect will be increased. The tests indicate that the second effect is not materially changed when two ground wires are used instead of one. These tests with traveling waves show that ground wires on a transmission line under a discharging cloud afford about that degree of protection indicated by the usual theoretical formula.

Copper *vs.* Steel Ground Wire

A limited number of tests was made in an effort to determine the relative merits of copper and steel conductors acting as ground wires. The tests made, which relate only to traveling wave effects at comparatively low voltages, do not show any marked difference between copper and steel ground wires. One set of test results is given in Fig. 11, showing the voltage measured back along the grounded conductor from a grounded point. These waves show the copper to have slightly lower reflected potentials than the steel wire but the difference is too small to be more than an indication. This curve is also interesting in that it shows that potentials as high as 200 kv. may exist a distance of 2000 ft. away from the grounded point, although much steeper gradients than this can no doubt be obtained with steeper waves. In the case under consideration the grounded point was at tower 55, the steep wave impulse being applied at a point over 5 mi. away.

For protection from direct strokes of lightning the copper ground wire may have some advantage from the standpoint of current carrying ability. Not enough is yet known about the direct stroke condition to make any definite statements regarding the relative merits of copper and steel.

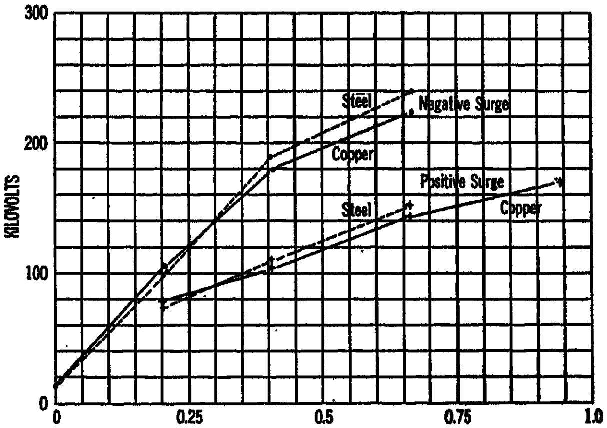

Fig. 11—Surge Voltage on a Steel or Copper Conductor Measured from a Grounded Point Back Toward the Impulse Generator

Effect of Ground Resistance

In studying ground wires it is worth while to give some thought to the value of ground resistance under impulse conditions compared to grounds measured in the steady state. The effect of two ground resistances in parallel but varying distances apart will also be briefly considered.

Measurements of tower footing resistance were made with a megger constructed especially for this purpose. The resistances were found to vary from 12 to more than 1500 ohms. The resistances of tower footings over the section of line used in most of the tests described in this paper are given in Fig. 12.

To measure a ground resistance under impulse conditions, a pipe was driven into the ground approximately 200 ft. from the impulse generator. An impulse current was sent from the generator through a water tube resistance into the pipe ground. Sphere-gap measurements were made across the water tube and between the pipe ground and a stake in the ground far enough away not to be influenced by the current flowing in the vicinity of the pipe ground. In every case the transient resistance was less than that measured under steady state conditions, which is in agreement with cathode ray oscillograph tests made at Pittsfield.[12] The results of one such test is given in Fig. 13, which shows that the ground resistance with a transient current of 550 amperes is about 67 per cent of the steady state resistance.

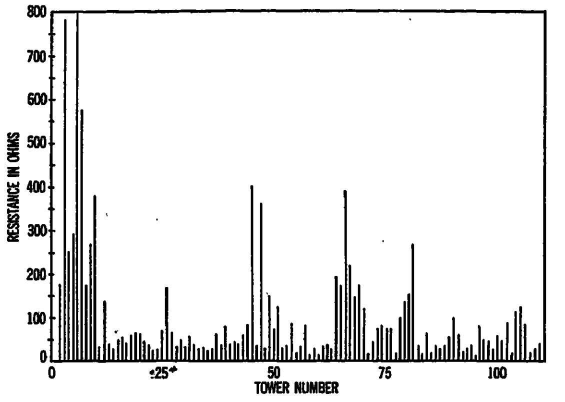

FIG. 12—TOWER FOOTING RESISTANCE ON 10-MI. SECTION OF S-19 LINE

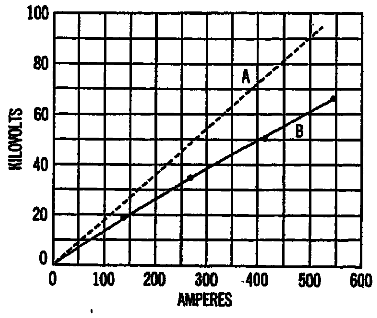

FIG. 13—VOLT-AMPERE CURVES OF A GROUND RESISTANCE

When a surge passes a point which is grounded through a resistance the voltage is reduced according to the relation $E/E_o = 2\,R/(2\,R + Z)$, where E_o is the initial voltage and E the transmitted voltage. The per cent reduction due to varying values of resistance is shown by Fig. 14. When a wave passes two grounded points sufficiently far apart the reduction is correctly represented by applying this formula twice, once to the original wave and once to the transmitted wave which passes the first resistance. When this transmitted wave reaches the second resistance there will be a new transmitted wave beyond the second resistance and also a negative reflection. traveling back toward the first resistance. Another reflection of reversed polarity will take place which will have the same polarity as the original wave. This wave will travel to the second resistance and the same operation will be repeated. If a rectangular wave is considered each of the two resistances will operate independently if the distance between them is equal to more than half the wave length. If the distance is less, the transmitted potential will be correspondingly higher until the condition is reached of a single resistance of half value when the distance between them is reduced to zero. This relation is shown in Fig. 15, which shows that if two resistances equal to one-eighth of the surge impedance are connected between line and ground farther apart than half the wavelength only 4 per cent of the wave is transmitted, while if the same two resistances are connected at the same point the transmitted wave will be 11 per cent. This does not mean, however, that it is better to ground a ground wire at a few points far apart rather than many points close together. but it does indicate

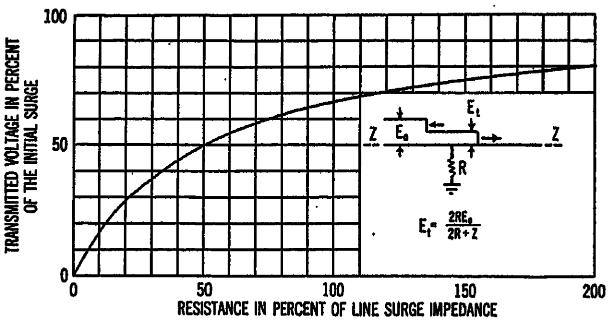

FIG. 14—TRANSMITTED SURGE VOLTAGE BEYOND A RESISTANCE TO GROUND ON A CONTINUOUS LINE

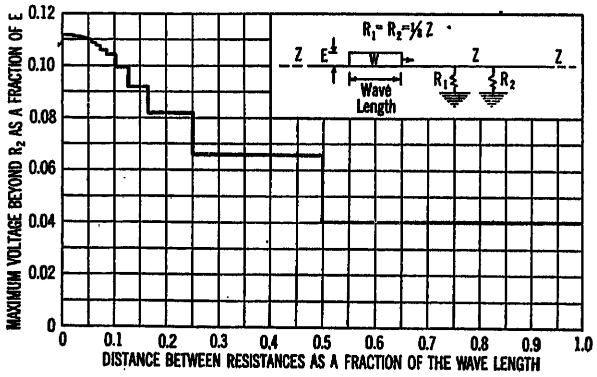

FIG. 15—FRACTION OF RECTANGULAR WAVE TRANSMITTED BEYOND TWO GROUND RESISTANCES

that two grounds of, for example, 50 ohms each are better if separated some distance apart than one ground of half the resistance in one location. This seems to indicate the desirability of keeping the ground resistance of towers uniformly as low as possible rather than having wide variations in tower footing resistance, even though some are low, but with a high average.

The following tabulation shows how distance between resistances affects the transmitted wave when using surges having a front of approximately 10 microseconds.

TABLE II

EFFECT OF DISTANCE BETWEEN RESISTANCE TO GROUND ON THE PROPAGATION OF A SURGE

The surge voltage with no resistance connected was 405 kv. R_1, R_2, R_3 were tower footings having resistances ranging between 58 and 66 ohms (megger).

Voltage meas. at	Spacing between resistances			
	Zero*	500 ft.	5000 ft.	Distant*
R_1	56 kv.	56 kv.	51 kv.	56 kv.
R_2	30	21	10	7.7
R_3	10.3	8	3	1.1

*Calculated assuming R/Z = 8 per cent

The test values which were measured at spacings of 500 and 5000 ft. lie in the range that would be expected from the theory.

MEASUREMENT OF SURGE IMPEDANCE

Some difficulty has been experienced in measuring the surge impedance of the transmission line conductors.

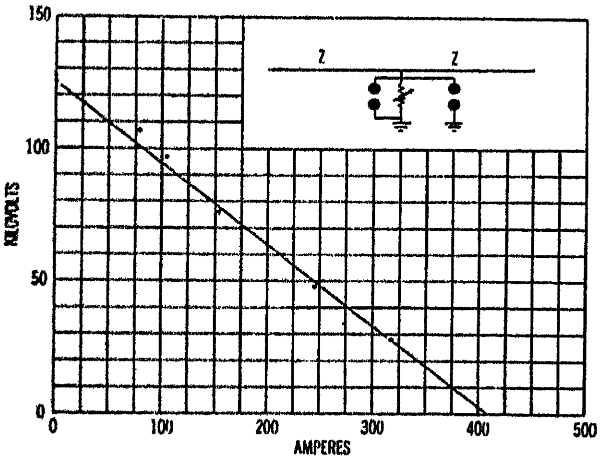

FIG. 16 - DETERMINATION OF SURGE IMPEDANCE BY MEASUREMENT OF VOLTAGE AND CURRENT TO GROUND WITH A VARIABLE RESISTANCE AND SPHERE-GAPS

The method finally adopted was to connect a variable resistance between line and ground at a point sufficiently remote from the generator to be sure that traveling wave conditions were established and plot a curve between amperes through the resistance and the potential to ground as in Fig. 16. The current values are determined from the potential measured across the grounding resistor, while the voltage measurements are taken between the line and a separate ground at a point such that the $I\,R$ drop in the resistance of the ground is included. If the measurement is not made at the end of a line the voltage at zero current is to be divided by one-half of the current at zero volts, which will give the surge impedance. If the measurement is made at the end of a line remote from the generator the voltage at zero current divided by the current at zero voltage will give the surge impedance. The advantage with this method lies in the possibility of determining the curve quite accurately by taking measurements with several resistances without being forced to measure the current at zero potential.

CONCLUSIONS

A million-volt impulse generator of sufficient capacity for field use with limited power facilities available has been shown to be satisfactory in use, and still higher potentials can be obtained when necessary.

The tests have shown the following:

1. Surges attenuate less rapidly when other near-by parallel conductors are carrying similar surges.

2. In general positive surges attenuate more rapidly than do negative surges, although in some tests but little difference was found.

3. Short waves, such as a wave chopped on the front by an insulator flashover, attenuate more rapidly than do longer waves of the same crest value.

4. An additional constant has been introduced into the formula for attenuation which seems to give better agreement with the test results.

5. The presence of the ground wire reduces the attenuation considerably for the negative wave and to a lesser degree for the positive impulse. This decrease in attenuation is important if found to be universally true, for it tends to cancel the reduction of induced potential as a result of a discharging cloud after the resulting traveling wave travels a distance of several miles. This of course does not affect the reduction in potential due to the ground wire at the point of inception.

6. The ground wire theory of reduction of induced potentials was checked using traveling waves. The results agree with the protective ratio for the ground wire given by the usual calculation. For the conductor arrangement used, the calculated reduction was 21 and 23 per cent, while the test showed 20 per cent reduction for the total ground wire effect.

7. The second ground wire effect was also checked with traveling waves and it was found that when entering or leaving a section with one or two ground wires the surge voltage was decreased or increased 8 per cent respectively. These approximate the calculated value based on the change in surge impedance. These results indicate that very little is to be gained by putting up additional ground wires near a station to reduce an incoming impulse. From the standpoint of additional protection from lightning discharges which originate close to the station their use may be justified.

8. The test indicates that there is little difference

between steel and copper conductors from the standpoint of their ability to transmit impulses.

9. The theory of voltage reduction due to two resistances varying distances apart connected between conductor and ground when subjected to traveling waves was checked by test. This work shows that if but two grounds of equal value are to be used the greatest reduction in wave transmitted beyond the resistances occurs when the resistances are at least half a wavelength apart. Thus two resistances of 40 ohms a half mile apart will more effectively ground the line than one resistance of 20 ohms. This seems to indicate that uniform low resistance is preferable to widely varying resistances.

10. Transient ground resistance is less than the resistance measured under steady state conditions.

11. The surge impedance of a transmission line may be satisfactorily determined from a series of voltage and current readings with varying values of resistance between line and ground.

The authors wish to express their appreciation to the officials of both the Consumers Power Company and the General Electric Company who have made this joint investigation possible. Much credit is due to Messrs. Brune, Eaton, Wade, and others who were responsible for conducting the tests in the field and to Messrs. Boehne, Brinton, and Brownlee who also assisted in the preparation of the paper.

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Discussion

For discussion of this paper see page 928.

Lightning Investigation on 220-Kv. System
of the Pennsylvania Power & Light Company (1928 and 1929)

BY NICOLAS N. SMELOFF[1] and A. L. PRICE[2]
Associate, A. I. E. E. Non-Member

Synopsis.—The purpose of this paper is to describe the results of a lightning investigation conducted during 1928 and 1929 on 114 circuit miles of a 220-kv. system located in a territory where severe lightning storms are frequent. During the 1929 investigation, which was a continuation and expansion of previous years' work, a number of devices was successfully used, such as surge voltage recorders, cathode ray oscillographs, electric field intensity recorders, and lightning stroke recorders. Some of these devices are new.

Many valuable data on magnitude and wave shape of actual lightning surges were obtained, proof of the existence of both single- and multiple-phase faults due to lightning; some data on the shielding effect of overhead ground wires and qualitative data on the nature and time of discharge of lightning strokes, together with the atmospheric gradients resulting therefrom, were also obtained.

GENERAL REVIEW

INVESTIGATIONS of lightning with particular reference to its effect on the transmission line and associated station apparatus comprising the 220-kv. system of the Pennsylvania Power & Light Company have been in progress since 1926. Data obtained in 1926 and 1927 are available in a paper[3] presented before the A. I. E. E. in 1928.

The investigation conducted in 1926 and 1927 emphasized the need of greater knowledge of the basic phenomena of lightning with the result that during 1928 and 1929 the study was extended to include the wave shape of lightning surges on transmission lines and experimental antennas, more accurate storm observations, supplemented by measurements of electric field intensities and their rates of change, and data on the causes of the surges due to induction or conduction from charged clouds. The operating records were secured in more detail, including the location and position of flashed insulators and tripout records regarding single- and multiple-phase failures. The broadening of the scope of this investigation was due not only to improvement in technique of cathode ray oscillograph operation with regard to its application in the field, but also due to the application of entirely new devices developed for this investigation.

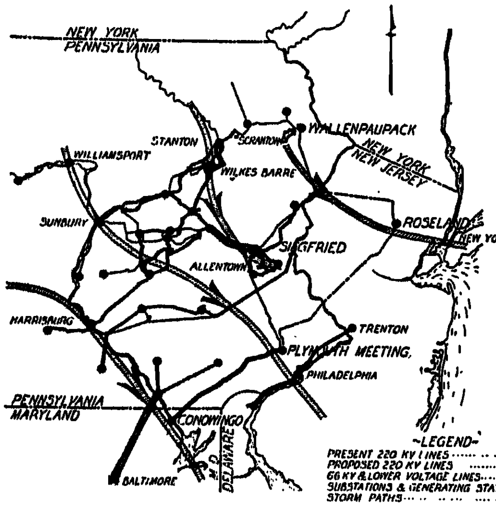
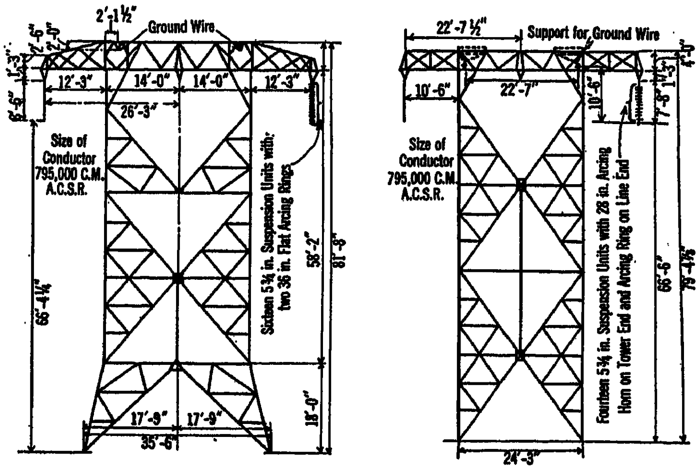


FIG. 1—SYSTEM DIAGRAM AND WEATHER MAP

In nature and amount of data collected, the results of the investigation during the past two years are very gratifying. They permit a better understanding of what is needed for a systematic solution of transmission line lightning problems.

LINE DATA

The 220-kv. system of the Pennsylvania Power & Light Company consists of the Wallenpaupack-Siegfried and Plymouth-Siegfried lines. These lines, together with future interconnections, are shown in Fig. 1. The lightning study was confined to these two lines, particularly the Wallenpaupack-Siegfried line.


2'-1½"
Ground Wire
12'-3"
14'-0"
14'-0"
12'-3"
26'-3"
Size of Conductor 795,000 C.M. A.C.S.R.
Sixteen 5¾ in. Suspension Units with two 36 in. Flat Arcing Rings
58'-2"
81'-8"
66'-4¼"
18'-0"
17'-9"
17'-9"
35'-6"
22'-7½"
Support for Ground Wire
10'-6"
22'-7"
10'-6"
Size of Conductor 795,000 C.M. A.C.S.R.
Fourteen 5¾ in. Suspension Units with 28 in. Arcing Horn on Tower End and Arcing Ring on Line End
66'-6"
79'-4⅞"
24'-3"

Plymouth-Siegfried Line Wallenpaupack-Siegfried Line

FIG. 2—STANDARD TYPE "A" SUSPENSION TOWERS

General characteristics of the Wallenpaupack-Siegfried line were published in a previous paper.[3] Early in 1929, a so-called "counterpoise" was installed for a distance of about 2½ miles along a section of the line where flashed insulators had been particularly numerous

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[1] System Planning Engineer, Pennsylvania Power & Light Co., Allentown, Pa.

[2] General Engineering Laboratory, General Electric Company, Schenectady, N. Y.

[3] For all references see Bibliography.

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(see Figs. 3 and 18). This counterpoise was a buried No. 00 copper cable which connected the tower footings. In addition to the counterpoise, the insulator assemblies on the outside conductors were increased during the 1929 lightning season, from 14 to 16 units over the 40-mile section of the line adjacent to Wallenpaupack.

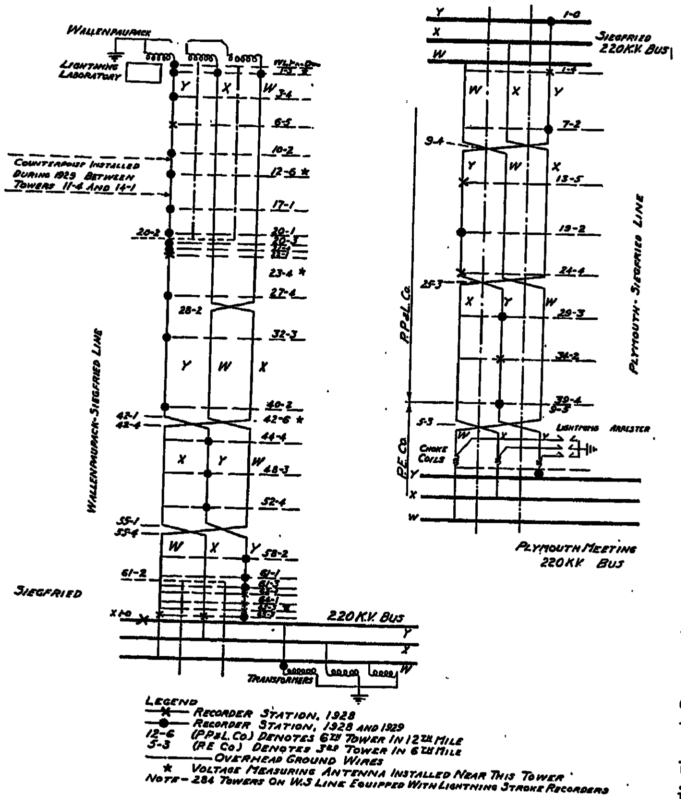

Wallenpaupack
Lightning Laboratory
Counterpoise Installed During 1929 Between Towers 11-4 And 14-1
Y
X
W
Wallenpaupack-Siegfried Line
Siegfried
220 K.V. Bus
Transformers
Siegfried 220 K.V. Bus
P.P.&L. Co.
P.E. Co.
Choke Coils
Lightning Arrester
Plymouth-Siegfried Line
Plymouth Meeting 220 KV Bus
Legend
Recorder Station, 1928
Recorder Station, 1928 and 1929
12-6 (P.P.&L. Co.) Denotes 6th Tower In 12th Mile
5-3 (P.E. Co.) Denotes 3rd Tower In 6th Mile
Overhead Ground Wires
* Voltage Measuring Antenna Installed Near This Tower
Note—284 Towers On W.S Line Equipped With Lightning Stroke Recorders

Fig. 3—One Line Diagram of Wallenpaupack-Siegfried and Siegfried-Plymouth Lines

Showing locations of lightning laboratory, surge voltage recorders, antennas, overhead ground wires, and counterpoise

The terrain crossed by the Plymouth-Siegfried line is for the most part rolling farm land similar to the section of the Wallenpaupack-Siegfried line near Siegfried. This line is equipped with two overhead ground wires, and has 16-unit insulator assemblies with top and bottom 36-in. grading shields throughout. Locke 7500 insulators, the same as used on the Wallenpaupack-Siegfried line, are installed on the section owned by the Pennsylvania Power & Light Company. Fig. 2 shows typical towers for both lines.

Facilities

This investigation was conducted, and facilities made available, cooperatively by the General Electric Company and the Pennsylvania Power & Light Company. The Public Service Electric and Gas Company of New Jersey and the Philadelphia Electric Company cooperated in supplying lightning weather data and in the general analysis of all data obtained. The Philadelphia Electric Company also supplied operating data on its section of the Plymouth-Siegfried line.

Apparatus employed and the scope of the investigation are indicated in Table I. Locations of measuring instruments with reference to the transmission lines are shown in Fig. 3.

In 1928, in addition to the previously used surge voltage recorders, a lightning laboratory was established at Wallenpaupack. It consisted of a cathode ray oscillograph installation coupled to phase Y of the Wallenpaupack-Siegfried line and of an antenna station. These antennas, each 300 ft. long, were arranged in two parallel spans located 50 ft. apart. One span consisted of four horizontal wires vertically arranged on 110-ft. rope guyed cedar poles at elevations of 12½, 25, 50, and 100 feet above ground. The second span consisted of a wire 50 ft. high with a grounded antenna 12 ft. above it which simulated a ground wire. Each antenna was connected at one end to earth through a two-megohm water resistor, the other end was connected through a potentiometer to a surge voltage recorder. In the case of the grounded antenna, the surge voltage recorder was coupled to the ungrounded end.

TABLE I
MEASURING DEVICES AND FACILITIES

	No. of instruments	
A—Instruments used on Lines	1928	1929
Cathode ray oscillograph[4]	1	1
Surge voltage recorders[5]	39*	28
Lightning stroke recorders[6] on towers	0	284
High-speed phase current recording ammeters	0	6
B—Instruments Coupled to Antennas		
Cathode ray oscillograph	0	1
Surge voltage recorders	7	12
C—Weather Data		
Weather observations from operating stations	29	160
Weather observations including location of lightning strokes, from Fire Towers and Wallenpaupack Lightning Laboratory	1	4
Radio set weather indicator at Wallenpaupack Lightning Laboratory	0	1
Electric field intensity recorder[6]	0	1
Rate of change of field recorder[6]	0	1
D—Operating Data		
Overhead (tower climbing) patrols—Wlpk.-Sieg. Line.	5	20
Siegfried-Plymouth Line	2	3
Operating records from 220-kv. operating stations		

*Excludes 6 surge voltage recorders used for measurement of wave shape and duration of surge voltages.

In 1929, additional facilities were provided. A second cathode ray oscillograph was installed and coupled to the 50-ft. antenna without overhead ground wire. These two cathode ray oscillographs at the Wallenpaupack lightning laboratory (Fig. 4) were manned by four trained operators.

Fig. 5 shows the circuit employed in obtaining records with the cathode ray oscillograph coupled to the transmission line. The surge voltage, upon its arrival, automatically operated the oscillograph by means of a three-electrode trip gap connected to the insulator string potentiometer shown at the left of the diagram. The deflection plates were connected to a potentiometer consisting of an insulator string with grading shields. In order to secure a record over an appreciable period of time, a logarithmic sweep was used to elongate the more important portion of the time axis. Constants shown in Fig. 5 are for a 40-microsecond record, but these were changed to secure longer records during a portion of the season.

During both years, surge voltage recorders were coupled to the transmission line by means of insulator string potentiometers with grading shields. The potentiometer shown to the right in Fig. 5 was the type generally used except that the middle grading shield was omitted. In 1929, four antenna stations were installed at the locations indicated in Fig. 3. The installations near towers 23-4, 42-6, and 65-3 consisted of 300-ft. spans of a single wire 50 ft. above ground. Near tower 12-6 two antennas were installed, on the same poles, at heights of 25 and 50 ft. above ground. Surge voltage recorders were coupled to all of these antennas.

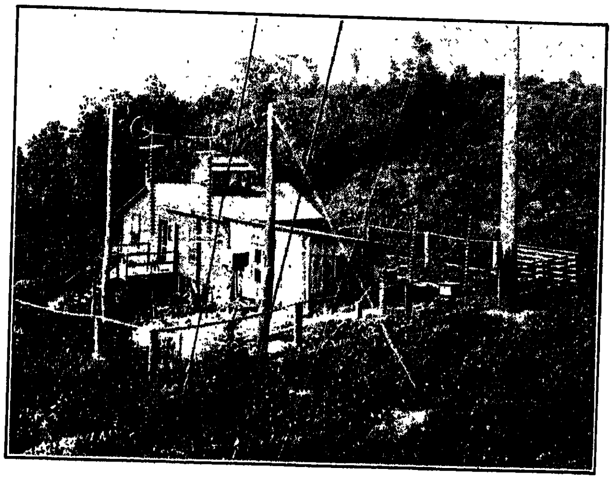
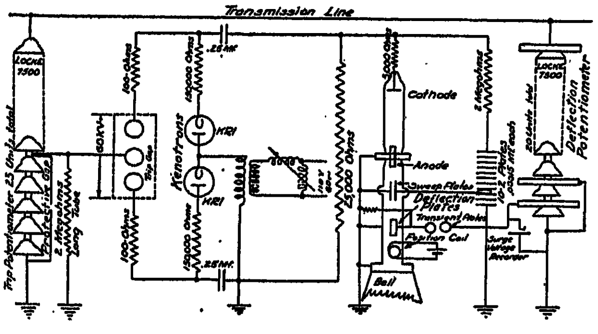


FIG. 4—PHOTOGRAPH OF WALLENPAUPACK LIGHTNING LABORATORY

Three newly developed instruments, the electric field intensity recorder, the rate of change of electric field intensity recorder, and lightning stroke recorders were made available and first used in 1929.



Logarithmic Sweep at 40 microseconds duration

FIG. 5—CATHODE RAY OSCILLOGRAPH CIRCUITS—WALLENPAUPACK LIGHTNING LABORATORY 1929

In order to determine whether faults on the 220-kv. circuits were single- or multiphase, high-speed phase-current recording ammeters were installed at Wallenpaupack and Siegfried.

LIGHTNING WEATHER DATA

In order to gain a more general insight into the lightning storm conditions in the territory under investigation, and to supplement the data collected by the power company, information regarding lightning storms was obtained from neighboring power companies.

Lightning weather observations were secured from approximately 160 observation points in the operating territories of all the power companies in a district from Altoona east to New York and from Wallenpaupack south to Washington, in addition to reports from the lightning laboratory and several fire towers in New Jersey and Pennsylvania. Typical storm paths are shown in Fig. 1, which indicates the storm paths followed on June 19, 1929. Generally, the storms move from west to east or from northwest to east southeast.

RESULTS

Cathode Ray Oscillograms. The first oscillogram of a lightning surge was obtained in 1928 and has been previously described.[7]

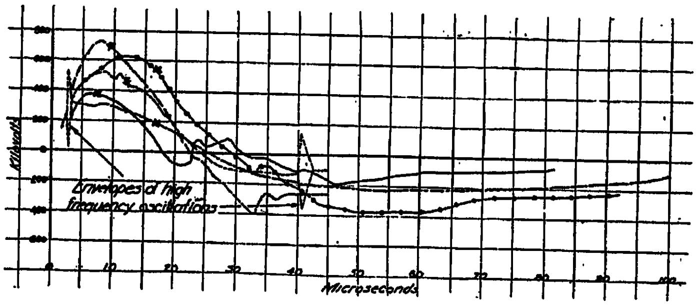


FIG. 6—WAVE SHAPES OF LIGHTNING SURGES—GROUP 1

During the 1929 lightning season cathode ray oscillograms of 95 voltage surges on phase *Y* of the Wallenpaupack-Siegfried line were obtained at the lightning laboratory at tower 1-3, and 11 cathode-ray oscillograms were obtained of voltage surges on the antenna at the lightning laboratory.

Of the 95 oscillograms obtained of voltage surges on the line, 50 measured under 100 kv. at the cathode-ray oscillograph, 30 measured between 100 and 300 kv., and 15 measured above 300 kv. The cathode ray oscillograph connections allowed measurements of voltages superimposed on the normal 60-cycle wave, while the surge voltage recorders measured total voltage to ground.

The wave shapes of the 15 surges above 300 kv. and that of the surge obtained in 1928 have been classified into three groups. These are shown in Figs. 6, 7, and 8 respectively.

Group 1—Fig. 6. The general characteristics of the wave shapes shown in this group are those common throughout all the oscillograms obtained. These waves rise to 75 per cent of crest voltage value in from 2 to 5 microseconds, reach crest value in 3 to 11 microseconds, decrease to 50 per cent crest value in 14 to 20 microseconds and pass through zero in 18 to 30 microseconds.

After passing through zero, these surges rise to a low value of voltage of opposite polarity to that of the first loop and maintain this low voltage until the end of the record, 45 to 90 microseconds. The first loop of the voltage surges in this group is of positive polarity.

Group 2—Fig. 7. Wave shapes 1 and 2 in this group are affected by the arcover of the insulation nearby and for this reason have characteristics not noted in other waves. Wave-fronts range from 1.6 to 8 microseconds and duration of first loops from 29 to more than 50 microseconds. In general, the wave-fronts rise more rapidly and the wave tails decrease in voltage more slowly than wave shapes in group 1.

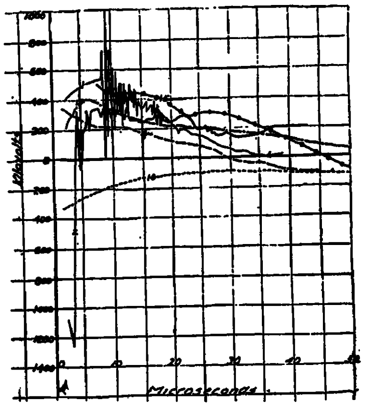


FIG. 7—WAVE SHAPES OF LIGHTNING SURGES—GROUP 2

Group 3—Fig. 8. Wave shapes in this group are of composite nature in that they have more than one peak value without a polarity reversal. A rather long period (at least 30 or 40 microseconds) of high sustained voltage results from this double peaked characteristic. Negative polarity predominates in this group.

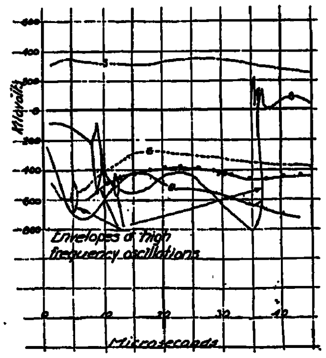


FIG. 8—WAVE SHAPES OF LIGHTNING SURGES—GROUP 3

The actual oscillograms of one surge from each of these groups are shown in Figs. 9, 10, and 11 respectively. Pertinent data relative to these surges is as follows:

Oscillogram, Fig. 9 (Group 1, wave shape 7 of Fig. 6). This oscillogram was obtained at the Wallenpaupack lightning laboratory at 7:39 p. m. on June 19, 1929, apparently coincident with the tripping of the Wallenpaupack-Siegfried line. Fig. 12 shows the profile of voltage in times-normal (180 kv.) which occurred at points on phase *Y* along the transmission line. This voltage profile is an envelope of two surges of opposite polarity too near together in time to be separated on the surge voltage recorder films. It indicates that the highest voltage of positive polarity which correlates with the oscillogram, Fig. 9, was at the Wallenpaupack Station with the next highest voltage about 30 miles away.

A voltage at this time of 15 kv. was measured on the 60-ft. grounded antenna at the lightning laboratory.

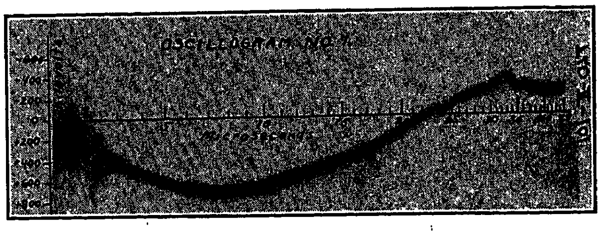


FIG. 9—OSCILLOGRAM OF LIGHTNING SURGE—WAVE SHAPE 7 OF GROUP 1, FIG. 6

The field intensity recorder indicated a voltage gradient of 12 kv. per foot.

A cloud-to-ground stroke of indefinite distance but over 10 miles south and 13 degrees east of the line was seen from the laboratory at the instant this oscillogram was obtained. Lightning was reported from several points between Wallenpaupack and Siegfried. Storm paths for the day are shown in Fig. 1.

Oscillogram, Fig. 10 (Group 2, wave shape 2 of Fig. 7). Oscillogram 2 was obtained at 1:39 p. m. on May 29, 1929, coincident with a tripout of the Wallenpaupack-Siegfried line.

Flashed insulators were found on all three phases at tower 9-3, two days after this tripout, the insulation on that structure being 16 units on phases *Y* and *W* and 14 units on phase *X*. Fig. 13, which shows the profile of voltage along the line on Phase *Y*, checks this location as being the point of highest voltage.

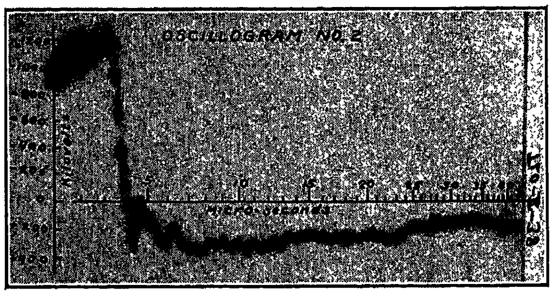


FIG. 10—OSCILLOGRAM OF LIGHTNING SURGE—WAVE SHAPE 2 OF GROUP 2, FIG. 7

A voltage of 39 kv. (negative) was recorded on the 60-ft. grounded antenna at Wallenpaupack.

The field intensity recorder indicated a voltage gradient of 5 kv. per foot. At the instant the oscillogram was obtained, the observer saw a stroke approximately 4.2 miles southeast of the laboratory. Two storms were in progress, the one had its center between the sixth and tenth mile from Wallenpaupack, the other 3 to 5 miles northeast of the laboratory. Storm clouds were not nearer to the laboratory than 1½ miles at the time the oscillogram was obtained.

Observers saw all three phases flash over the dead-end protective gaps at the Wallenpaupack station, which is 840 feet north of the lightning laboratory. This flashover accounts for the sudden reversal of polarity shown in this oscillogram (Fig. 10) and is interesting as a comparison with the oscillogram obtained in 1928, shown as wave shape No. 1 in this same group (Fig. 7). In the case of wave shape No. 1 a flashover occurred over the protection gap on phase *X* at tower 1-2 which is 460 ft. from the lightning laboratory. When the phase to which the oscillograph was coupled, flashed over, there was a sudden reversal of polarity, the voltage rising from 1260 kv. negative to 310 kv. positive in one microsecond; however, when the flashover was on an adjacent phase only, there were oscillations of about 2000 kilocycles superimposed on the main wave without reversal of polarity.

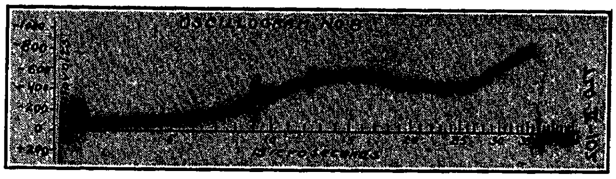


Fig. 11—Oscillogram of Lightning Surge—Wave Shape 8 of Group 3, Fig. 8

Oscillogram Fig. 11 (Group 3, wave shape 8 of Fig. 8). At 7:42½ p. m., 3½ minutes after oscillogram Fig. 9 was obtained, oscillogram Fig. 11 was recorded at the same place. Records indicate that the line was not

TABLE II

CATHODE RAY OSCILLOGRAMS OF TRANSMISSION LINE SURGES, CREST VOLTAGES OVER 100 KV. (YEAR 1929)

Kv. max.	Polarity of first loop	Nature of wave	Time in microseconds to reach: At least 75% of max. voltage recorded	Max. voltage recorded	50% max. on tail of wave	Zero of first loop	Original disturbance on line: Distance in miles from osc.	Kv. measured near origin
1260	Neg.	Osc.	3.3	3.3	3.7	4	10	2400
810	Nog.	Osc.	36	36	?	37	21	1360
740	Pos.	Osc.	7	7	16	22	0–3	690
740	Neg.	Uni.	6.5	6.5	43 +	43 +	0–3	640
630	Pos.	Osc.	11	11	20	29	0–3	860
600	Neg.	Uni.	0– 1.6	4	14.5	45 +	0–3	550
600	Neg.	Uni.	0– 7.5	0– 7.5	69 +	69 +	40	2400
530	Pos.	Osc.	3	7– 8	18	24	0–3	580
500	Pos.	Osc.	0– 8	0– 8	32	45	0–3	470
390	Pos.	Osc.	6	6	14	18	0–27	480
390	Pos.	Osc.	6	6	35	55	?	?
360	Pos.	Uni.	0– 1.6	4	45 +	45 +	0–3	400
330	Pos.	Osc.	1.6	1.6	16	29	0–3	300
330	Neg.	Uni.	?	?	12	75	58	2400
330	Pos.	Osc.	?	?	18	25	?	?
300	Pos.	Osc.	8	8	24	45	?	?
280	Pos.	Osc.	4	4	11	20	32	610
260	Pos.	Uni.	0– 1	5	26	45 +	0–3	300
260	Pos.	Osc.	5	5	27	36	?	?
260	Pos.	Osc.	2	5	25	35	44	2400
260	Pos.	Osc.	0– 2	19	40	42	61	2400
250	Pos.	Osc.	0– 1.3	3	5.5	9.5	?	?
245	Pos.	Uni.	0– 1.2	0– 1.2	22	33 +	?	?
210	Pos.	Uni.	0– 5	0– 5	85 +	85 +	17	720
210	Pos.	Uni.	9	9	10 +	10 +	?	?
205	Pos.	Uni.	0– 3.5	0– 3.5	13	89 +	0–3	300
180	Pos.	Osc.	5	5	23	32	?	?
180	Pos.	Uni.	0– 1.5	5	20	45 +	?	?
180	Pos.	Osc.	0– 1	6	27	35	?	?
180	Pos.	Osc.	2.5	4.5	18	26	?	?
180	Pos.	Osc.	0–17	0–17	28	40	48	1100
170	Pos.	Uni.	0– 1	20	43 +	43 +	44	2400
160	Pos.	Osc.	0– 1	5	13	24	?	?
155	Neg.	Osc.	17	17	24	31	40	2400
150	Neg.	Uni.	0– 1.5	4	45 +	45 +	?	?
140	Neg.	Uni.	0– 1.5	20–28	45 +	45 +	?	?
140	Neg.	Uni.	4	32	38	43 +	44	2400
135	Neg.	Osc.	0– 2	0– 2	6	10	?	?
130	Pos.	Osc.	3	3	11	33	?	?
130	Pos.	Uni.	0– 1	20	81 +	81 +	?	?
120	Neg.	Uni.	0– 2	80	156 +	156 +	?	?
120	Pos.	Osc.	6	6	12	15	?	?
110	Pos.	Uni.	0– 4.5	0– 4.5	9.5	14	3	2400
105	Neg.	Uni.	0– 1.5	40	96 +	96 +	?	?
105	Pos.	Uni.	0– 2	82	82 +	82 +	?	?

Osc.—Oscillatory highly damped.
Uni.—Unidirectional.
? —Point of origin of surge not definite from surge voltage recorder records.

closed until 3 or 4 minutes after the previous tripout at 7:40 p. m. so it is very probable that oscillogram 8 was obtained while the line was deenergized.

On June 21, two days after this oscillogram was obtained, flashed insulators were found on phase Y at tower 23-2, phase X at tower 23-4, and phase W at towers 23-2, 23-3, and 23-4. The negative polarity of the voltage profile, Fig. 12, which correlates to this oscillogram, indicates that the highest voltage for this surge was at this location.

Summary of Cathode Ray Oscillograms of Transmission Line Surges. Wave shape data relative to the transmission line surges measuring above 100 kv. as determined from the cathode ray oscillograms are shown in Table II. These are arranged in decreasing order of crest voltage. The oscillograms have various wave shape characteristics and for this reason in Table II under the heading "Time in microseconds to reach" four columns are given: First, the lapse of time in the initial voltage rise (in several cases this is given between zero and a definite time value, the portion of the wave previous to this time not being recorded); second, the time required to reach the maximum voltage recorded; third, the time between the beginning of the surge and half maximum voltage on the tail of the wave; and fourth, the time duration of the first loop of the wave.

The last two columns in Table II show the approximate distance from the oscillograph to the source of the surge on the transmission line as judged by the maximum kilovolt measured by the surge voltage recorders.

Voltages reach 75 per cent of their crest in an interval of time ranging from less than 1.0 to 36 microseconds, then 100 per cent of crest value in 1.2 or less to 82 microseconds, decrease to 50 per cent of crest value in 3.7 to 156 microseconds, and reach zero of the first loop in 4 to 156 microseconds. 50 per cent of surges reach at least 75 per cent of their value in less than 3 microseconds.

A majority of the surges have a wave front of 3 to 8 microseconds and a duration of the first loop of 15 to 40 microseconds. Approximately 70 per cent are positive in polarity and 30 per cent negative for the first loop. Within the time range of the oscillograms approximately 60 per cent are slightly oscillatory and 40 per cent unidirectional.

All of the extremely high voltages which were measured on phase Y of the transmission line by surge voltage recorders were 3 or more miles distant from the lightning laboratory, and, therefore, have attenuated considerably. It should also be remembered that the oscillograph was coupled to the transmission line at a point 840 ft. from its terminal where reflections may be pronounced.

SURGE VOLTAGE RECORDER DATA

Fig. 14 shows that about 50 per cent of the lightning surges on the Wallenpaupack-Siegfried line and 15 per cent of those on the Plymouth-Siegfried line, were above 5 times normal. (900 kv.)

Of the 45 surge voltages measured on the Wallenpaupack-Siegfried line during 1929, apparently 44 originated on this line, while one was propagated from the Plymouth-Siegfried line. Fifteen of the 21 surges registered on the Plymouth-Siegfried line apparently originated on the Wallenpaupack-Siegfried line. A similar ratio of surge voltage origin held for the 1928 lightning season.

While voltages of about 2400 kv. were registered on both lines, not all of the high voltages were coincident with tripouts. This is shown in Fig. 15.

Nature of Surge Voltages. The voltages, as indicated by Lichtenberg figures, are highly damped, ($H\ D$), and at the point of highest registration are frequently unidirectional; but, if the voltage along the entire line be considered, the surge is usually classified as oscillatory. Table III indicates the polarity characteristics of surges obtained on these two 220-kv. lines, the surge being classified as oscillatory if voltages on the line of both polarities were registered coincidentally.

TABLE III
POLARITY CHARACTERISTICS OF LIGHTNING SURGES—YEARS 1928 AND 1929

	Wallenpaupack-Siegfried		Siegfried-Plymouth	
	Number	Maximum voltage	Number	Maximum voltage
Polarity				
Undirectional				
Positive	9	13.3-15.5	3	12.5-14.5
Negative	4	4.2	2	4.5
Oscillatory				
H. C. V. *positive	16	13.3-15.5	4	10.1
H. C. V. *negative	37	13.3-15.5	17	12.5-14.5
H. C. V. *positive and negative equal	1	1.6	6	1.6

*H. C. V.—Highest crest voltage.

Table III indicates a preponderance of oscillatory surges, either polarity high, but with a tendency toward a greater number with high negative voltage.

Attenuation. Figs. 12, 13, and 16 are typical voltage profiles of surges measured along the transmission line. Many profiles such as shown in Fig. 16 were recorded; they represent profiles with very rapid attenuation. Such voltage profiles are sometimes obtained with difficulty because of the rapid succession of surges on the line. In several instances, surge voltage recorder registrations show superimposed Lichtenberg figures which indicate that at least two surges occurred within a few minutes time.

Table No. 4 gives values of the constant K in the attenuation equation[8] $e = \dfrac{E_0}{K\,s\,E_0 + 1}$

where

E_0 = maximum voltage recorded—kv.
e = voltage at any point—kv.
s = distance in miles between points where E_0 and e are recorded
K = A constant
Attenuation $= -\,K\,e^2$

The constant K in the above equation is calculated from surge voltage recorder measurements at the point of maximum registration and the measurement at the nearest adjacent recorder station. Only those measurements which were not obscured by several Lichtenberg figures are tabulated.

Of the 10 more slowly attenuated surges, those having an attenuation constant of 0.0003 or less, 9 were of negative polarity. In the range of K greater than 0.0005 the majority of the surges were of positive polarity. These data indicate that surge voltages on this line of negative polarity, attenuate more slowly than do those of positive polarity.

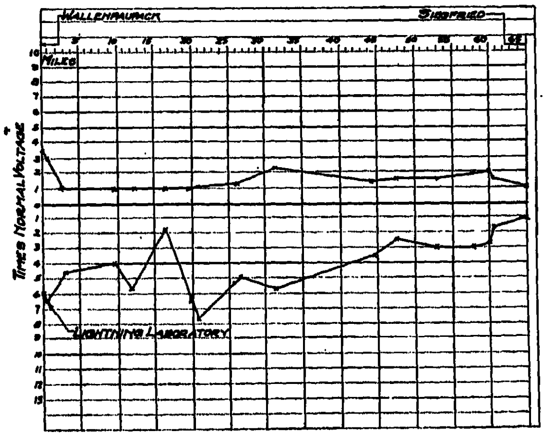


FIG. 12—VOLTAGE PROFILE OF LIGHTNING SURGE NO. 28

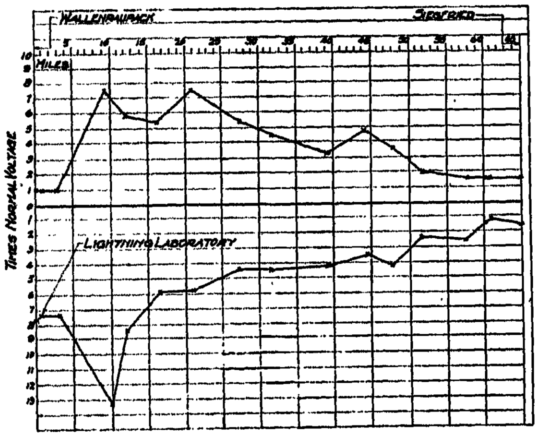


FIG. 13—VOLTAGE PROFILE OF LIGHTNING SURGE NO. 15

Effects of Lack of Continuity of Overhead Ground Wires. As shown in Fig. 3, the section of the Wallenpaupack-Siegfried line between towers 20-2 and 61-2 was the only portion of this system not protected by overhead ground wires during the period under study. Surge voltage recorders were coupled to phase Y one tower either side of the end of the overhead ground wires for studying the effects on the surges at these transition points. During 1928 the average of 10 surges which passed one or both these locations at voltage values in excess of 300 kv., averaged 430 kv. on the side of the open line and 360 kv. under the overhead ground wire. In 1929 an average of 15 surges was 580 kv. on the open line and 700 kv. under the overhead ground wire. The number of these surges originating under the overhead ground wire and on the open line was about equal.

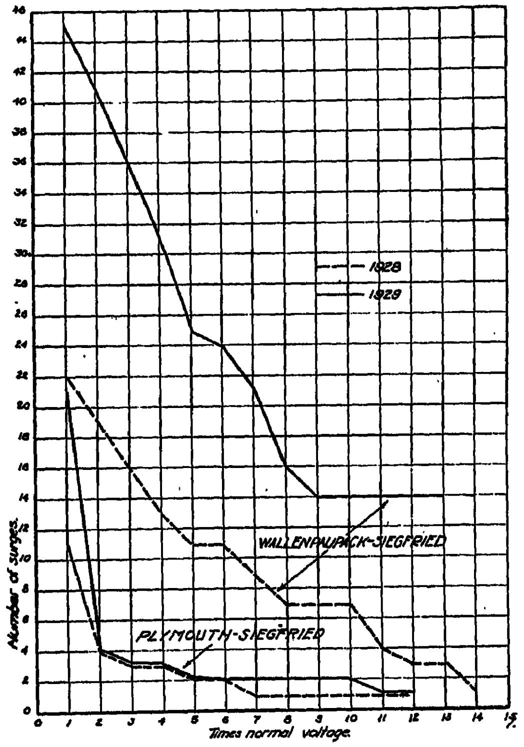


FIG. 14—NUMBER AND CREST VALUE OF LIGHTNING SURGES

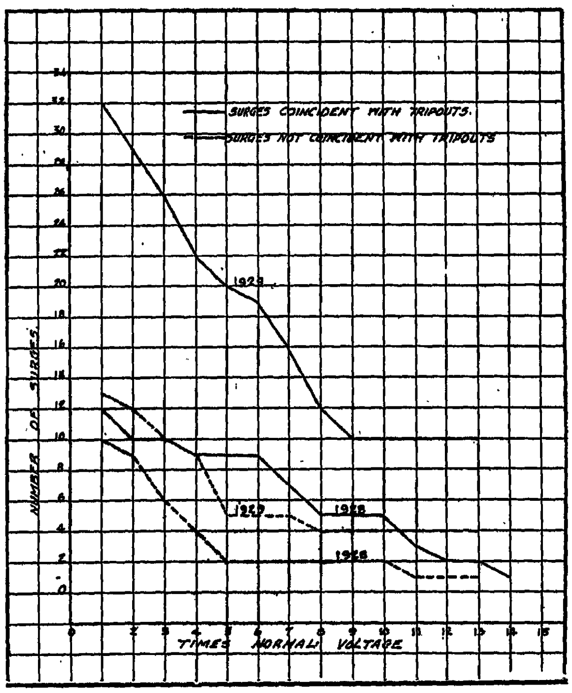


FIG. 15—NUMBER AND CREST VALUE OF LIGHTNING SURGES ON WALLENPAUPACK-SIEGFRIED LINE

Showing relation of surge voltages to circuit tripouts

The above data indicate that no substantial change in voltage occurs near the end of the overhead ground wires when a surge passes this point.

ANTENNAS

Data obtained from the antennas were not numerous but are important. The cathode ray oscillograph coupled to the 50-ft. antenna at the lightning laboratory recorded 11 oscillograms of small magnitude, all of which were straight lines. This fact indicates that the antenna potential was constant throughout the period of registration. The high damping resistance between antenna and ground undoubtedly accounts for the long period of continuous voltage as indicated by the cathode ray oscillograms obtained.

One complete record was obtained at the laboratory at 2:33 p. m., on June 19, 1929, when a surge was measured on all antennas at the lightning laboratory. The results are shown plotted in Fig. 17. The voltage on the 50-ft. antenna under the ground wire was about one-half that on the 50-ft. antenna not protected with ground wire. Voltage on the transmission line at the lightning laboratory was less than 300 kv.

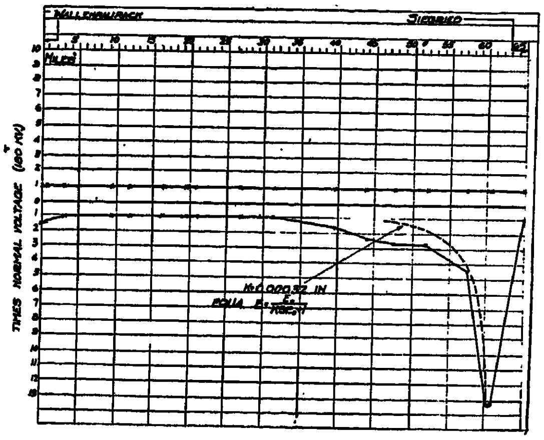


FIG. 16—VOLTAGE PROFILE OF SURGE NO. 76

At this time a voltage gradient of 53 kv. per foot was indicated by the field intensity recorder, the storm clouds being directly over the lightning laboratory with lightning strokes occurring as near as one-half mile.

The cathode ray oscillograph coupled to the 50-ft. antenna was actuated 7 times during a 3½ minute interval just prior to 2:33 p. m. On three occasions, oscillograms of small deflection but with the straight line characteristic previously described and with crest voltages of + 210, + 150, and − 150 were obtained. These oscillograms do not correlate with the surge shown in Fig. 17 which appears to have occurred before the oscillograph actuating circuit was reenergized after a previous operation.

Lichtenberg figure registrations at this time were of type *II*[9] indicating a time to reach crest in the range of 10 to 1,000,000 microseconds. Antennas located along the line produced only three small magnitude voltage records during the entire season, all being on the antennas near tower 12-6.

DIRECT STROKES

Lightning stroke recorders[6] developed in the High-Voltage Engineering Laboratory of the General Electric Company were adjusted to register only currents above 50,000 amperes, and installed on 91 per cent of the towers of the Wallenpaupack-Siegfried line by the end of the 1929 lightning season. From 174 to 284 of these instruments were in service between July 17 and October 1, during which period the Wallenpaupack-Siegfried line tripped out 22 times and 28 lightning surge voltages were measured. Lightning stroke recorders indicated two direct strokes, one at tower 40-2 and one at tower 12-5. At tower 40-2 the current as estimated from the lightning stroke recorder registrations was 60,000 amperes, and was due to a negative cloud discharge to ground. A positive voltage of 2400 to 2800 kv. was registered by the surge voltage recorder connected to phase *Y* at the same tower, but there was no evidence of insulators being flashed at this or immediately adjacent towers. Flashed insulators found at tower 41-1 (fourth tower from tower 40-2) may have been damaged at this time. Judged by the phase current recording ammeters, the coincident tripout was due to a fault on phase *X*.

At tower 12-5 on a portion of the line where the counterpoise was installed, a negative cloud discharge of about 100,000 amperes was indicated by a lightning stroke recorder. The nearest surge voltage recorder in operation at the time, the one connected to phase *Y* at tower 10-2, registered a positive voltage of 350 kv. There was no coincident tripout of the line.

During the 1929 lightning season, 24 cloud-to-ground strokes were observed within a one mile radius of the Wallenpaupack lightning laboratory, none of which are known to have hit the transmission line.

STORM INTENSITY

Two instruments, an electric field intensity recorder for measuring the electrostatic field intensities and a time rate of change of electric field intensity recorder for measuring the maximum rate of change of electrostatic fields, were developed in the General Engineering Laboratory of the General Electric Company and placed at the lightning laboratory during 1929 to obtain field experience in operation. Many records were obtained with these instruments and are included in another paper presented before the Institute.[6]

Voltage gradients measured, averaged from 55 kv. per foot for strokes occurring near the laboratory to about 8 kv. per foot for strokes 8 miles distant. The maximum registration was 85 kv. per foot.

Records of 33 storms indicated that lightning fields during any single storm were usually of one predominant polarity either positive or negative. About 20 per cent of these storms were of mixed polarity, the balance was about equally divided between those predominantly positive and predominantly negative.

One record of particular interest which correlates with the data obtained by surge voltage recorders coupled to antennas at the lightning laboratory indicated a voltage gradient of 53 kv. per ft. This is shown in Fig. 17.

TABLE IV
ATTENUATION CONSTANTS OF LIGHTNING SURGES

No of surges having attenuation constant *K* within limits indicated in Column 2			
Neg. polarity	Pos. polarity	Total	Limits of *K*
9	1	10	Less than 0.0003
6	4	10	Between 0.0003 and 0.0005
8	12	20	Greater than 0.0005

	Pos.	Neg.
Maximum *K*......	0.012	0.0086
Minimum *K*......	0.000012	0.000065

FLASHED INSULATORS

Fig. 18 indicates that on the Wallenpaupack-Siegfried line a total of 92 insulator assemblies was flashed in 1929, and 171 assemblies during 1926 to 1928 inclusive. These figures indicate an average of 2½ assemblies flashed per tripout due to lightning in 1929 and 4½ assemblies flashed per tripout for the three previous years.

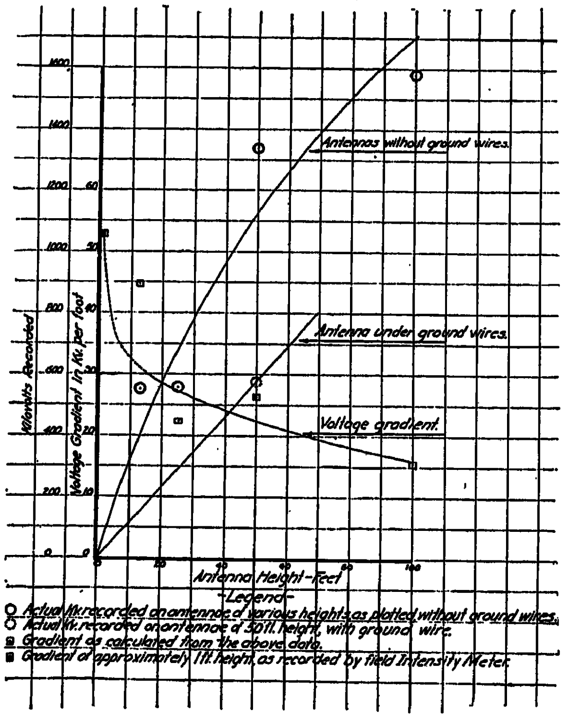


FIG. 17—VOLTAGES ON ANTENNAS—SURGE NO. 23

Most of the insulators are flashed on the outside phases as is shown in Table V, and of those marked, only a portion have glaze burns by dynamic arc, the others apparently being flashed by impulse currents only.

In Fig. 18 the number of surge voltages exceeding 5 and 10 times normal at surge voltage recorder stations indicates comparable voltages for 1928 and 1929, but actually there were more than twice as many individual surges of high magnitude in 1929. It appears that a greater percentage of the surges in 1929 was of a type that registered high voltages at one or two recorder stations only.

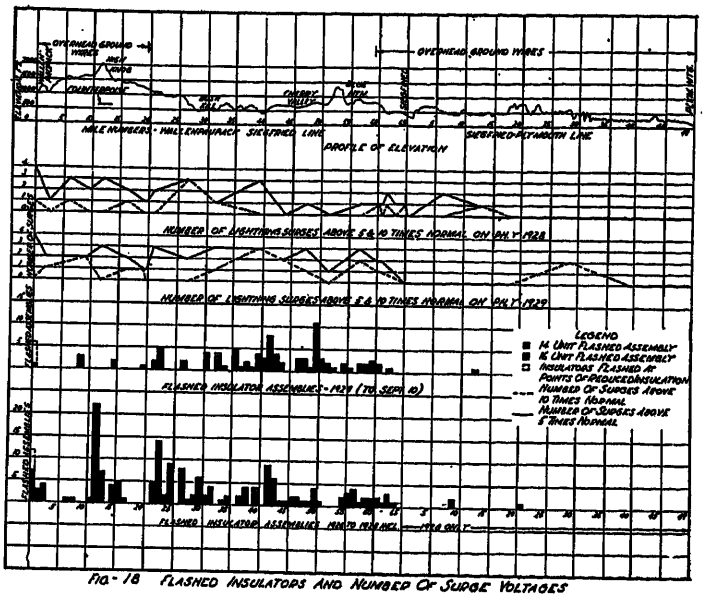


FIG. 18—FLASHED INSULATORS AND NUMBER OF SURGE VOLTAGES

In the section of the line (Fig. 18) where the counterpoise was installed no flashed insulators were found in 1929, although this section of the line had a large number of insulators flashed in previous years. It is, of course, too early to conclude whether the improvement came from the counterpoise installation or whether it represents merely different storm paths during the present year.

On the Wallenpaupack-Siegfried line during 1929, exclusive of flashed assemblies at points of reduced insulation near terminal apparatus, only about 10 per cent of the insulators were flashed under 25 miles of overhead ground wire as against 90 per cent flashed in 40 miles of line unprotected by overhead ground wire. Owing to the lesser number of overhead patrols in previous years, comparison similar to the above cannot be made.

TABLE V
FLASHED INSULATORS ON WALLENPAUPACK-SIEGFRIED LINE

	1926 to 1928 incl.			1929		
Phase	Glaze burns	Flash tracks	Total	Glaze burns	Flash tracks	Total
East..............	..	..	59	26	12	38
Middle..............	..	..	30	11	10	21
West..............	..	..	82	28	5	33
Total..............			171	65	27	92

Flashed insulators on the Plymouth-Siegfried line have averaged about 1½ insulator assemblies flashed for each tripout attributed to lightning.

TRIPOUTS

From Table VI it will be seen that the tripouts attributed to lightning in 1929 were nearly three times the number of those occurring in 1928.

As shown in Fig. 14, the number of surge voltages on the Wallenpaupack-Siegfried line in 1929 was slightly more than twice the number in 1928, indicating that the increase in the number of tripouts was due to the greater lightning hazard to which the Wallenpaupack-Siegfried line was exposed, during the present year.

TABLE VI
220-KV. TRIPOUTS DUE TO LIGHTNING

Line	1928	1929
Wallenpaupack-Siegfried	14	38
Siegfried-Plymouth	2	1

The relative magnitudes of the phase fault currents indicate that of the last 19 tripouts of the Wallenpaupack-Siegfried line, five were due to single-phase-to-ground faults, and not less than seven were multiphase faults. The remaining seven were due either to single or multiphase faults.

SWITCHING SURGES

Incidental to the lightning investigation, numerous switching surge voltages were measured. The upper voltage limit for these surges appears to be about 900 kv. (5 times normal). Most switching surges are under 3 times normal. During 1929 the upper limit for switching surge voltages on the Wallenpaupack-Siegfried line was 2.8 times normal.

The interruption of short-circuit currents did not give rise to greater voltages than other switching operations nor were energizing or deenergizing surge voltages appreciably different.

CONCLUSIONS

1. Records obtained from storm reports show that in general storms cross this system from west to east or from northwest to east southeast.
2. Wave fronts of lightning surges recorded on the transmission line have a range from a fraction of a microsecond to 82 microseconds. A common characteristic was a front between 3 and 8 microseconds, a duration of the first loop of 15 to 40 microseconds, and a slightly oscillatory nature. Positive polarities predominate.
3. Not all high voltages are accompanied by tripouts of the circuit.
4. Lightning surges may be unidirectional (either polarity high) or slightly oscillatory.
5. Surge voltages of positive polarity attenuate more rapidly than those of negative polarity.
6. The quantitative value of protection afforded by overhead ground wires cannot be deduced from the limited data. One record on antennas indicates a protection of about 50 per cent against induced voltages.
7. Preliminary data indicate that high voltages due to direct strokes having currents of over 100,000 amperes, are infrequent.
8. An average of from 3 to 5 insulator assemblies were flashed per tripout of the circuit and about 70 per cent of these show glaze burns by dynamic arcs. Most of the flashed assemblies were on the outside phases.
9. A considerable percentage of tripouts are due to multiphase faults.

ACKNOWLEDGMENTS

Acknowledgment is made to Messrs. E. Bell, E. A. Evans, G. J. Gross, H. B. Newsom, N. Rohats, and others[10] for field work, analysis, and correlation of data, and to Messrs. C. M. Foust, C. A. Jordan, E. S. Lee, W. W. Lewis, H. S. Phelps, H. K. Sels, and A. E. Silver for general direction of this investigation.

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10. Personnel of the General Electric Company and Pennsylvania Power & Light Company.

Discussion

For discussion of this paper see page 928.

Lightning Investigation on the Ohio Power Company's 132-Kv. System

BY PHILIP SPORN* and W. L. LLOYD, Jr.†
Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—Description is given of the Lightning Field Investigation on the 132-kv, Philo-Canton, 73-mile, two-circuit line made during 1929 with surge recorders, cathode ray oscillograph, lightning stroke recorders, and a lightning generator. Data are presented on the magnitude of lightning voltage surges, the attenuation of these surges as they travel along the line, and the difference in attenuation between positive and negative lightning surges. The preponderance of positive lightning surges is indicated; but the fewer negative surges produce the highest voltage on the line.

The lightning stroke recorders gave data on the polarity of the direct strokes, and the magnitude of the current in the lightning stroke.

Lightning arrester performance on lightning and switching surges is analyzed, and typical surges obtained by the cathode ray oscillograph are given for both cases.

The magnitude and shape of lightning waves occurring on the line are shown and these waves analyzed as to duration of front, tail, and total length. These data are discussed as they relate to the proximity and rate of discharge of clouds producing lightning surges.

General conclusions are drawn based on the data obtained in the investigation.

I. INTRODUCTION

THE importance of continuity of service on high-voltage transmission lines under our present day method of central station generation and transmission of power is too well known to require extended comment. Estimates have been made, and many records show, that from 60 per cent to 90 per cent of the total number of outages on overhead transmission lines are caused by the influence of lightning on the line itself or on the connected apparatus. Line outages, other than those due to lightning, are more or less under control, such, for example, as those caused by improper or erratic relay operation, obstructions in the right of way, etc. Lightning, however, remains as one element over which, at the present time, we do not have complete control, and concerning which we have had, up until a few years ago, only very little authentic information. It is for these reasons that the investigation work covered by this paper was undertaken.

Intensive field research work has been in progress since 1927 in an attempt to solve some of the lightning problems connected with the transmission lines covered by this paper. A report on some of this work on this same system has been published previously.[1]

While a considerable quantity of information was gathered in 1927 and 1928, it had resulted primarily in laying bare the problem as a whole, and in indicating more clearly further lines along which data and information were needed. The purpose of the investigation herein described was to continue the work started in 1927 and 1928 and to explore some of the newly indicated paths of needed knowledge.

The program laid out in 1929 consisted in general of:

1. Continuing the use of surge voltage recorders used in past seasons.
2. Installing a cathode ray oscillograph coupled to the line.
3. Installing lightning stroke recorders to obtain data on direct lightning strokes.

The use of the cathode ray oscillograph on this system is entirely new, as is also the use of lightning stroke recorders.

This lightning investigation was carried through on a cooperative basis by the General Electric Company, the Ohio Power Company, and the American Gas and Electric Company, each company supplying part of the equipment and personnel.

II. SYSTEM INVESTIGATED

The 1929 lightning investigation was conducted on the Philo-Canton 132-kv. lines of the Ohio Power Company, these being part of the system on which the investigation work was performed in 1927 and 1928. A map of the 132-kv. network, of which the Philo-Canton line forms one link, is shown in Fig. 1. A detailed description of this network has been given previously,[2] as has also a description of the line characteristics.[1] In brief, the line is a two-circuit steel tower line, the circuits being in vertical configuration and with one ground wire installed at the peak of the tower. The dimensions of the standard tower are shown in Fig. 2. The insulator strings are equipped with grading shields on the line side and with arcing horns on the ground side of all insulator strings. The records for this line during the lightning seasons of 1925, 1926, and 1927 have already been published,[1,2,3,4] In choosing a transmission system for carrying on this year's work, it was felt that the lightning experience background of the Philo-Canton Line would be of distinct advantage in continuing the work which was started in 1927. Another

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*American Gas and Electric Company, New York.

†General Electric Company, Pittsfield, Mass.

[1] *Lightning and Other Experiences with 132-Kv. Steel Tower Transmission Lines*, M. L. Sindeband and P. Sporn, A. I. E. E. TRANS., Vol. 45, 1926, p. 770.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 27-31, 1930.

[2] *1926 Lightning Experience on 132-Kv. Transmission Lines*, Philip Sporn, A. I. E. E. TRANS., Vol. 47, p. 668.

[3] *1927 Lightning Experience on 132-Kv. Transmission Lines of the American Gas and Electric Company*, Philip Sporn, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 480.

[4] *Surge-Voltage Investigations 132-Kv. Transmission Lines of the American Gas and Electric Company*, Philip Sporn, A. I. E. E. Quarterly TRANS., Vol. 47, October, 1928, p. 1132.

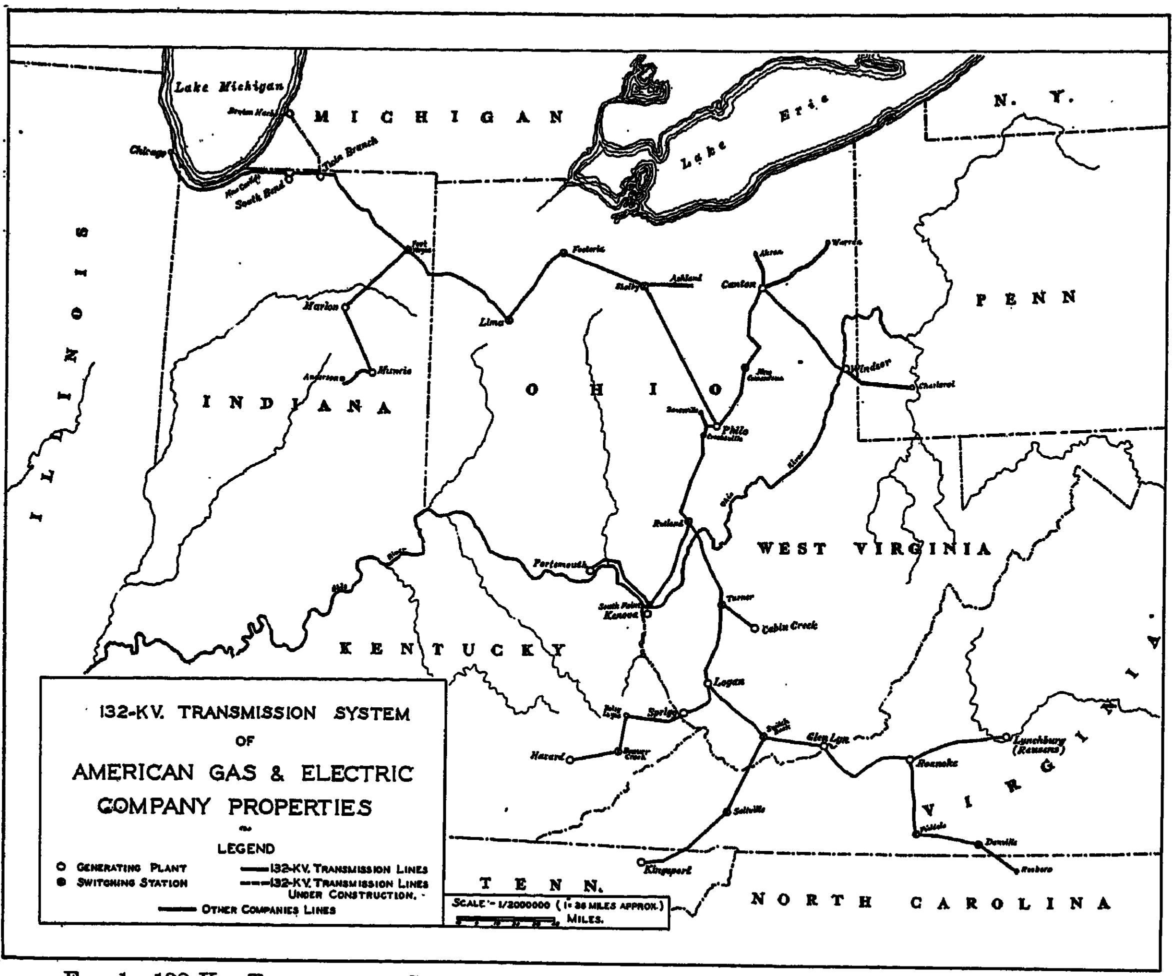
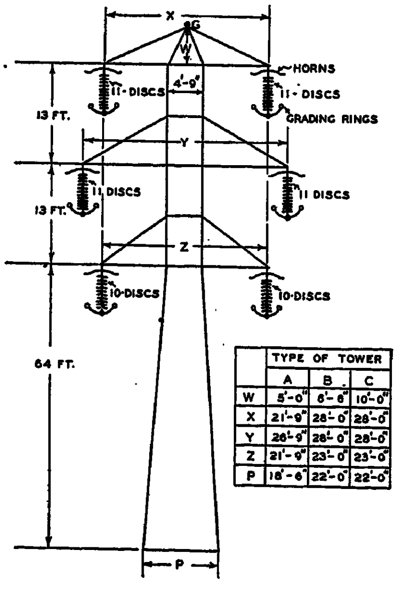


FIG. 1—132-KV. TRANSMISSION SYSTEM OF AMERICAN GAS AND ELECTRIC COMPANY PROPERTIES



	TYPE OF TOWER		
	A	B	C
W	5'-0"	6'-6"	10'-0"
X	21'-9"	28'-0"	28'-0"
Y	26'-9"	28'-0"	28'-0"
Z	21'-9"	23'-0"	23'-0"
P	18'-6"	22'-0"	22'-0"

FIG. 2—STANDARD TOWER OF PHILO-CANTON 132-KV. LINE

feature presented by this line is that it had a substation tapped to both circuits at about the midpoint, thus offering the possibility of tests on the effect or influence of non-dead-end station apparatus on line surges.

III. SCOPE OF INVESTIGATION

The locations of surge recorders, cathode ray oscillograph and lightning stroke recorders are shown in Fig. 3 and accompanying table. Surge recorders were located at the Canton Station to determine the effectiveness of lightning arrester operation. On a section of the line about 16 miles long surge recorders were located at approximately one mile intervals to determine the attenuation of surges originating on or near this section of line. This 16-mile section was chosen adjacent to the Newcomerstown substation which is at the approximate center of the line, and where both circuits are tapped to a common bus. The surge recorders, in addition to supplying attenuation data, served to supply confirming information for the cathode ray oscillograph which was located at the Newcomerstown substation. At Newcomerstown there were also placed surge recorders on the lightning arresters with the idea of studying lightning arrester performance and getting data which could be correlated with the cathode ray oscillograph records.

By the use of the cathode ray oscillograph it was expected to obtain information on crest value of lightning surges occurring on the line, and in addition, the shape, duration, and reflection characteristics of surges.

This location at Newcomerstown substation was selected for the oscillograph as it offered an opportunity of studying the performance of lightning waves on reaching a substation where there is considerable station equipment, such as post type insulators, bushings, transformers, and nearby steel work. Since in the past, and during the present year, cathode ray oscillographs at other points, with the exception of one, where lightning investigations have been or were being conducted, have been located on parts of the line quite remote from station equipment, it was felt this installation might furnish additional pertinent data due to its proximity to station equipment.

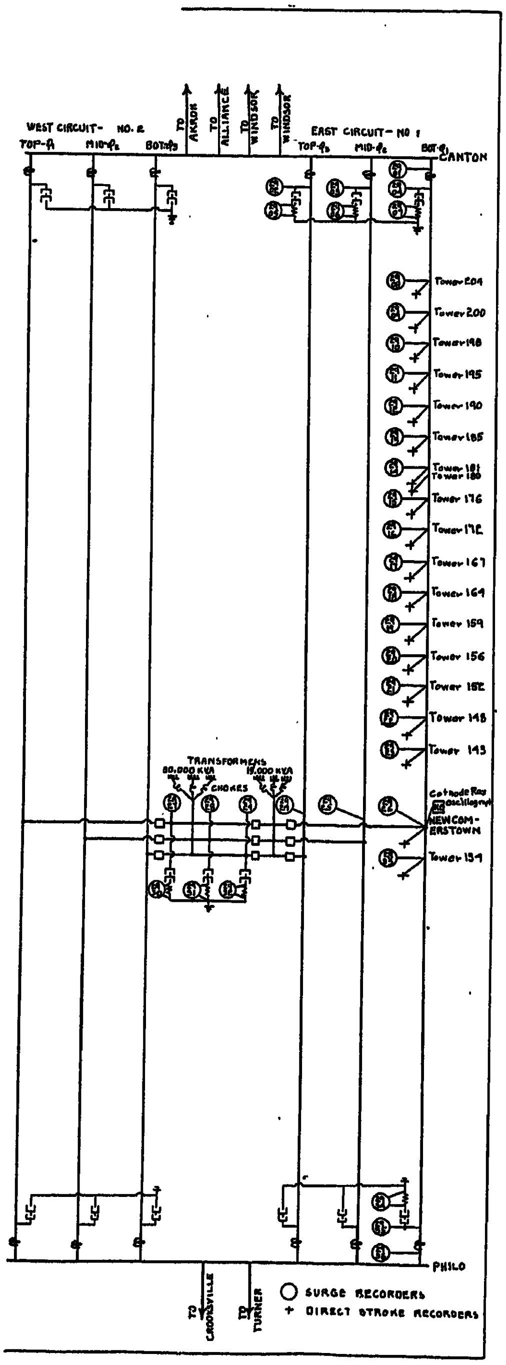


FIG. 3—LOCATION OF SURGE RECORDERS, LIGHTNING STROKE RECORDERS, AND CATHODE RAY OSCILLOGRAPH

On the Philo-Canton 132-kv. line of the Ohio Power Co., 1929

LIGHTNING INVESTIGATION—1929

Philo-Canton 132-Kv. Line

Surge Recorder Station No.	Location	Miles from Canton	Miles from back station	Instrument connections	Date of installation
2901	Canton	..	..	Bot. φ_1—bus side choke coil	5/ 6/29
2902	Canton	..	..	Top φ_3—line side choke coil	"
2903	Canton	..	..	Mid. φ_2—line side choke coil	"
2904	Canton	..	..	Bot. φ_1—line side choke coil	"
2905	Canton	..	..	L. A. amps.—Top φ_3 35.3 ω	"
2906	Canton	..	..	L. A. amps.—Mid. φ_2 34.3 ω	"
2907	Canton	..	..	L. A. amps.—Bot. φ_1 35.7 ω	"
2908	Tower 204	23.3	23.3	Bot. φ_1	5/ 7/29
2909	" 200	24.1	0.80	Bot. φ_1	"
2910	" 198	24.87	0.77	Bot. φ_1	"
2911	" 195	25.64	0.77	Bot. φ_1	"
2912	" 190	27.15	1.51	Bot. φ_1	"
2913	" 185	28.65	1.50	Bot. φ_1	"
2914	" 181	29.74	1.09	Bot. φ_1	"
2915	" 176	30.97	1.23	Bot. φ_1	"
2916	" 172	32.13	1.16	Bot. φ_1	5/ 8/29
2917	" 167	33.21	1.08	Bot. φ_1	"
2918	" 164	34.19	0.98	Bot. φ_1	"
2919	" 159	35.26	1.07	Bot. φ_1	"
2920	" 156	36.27	0.91	Bot. φ_1	"
2921	" 152	37.21	0.94	Bot. φ_1	"
2922	" 148	38.33	1.12	Bot. φ_1	"
2923	" 143	39.26	0.93	Bot. φ_1	"
2924	Newc't'wn	40.09	0.83	Top φ_1—Circuit No. 1	5/23/29
2925	"		0.83	Mid. φ_2—Circuit No. 1	"
2926	"		0.83	Bot. φ_3—Circuit No. 1	"
2927	"		0.83	Bus at L. A. φ_1	5/12/29
2928	"		0.83	Bus at L. A. φ_2	"
2929	"		0.83	Bus at L. A. φ_3	"
2930	"		0.83	L. A. amps. φ_1 33.4 ω	5/23/29
2931	"		0.83	L. A. amps. φ_2 32.5 ω	"
2932	"		0.83	L. A. amps. φ_3 30.3 ω	6/ 6/29
2933	Tower 134	41.09	0.98	Bot. φ_1	5/12/29
2934	Philo	73.3	32.22	L. A. amps. φ_1 37.0 ω	6/13/29
2935	"	73.3	32.22	Bot. φ_1—line side choke coil	5/ 9/29
2936	"	73.3	32.22	Bot. φ_1—bus side choke coil	"
K-2	Newc't'wn	40.09		Bot. φ_1—Circuit No. 1	5/22/29
	Tower 180				8/19/29

The lightning laboratory, housing the oscillograph, together with coupling potentiometers, are shown in Fig. 4. The oscillograph itself, ready for operation, is shown in Fig. 5.

Since the moment a lightning surge will appear at the measuring station cannot be predicted, it is necessary to have the incoming surge itself place the cathode ray oscillograph in operation. This consists in applying a high negative voltage to the cathode to obtain the stream of electrons making up the cathode ray and in applying a gradually increasing voltage to the sweep plates of the oscillograph to obtain a deflection of the cathode ray on the photographic film proportional to the time and at right angles to the deflection produced by the lightning surge. These operations must be carried on in an extremely brief interval of time if the front of the in-coming voltage surge is to be fully

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5. *Lightning Investigation of Transmission Lines*, by W. W. Lewis and C. Foust, see p. 917.

determined. A circuit for accomplishing this was devised by engineers of the General Electric Company.[6] The diagrammatic connections of this circuit are shown in Fig. 6. A tap on the initiating potentiometer string, causes the three-electrode sphere-gap to discharge and carry out the above operations whether the incoming lightning surge is positive or negative.

The lightning stroke recorder is an entirely new development. The purpose of this instrument, which was proposed by F. W. Peek, Jr., and developed by W. L. Lloyd, Jr., is to tell when and where a direct stroke of lightning to a transmission line takes place. There was reason to believe that the steel towers or ground wires of certain lines, at least, might be struck without always producing a 60-cycle dynamic short circuit or outage of the line. It was desired to know the percentage of direct strokes causing tripouts as compared with the percentage not causing tripouts, also the percentage of tripouts produced by direct strokes as compared with the percentage produced by induced voltage surges. The polarity of the cloud and the magnitude of the current in the lightning discharge were of considerable theoretical interest. Since lightning might strike anywhere at any time the instrument had to be simple enough in construction and operation to permit its use on several towers along a line without attendants.

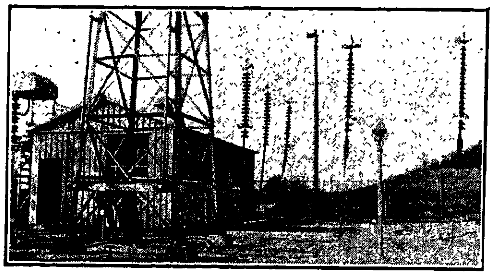

FIG. 4—CATHODE RAY OSCILLOGRAPH LIGHTNING MEASUREMENT STATION

At the Newcomerstown Substation of the Philo-Canton line

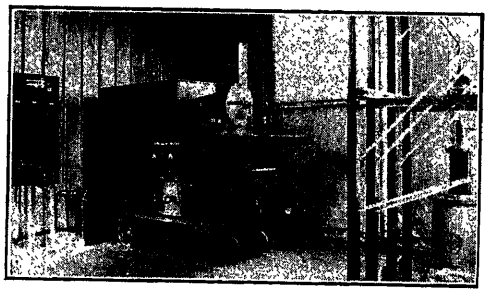

FIG. 5—PORTABLE CATHODE RAY OSCILLOGRAPH

At the Lightning Measurement Station in Newcomerstown, Ohio

The instrument consists essentially of a small size surge recorder, without timing mechanism, which is placed across a section of the tower steel. The theory of this instrument is that should the tower be struck by a direct stroke of lightning the current through the tower steel work will be of sufficiently high magnitude to produce a voltage drop which can be recorded by Lichtenberg figures on a photographic film inserted between the electrodes. In the case of induced strokes, however, the current in the tower would be so small that no Lichtenberg figure would appear on the lightning stroke recorder film. The photographic film is in a specially prepared light-proof package permitting daylight loading of the instrument. Films were changed once a week during the lightning season. Laboratory tests, using the steepest fronts and current up to the maximum which could be obtained from the lightning generator, indicated the size of the film and electrodes necessary, the nature of the records which would be obtained, the satisfactory operation of the instrument in accordance with the above theory, and furnished an approximate calibration curve for determining the magnitude of current in the lightning stroke. An assembled and disassembled view of the instrument is shown in Fig. 7. A field installation is shown in Fig. 8.

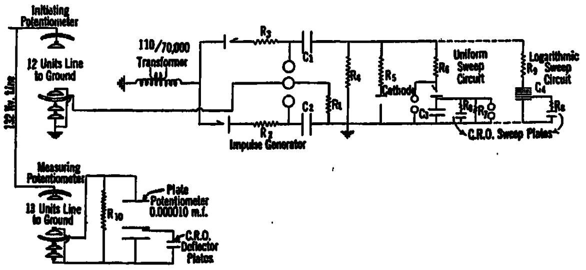

FIG. 6—DIAGRAM OF CONNECTIONS FOR THE AUTOMATIC OPERATION OF THE CATHODE RAY OSCILLOGRAPH BY THE INCOMING LIGHTNING SURGE

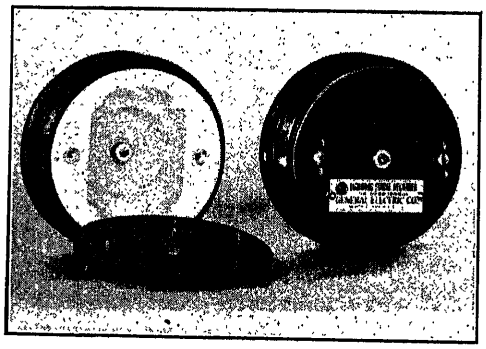

FIG. 7—INTERIOR AND EXTERIOR VIEWS OF THE LIGHTNING STROKE RECORDER

These recorders were located on each tower where a surge recorder was located. One recorder was also located at Newcomerstown substation on the steel structure adjacent to the lightning laboratory. By placing the lightning stroke recorders at the same location as the surge recorders, it was possible to correlate the data obtained from both the lightning stroke recorders and surge recorders.

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[6] *Lightning—Progress in Lightning Research in the Field and in the Laboratory*, F. W. Peek, Jr., A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 436.

IV. OPERATION OF OSCILLOGRAPH STATION AND OF INSTRUMENTS

Surge Recorders. The surge recorder investigation, covering the lightning season of 1929, extended from approximately May 1 to October 15, the exact dates when surge recorders were placed in service being given in Fig. 3.

Cathode Ray Oscillograph. The cathode ray oscillograph was installed May 22; from then until October 15 it was placed in operation whenever lightning storms were reported on the system between Philo and Canton. This necessitated the constant attendance of an oscillograph operator at the lightning measurement station when storms were at all probable during this time. The oscillograph was placed in operation by the surge to be measured, but it was necessary for the operators to adjust the potentiometer according to the proximity of the storm, to keep the proper vacuum, to turn the photograph film after each exposure and replace the film after every twelve exposures. The operators also made observations of the storms and noted the time of each exposure.

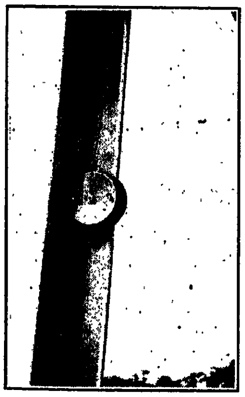

FIG. 8—FIELD INSTALLATION OF LIGHTNING STROKE RECORDER

Showing recorder on tower leg

Lightning Stroke Recorders. The lightning stroke recorders were placed in operation July 16. Thereafter, the film packs were changed every week until October 1. After development of the films, an examination was made to determine the indication of a direct stroke.

V. SURGE RECORDER DATA AND ANALYSIS

To better analyze the surge data obtained, all surges have been classified as to cause under six headings, as follows:

1. Those due to lightning only, where it was known a lightning storm was in progress over the system and no switching occurred.
2. Those due to lightning and switching, where the circuit tripped during a lightning storm, and a record was obtained.
3. Those due to switching operations on the line, where the character of switching surges were known, that is whether they were energizing or deenergizing surges.
4. Switching surges occurring so rapidly that the energizing and deenergizing characteristics could not be separated.
5. Surges of unknown origin, where no cause could be assigned.
6. Surges of doubtful origin, where the recorded surge was believed to be due to electrostatic interference on the potentiometers and not primarily associated with characteristic surges causing disturbance.

A total of 115 surges was recorded during the season; these surges are classified, as above described, in Table I.

TABLE I
TOTAL NUMBER OF SURGES

Cause of surge	Total No.	Selected No.
Lightning	41	24
Lightning and switching	7	7
Switching—Energizing No. 1	3	3
" " No. 2	0	0
" Deenergizing No. 1	19	19
" " No. 2	1	1
Mixed energ. and deenerg. No. 1 & No. 2	10	10
Unknown	27	0
Doubtful	7	0
	115	64

The "Selected" number of surges in the second column represent the 115 surges after deducting those which recorded on the three bus recorders at Newcomerstown only. These three recorders appeared to be influenced by stray fields in the station, so that when these recorders only registered, the records were considered of doubtful significance.

A further study of these surges was made to determine their polarities and this analysis is given in Table II.

The maximum voltage recorded (in times normal—

TABLE II
PREDOMINATING NATURE OF SURGES WITH MAXIMUM RECORDED VOLTAGES
(Times Normal)

Cause of surge	Positive		Negative		Oscillatory	
	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.
Lightning	8.4	..	..	2.1	4.2	Tr.
Lightning and switching	6.5	..	7.1	14.3	3.2	Tr.
Switching–Energizing No. 1.	2.8	..	..	2.4	2.3	2.7
" " No. 2.	..	..	..	..	..	..
" Deenergizing No. 1	..	..	..	..	4.9	2.8
" " No. 2	Tr.	..	..	..	..	..
Mixed energ. and deenerg. No. 1 and No. 2	3.0	..	..	..	2.7	4.5
Unknown	4.9	..	..	2.4	3.8	Tr.
Doubtful	4.2	..	..	3.6	3.4	Tr.

"Tr." is trace—slightly above 108 kv.

normal being 108 kv.) is given for the maximum surge under each classification. The maximum crest value of any pure lightning surge is 8.4 times normal positive. The maximum voltage recorded on combined lightning and switching surges was 14.3 times normal negative with a positive indication for this same surge of 7.1 times normal, occurring at the same place the 14.3 occurred. There was also recorded a positive surge of 6.5 times normal at the time of a lightning storm when the line was known to have tripped out.

To indicate the relative frequency and magnitude of lightning surges on the line Fig. 9 has been drawn. This shows that of 31 lightning surges, about 25 per cent were above 4½ times normal, 10 per cent above 6 times normal, 6.5 per cent above 8 times normal, and 3 per cent, or 1 surge, above 14 times normal. The general character of this curve checks and confirms quite well data obtained previously and last year on the same question on other systems. Under switching conditions, the maximum value of surge recorded was 4.9 times normal on deenergizing the line, and 2.7 times normal on energizing the line. Both of these surges were highly damped oscillations. The maximum surge classified as unknown and doubtful reached a value of 4.9 positive times normal but both groups are considered unimportant for purposes of this analysis.

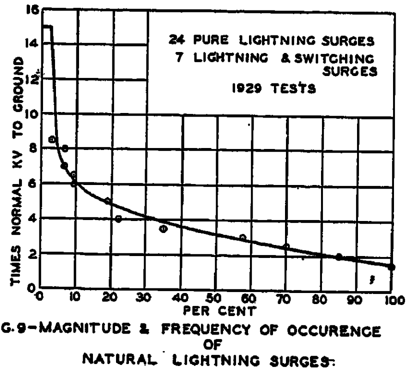

FIG. 9—MAGNITUDE AND FREQUENCY OF OCCURRENCE OF NATURAL LIGHTNING SURGES

All surges have further been analyzed as to characteristics, that is, whether all positive or negative or predominantly of one polarity. Some surges were obtained this year, as in the past, where most instruments recorded one polarity, but the opposite polarity was also registered at other instruments at apparently the same time. Such surges have been classified as unidirectional (positive or negative) with an indication of the prevalent polarity. This analysis is given in Table III.

The important fact brought out by Table III is that 61.0 per cent of the lightning surges were pure positives, 12.2 per cent predominantly positive, 14.7 per cent pure negative, 4.9 per cent predominantly negative, and 7.2 per cent highly damped oscillations. Further, switching surges are mostly oscillatory, with a few unidirectional surges indicated; these however were of small magnitude.

If the number of positive surges only were taken as an indication, it could be concluded that the greater portion of surges is of induced origin; on the other hand, the highest surge recorded (in point of voltage) was negative. The only safe conclusion at the present, therefore, is that both positive and negative surges can be of an appreciable magnitude. More light may be thrown on this by additional direct stroke recorder data.

TABLE III
PREDOMINATING NATURE OF SURGES
(Number Recorded)

Cause of surge	Positive		Negative		Oscil-latory	Total
	Pure	Pre-dom.	Pure	Pre-dom.		
Lightning	25	5	6	2	3	41
Lightning and switching	1	2	..	2	2	7
Switching–Energizing No. 1.	1	..	1	..	1	3
" " No. 2.	..	..	..	..	..	..
Switching Deenergizing No. 1	..	..	..	..	19	19
" " No. 2	1	..	..	..	..	1
Mixed energ. and deenerg. No. 1 and No. 2	4	..	..	..	6	10
Unknown	11	5	5	1	5	27
Doubtful	1	2	..	1	3	7

Attenuation Data. Attenuation data on natural lightning were obtained for two different surges. During the year tests were also run using artificial lightning and the attenuation checked with Lichtenberg figures secured on the surge recorders which were in normal operation throughout the season. A study was also made with the cathode ray oscillograph of the attenuation of artificial lightning surges along the line and reflections from the open and closed ends of the line.

Lightning Arrester Tests. The lightning arrester surge recorders yielded 53 surge records. In most cases a correlating voltage at the terminals of the lightning arrester was obtained, where current discharges through the arrester were recorded. The summary of these data is shown in Table IV. The data in Table IV shows that the lightning arrester, in the case of switching surges, has discharged a current of 226 amperes per leg at 450 kv. Since it is known switching surges do not exceed some 550 kv. on these lines, there are no definite data to show that the arrester has held the voltage down appreciably on switching surges. The arrester has, however, discharged a maximum current of 226 amperes per leg at 450 kv. which is a point definitely in favor

TABLE IV
LIGHTNING ARRESTER OPERATION

	Cause of surge	
	Lightning	Switching
No. of surges recorded	12	41
Max. Current–amperes	224	243
Max. Correlating voltage–kv	(1)	300
Max. Recorded voltage–kv	380	450
Max. Correlating current–amperes	105	226

(1.) 225 kv. or less (no record on S. V. R.)

of the lightning arrester. Of course this would not effect its economic justification, if the system apparatus is built to withstand an indefinite number of surges of switching origin and magnitude without failure.

The maximum lightning voltage surge on the arrester recorded 380 kv. with a correlating current of 105 amperes. This again shows the arrester is performing, but a quantitative measure of its performance is lacking. That is, there are no data available to show how much the discharge current of 105 amperes recorded helped to bring down the lightning voltage which might have existed without the arrester, or how much the arrester aided in holding down the voltage to 380 kv.

TABLE V
CHARACTERISTICS OF THE LIGHTNING SURGES MEASURED WITH THE CATHODE RAY OSCILLOGRAPH ON THE PHILO-CANTON LINE DURING 1929

No.	Date	Max. kv.	Polarity	Time in microseconds To 75 % On rising front	 To max. 	 To 50 % On falling tail	 To zero
Fast or steep-fronted surges:							
341	6/30	97	+	0.2*	1*	20	70*
457A	8/23	245	+	0.5*	4	12	55
462	"	190	+	0.2*	1*	19	80
			Average	0.3*	2*	17	70*
Slow or slanting-fronted surges:							
457B	8/23	70	+	8	16	37	65
458	"	70	+	8	16	37	48
463	"	55	+	4	20	38	65
464	"	70	+	7	15	52	95*
			Average	7	17	41	68*
Very slow or slanting-fronted surges:							
459	8/23	85	+	10	30	45	65
461	"	85	+	10	30	42	60
460	"	108	+	13	28	43	55
			Average	11	29	43	60
Average of ten surges:				6	16	34	66

*Estimated approximate values

Cathode Ray Oscillograph Data and Analysis

Lightning Surges. During the time the cathode ray oscillograph was in operation 29 storms occurred in the vicinity of Newcomerstown. Ten oscillograms of lightning were obtained. Although the oscillograph was set to record with equal sensitiveness both positive and negative surges, all ten surges recorded were positive. Table V gives the characteristics of these surges. Fig. 10 shows typical oscillograms from among these ten.

From Table V it will be seen that the time to reach 75 per cent of maximum voltage on the front of the lightning wave varied from ¼ of a microsecond for the fastest waves to 13 microseconds for the slowest waves, while the total front of the waves varied from one to 30 microseconds. The length of the waves, to half maximum value on the tail, varied from 12 microseconds to 52 microseconds while the total length of the waves, to zero, varied from 48 microseconds to approximately 95 microseconds. All waves recorded were of low voltage. Fig. 11 gives the average of the three steepest surges, the average of the three least steep and the average of all ten surges plotted in rectangular coordinates.

Since all ten surges were of low voltage and positive polarity it would appear that they were induced voltage surges rather than surges resulting from direct hits. Furthermore, the surge voltage recorders indicated no simultaneous high voltages, the lightning stroke recorders no simultaneous direct hits, and no accompanying tripout occurred.

Since the surges recorded by the cathode ray oscillograph were so low in voltage, it is not surprising to find that all of the surges were not of the steepest front. Theoretical study and calculations indicate that for a 1000-ft. cloud and a cloud discharge rate of ten microseconds, less than 15 per cent of the theoretical maximum voltage should appear on a line by induction.[7] Thus, whereas a voltage of 4000 kv. could appear on a highly insulated line 40 ft. above earth, if an overhead

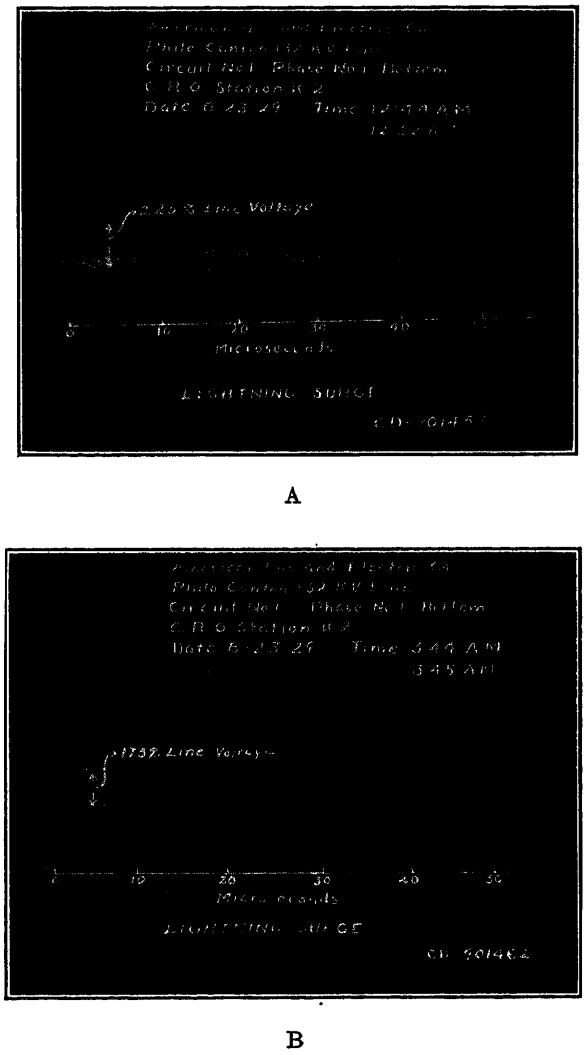

FIG. 10—CATHODE RAY OSCILLOGRAMS OF NATURAL LIGHTNING SURGES

cloud were to discharge instantly, less than 15 per cent of this voltage or approximately 500 kv. is the maximum induced voltage which can appear with a 1000-ft. overhead cloud discharging in ten microseconds. The highest voltage measured was well below this value indicating the cloud at a distance rather than directly overhead. Since the fronts of the seven slowest surges are not steep, high voltages are not to be expected or, conversely, since the voltages are low it is not surprising that all of the surges were not of the steepest front. The three steepest surges were apparently the result of rapid discharges of a distant cloud since these, also, were of low magnitude.

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[7] "Lightning," F. W. Peek, Jr., *General Electric Review*, Volume 32, No. 11, November 1929, p. 603, (see Figs. 30 and 31).

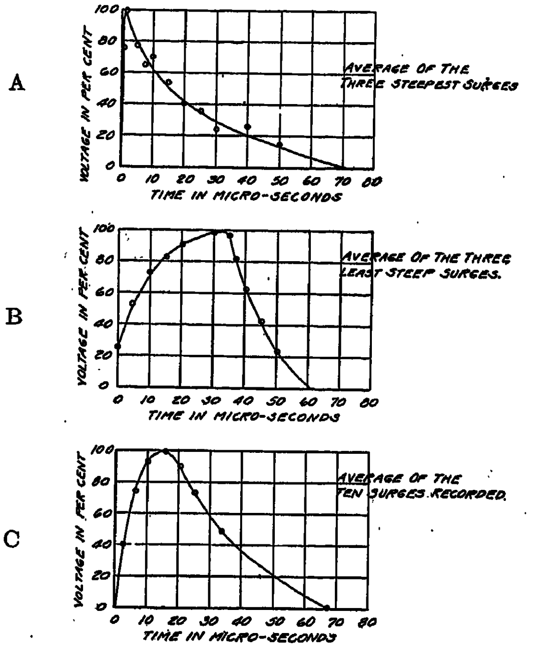

Fig. 11—Wave Shapes of Natural Lightning

Recorded during 1929 by the cathode ray oscillograph at Newcomerstown, Ohio

The relative order of magnitude of induced voltage surges may be given by the following tabulation.

	Rapid cloud discharge	Slow cloud discharge
Near cloud.......	High induced voltage	Low induced voltage
Distant cloud.....	Low induced voltage	Very low or zero

According to the above classification, the ten surges recorded by the cathode ray oscillograph during 1929 may be grouped as follows:

	Rapid cloud discharge	Slow cloud discharge
Near cloud.......	None	Seven
Distant cloud.....	Three	None[8]

From the above discussion it would appear probable that a rapid cloud discharge producing a wave of the type represented by the average of the three steepest waves would be the most important on high-voltage lines. See Fig. 11A.

Switching Surges. Several oscillograms of switching operations were obtained during the regular switching operations of the line (see Table VI). Four of these were obtained when the line was being deenergized and one when the line was being energized. The highest voltage recorded, 4.5 times normal line to neutral maximum voltage, was obtained when the line was being deenergized. The actual length of the line was 73.3 miles. The switching was done on the high-voltage side at either Philo or Canton, with the cathode ray oscillograph at Newcomerstown near the midpoint of the line. Fig. 12 gives an oscillogram of a typical switching surge in rectangular coordinates.

VII. LIGHTNING STROKE RECORDER DATA AND ANALYSIS

Two direct strokes to the line were indicated by the lightning stroke recorders. The first took place to tower 185 on July 25. Fig. 13 gives the record obtained from which it was determined that the cloud was negative and that a current of approximately 175,000 amperes had flowed down the tower during the stroke. This was the first record ever obtained of a stroke which was actually known to have struck a transmission line.

Fig. 14 gives the correlated data obtained in connection with the discharge. A positive voltage surge was released on the line. The surge voltage recorded, at the tower struck, measured a maximum voltage of 8.4 times normal positive, but it is of course possible that this value was higher by an amount at least equal to the limit of accuracy of the instrument. The surge attenuated rapidly on each side of the tower and after five miles was not further indicated by the surge voltage recorders. No tripout occurred.

The second measured direct stroke occurred at tower 159 during the storm of August 23. Fig. 15 gives the record obtained. A positive voltage of 6.5 times normal was recorded by the surge voltage recorder at the same tower while a positive voltage of 3.5 times normal was recorded at tower 156 one mile away. No other voltage measurements were obtained. The line tripped but no damage was done and voltage could be immediately restored to the line. The indication is that in spite of a direct stroke tripout, the arc jumped clear of the string from ring to horn, without damage to the porcelain insulators.

In this case, unfortunately, the film pack of the recorder had been improperly assembled so that only one record, rather than two, was obtained. A negative cloud was, however, again indicated. A lightning current in the tower of approximately 100,000 amperes was estimated.

The first of the three direct strokes which apparently occurred to the line during 1929 occurred on July 5 before the lightning stroke recorders were installed on

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[8] Possibly some of the above seven surges should be included here, depending upon what constitutes a "near" cloud.

TABLE VI

No.	Date	Switching	Max. kv.	Nature	Approx. frequency (cycles)	Indicated length in miles	Time to maximum	Duration of film
669	9/14	Off	327	Oscillatory	..	..	10,000 ms.	12,000 ms.
440	7/28	On	136	"	500	108	500	6,000
441	"	Off	490	"	500	108	500	6,000
338	6/29	Off	68	"	200	43	300	5,000
439	7/28	Off	136	"	..	..	..	6,000

the line. The surge voltage recorders indicated a negative surge voltage of 14.3 times normal voltage. A line tripout occurred without damage to the insulator strings.

VIII. Attenuation Data and Analysis

Two lightning surges of relatively high magnitude were recorded on the instruments located on the 16 mile attenuation set-up. One surge had a maximum value of 8.4 times normal positive, and the other a maximum of 14.3 times normal negative. The positive surge was a pure positive as indicated by all instruments; and the negative surge was recorded negative at all instruments except at the one instrument where the surge apparently originated, there being recorded here also a crest value of 7.1 times normal positive. The attenuation of the crest value of these surges is shown in Fig. 16.

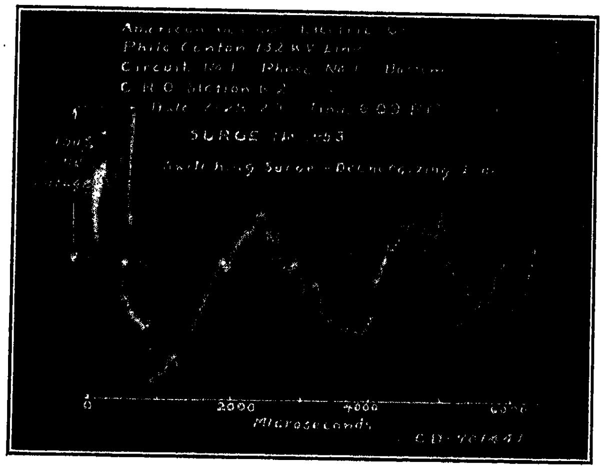
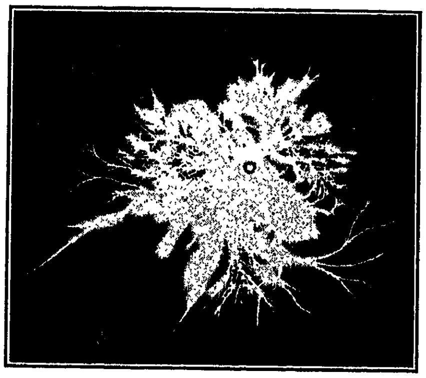


Fig. 12—Typical Switching Surge

Recorded by the cathode ray oscillograph upon deenergizing of the Philo-Canton line

The attenuation data on these two surges, together with three similar surges recorded on this line in 1928, have been grouped and plotted in Figs. 23 and 24. These will be discussed later. In Fig. 17 the two surges shown in Fig. 16 have been replotted on a single group of coordinates; that is, the data obtained on both sides of the point of apparent origin of the surge have been grouped together; and it will be seen that there is a very marked difference in the rate of attenuation of the positive and negative surges, the positive surge attenuating considerably faster than the negative surge; thus, whereas the positive surge at 8.3 times normal potential attenuates to 50 per cent of its value in 1.3 miles, it takes the negative surge 2.75 miles to reach a corresponding attenuation.

During the season surges from a 1,000,000-volt field lightning generator were placed on the three phases of the No. 1 circuit separately and in groups, and the magnitude of the crest voltage measured from Lichtenberg figures obtained from the surge recorders. Results of these tests are shown in Figs. 18, 19, and 20, where Fig. 18 represents the decay of the crest value when the surge was applied to one line wire only, Fig. 19 representing the attenuation when the surge was applied to two line wires in parallel, and Fig. 20, representing the attenuation when the surge was applied to the three line wires in parallel. The higher rate of voltage attenuation with the surge energy supplied to the three line wires instead of one is readily noticeable from the shape of the curves.



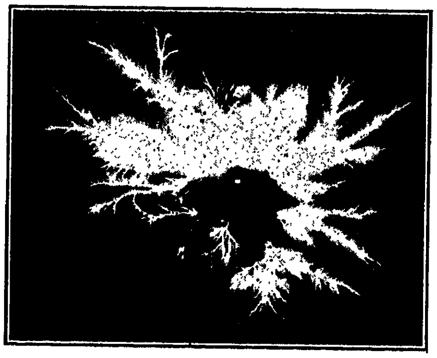


Fig. 13—The First Record Ever Obtained of the Current and Polarity of a Natural Lightning Stroke Hitting a Transmission Line Tower

The data from which the curves in Figs. 18, 19, and 20 have been plotted are actually indicated, and it will be apparent that the curves which have been drawn to represent that data are rather broad averages. It is believed, however, that there is sufficient background available for giving the curves the general shape that they have been given.

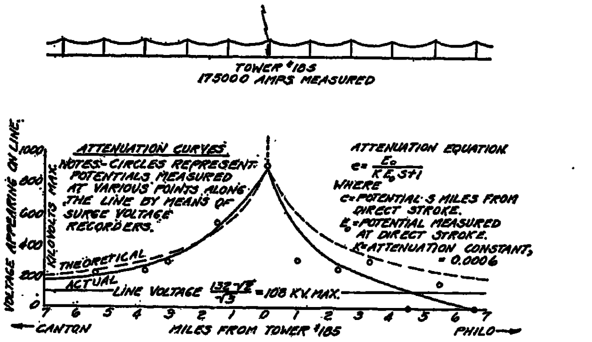


Fig. 14—Attenuation Curve of the Surge Resulting from a Direct Stroke of Natural Lightning as Measured by the Surge Voltage Recorder

Fig. 15—Record of the Direct Stroke of Natural Lightning which Struck Tower No. 159 of the Philo-Canton Line on August 25

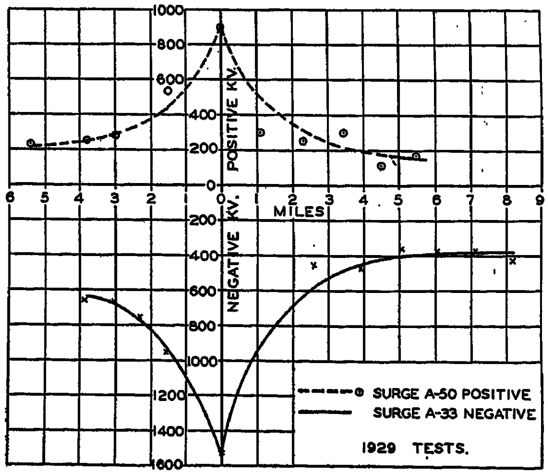


Fig. 16—Attenuation of Natural Lightning

Philo-Canton 132-kv. line No. 1—Bottom conductor—Phase 1

Referring again to Figs. 18, 19, and 20, it will be seen that with artificial lightning there is again a distinct difference in the rate of attenuation between positive and negative surges. For example, with a positive surge the crest voltage drops from 500 kv. to 200 kv. in nine miles when the surge from the lightning generator is applied to one wire only; from 500 kv. to 200 kv. in six miles, when it is applied to two line wires; and from 500 kv. to 200 kv. in five miles when it is applied to all three line wires. When the surge is negative, the same relative difference on the rate of attenuation with one, two, and three wires appears, the drop from 550 to 250 kv. occurring in 8½, 4¾, and 4½ miles respectively.

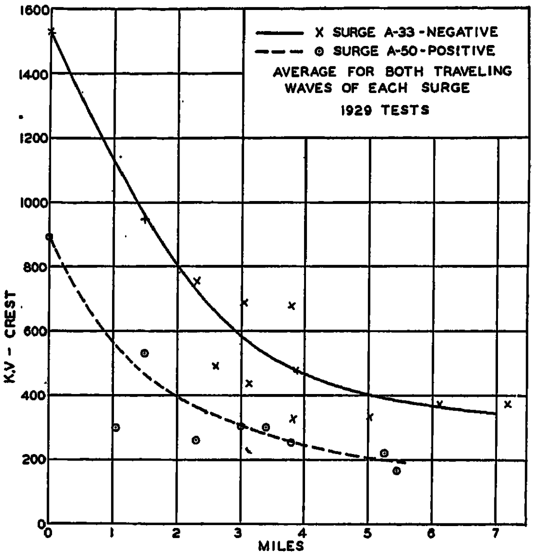


Fig. 17—Attenuation of Natural Lightning

Philo-Canton 132-kv. line No. 1—Phase 1

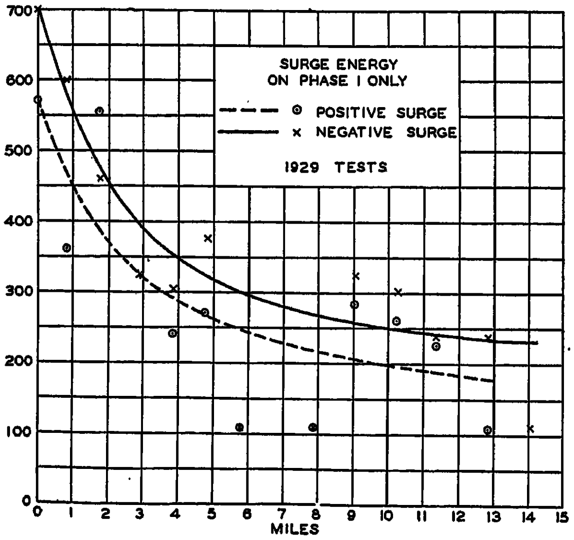


Fig. 18—Attenuation of Artificial Lightning

Philo-Canton 132-kv. line No. 1—Phase 1

The cathode ray oscillograph permitted the more precise study of attenuation and also the reflections taking place at the open and closed end of a line. Fig. 21 shows the repeated reflections from the open end of a

43-microsecond (7.96 mile) section of the line. Twice this time or 86 microseconds is required for the first reflection to return after traveling twice the length of the 7.96-mile section. A negative impulse of 427 kv. was applied in this case.

Fig. 22 shows the repeated reflections of a 427-kv. negative impulse from the closed end of the same length section of the line. Each reflection at the closed end is reversed in polarity, the second being of the same polarity as the original impulse, but lower due to attenuation along the line.

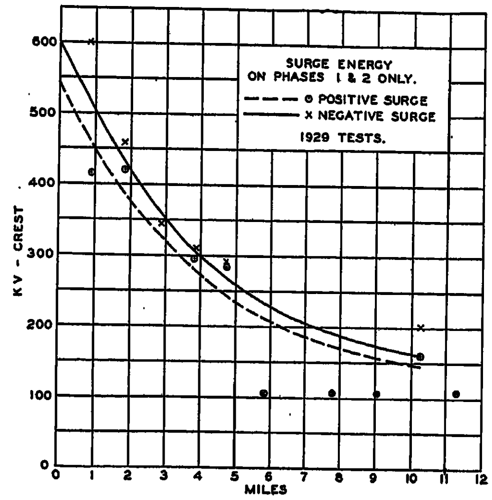


FIG. 19—ATTENUATION OF ARTIFICIAL LIGHTNING

Philo-Canton 132-kv. Line No. 1—Phase 1

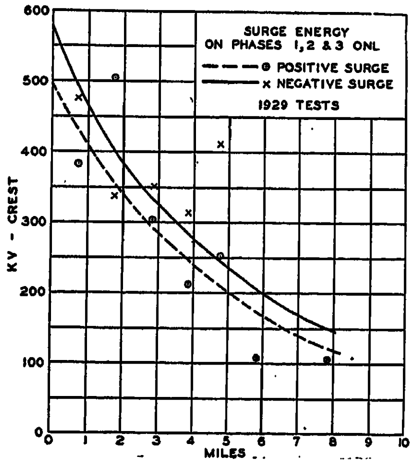


FIG. 20—ATTENUATION OF ARTIFICIAL LIGHTNING

Philo-Canton 132-kv. Line No. 1—Phase 1

Referring to Figs. 23 and 24 previously mentioned, Fig. 23 shows plotted the two surges previously indicated in Fig. 9, these being the two surges on which attenuation data was obtained in 1929, and also three other surges obtained during 1928. It will be noticed that four of the surges are positive and only one was negative. It will be noticed first that only the two highest surges resulted in line tripouts. Further, it will be noted that one of the surges, although it reached a positive value of 1450 kv., did not cause line tripout. Again it will be very definitely noticed that the positive surges show a higher rate of attenuation than the negative surges. This is shown particularly well in Fig. 24, where all the positive surges have been combined and plotted as a single curve and the negative surge shown in Fig. 23 has been replotted. In Fig. 24 also are indicated the curves plotted according to Foust and Menger's formula and using an average value of K. It will be seen that the formulas fit the data obtained quite well. Further, the positive constant K is almost twice that of the negative constant, the value of one being 0.000574 and of the other 0.000307.

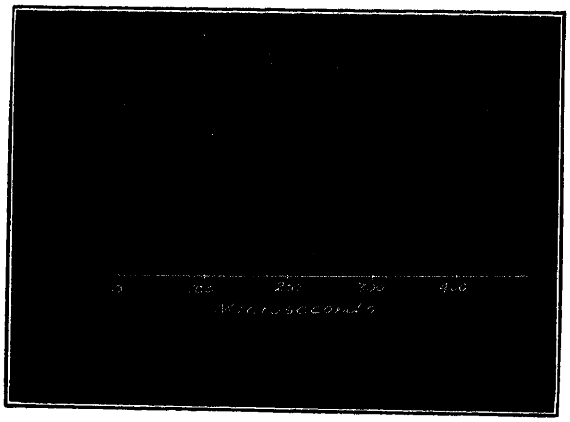


FIG. 21—CATHODE RAY OSCILLOGRAM SHOWING ATTENUATION OF REFLECTIONS FROM THE OPEN END OF A TRANSMISSION LINE

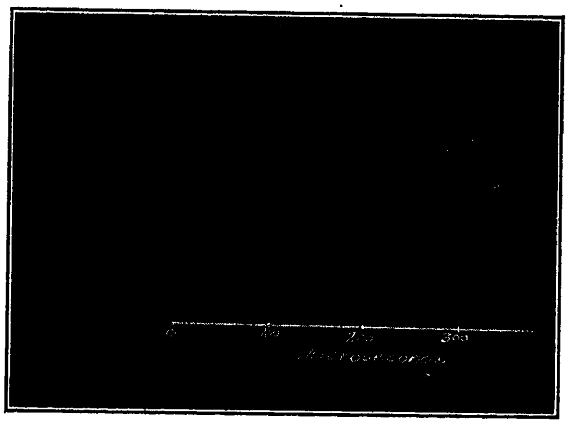


FIG. 22—CATHODE RAY OSCILLOGRAM SHOWING THE ATTENUATION OF THE REFLECTIONS FROM THE CLOSED, OR GROUNDED, END OF A TRANSMISSION LINE

CONCLUSIONS

The field studies carried out during the last year on the Philo-Canton line and in the field laboratory indicate the following:

1. The lightning surges on the system are predominantly positive, indicating that they come from induced surges and are not direct strokes

2. The maximum positive lightning surge recorded was 8.4 times normal; the maximum negative surge recorded was 14.3 times normal; the maximum de-energizing switching surge was 4.9 times normal and the maximum energizing surge was 2.7 times normal.

3. Records were obtained showing lightning arresters·discharging current with correlated voltages, and fairly high voltages existing at lightning arresters with correlated current discharges. No specific data were obtained to indicate the extent of benefit derived from lightning arresters functioning.

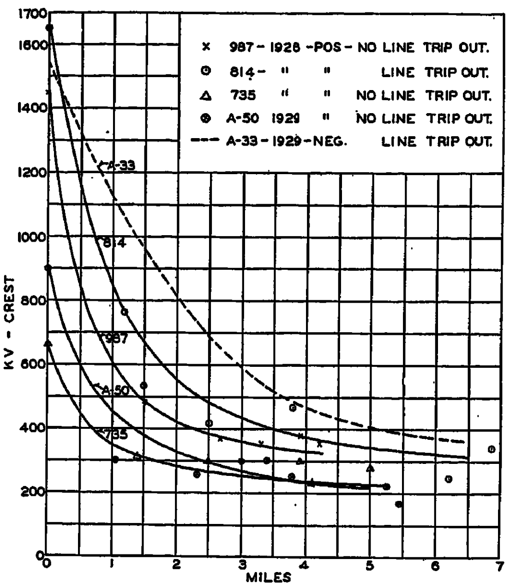


Fig. 23—Attenuation of Natural Lightning
Philo-Canton 132-kv. Line No. 1—Phase No. 1

4. Data were obtained showing the attenuation different for positive and negative surges and the attenuation for both was shown to be rather steep. Constants of 0.000574 and 0.000307 for positive and negative surges respectively in the formula of Foust and Menger were obtained, indicating much more rapid attenuation for the positive surges.

5. Two direct strokes were recorded. Both were of negative polarity (negative cloud). Currents of 175,000 amperes and 100,000 amperes were measured in the tower. Induced voltages were simultaneously recorded on the phase conductors. Only one, however, produced a dynamic tripout of the line. The potential of this induced voltage surge attenuated rapidly and in accordance with the Foust and Menger formula.

6. Ten cathode ray oscillograms of induced natural lightning surges were obtained. All were of low voltage and of positive potential. Attenuation curves applying to these oscillograms could not be obtained since the voltages were too low to be recorded by the nearest surge voltage recorders. Oscillograms of the surges resulting from the two direct strokes could not be obtained since the direct hits occurred at too great a distance from the cathode ray oscillograph station.

7. Of the ten oscillograms of natural lightning obtained three were of steep wave-front (0.5 microsecond or less to 75 per cent of maximum voltage); four were of slow-wave-front (4 to 8 microseconds to 75 per cent); three of very slow front (10 to 13 microseconds). Since a slanting wave-front results from a slow cloud discharge and a slow cloud discharge cannot produce a high induced voltage, it is not surprising that these low voltage surges included seven having relatively slow fronts. The three steep-fronted surges were also of low voltage, in spite of the rapid cloud discharge, being probably the result of the discharge of a distant rather than of a near cloud.

8. *The average of the three steepest waves probably represents a type that is most likely to reach high enough voltages to cause spark-overs on high voltage lines.*

9. Data were obtained showing that a direct stroke can result in line outage; that it is possible for a direct stroke to cause no line outage; and that it is possible to have a direct stroke away from the line result in an outage on the transmission line.

Acknowledgment is due to the executives of the General Electric Company, the Ohio Power Company, and the American Gas and Electric Company who made this investigation possible; to various members of the General Electric Company, the Ohio Power Company, and the American Gas and Electric Company for their cooperation and help in the design, installation, and operation of the apparatus and equipment, and for their help in supervising, correlating, and working up the data; and in particular to Messrs. H. L. Rorden, J. C. Dowell, S. D. Day, and George Lippert, who actually carried out and were in charge of the various tests and field investigations.

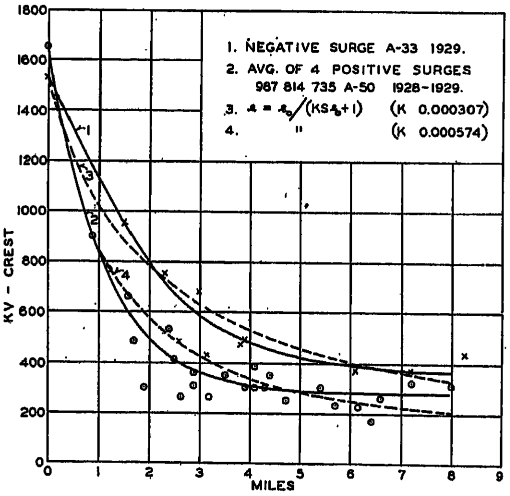


Fig. 24—Attenuation of Natural Lightning
Philo-Canton Line No. 1—Phase No. 1

Discussion

For discussion of this paper see page 928.

Lightning Investigation on Transmission Lines

W. W. LEWIS* and C. M. FOUST*
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The surge voltage investigations for 1926 and 1927 are briefly reviewed, and data given for 1928 and 1929. The cathode ray oscillograph has played a prominent part in the last two years work, and about 115 oscillograms have been obtained. New instruments were introduced in 1929 as follows: Lightning stroke recorders, field intensity recorder, and rate of change of field intensity recorder.

A vast amount of progress has been made in the solution of the lightning problem. Personnel, equipment, and technique are available for the complete solution of the problem, by means of field and laboratory studies carried on simultaneously and supplementing each other.

* * * * *

A *Symposium on Surge Voltage Investigation* covering mainly the work done in 1926 and 1927, was presented at the annual convention of the A. I. E. E. at Denver in June 1928.[1]

Tentative conclusions were drawn as to polarity and magnitude of surges, wave shape, and attenuation, and it was stated that in the coming year special efforts would be made to obtain data on wave shape, attenuation, effect of ground wires, and effect of lightning arresters and choke coils.

Two years' additional data have now been accumulated and we can answer some of the questions which were unanswered at the Denver Convention.

In 1928 work was continued on the Pennsylvania 220-kv. interconnected system, the Consumers Power Co. 140-kv. system, the Ohio Power Co. 132-kv. system, and the New England Power Co. 110-kv. system. The cathode ray oscillograph was introduced and a laboratory installed at Wallenpaupack to obtain oscillograms of the surges produced on transmission lines by natural lightning.

Mr. McEachron also began in 1928 his work on the Turners Falls 66-kv. transmission line, in the course of which he placed artificial lightning surges up to 400 kv. on the transmission line and studied their effects by means of the cathode ray oscillograph.[2]

In 1929 the work was discontinued on the New England Power system, but continued on the Pennsylvania, Consumers, and Ohio systems. A second cathode ray oscillograph was installed at Wallenpaupack and connected to a 300-ft. long antenna, 50 feet above the ground. A cathode ray oscillograph was installed at Newcomerstown on the Ohio system and connected to the 132-kv. line. Mr. McEachron continued his work with artificial lightning on the Turners Falls system and in addition conducted an extensive investigation with a 1000-kv. portable lightning generator on a 70-kv. line of the Consumers Power Co. in Michigan. The artificial lightning investigation is described in a paper at this convention.[3]

In the 1929 investigation new instruments were introduced as follows: Lightning stroke recorders, field intensity recorder, and rate of change of field intensity recorder.

In the present paper the 1928 and 1929 work will be discussed and an effort will be made to summarize the present status of the investigation and to outline the future work.

I. CREST VALUES OF SURGE VOLTAGES DUE TO LIGHTNING AND SWITCHING

Lightning. In Table I are listed the maximum voltages due to lightning on three systems for the year 1926, 1927, 1928, and 1929, as measured by the surge voltage recorder. (The locations of the surge voltage recorders are indicated in diagrams in companion papers[4] and in the Denver papers previously mentioned.)[1] The voltages are given both in kilovolts and in the number of times normal crest line-to-neutral voltage. For comparison also are given the approximate insulator impulse flashover values as measured by the five-microsecond "Pittsfield Wave." This wave has the following general description:

Rises to its crest value in about ¼ microsecond, then decreases to 50 per cent crest value in about five microseconds.[5] The values given are for flashover on the tail of the wave.

In the case of the Pennsylvania Power & Light Company's system the line under investigation is that formerly known as the Wallenpaupack-Siegfried line, (now known as Wallenpaupack Tap and Siegfried-Roseland Line). The length is 65 miles.

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*Both of General Electric Co., Schenectady, N. Y.

[1] *Symposium on Surge Voltage Investigation*, by W. W. Lewis, E. W. Dillard, J. G. Hemstreet and J. R. Eaton, P. Sporn, and N. N. Smeloff, A. I. E. E. TRANS., Vol. 47, Oct. 1928, p. 1111.

[2] *Cathode Ray Oscillograph Study of Artificial Lightning Surges on the Turners Falls Transmission Line*, by K. B. McEachron and V. E. Goodwin, A. I. E. E. TRANS., Vol. 48, 1929.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 27-31, 1930.

[3] *Study of Traveling Waves on Transmission Lines with Artificial Lightning Surges*, by K. B. McEachron, J. G. Hemstreet, and W. J. Rudge, part of symposium on the subject of Lightning Investigation, see p. 885.

[4] *Lightning Investigation on the Ohio Power Co. System*, by Philip Sporn and W. L. Lloyd, see p. 905; *Lightning Investigation on the 220-Kv. System of Pennsylvania Power & Light Co.*, by N. N. Smeloff and A. L. Price, see p. 895.

[5] *Progress in Lightning Research in the Field and in the Laboratory*, F. W. Peek, Jr., A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 436.

The conductors are horizontally arranged. Up to the year 1929 the insulation consisted of 14 disks spaced 5¾ in. with grading rings at the line end only. In the year 1929, two additional disks were added on the outside conductors for a distance of 40 miles from the Wallenpaupack end. The conductors are of 795,000 cir. mils, aluminum, steel reinforced. There are two overhead ground wires of 184,000 cir. mils, aluminum, steel reinforced. The ground wires extend for 20 miles from the Wallenpaupack end and five miles from the Siegfried end. They were installed during the year 1927.

TABLE I

MAXIMUM VOLTAGE CAUSED BY LIGHTNING, AS MEASURED BY SURGE VOLTAGE RECORDER

System	Rated voltage (kv.)	Normal voltage crest (kv. line to neutral)	Approximate insulator flashover impulse (kv.)[1]	Maximum crest voltage caused by lightning (kv.)				Number of times normal voltage			
				1926	1927	1928	1929	1926	1927	1928	1929
Pennsylvania Power and Light Co..............	220	180	1950 2210[2]	2100 +	2100	2700 +	2400 +	11.7	11.7	15.00	13.3
Ohio Power Co..........	132	108	1180 1280[3]	....	1200	1640	1545	....	11.1	15.2	14.3
New England Power Co..	110	90	1025 1150 1280 1400 [4]	240	900	740	....	2.7	10.0	8.2	....

1. Based on 5-microsecond Pittsfield Wave.
2. Part of line changed from 14 to 16 disks (spaced 5¾ in.) in 1929.
3. Bottom conductor had 10 disks (spaced 4¾ in.), middle and top conductor 11 disks, for all years.
4. Up to 1928 all conductors had 8 disks (spaced 5 in.); in 1928 outside conductor had 10 disks and grading rings, next conductor 9 disks and rings, and inside conductor 8 disks and rings.

NOTE: Maximum voltage on Ohio Power system in 1927 recorded on middle conductor, in 1928 and 1929 on bottom conductor.

On the Ohio Power Company's system the lines under investigation are the two circuits vertically arranged on the same towers, extending from Philo to Canton, a distance of 73 miles. The insulation consists of 11 disks on top and middle and 10 disks on bottom conductors, all spaced 4¾ in. The strings are equipped with grading rings at the line end only. The conductors are of 336,400 circular mils, aluminum, steel reinforced. Extending the total length of the line there is one overhead ground wire of 159,000 cir. mils, aluminum, steel reinforced.

On the New England Power Co. system there are two circuits, with all conductors horizontally arranged. The conductors are of 4/0 copper cable (212,000 cir. mils). Previous to 1928 there was one ground wire, and in 1928 a second ground wire was added. The ground wires are of 7/16 in. steel cable.

Prior to 1928 all conductors were insulated with eight disks spaced 5 in., with no grading rings. In 1928 the insulation was increased to 10 disks on the two outside conductors and to 9 on the conductors adjacent to the two outside ones, while 8 disks were retained on the two innermost conductors. Grading rings were added to all strings at both line and ground ends.

It will be noted from Table I that maximum voltages have been recorded which exceed the approximate impulse insulator flashover kv. This may mean that occasionally actual surges have required a much shorter time to reach flashover than with the Pittsfield wave on which the assigned impulse flashovers are based. However, the accuracy of voltage registration by means of Lichtenberg fixtures should be kept in mind in considering these extreme registrations.[6]

In Table II are given the height of line at tower and the average height of conductor, also the kilovolts per foot of height, as measured by surge voltage recorders in 1926-27-28-29.

On the special antennas used in connection with the field intensity recorder a potential gradient of 280 kv. per meter (85.5 kv. per foot) was measured. This is beginning to approach the figure of 100 kv. per foot, which it has been customary to use as the gradient possible in the worst case. It is probable that the gradient on the transmission line is set by the flashover of the line insulators, and that this gradient will increase as the line insulation is increased.

TABLE II

HIGHEST MEASURED VOLTAGE CAUSED BY LIGHTNING IN KILOVOLTS PER FOOT, BASED ON AVERAGE HEIGHT OF CONDUCTOR

	Height of line at tower (feet)	Average height of conductor (feet)	Kv. per foot of height			
			1926	1927	1928	1929
Pennsylvania Power and Light Co..............	66	52	40	40	52	46
Ohio Power Co..........	59*	48*	..	20	34	32
New England Power Co..	49	41	6	22	18	..

*Bottom conductor.

In Figs. 1, 2, and 3 are plotted the surge voltages caused by lightning during the years 1927, 1928, and 1929 respectively. The abscissas represent times normal crest line-to-neutral voltage, and the ordinates represent percentage of the total number exceeding the value indicated by the abscissas.

The curves for the Pennsylvania system show a much larger percentage of high surges than for the Ohio and New England systems. The tendency of the Pennsylvania system to record higher voltages may be due to some property of the line insulation and construction (the Pennsylvania line is more highly insulated than the others and has ground wires over only a portion of the line), it may be caused by the chance of more strokes and strokes of greater severity near the line, or it may be due to the fact that recorders are placed closer together over the entire line than on the other systems.

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[6] *Measurement of Surge Voltages on Transmission Lines due to Lightning*, by E. S. Lee and C. M. Foust, A. I. E. E. TRANS., Vol. 46, 1927, p. 339.

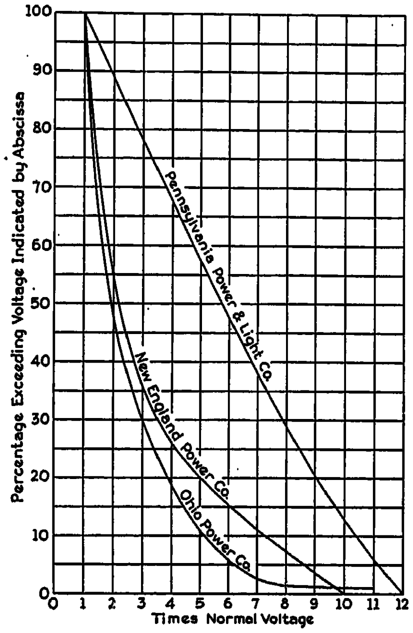

FIG. 1—VALUES OF SURGE VOLTAGES CAUSED BY LIGHTNING, 1927

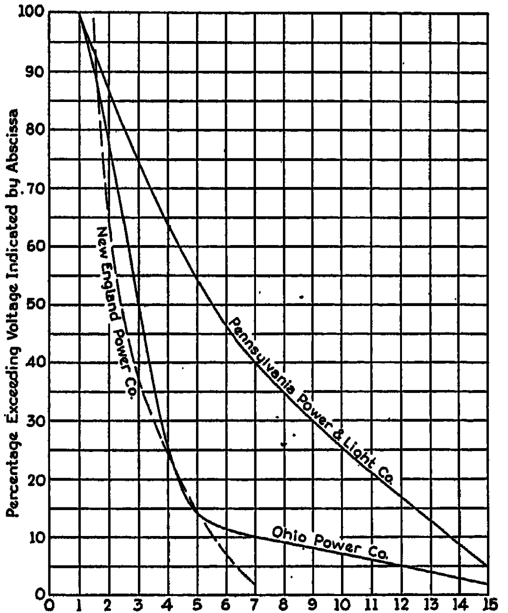

FIG. 2—VALUES OF SURGE VOLTAGES CAUSED BY LIGHTNING, 1928

The year 1929 shows a tendency for the number of surge voltages of high magnitude on the Pennsylvania system to be much greater than in previous years. In fact, in the year 1929 there were 14 surges over 13.3 times normal (2400 kv.) out of a total of 45 surges which exceeded 300 kv. The indication from the measurements and from line operation is that this line is in an unusually severe lightning territory.

Switching. Table III gives the maximum voltage caused by switching, as measured during the years 1926, 1927, 1928, and 1929, also the approximate 60-cycle flashover of the line insulators.

In spite of the high values occasionally reached, the majority of all switching surges are less than 3.5 times normal. It is probable that slightly more than the 60-cycle flashover voltage is required to flash over during a switching surge (estimated impulse ratio 1.2). It is apparent from the data that the 60-cycle flashover of line insulation cannot be neglected, especially if the insulators are liable to be subjected to rain at the time of switching.

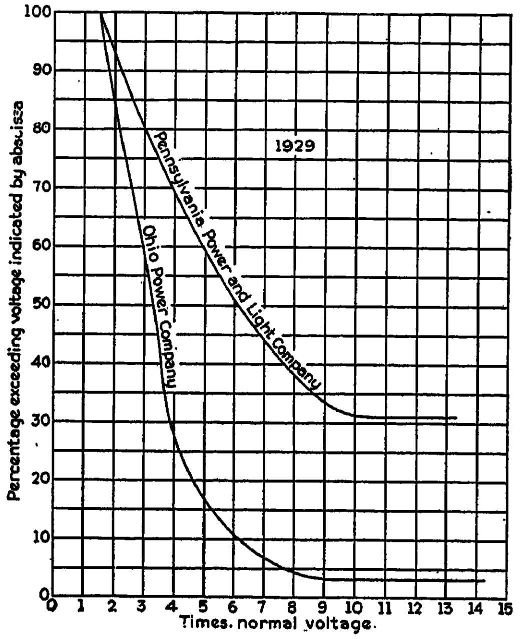

FIG. 3—VALUES OF SURGE VOLTAGES CAUSED BY LIGHTNING, 1929

In general it was noted that deenergizing was somewhat more severe than energizing. For example, on the Ohio system in 1929, there were measured 20 surges caused by deenergizing with a maximum value of 4.9 times normal, and only three surges caused by energizing with a maximum of 2.8 times normal. Switching load on and off gave a very mild overvoltage disturbance (of the magnitude of 1.2 times normal on the Ohio Power system).

II. TRIPOUTS CAUSED BY LIGHTNING, AND FLASHED INSULATOR STRINGS

The number of surges caused by lightning, as measured by the surge voltage recorder, is given in Table IV for the years 1926, 1927, 1928, and 1929 for the three systems under discussion, also the number of line

TABLE III

MAXIMUM VOLTAGE CAUSED BY SWITCHING, AS MEASURED BY SURGE VOLTAGE RECORDER

System	Rated voltage (kv.)	Normal voltage crest (kv. line to neutral)	Approximate insulator flashover 60 cycles (kv. crest)	Maximum crest voltage due to switching				Number of times normal voltage			
				1926	1927	1928	1929	1926	1927	1928	1929
Pennsylvania Power and Light Co..............	220	180	1025 1150[1]	630	720	845	450	3.5	4.0	4.7	2.5
Ohio Power Co...........	132	108	650 700[2]	....	560	595	530	....	5.2	5.5	4.9
New England Power Co..	110	90	565 510 } [3] 565 615	207	360	370	....	2.3	4.0	4.1	....

1. Part of the line changed from 14 to 16 disks (spaced 5¾ in.) in 1929.
2. Bottom conductor had 10 disks (spaced 4¾ in.); middle and top conductors, 11 disks, for all years.
3. Up to 1928 all conductors had 8 disks (spaced 5 in.); in 1928 outside conductor had 10 disks and grading rings, next conductor 9 disks and rings, and inside conductor 8 disks and rings.

tripouts. The data on the number of surges are given for the portion of the year indicated in the last column of the table, while the data for the tripouts cover the whole year. It is evident that there are many more surges recorded than there are tripouts.

The data indicate that for the four years there was an average of about 29 tripouts per one hundred miles per year on the Pennsylvania system, 14 on the Ohio system, and 24 on the New England system.

Data compiled on these systems also show that there are as many as 1 to 5 insulator strings or assemblies flashed over for each tripout. In other words, each stroke which causes a flashover usually affects more than one string or assembly, either on the same tower or on adjacent towers.

TABLE IV

NUMBER OF SURGES AND NUMBER OF TRIPOUTS CAUSED BY LIGHTNING

System	Rated voltage kv.	Line length miles	Number of surges caused by lightning measured by surge voltage recorders				Number of line tripouts caused by lightning				Period during which surge voltages were measured
			1926	1927	1928	1929	1926	1927	1928	1929	
Pennsylvania Power and Light Co......................	220	65	33	48	28	45	16	7*	14	38	1926 July 20–Oct. 4 1927 Mar. 12–Nov. 2 1928 April 15–Oct. 1 1929 April 17–Oct. 1
Ohio Power Co.............	132	73	..	166	46	31	15	4	12	11	1927 May 11–Oct. 16 1928 April 29–Oct. 20 1929 May 6–Oct. 1
New England Pr. Co.........	110	74	14	104	47	..	11	43	5	13	1926 July 28–Sept. 12 1927 June 6–Oct. 10 1928 June 23–Oct. 8

*Line out of service two months during lightning season, installing ground wires.

NOTE: Tripouts are given for the whole year while surges were recorded for only a portion of the year as noted in last column. Ohio Power and New England tripouts are for two circuits.

Statistics also indicate that where there are two circuits on a tower, from 5 to 25 per cent of the strokes affect insulator strings on both circuits.

In case of circuits with conductors horizontally arranged, or two circuits with some horizontal distance between them, statistics seem to indicate that the conductor or circuit on the side from which the storms generally approach suffer the greatest number of insulators damaged. For example, on the Pennsylvania Power & Light Co. 220-kv. line from Wallenpaupack to Siegfried, during the four years 1926, 1927, 1928, and 1929, there were 115 insulator strings flashed over on the west or storm conductor, 51 on the middle, and 97 on the east conductor.

On the Hartford Electric Light Co. system, a 66-kv. line 33 miles long with two circuits vertically arranged on the same towers ran generally northwest. The southwest circuit is on the windward side and the side towards approaching thunder storms. Three years operation of the line showed that trouble from insulator flashover and failure was more than twice as great on the southwest circuit as on the other or lee circuit.

III. WAVE FRONT AND WAVE SHAPE

An analysis of 150 Lichtenberg figures obtained by surge voltage recorder in 1927 on three different systems gave the following rough summary as to the duration of wave front of the surges producing the figures: One surge only less than 1 microsecond, about 20 per cent of all surges between 1 and 10 microseconds, about 20 per cent between 10 and 100, and about 60 per cent over 100 microseconds.

A cathode ray oscillograph was installed in the lightning laboratory on the Pennsylvania System at Wallenpaupack (tower 1-3) during the latter part of 1928 season, and one oscillogram was obtained of a lightning surge.[7] The surge was positive in polarity, rose to maximum in 8 microseconds, reduced to 50 per cent of that value in 17 microseconds and to zero in 35 microseconds. The crest voltage of the main wave was approximately 600 kv. At a point 17 miles from the laboratory a value of 2100 kv. was measured by surge voltage recorder.

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[7] "Transmission Line Insulation and Field Tests Pertaining to Lightning," W. W. Lewis, *General Electric Review*, July 1929.

In 1929 the cathode ray oscillograph installation (Fig. 4) was continued at Wallenpaupack and in the course of the season 95 surges were recorded. Of these, there were 50 surges under 100 kv., 30 between 100 and 300 kv., and 15 over 300 kv. at the laboratory.

Table V summarizes the data for the oscillograms taken up to August 11, 1929, with voltage magnitudes between 100 and 300 kv. These are typical of all surges recorded in this voltage classification.

The oscillograms obtained indicate that the surges have various wave shape characteristics. For this reason, in Table V under the heading, "Time in Microseconds," four columns are given: First, the time consumed in the initial voltage rise. In several cases this is given between zero and a definite time value, the portion of the wave previous to this time not being recorded. Second, the time required to reach the maximum voltage recorded. Third, the time between the beginning of the surge and half maximum voltage value on the tail of the wave, and fourth the time duration of the first loop of the wave.

Fig. 4—Interior View of Wallenpaupack Laboratory

Showing two General Electric portable cathode ray oscillographs

In Table VI are summarized the data for the 15 oscillograms with voltage magnitude exceeding 300 kv. It will be noted that the voltage magnitudes in this classification range from 330 to 1260 kv. The wave front ranges from 1.6 to 36 microseconds. Two of the oscillograms obtained could not be analyzed for wave front, because of improper adjustment in the oscillograph arrangement. The total duration varied from 4 to 75 microseconds. Approximately 60 per cent were positive and 40 per cent negative for the first loop. Within the time range of the oscillograms obtained, two-thirds of the waves were oscillatory and one-third unidirectional.

In considering the magnitudes of these waves, and their front and duration, it should be remembered that all of the waves were measured at the terminal of the transmission line, and the voltage values recorded were influenced by reflection. In most cases the waves originated on the line many miles from the terminal. The waves therefore have been subjected to considerable attenuation in magnitude and change in form. Taking this into account and the extremely high magnitudes measured on the line by surge voltage recorders, it is reasonable to assume that many of the waves at their origin have fronts steeper than those measured by the cathode ray oscillograph.

TABLE V

PENNSYLVANIA POWER & LIGHT CO. CATHODE RAY OSCILLOGRAMS 100 TO 300 KV., YEAR 1929, UP TO AUGUST 11

Oscillogram number	Kv. max.	Polarity of first loop	Nature of wave	Time in microseconds to reach: At least 75% of max. voltage recorded	Max. voltage recorded	50% max. on tail of wave	Zero of first loop
901178	245	Pos.	Uni.	0–1.2	0–1.2	22	33 +
179	160	Pos.	Osc.	0–1	5	13	24
183	180	Pos.	Osc.	5	5	23	32
187	260	Pos.	Uni.	0–1	5	26	45 +
189a	180	Pos.	Uni.	0–1.5	5	20	45 +
189b	140	Neg.	Uni.	0–1.5	10–28	45 +	45 +
190	280	Pos.	Osc.	4	4	11	20
191b	250	Pos.	Osc.	0–1.3	3	5.5	9.5
197	180	Pos.	Osc.	0–1	6	27	35
198	150	Neg.	Uni.	0–1.5	4	45 +	45 +
199	180	Pos.	Osc.	2.5	4.5	18	26
200	260	Pos.	Osc.	5	5	27	36
201a	130	Pos.	Osc.	3	3	11	33
203	260	Pos.	Osc.	2	5	25	35
204	170	Pos.	Uni.	0–1	20	43 +	43 +
214	135	Neg.	Osc.	0–2	0–2	6	10

Osc. = Oscillatory.
Uni. = Unidirectional.

TABLE VI

PENNSYLVANIA POWER AND LIGHT CO. CATHODE RAY OSCILLOGRAMS OVER 300 KV., YEAR 1929

Oscillogram number	Kv. max.	Polarity of first loop	Nature of wave	Time in microseconds to reach: Max. voltage recorded	50% max. on tail of wave	Zero of first loop
901180	1260	Neg.	Osc.	3.3	3.7	4 Flashover
185	360	Pos.	Uni.	4	45 +	45 +
186	330	Pos.	Osc.	1.6	16	20
188	530	Pos.	Osc.	7–8	18	24
191	600	Neg.	Uni.	4	14.5	45 +
193	740	Pos.	Osc.	7	16	22
194	810	Neg.	Osc.	36	?	37 Flashover
201	740	Neg.	Uni.	6.5	43 +	43 +
220	330	Neg.	Uni.	?	12	75
231	330	Pos.	Osc.	?	18	25
249	390	Pos.	Osc.	6	14	18
251	390	Pos.	Osc.	6	35	55
272	600	Neg.	Uni.	< 7.5	69 +	69 +
275	500	Pos.	Osc.	< 8	32	45
285	630	Pos.	Osc.	11	20	29

Osc. = Oscillatory.
Uni. = Unidirectional.

A cathode ray oscillograph was installed in 1929 on the Ohio Power Company system at Newcomerstown. Ten lightning surges were recorded, of which only three were equal to or greater than normal (108 kv.). All the recorded surges were positive and all unidirectional. The wave fronts varied from 4 to 30 microseconds and the total duration from 45 to 60 + microseconds.

In the Consumers Power Co. investigation for 1929[8] seven cathode ray oscillograms of lightning surges 200 kv. and above were obtained. The wave front varied from 0.5 to 7 microseconds. The total duration of the first loop varied from 0.7 to 40 + microseconds. Four of the surges were of negative polarity and three positive. Five of the waves were unidirectional and two oscillatory.

IV. ATTENUATION

In 1927 plots were made of two surges which appeared at more than one station on the Pennsylvania Power & Light Co. system and six surges on the New England Power Co. system, with distance from an assumed origin as abscissas and voltage as ordinates.[9] An average curve was drawn and equations determined for the variation of voltage with distance and the rate of change of voltage with distance, or attenuation. These equations are as follows:

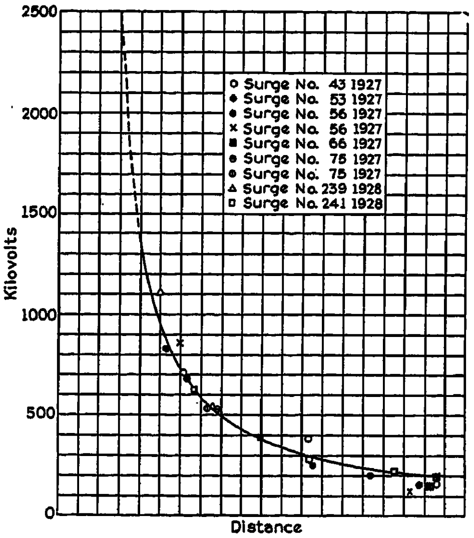

FIG. 5—CURVE SHOWING VARIATION OF VOLTAGE WITH DISTANCE FOR VARIOUS SURGES

Occurring on the New England Power Co. system in 1927 and 1928. Factor k from curve = 0.00016

$$e = \frac{e_o}{k\, s\, e_o + 1} \qquad (1)$$

$$A = -\, k\, e^2 \qquad (2)$$

in which

e_o = initial surge voltage at the point where the surge originated.

k = a proportionality factor which is found empirically.

s = distance in miles from the origin of the surge.

e = voltage at any distance s.

A = the attenuation in kilovolts per mile.

The factor k for the surges investigated was found to be 0.00016.

The curve referred to has been replotted with the two surges from the Pennsylvania system omitted, thus confining the data to the 6 surges found on the New England Power system in 1927, together with 2 additional surges obtained on this system in 1928 (see Fig. 5).

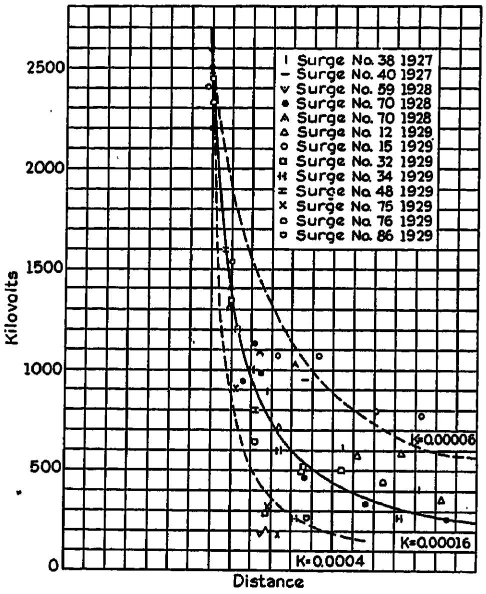

FIG. 6—CURVE SHOWING VARIATION OF VOLTAGE WITH DISTANCE FOR VARIOUS SURGES

Occurring on the Pennsylvania Power and Light Co. system in 1927, 1928, and 1929

Surges on the Pennsylvania system for 1927, 1928, and 1929 have been plotted in Fig. 6 and surges obtained on the Ohio Power system for 1928 and 1929 have been plotted in Fig. 7.

These data all tend to confirm the validity of Equations (1) and (2) derived in 1927.[9] There is, however, considerable difference in the factor k for the various systems. This factor from the New England surges is 0.00016. On the Pennsylvania system there is a considerable range in the points of the various surges, but a fair average curve through these points again gives a factor k of 0.00016. On the Ohio Power Co. system an average curve drawn through the points of the surges for 1928 and 1929 has a factor k of 0.00057. The last named system thus shows a much faster attenuation than the other two.

A few tests were made on the Ohio System with surges from a 1000-kv. lightning generator. The potentials at various points were measured by means of surge voltage recorders. The results are plotted on

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[8] *Lightning Voltages on Transmission Lines*, by R. H. George and J. R. Eaton, part of symposium at this convention on the subject of Lightning Investigation. See p. 877.

[9] *Surge Voltage Investigation on Transmission Lines*, by W. W. Lewis, A. I. E. E. TRANS., Vol. 47, No. 4, 1928.

Fig. 8, the full curve representing an average of the various test points. The dotted curve is taken from Fig. 7, and represents the average attenuation measured with natural lightning surges.

Unfortunately, the surges placed on the transmission line from the lightning generator were not of sufficient amplitude to permit the checking of attenuation over the voltage range of most interest in connection with natural lightning surges. However, in the limited range of these tests, the attenuation of the artificial surges appears to be of the same order as that of the natural surges.

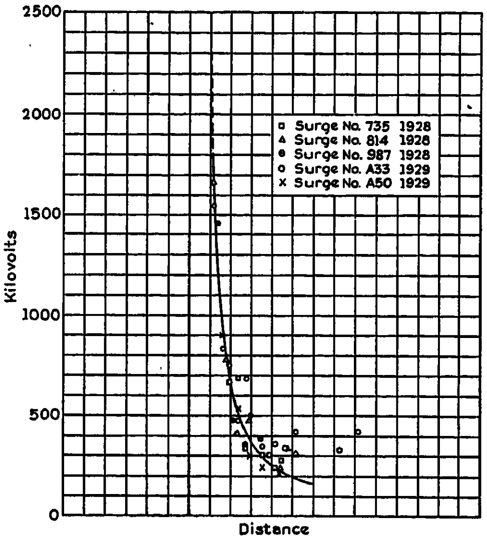

FIG. 7—CURVE SHOWING VARIATION OF VOLTAGE WITH DISTANCE FOR VARIOUS SURGES

Occurring on the Ohio Power Co. system in 1928 and 1929. Factor k from curve = 0.00057

Mr. McEachron in his work in Michigan[3] found factors k of the order of 0.00039 to 0.00049 when an artificial surge was placed on three conductors in multiple. With natural surges it is probable that three conductors in multiple are also involved, so that attenuation factors should be compared on this basis. Mr. McEachron also found it necessary to introduce an exponent to modify the distance s in the denominator of formula (1). This exponent, however, does not differ greatly from unity, and for practical purposes it can probably be neglected.

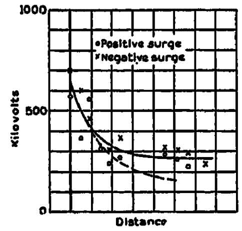

FIG. 8—CURVE SHOWING VARIATION OF VOLTAGE WITH DISTANCE FOR ARTIFICIAL SURGES PRODUCED BY LIGHTNING GENERATOR

Ohio Power Co. system, 1929. Dotted curve is from Fig. 6

It was found in the Michigan tests that k varied with steepness of wave front, was different for positive and negative surges, and was different for a line with overhead ground wires and for a line without overhead ground wires.

A factor k of 0.00069 was found on the Turners Falls 66-kv. line when artificial surges were placed on a single conductor.[3]

While the attenuation factors from the various systems indicate that the wave attenuates faster with smaller conductors, it appears that other line and wave characteristics, such as those mentioned in connection with the Michigan tests, have considerable influence in affecting this factor k.

V. EFFECT OF OVERHEAD GROUND WIRES IN REDUCING SURGE VOLTAGES

Ohio Power Company System. In 1927 approximately 50 surges were measured simultaneously on top, middle, and bottom conductors of the vertically arranged circuits; in 1928 five surges were recorded, and in 1929 one surge. This line is a double circuit line with one overhead ground wire above and midway between the two circuits.

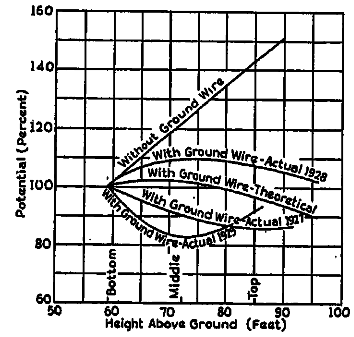

FIG. 9—CURVES ILLUSTRATING GROUND WIRE PROTECTION

Ohio Power Co. system, 1927, 1928, and 1929

In Table VII is given the theoretical potential of the three conductors without ground wire, expressed in per cent, on the assumption that the bottom conductor is 100 per cent and that the other conductors have potentials proportionate to their heights above ground. Then there is given the theoretical potential with ground wire, still assuming that the bottom conductor is 100 per cent and using the protective ratios 0.42, 0.52, and 0.62 for top, middle, and bottom conductors, respectively.[10] Finally, is given the average of the surge voltage recorder readings expressed in per cent, with the bottom conductor as 100 per cent.

In case of the top conductor two figures are given under the column of measured potential; the first is a comparison of the surges on top and middle conductors and the second of the surges on top and bottom conductors. The results are illustrated graphically in Fig. 9.

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[10] *Relation between Transmission Line Insulation and Transformer Insulation*, by W. W. Lewis, A. I. E. E. TRANS., Vol. 47, No. 4, 1928, p. 992.

TABLE VII
EFFECT OF OVERHEAD GROUND WIRE, OHIO POWER COMPANY

Conductor	Approximate height at tower (feet)	Theoretical potential without ground wire (per cent)	Theoretical potential with ground wire* (per cent)	Measured potential with ground wire average of readings (per cent)			
				1927	1928	1929	Weighted average
Top	85	143	97	83–89†	99–116†	92	88
Middle	72	121	102	91	110	83	93
Bottom	59	100	100	100	100	100	100

*Calculated by means of tables in Footnote reference (10).
†First figure is comparison of top to middle, and second figure comparison top to bottom conductor.
1927 figures average of 51 to 54 surges.
1928 figures average of 5 surges.
1929 figures are one surge only.

TABLE VIII
EFFECT OF OVERHEAD GROUND WIRE, NEW ENGLAND POWER COMPANY
Surge Voltages Having Their Highest Voltage at Barre

	1927						1928					
			G						*G*		*G*	
Conductor position	1	2	3	4	5	6	1	2	3	4	5	6
Average of surge voltages (number of times normal)	4.8	3.1	4.2	3.3	4.4	6.4	4.2	6.4	2.7	2.1	3.4	4.3
Surge voltages in per cent (assuming conductor 6 as 100 %)	75	49	66	52	69	100	98	146	63	49	79	100
Theoretical protective ratio* in per cent	74	64	64	74	89	100	100	84	72	72	84	100
Insulator flashovers	22	16	21	17	17	23	5	4	4	1	4	6
Lightning storms	22						20					
Insulator flashovers	116						24					
Tripouts	43						5					

*From Footnote Reference 10.
G = Ground wire.

The last column of Table VII gives the weighted average for the three years. On account of the large number of surges recorded in 1927 compared with two later years, the weighted average figures give a curve practically the same as the 1927 curve.

NEW ENGLAND POWER COMPANY'S SYSTEM

This transmission line consists of two circuits horizontally arranged. The conductors may be designated 1, 2, 3, 4, 5, 6. In 1927 there was one ground wire above and between conductors 2 and 3. Instruments were installed on all six conductors at two places, Griswoldville and Barre. The readings on these instruments did not necessarily indicate the potential due to the conductor position at that point, because the conductors were frequently transposed. The fairest attempt to evaluate the effect of the ground wire was to select those surges which had their highest values at one of the stations, and therefore, presumably originated near that station. For this purpose five surges which had their highest values at Barre were studied. The average values of the surges expressed in times normal are given in Table VIII.

In 1928 a second ground wire was placed above and between conductors 4 and 5. The average values of two surges having their highest values at Barre are given in Table VIII.

Fig. 10 illustrates graphically, the effect of overhead ground wires for the years 1927 and 1928.

The number of insulator flashovers, storms, and tripouts for 1927 and 1928 are also given in Table VIII.

In 1929 there were 13 tripouts on this line, and a total of 38 flashovers, divided among conductors 1 to 6 respectively as follows: 7, 7, 7, 2, 6, 9.

On Fig. 11 are shown the number of insulator string flashovers for each conductor, for the years 1924 to 1929.

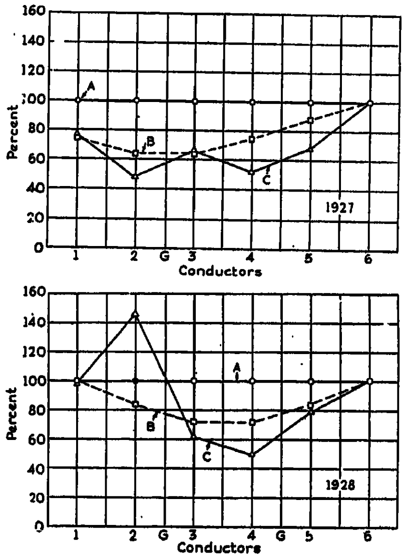

FIG. 10—CURVES ILLUSTRATING GROUND WIRE PROTECTION

New England Power Co. system, 1927 and 1928
A. Without ground wires
B. With ground wires (estimated)
C. With ground wires (measured by surge voltage recorder)

In this connection it must be remembered that in addition to adding the second ground wire in 1928, the line insulation was also considerably improved, being increased from 8 disks on all conductors to 10, 9, 8, 8, 9

10 disks. Grading rings were also added at both line and ground ends of all strings.

General. A similar improvement was shown on the Ohio Power Co. system previously discussed. In 1925 there were 88 tripouts. In 1926 a ground wire was added and grading rings at the line ends of all strings. The tripouts, as shown by Table IV, have not exceeded 15 per year since that time. There is no doubt that the ground wire should be credited with some of this improvement in operation.

Some data were secured on antenna at Wallenpaupack during a lightning stroke. An antenna 50 feet from the ground with a grounded conductor 10 feet above, registered a potential approximately one-half that recorded on a similar 50-foot antenna without ground wire.

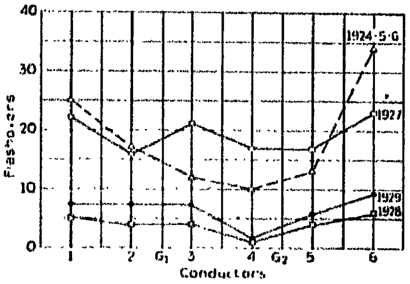

FIG. 11—CURVES SHOWING INSULATOR STRING FLASHOVER ON NEW ENGLAND POWER CO. SYSTEM, 1924 TO 1929

Ground wire G_2 was installed in 1928

The Overhead Systems Committee of the Great Lakes Division, National Electric Light Association,[11] made a study of the operation of high-voltage transmission lines during 1927. The lines without overhead ground wires (728 miles) had 12.2 interruptions per 100 miles due to lightning, while the lines with overhead ground wires (1511 miles) had 5.9 interruptions per 100 miles due to lightning.

A similar study was made for 1928 operation.[11] The lines without overhead ground wires (719 miles) had 16.8 interruptions per 100 miles, and the lines with overhead ground wires (1576 miles) had 8.1 interruptions per 100 miles. The data for the two years indicate approximately twice as many interruptions per 100 miles on lines without ground wires as on lines with ground wires.

The Texas Power & Light Company in 1925 operated 726 miles of 60-kv. line with overhead ground wire. On these lines they had 28 interruptions or approximately 4 interruptions per 100 miles. During the same year they operated 491 miles of 66-kv. line without overhead ground wire. The interruptions were 75 or about 15 per 100 miles. In 1926 they had 800 miles with ground wire, and the interruptions were 64 or 8 per 100 miles. In the same year they had 784 miles without ground wire. The interruptions were 220 or 28 per 100 miles.

All these data, though incomplete, give corroborative evidence of the value of the overhead ground wire in improving the continuity of service.

VI. LIGHTNING STROKE RECORDERS

In 1929 lightning stroke recorders were placed in service on 284 of the approximately 300 towers of the Wallenpaupack-Siegfried line and on about 20 towers of the Philo-Canton line.

Fig. 12 is a view of the recorder. This recorder is a small Lichtenberg camera, which is placed across a portion of a leg of a transmission tower. The development of the instrument and its application are due to Mr. W. L. Lloyd of the Pittsfield High-Voltage Laboratory.

The film for this instrument consists of two sheets separated by tinfoil and wrapped in treated paper, the whole forming a lightproof and water proof packet, so that the instrument can be readily loaded in the field. The film is inserted in the camera and pressure is applied till the film is held firmly in place, with the electrodes against the outside wrapping of the packet.

The theory of the device is that a direct stroke hitting the tower or flashing over an insulator will cause sufficient current to flow down the tower leg to give an appreciable drop, and consequently a Lichtenberg figure on the film. The current flowing when an insulator flashes over due to an induced stroke will not be sufficient to cause enough drop in the tower leg to give a figure on the film.

Four figures were obtained, two on the Wallenpaupack-Siegfried line and two on the Philo-Canton line.

The first Pennsylvania record was obtained on August 27 at tower 26-2 (Siegfried-Roseland Section). This record was interpreted to indicate about 60,000 amperes negative polarity. At the same time the surge voltage recorder at this tower indicated a positive surge of 2400-2800 kv. (13.3 to 15.5 times normal). An oscillogram of low-voltage amplitude was taken of this surge at the lightning laboratory (about 40 miles from origin of surge). Tripout 33 was associated with this surge.

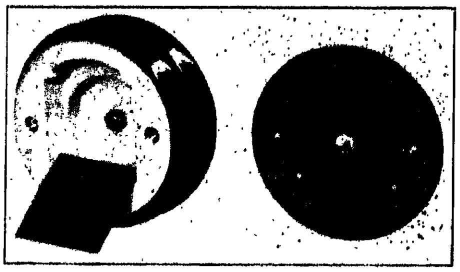

FIG. 12—VIEW OF LIGHTNING STROKE RECORDER

The second Pennsylvania record was obtained Sept. 10 at tower 12-5 (Wallenpaupack Tap section). This tower is at High Knob, in the section of line equipped with a counterpoise.[4] The record was interpreted to indicate about 100,000 amperes negative polarity. There was no surge voltage recorder at this tower and the recorder at the next tower, 12-6, was out of service. The recorder at tower 10-2 (the nearest recorder north) indicated a positive surge of 350 kv., which it is thought correlates with the surge indicated by the lightning stroke recorder. The surge voltage recorder at tower 17-1 (the nearest recorder south) gave no indication. No tripout accompanied this surge.

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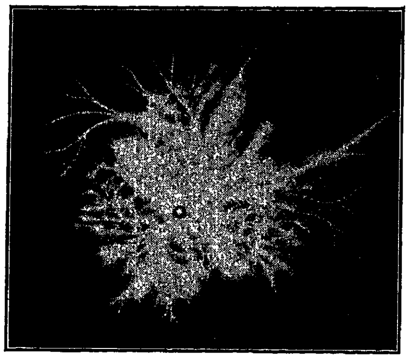
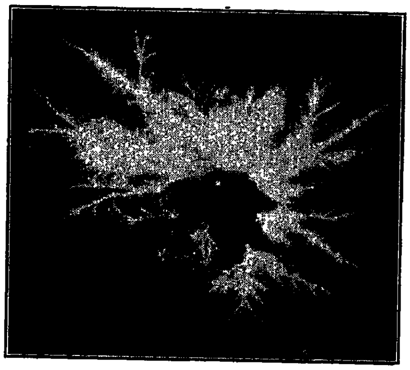
[11] Report of Overhead Systems Committee, Great Lakes Division, National Electric Light Association, Sept. 1928. Also report of same committee Oct. 1929.

The first Ohio record was obtained on July 25 at 3:20 p. m. on tower 185 and indicated approximately 175,000 amperes negative polarity. (Fig. 13.) At the same time, the surge voltage recorder on the transmission conductor on this tower indicated a positive surge of 8.4 times normal (surge A-50). There was no tripout. A good attenuation curve was obtained of this surge by means of surge voltage recorders, (Fig. 14).



Top Film



Bottom Film

Fig. 13—Figure Obtained with Lightning Stroke Recorder on Ohio Power Co. System

July 25, 1929, Surge A-50

The second Ohio record was thought to have been obtained on August 23 at 2:58 a. m. This was at tower 159. The recorder indicated a surge of approximately 100,000 amperes negative polarity. At the same time, the line surge voltage recorder indicated a positive surge of 6.5 times normal at this tower (surge A-72). This surge was accompanied by a tripout.

There is a large field for an instrument of this sort, but the records obtained so far are too few to establish any conclusions as to the value of the instrument for the purpose intended. It is planned to continue the operation of these instruments for another year.

VII. Field Intensity Recorder and Rate of Change of Field Intensity Recorder

Two instruments were built and placed in operation at the Wallenpaupack laboratory, in an effort to obtain some measurements with regard to electric field conditions to which transmission conductors might be subjected. These instruments are called respectively a field intensity recorder and a rate of change of field intensity recorder.

The instruments were developed and applied under the supervision of Mr. H. L. Marvin of the General Engineering Laboratory, Schenectady, and the data obtained were analyzed by Mr. Marvin.

The purpose of the field intensity recorder is to obtain a photographic record on motion picture film of the vertical component of electric field intensity (voltage gradient) versus time. The purpose of the rate of change recorder is to obtain a photographic record of the vertical component of the time rate of change of electric field intensity. The adjustment is made so that only relatively large values such as occur during thunder storms are recorded. The data so obtained are expected to be helpful in two ways: (a) to acquire further knowledge of the nature of lightning storms; (b) in measuring the relative severity of thunder storms.

Instruments were installed in a small building near the lightning laboratory at Wallenpaupack and began operating on May 28, 1929, and continued operation throughout the lightning season. During that time the recorders went through more than 33 storm periods. Each recorder used approximately 1400 ft. of motion picture film. The field intensity recorder operated satisfactorily from the first, but the rate of change recorder produced blank records until after August 5, at which time changes were made in the instrument to increase sensitivity and adjust for slower rates of change. Following these changes records were obtained on this instrument also.

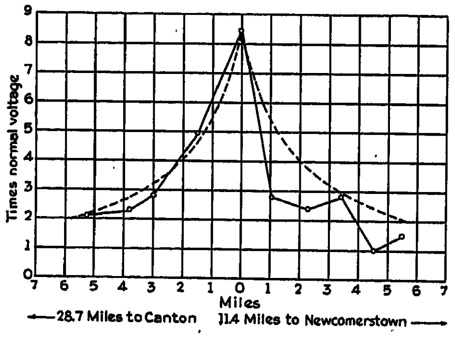


Fig. 14—Attenuation of Surge A-50, which Occurred on Ohio Power Co. System July 25, 1929

(See Fig. 13 for lightning stroke recorder registration.) Dotted curve shows attenuation calculated from Equation (1), with $K = 0.0006$

Fig. 15 shows a view of the field intensity recorder and Fig. 16 shows a typical record taken with this instrument, also a record obtained at the same time with the rate of change of field recorder.

Records obtained from the field intensity recorder were measured, and the data plotted for each storm on cross-section paper, the abscissas representing time and the ordinates kilovolts per meter. Fig. 17 shows the data for the storm of June 19, 1929.

A study of the records indicates that the readings for any one storm are usually of one predominant polarity, which may be either positive or negative. Of 33 storm periods studied, about 40 per cent gave readinas predominately positive, about 40 per cent predominantly negative, and about 20 per cent mixed positive and negative.

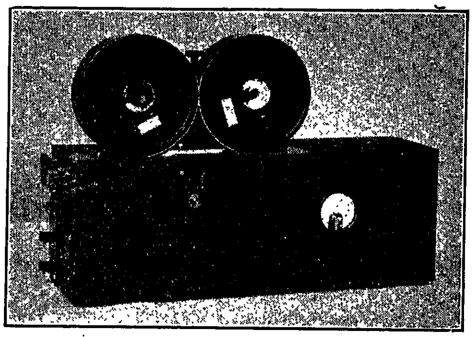


Fig. 15—View of Field Intensity Recorder

In Fig. 18 are plotted as abscissas kilovolts per meter against number of strokes as ordinates, as measured by the field intensity recorder.

In Fig. 19 an attempt is made to correlate the potential gradient with distance of the stroke from the laboratory. The distance was determined roughly by an observer in the cupola of the laboratory, with the assistance of a stop watch and a device for sighting on the stroke. Of 253 strokes over 15 kilovolts per meter recorded, it was possible to correlate in this manner only 81 strokes or 32 per cent of those occurring. The other strokes were too uncertain in correlation or else no satisfactory distance measurement was obtained. It will be noticed that the correlation is very rough but that there is a tendency for a curve showing greater field intensity with shorter distance from the laboratory.

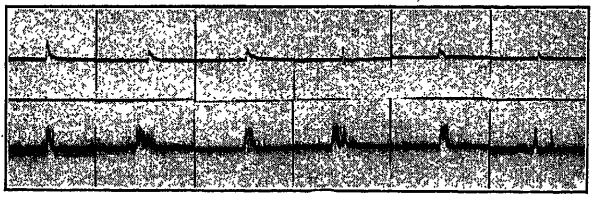


Fig. 16—Typical Record Obtained with Field Intensity Recorder (Upper) and with Rate of Change of Field Recorder (Lower)

Up to the time of writing this paper it has not been possible to determine the meaning of the records obtained from the rate of change of field intensity recorder, and their discussion will be left until a later date.

VIII. Conclusions

A vast amount of progress has been made in the study of the effects of lightning on transmission lines in the past two years.

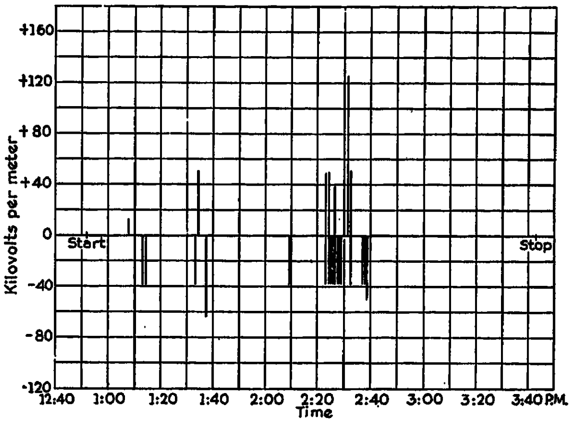


Fig. 17—Field Intensity Data Obtained by the Field Intensity Recorder during the Storm of June 19, 1929

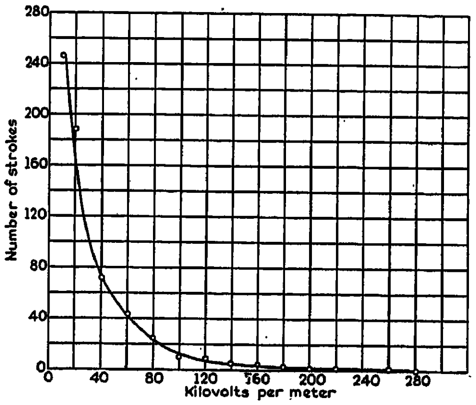


Fig. 18—Curve Showing Field Gradient against Number of Strokes, as Obtained by the Field Intensity Recorder During the 1929 Season

Ordinates represent number of strokes giving intensity greater than specified value

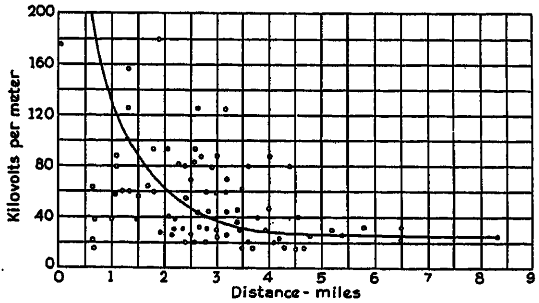


Fig. 19—Correlation of Potential Gradient with Distance of Stroke from Wallenpaupack Laboratory, Season of 1929

Upper limits of 15.2 times normal voltage for lightning surges and 5.5 times normal for switching surges have been reached.

Gradients as high as 52 kv. per foot of height have been recorded on transmission lines, and 85 kv. per foot on special antenna.

Tripouts have occurred on the lines under investigation as high as 29 per 100 miles per year, with 3.6 flashed insulator assemblies per tripout.

About 115 cathode ray oscillograms of natural lightning were obtained on the Pennsylvania, Ohio, and Consumers Power systems, ranging from low values to 1260 kv. Wave-fronts varied from less than one to 36 microseconds and total duration of first loop from about one to 75 microseconds. Unidirectional waves predominated and positive polarity.

The formulas developed from the 1927 results for attenuation[9] have been substantiated, both by results from natural lightning and from artificial lightning. The factor k for natural lightning has been found to average 0.00016 on the New England and Pennsylvania systems. For Ohio the factor is 0.00057. Artificial lightning surges on the Michigan system gave a factor of the order of 0.00044 and on the Turners Falls system of the order of 0.00069.

Attenuation apparently varies with the size of conductor, with the steepness of wave-front, with the polarity of the wave, with the presence or absence of ground wires, and possibly other factors.

The value of overhead ground wires in reducing induced potential and in reducing tripouts has been demonstrated by test and by operating experience.

Lightning stroke recorders have been introduced and a few records obtained. These give promise of much usefulness in the future.

Field intensity recorders and rate of change of field intensity recorders have been used and data of considerable value obtained. Gradients as high as 85 kv. per foot have been recorded. These instruments promise to yield a good deal of information about the habits of storms.

The intensive observation of storm conditions at the Pennsylvania lightning laboratory indicated that in 1929 there were 24 cloud-to-ground strokes within a one mile radius in this vicinity.

In the future work an attempt will be made to differentiate between the surges and flashovers due to induced strokes and direct strokes. An attempt will be made to record by cathode ray oscillograph the wave shapes of some of the extremely high voltages which have been found by surge voltage recorder. The effects of counterpoises or grounded conductors under the transmission line will be studied. Attenuation will be further studied both with natural and artificial lightning. Devices to prevent flashover (line type arresters) will probably be tried out experimentally during the coming year.

Discussion

PAPERS ON LIGHTNING INVESTIGATIONS

(Cox and Beck, Torok, Conwell and Fortescue, George and Eaton, McEachron, Rudge, and Hemstreet, Smeloff and Price, Sporn and Lloyd, Jr., Lewis and Foust)

New York, N. Y., January 29, 1930

A. E. Silver: Of the unsolved transmission problems today, lightning is outstanding. Bringing it under intelligent control is an economic necessity in power system development.

The outstanding item of progress reflected by the very valuable group of papers comprising this symposium is the arrival at that state of mind where the accepted goal is to conquer natural lightning in its worst effects on transmission lines. It is evident that investigators have now firmly resolved, and with confidence, to discover the fundamentals of lightning sufficiently to enable designing practical transmission lines whose service will be immune to lightning effects, *i. e.*, lines proof against direct as well as induced strokes.

Contrast this with five years ago, when transmission engineers and the few others who were then studying lightning effects accepted direct strokes as being of a magnitude beyond practical control, thinking that when they occurred service interruptions must be endured. Even two years ago only the most optimistic had the courage to resolve that the problem of withstanding direct strokes on transmission lines must be solved.

Again, large progress is reflected in determining propagating and terminal effects of lightning surges once established on the line. Many data have already been obtained, the necessary equipment and technique seem to be well in hand, and present interest seems to assure carrying on until sufficient data have been gathered and analyzed to establish laws and formulas for these phenomena. It is already well established that the insulator spillover interrupting service usually occurs very close to the point where the lightning surge originates on the line.

It is essential that progress in this branch of the problem be not confused with the far more fundamental and difficult task of discovering the mechanism and laws of the lightning stroke in its origin and in its establishment on the transmission line. In part, distinctly different equipment and technique are required, and these are not yet out of the development stage although now yielding some of the necessary data.

Thus far, in analyzing data obtained, major weight has been given to the factors of surge propagation on the line with a tendency in the papers to overlook segregation between the two phases of the problem. For example, I refer to the paper by Messrs. Lewis and Foust. In Tables V and VI, 31 surges of the 95 recorded at the Wallenpaupack Laboratory during the 1929 season are classified by measurements at the cathode ray oscillograph. The classification does not distinguish between waves measured at the point of origin on the transmission line and those measured at a distance from the origin. Rapid attenuation and change of wave shape in propagation is accepted. Clearly, to take the average of any substantial number of these waves as a conception of the characteristics of natural lightning surges, as they exist at the point of origin where they must be controlled to avoid service interruption, is misleading, and tends to wrong working assumptions.

If waves measured at the origin were segregated, then of the 31 listed in Tables V and VI, it is probable that not a single wave could be so classified. This is impressive as an index of the inherent slow rate of gathering the necessary fundamental data, at least until the factor of distance can be incorporated in measuring devices. Also, it should be an incentive to expediting the obtaining and analysis of propagation data to deduce the laws for correcting surge measurements back to the origin wherever along the line they may have been recorded, provided this proves to be a practical procedure.

I repeat, for emphasis, the caution that throughout our investigation work, we all keep clearly in mind as the main objective the protection of service from lightning spillover, which, in general, does not result from the lightning surge after it has traveled away from the point of its origin on the line.

In the matter of transmission line design to render harmless the effects of lightning surges, valuable aid is made available in the papers by Messrs. Conwell and Fortescue, and Messrs. McEachron, Hemstreet, and Rudge. These not only present valuable data but lay the ground work for even more valuable results as the field tests are continued. Laboratory methods so applied in the field on actual lines, together with the experience to be gained from thoughtful trial of improved types and observation on the part of operating men, give definite promise of rapidly bringing the application of overhead ground wires and their efficient connection to earth from the realm of conjecture to that of intelligent design.

Prevailing theory, field experience, although confused, and experimental results, already afford ample grounds for the belief that when lightning-proof transmission lines have been reduced to a basis of scientific design, the effective application and proportioning of overhead ground wires and earth connections will be a major factor in at least one method of solution.

D. W. Roper: In connection with our investigations on 2400-volt lightning arresters, we have found such informal discussions very desirable and valuable.

In the several papers presented, there is a number of conflicting opinions and theories and all these points must be ironed out and some agreement reached before the proper conclusion can be obtained.

The Transmission Distribution Committee, of which Mr. Woodrow is chairman, and the subcommittee on Lightning and Insulators, of which Mr. Sporn is chairman, are to be congratulated on this very excellent meeting and for the results which are here presented. If they care to arrange for further informal discussions to summarize and digest the information that has been presented, I assure them in advance of the cooperation and support of the Commonwealth Edison Company and affiliated companies.

E. P. Peck: All of these lightning studies are made for the purpose of furnishing data which will permit the reduction of interruptions to service due to lightning. An attempt has been made to correlate the results of these tests and of additional information on circuit characteristics, and arrange the data in a systematic way to permit the predetermination of the probable number of interruptions due to lightning on any given transmission line. A chart has been prepared which purports to show the anticipated number of trip-outs due to lightning per 100 circuit miles per year, when the electrical characteristics of the transmission line and the lightning characteristics of the territory through which the line runs are known. All of the reports of tests made throughout the country that were available, were analyzed to determine the effect of different characteristics on the line performance. The chart was prepared from the data obtained by this analysis.

The method is not expected to give results which have engineering accuracy, but rather the order of accuracy expected from rain fall or stream run-off records. The answers should not be considered useless, even though they are supposed to furnish average results over a long term of years. Since the charts are based on data which were available in the early part of 1929, and which data were somewhat inaccurate and incomplete, it must be expected that the charts will require revision after better data are obtained. Even in its present state, however, this method appears to offer one of the best means of determining the effect of changing any of the characteristics of the circuit, such as number of insulators, height of conductors, or number of ground wires.

To use the charts, it is necessary for the investigator to have records on his own system or in his territory, of the number of lightning flashovers per year. Data are also needed on the voltages impressed on the line by lightning impulses and the number of times these voltages occur. With this information and the charts, we believe it possible to determine with reasonable accuracy the improvement that may be obtained in interruptions per year due to lightning by making any definite change in the number of insulators or others characteristics of circuits.

L. V. Bewley: I wish to discuss the characteristics of traveling waves caused by induced strokes. This discussion will be based on the principles established in my paper *Traveling Waves Due to Lightning*, A. I. E. E. TRANS., Vol. 48, July 1929, p. 1050. By studying these inherent characteristics we are in a much better position to decide whether the observed traveling waves originated from induced or direct strokes. The shape of a traveling wave due to an induced potential depends upon four factors. First and most important, the law of cloud discharge; second, but not nearly so important, the distribution of bound charge; third, the distortion and attenuation which the wave suffers due to losses as it moves along the line; and fourth, the change in wave shape caused by its behavior at a transition point. The cloud discharge may be divided into two stages. The incipient or preliminary stage, probably lasting for several hundred microseconds, during which time there is undoubtedly an intensification of the field gradient, and a migration of the bound charges held by adjacent cloud fields toward the center of disturbance. Possibly, therefore, the actual field gradient at the instant of rupture is much greater than under static equilibrium conditions. The final stage of cloud discharge probably lasts up to a hundred microseconds. The corresponding distribution of bound charge is therefore also divided into an initial stage corresponding to the static field prior to discharge, and a transient stage caused by the migration of adjacent bound charges towards the center of disturbance. Line distortion is caused by corona, and to a much less extent by skin effect. If you will refer, for instance, to Figs. 5, 6, and 7 of Conwell and Fortescue's paper, you will see that, in the short course of a few miles, there is noticeable distortion in the traveling wave, although such excessive distortion is rather unusual. But the point is, when picture of a traveling wave on a transmission line is taken with a cathode ray oscillograph, it is not known, as a rule, how far it has traveled, and therefore it is impossible to determine whether the wave shape under observation is the same as that at the point of origin. This is very important when drawing conclusions from the wave-front.

The following criteria of induced strokes are of interest:

1. If a wave has not suffered distortion or elongation due to reflection at a transition point, then its length in microseconds is equal to the length of the bound charge in thousands of feet plus the time of cloud discharge in microseconds.
2. The front of the traveling wave is equal to the length of the bound charge, or to the time of cloud discharge, depending upon which is the lesser.
3. The general outline of a traveling wave is established by the law of cloud discharge rather than by the distribution of bound charge and, therefore, a rectangular bound charge may be assumed for estimating purposes.
4. The *rate* of rise of potential under the center of disturbance is practically double that due to the passage of a traveling wave front, although the maximum potential is only a few per cent higher than the crest of the traveling wave. This is an important point because it has a bearing on the arc-over characteristic of the insulator at the point of disturbance.
5. The longer the time of cloud discharge the more depressed the crest. If, for instance, a 1000-ft. bound charge released instantaneously gives a 100 per cent crest, then a 10-microsecond discharge gives only about a 10 per cent crest.

Now consider a few of the oscillograms given in this group of lightning papers, in the light of the above criteria. Take, for instance, the lower right hand surge in Fig. 7 of the paper by Cox and Beck. This surge has approximately a 6-microsecond front and an effective length of around 35 microseconds. If that wave is just as it was at its origin, and has not suffered distortion as it developed, then it can be accounted for as follows: 5000-ft. bound charge, exponential cloud discharge to practically zero in 30 microseconds, and an initial gradient of 25 kv./ft. The 5000 ft. bound charge probably is exorbitant. It does not seem intuitionally reasonable to talk about bound charges of that extent, but on the other hand there is no definite proof known to the writer that such long charges are impossible. However, it is more probable that this wave which they give has experienced considerable slowing up of its front between the point where it originated and the point where it was measured. On such a basis it could be accounted for by a much shorter bound charge released in about 35 microseconds, but in that case a field gradient of from 50 to 75 kv./ft. would correspond.

The highest record obtained by George and Eaton is shown in their Fig. 9B. That wave could have originated from a 3500-ft. bound charge released in 22 microseconds, and an initial field gradient of 50 kv./ft. Here again, the wave has probably been distorted on its front, and no doubt came from a much shorter bound charge. In the oscillograms obtained by these authors there is a little flat place just back of the crest in all of them. This same characteristic has been found by other investigators. It would be of interest to know whether the authors assign any sound reason for this peculiarity.

McEachron, Hemstreet, and Rudge describe some interesting tests with traveling waves to determine the protective ratio afforded by ground wires. It is rather interesting that the same protective ratio is found with fully developed traveling waves, as by the purely static methods, such as the conventional calculations, tests on model transmission lines, and tests on antennas. Within a short time a paper will be submitted to the Institute in which it will be shown that the protective ratio provided by ideal ground wires is entirely independent of the law of cloud discharge, of the distribution of bound charge, and of the formation of traveling waves; and that the protective ratio found by traveling waves is theoretically identical with that found by Petersen's method or tests on models and antennas. It is worth remarking that ground wires are distinctly beneficial whether the trouble is due to induced or direct strokes, although there is some difference in the most efficient way to employ ground wires, depending upon whether the protection is for induced or direct strokes. As far as the use of ground wires for direct strokes is concerned, good grounds are essential, for, as the pioneer work of Cox, Slepian, Fortescue, and Atherton, of the Westinghouse Company, has shown, the lower the ground resistance the lower the voltages on the ground wires, line wires, and across the insulator strings; and the fewer the number of spans subjected to dangerous voltages.

H. S. Randall: The papers covering the intensive Lightning Investigation on Transmission Lines carried on during the last year recall to mind one of the first surge investigations made on an actual transmission line with a surge generator at one end and a Dufour cathode ray oscillograph at the other.

In 1927 the Duquesne Light had a two circuit 22-kv. line approximately 5 miles long and of standard Duquesne Light construction consisting of 37-kv. insulator, 4/0 copper mounted on wooden poles, and crossarms which was temporarily out of service.

Permission was given the Westinghouse Company to install a surge generator at one end and a Dufour cathode ray oscillograph at the other. The set-up was in operation during the summer. It was hoped to obtain a few actual lightning surge records but two things prevented: first, the present method of tripping the cathode-ray oscillograph, getting it onto the firing line at the right time, was not developed, and, second, there was no lightning in this district during the test period.

Up to this time many calculations had been made on the reactions occurring on a transmission line when subjected to surge voltages and laboratory tests had been made on laboratory circuits with lumped constants only. It was hoped to be able to obtain a direct check on the results obtained in the laboratory.

Among the phenomena predicted by these theories which were observed during this pioneer surge test were:

1. Traveling waves move with the velocity of light.
2. Traveling waves on reaching open end of the line reflect back on the line with same polarity.
3. Traveling waves on reaching grounded end of the line reflect back on the line with reverse polarity.
4. The surge impedance for a single conductor is in the order of 400 to 500 ohms.

An attempt was made to determine the effects of lightning arrester, choke coils, capacitors, and resistors, on the surge voltages. A 40-kv. surge was applied to the open line and there were many repeated applications of the surge due to the reflections. With a 3-kv. lightning arrester at the end of the line, the surge was limited to a much lower value and the arrester absorbed sufficient energy so that there were only two impulses.

A choke coil was inserted ahead of a transformer at the end of the line and steep surges applied to the line. The tests showed that although the voltage on the transformer was not reduced, the wave-front was sloped off. No attempts were made to go further with this study but at the present time several thorough studies have been made on the effect of choke coils.

A capacitor was installed at the end of the line and this modified the surge due to the time it took to charge the capacitor. A 40-kv. surge was applied but the voltage across the capacitor rose to only 6.5 kv. with a sloping wave-front. There was a number of reflections as the capacitor does not absorb the energy of the surge.

With a resistance equal to the surge impedance of the line installed at the end of the line there was no reflection or folding back on itself. With the resistance greater than the surge impedance a wave of same polarity was reflected—with a smaller resistance the reflection was of opposite polarity.

The amount of absorption depends on the relation of resistance to surge impedance, there being no absorption when the line is open or grounded through zero resistance.

Compared with the last year's surge investigations the report of this one reads like a school primer, "but we must learn to creep before we walk" and getting started in the right direction has been a big help in arriving at the present point in surge investigations.

L. A. Terven: Thanks to the engineering investigations which have been possible through improved equipment for which the manufacturing companies in the United States are so largely responsible, the West Penn Power Company has been able to make several practical applications of the principles of lightning protection to the solution of specific problems. As an example, a case might be mentioned where a short 2300-volt line feeding a manufacturing establishment was subject to numerous induced surges from a 25,000-volt line which paralleled it on the same pole structure for about 670 ft. The 25-kv. line then crossed the river adjacent to the manufacturing plant at a rather high elevation above the ground plane.

From actual klydonograph measurements, the quantitative values of the surge were learned. Additional lightning arresters were then installed and all ground connections were improved. In fact, the ground for the lightning arrester at the consumer's premises was extended into the river bed. An idle line was used as a grounded system of shield wires between the 25-kv. line and the 2300-volt line. The shield wires also extended across the river, which was outside of the parallel, and a branch followed the 2300-volt line for some 200 ft., after the latter had left the exposure. Subsequent investigations showed a marked reduction in the amplitude of the surges experienced and service over this line has been quite satisfactory since the work was done in 1927.

R. N. Southgate: It has been very gratifying to view the progress which has been made during the past year in studying the attenuation of real and artificial surges along transmission lines.

Before proceeding with a discussion of attenuation a distinction should be made between attenuation, as a reduction in the magnitude of the surge, and distortion as a change in shape of the surge. Both attenuation and distortion of wave shape are brought about by the same factors: namely, the series resistance of the line and the shunt leakage such as corona or leaky insulation. These factors are perhaps too interlinked to be segregated definitely into two different groups.

Considering first the effect of series resistance, a transmission line having an appreciable resistance to transients may be represented by an ideal transmission line having no resistance divided into segments by small series resistors. A surge on such a line will behave as follows: The wave travels over the first segment of line unaltered until it strikes the first resistor, then a change in surge impedance causes a reduction in the magnitude of the wave passing through the resistor, as a positive reflection will travel in a reverse direction. This process is repeated when the next resistor is reached but the reflection from the next resistor must pass through the first resistor. Thus we can see that an original wave of definite length will be replaced by a composite of backward and forward reflections added to the remaining portion of the original wave. Therefore, in addition to a reduction in magnitude, both the front and tail of the wave will be distorted as the wave is propagated along the line.

The effects of shunt leakage resistance may be treated similarly so as to show the continuous leakage to ground of the surge and the reflections on the original wave. However, when the voltage impressed upon the line is raised to a certain point a much more important factor, called corona, appears for consideration. The formation of corona or breakdown of air takes place when the electrostatic stress about the wire reaches a value at which extensive ionization by collision takes place. The effect is to move the charge to a specific distance out from the wire and concentric with the wire. The formation of this space charge though rapid is not instantaneous so that in a schematic diagram this effect may be drawn as follows: A condenser is connected to each incremental length of line conductor through a sphere-gap and a resistance. When the traveling wave reaches a definite voltage these gaps break down and charge the condenser through the resistance. Initially the current delivered to these condensers will be rather large, then as the condenser comes nearly to the potential of the line the current will diminish and finally at the crest of the wave only the leakage current will flow. As the voltage of the line diminishes, the charge in this condenser will flow back into the conductor when the difference in potential between the condenser and the conductor is sufficiently large to cause the gap to break down, again completing the discharge process. It is noted that the effects of corona are to slope off the front of the wave, reduce the crest of the wave, increase the tail, and lengthen the entire surge.

It is interesting to observe the general agreement in the papers presented with regard to the greater attenuation of positive surges and surges having steep wave-fronts. Also the greater part of the attenuation is charged to corona loss rather than to resistance, which accounts for the fact that it varies greatly with the original voltage and polarity of surge.

At the midsummer convention in Denver in 1928, Mr. W. W. Lewis presented a formula derived by Messrs. Foust and Menger which seems to express fairly satisfactorily the general law of attenuation on a given line. In the light of additional data, it now appeared necessary to modify the factor k or introduce other variables to conform with the condition, size, and spacing of the conductors as well as the nature of the weather and shape of the surge. Any formula that will accurately express attenuation under all conditions promises to be quite a complex function.

However, some of the general laws or characteristics controlling the attenuation of a surge will be useful in determining the protection features that must be incorporated at terminal points. While there is some indication that the attenuation of surges may be materially different under ground wires, it may be possible to apply greater protection to lines near substations so a surge originating at remote points on the system will be attenuated to a harmless quantity by the time it reaches the substation. It therefore becomes necessary to pay particular attention to the origin of surges on the line next to the substation in order to coordinate the apparatus insulation with that of the line. The reverse corollary to this statement is that attenuation makes it possible to over-insulate remote portions of the line which are particularly susceptible to disturbances and decrease the outages per year.

E. E. George: It seems that a new field is opened up by equipment to record and measure direct lightning strokes. In the past research has been pretty well confined to induced strokes. Some of the operating and transmission men have had an idea that a good deal of damage on high-voltage systems, particularly the burndowns, was caused from direct strokes.

About a year and a half ago attention was particularly directed to this again in making arc tests, because it was never possible to get any very extensive damage to insulators or conductors for the currents that would be on the system in any reasonable time, that is, the time we used to consider good relaying. There is nothing to indicate that, if the kv-a. could be doubled or trebled, the conductor and insulators would be injured, yet it was known that, in many of the burndowns, although the line apparently relayed instantaneously, the insulators were shattered and the conductor was badly burned.

It is hard to reconcile this with the fact that lightning is supposed to be a matter of microseconds. Currents of the order of a hundred thousand amperes were never mentioned at that time, but it did seem that there must be some enormous melting or mechanical effect there. If the stroke of lightning could tear up an oak tree or set a barn on fire from top to bottom, certainly it ought to do some damage to copper conductors the size of one's little finger.

The evidence of burndowns from this time on was followed very closely and it was never possible to find any case in which the damage was attributed to a power arc. In other words, the burning down of the line seemed to be due to the initial shot of lightning.

One thing to keep in mind in such a system is the relative seriousness of trip outs and burndowns. A line 100 miles long may be out twenty times a year. The total duration of outs will be twenty minutes. Six of those outs may occur in the worst lightning months, say July. This means that for six minutes the customer has been interrupted. If from those twenty outs five per cent of burndowns, say one burndown a year, occur in the country where it is difficult to patrol and repair trouble, the line will be out from two to twenty hours. It may be a radial transmission line, affecting a good many customers. So the prevention of burndowns is of particular interest.

It has been said if a lightning proof line can be made customers will receive perfect service. This statement should be qualified somewhat. A record of all the cases of trouble for the past year discloses that about 55 per cent are due to causes external to the system—wind, weather, natural phenomena, floods, and things of that kind, and in no way associated with the system or its operation. About 45 per cent are due to personnel failures, poor operation, poor design, equipment defects, poor maintenance, etc. Of the 55 per cent due to external causes a reasonable portion is due to lightning. We believe it is a fair statement to say that if lightning could be eliminated entirely the interruptions to customers probably could not be cut by 40 per cent.

In studying lightning one of the things to keep in mind is the limitations of the protective scheme. Phase-to-phase faults are hard to relay. It may not be so always. At the present time, the ground faults can be very well cleared up. If more than one phase is involved, clearing up is a little more difficult. If the fault involves three phases, it is still more serious. For that reason, some operating men are opposed to horizontal construction, if it means closer spacing between phases and higher insulation to ground.

The only other point in connection with direct strokes is that this device permits measuring strokes on steel towers. Maybe some day somebody can devise equipment to be put on wood poles. (Perhaps such a device ought to be pretty well insured beforehand.) The belief exists that it will be possible to decide soon whether or not wood construction is worth while on high-voltage lines.

E. J. Wade: A set of three oscillograms is shown in Fig. 9 of the Conwell and Fortescue paper which show no oscillations. I know from experience that obtaining such oscillograms is difficult where the impulse generator is close to the oscillograph. It is important to be sure that they have thought of all factors in the divider and connections which would make it absolutely certain that these oscillograms represent the actual conditions.

The importance of careful study of the potentiometer schemes which are in use should be emphasized. When a capacity potentiometer is used there is little question but that all the oscillations are reproduced which are important. Possibly additional oscillations are introduced if care is not taken, but when a resistance potentiometer is used, especially if it is of high resistance value, the oscillations are likely to be suppressed and result in error, not only in the calibration but in the time it takes the wave to reach maximum.

The cable resistance type potentiometer used in these tests is very convenient and if one can be sure that it is accurate, it is the proper thing to use. However, the capacities at each end which can be considered as in parallel with the resistances have a considerable effect on the front of the wave. We have been using in the laboratory for several years a resistance potentiometer without the cable in which the capacities are controlled so that it is like a capacity bridge. The capacities are adjusted to have the same ratio as the resistances. In this manner the ratio is controlled to a known value from the resistance measurements. The capacities are then balanced so that it is accurate for all wave-fronts.

The manner in which the capacities are balanced is interesting. Volt ampere curves are taken on a test piece which is known to be of constant resistance and the capacities are adjusted until no loop is obtained between the rising and falling current. A description of this laboratory potentiometer is given in *Electric Light and Power* for May, 1929.

Concerning Messrs. Conwell and Fortescue's paper, I should like to say something about the surge impedance measurement which they made. In speaking particularly of the tower impedance measurements, they apparently have combined what they call the surge impedance of the tower with the resistance of the ground. No attempt is made to separate them. If they are separated we can consider the ground resistance as either constant or falling slightly with increasing currents.

Mr. Beck made some tests in the laboratory, reported in the *Electrical World* in March, 1928. These tests were made on earth in a box, and he found no change due to impulse currents. Other tests were made on driven grounds, some by Towne, noted in the *General Electric Review* of November, 1928, and others which were made last summer in Michigan, indicate a reduction in resistance for high impulse currents. The reduction is not very great. These tests by Messrs. Conwell and Fortescue show an increase in impedance for impulses. Based on the tests just referred to this increase must be ascribed not to the ground resistance, but to the tower itself. It would seem that the impedance of a tower would depend on the design of the tower, contour of the ground around the tower, and the resistivity of the soil and ought to be nearly the same for different towers.

In Table I is shown an increase of from 15 to 55 ohms over the low-voltage resistance. This seems to be quite a wide variation for the different towers. I also wonder whether it is accurate to use the term "surge impedance" to describe these tests, because the height of the tower is very short compared to the length of the wave front, and there is time for a number of reflections while the voltage is rising so that the actual surge impedance of that short section of tower might not have much effect. Perhaps they can explain their tests so that it will clear up this matter.

On the surge impedance of the transmission line, Fig. 10, the two upper points at 200 kv. seem to be the only supporting data which indicate a falling off of impedance for high voltages. It is doubtful whether such data provide sufficient justification for saying that the surge impedance is lower for high voltages due to corona.

In Mr. Torok's paper, Fig. 17, the impulse breakdown at slow impulses, 20 microseconds, does not show a constant impulse ratio. For instance, at a setting of two times the diameter an impulse breakdown 25 per cent higher than the 60-cycle breakdown is found, while at seven times the diameter it is only 5 per cent higher. This is not only contrary to expectation, but the different curves are somewhat erratic. Is this due to an uncertain calibration or is it a characteristic of the sphere-gap?

W. J. Rudge: The oscillograms shown in Fig. 1 herewith, were taken to supplement the measurements given in the paper by McEachron, Hemstreet, and Rudge. The oscillograph was located 5.5 miles from the Croton end of the S-19 line and the impulse generator; 30 miles of line extended beyond the point of measurement.

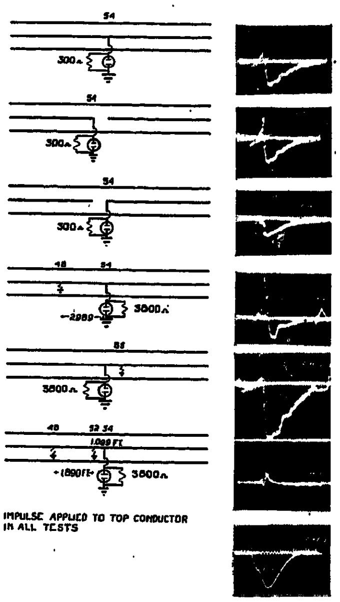


Fig. 1—Measurement of Induced Voltages for Various Conditions of Grounding

The negative wave shown by the bottom oscillogram, Fig. 1, was applied to the top conductor, the other conductors being insulated except for the measuring devices. Voltage which was induced on the middle conductor by the wave traveling on the top conductor is shown by the top oscillogram. A positive rise in voltage may be noted at the beginning of the wave. On the next oscillogram the line was opened and measurement of induced voltage made on the section near the impulse generator. The wave shape is essentially the same as before. The third oscillogram was taken on the section of line removed from the generator. It is to be noted that the positive rise in voltage is absent.

Comparison of these oscillograms indicates that where one conductor carrying a wave is paralleled by an insulated conductor, upon which an induced wave is traveling, the shape of the wave will be dependent on the position of measurement.

Fig. 2 shows a series of oscillograms which were taken with one conductor of the line used as a ground wire. Arrows are placed at the position where measurements were made.

The applied waves traveling on the bottom conductor are shown by the upper middle and right oscillograms. Voltage induced on a span of the ground wire by the wave shown on the upper oscillogram is shown by the oscillogram below.

An analysis of these oscillograms shows that a value of resistance which gives a damping decrement corresponding to that of the oscillations found on the middle oscillogram was nearly the same as the resistance obtained by taking the sum of the tower footing resistances at each end of the span on the ground wire.

The frequency of the oscillations checks up with the time which is required for a wave to travel twice the open length.

While these measurements are limited in scope they will serve as a guide to any one making a theoretical study of the ground wire problem.

O. Ackerman: Messrs. McEachron, Hemstreet, and Rudge state that "some difficulty has been experienced in measuring the surge impedance of the transmission line conductors."

Similar difficulties have been encountered in the transmission line experiments about which Mr. Conwell has reported. These difficulties are not only of an experimental nature as might be expected from the use of sphere-gaps, but they are rather fundamental.

The surge impedance was measured by surging the line over a resistance R and the voltage was recorded from both ends of this resistance to ground. The voltage drop across the resistance is proportional to the surge current. According to our conventional conception of surge impedance, this is the ratio between voltage and current at any point of the surge and this ratio should be constant.

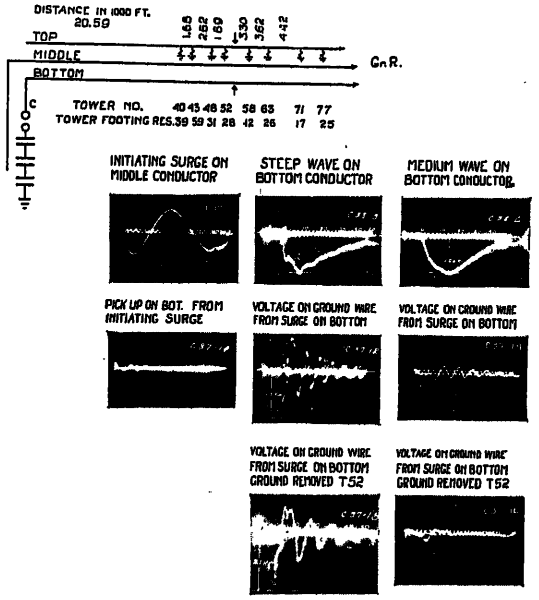

DISTANCE IN 1000 FT.
20.59
1.68
2.62
1.69
3.30
3.62
4.42
TOP
MIDDLE
BOTTOM
GnR.
C
TOWER NO. 40 43 48 52 58 63 71 77
TOWER FOOTING RES. 39 59 31 28 42 26 17 25
INITIATING SURGE ON MIDDLE CONDUCTOR
STEEP WAVE ON BOTTOM CONDUCTOR
MEDIUM WAVE ON BOTTOM CONDUCTOR
PICK UP ON BOT. FROM INITIATING SURGE
VOLTAGE ON GROUND WIRE FROM SURGE ON BOTTOM
VOLTAGE ON GROUND WIRE FROM SURGE ON BOTTOM
VOLTAGE ON GROUND WIRE FROM SURGE ON BOTTOM GROUND REMOVED T52
VOLTAGE ON GROUND WIRE FROM SURGE ON BOTTOM GROUND REMOVED T52

Fig. 2—Measurements on Ground Wire

A glance at the oscillogram in Fig. 3 shows that this is not the case. If the ratio were constant, the two curves ought to be similar. Figuring the ratio E/I at various points, it varies from 300 ohms to infinity. We were therefore at first quite at a loss as to how to calculate the surge impedance from such an oscillogram. We owe the solution of the problem to Dr. Slepian.

Obviously, resistance and corona play a big part in the problem. The effect of the line resistance as outlined by Mr. Southgate is nicely illustrated in this oscillogram. Reflected by the line resistance part of the surge current is flowing backward, spreading the wave over a longer stretch than that given by original impulse. Therefore, the current flow through the resistance comes to a stop and may even reverse before the voltage becomes zero.

This is part of the facts which necessitate dropping the simple term "surge impedance" for the term "impedance function" known from the operational calculus. The impedance function for a line with constant L, C, r, and g is given by the term under the radical sign. The sign p, can be considered as a parameter although it has a more definite meaning in operational calculus. Here we can take it as standing for the fact that Z is not a constant but depending on some variable p.

The impedance function of a transmission line with compilcated corona characteristics and skin effect is not necessarily given by the term under the radical sign, but it is always equal to this quotient of integrals which has been derived by Dr. Slepian. Leaving aside the exponential term, we obtain the impedance function by dividing $\int V\,dt$ by $\int I\,dt$. Instead of taking V/I at any particular point, we take an average in some way. The method yields satisfactory and consistent results. For large values of p, Z approaches $\sqrt{\dfrac{L}{C}}$. Knowing $\dfrac{L}{C}$ we can gain information about r and g out of relations such as under the radical sign, and thus will be able to tell more about attenuation and wave distortion not only in an empirical manner but from a more scientific standpoint.

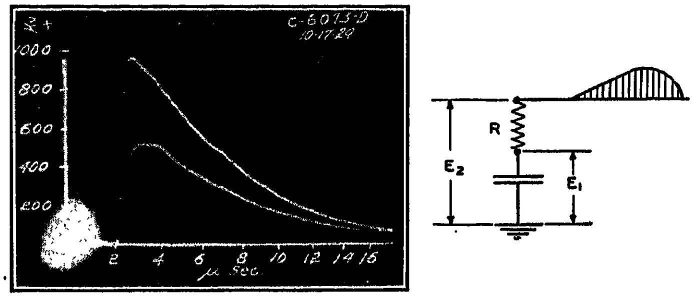


Fig. 3

$$Z_{(p)} = \frac{\int V_{(t)}\,\epsilon^{-pt}\,dt}{\int I_{(t)}\,\epsilon^{-pt}\,dt} = \sqrt{\frac{Lp+r}{Cp+g}}$$

Measuring Impedance Function $Z_{(p)}$ and Surge Impedance of Transmission Line with a Cathode Ray Oscillograph

F. W. Peek, Jr.: Of importance in these investigations are the results of the work of the investigators with the operating companies cooperating with the General Electric Company. On one line alone, a 220-kv. line of the Pennsylvania Power and Light Company, over 150 cathode ray oscillograms of natural lightning were obtained during the year. Other oscillograms of natural lightning were obtained on other lines, particularly on the lines of the Ohio Power Company. Next to the cathode ray oscillograms, the most important factor that has entered the investigation this year is the direct stroke indicator. Although this work was not started until late in the season a number of records was obtained of direct strokes of lightning on towers. Direct-stroke currents of the order of 175,000 amperes were recorded. These measurements are of extreme importance because a knowledge of actual current values will permit us to calculate the lightning voltages that appear across a tower or between ground wire and earth during a direct stroke. With this information it is possible to determine the tower design and conductor configuration necessary to make a transmission line lightning proof against direct strokes.

Next in importance are the data obtained on ground wires and rods described by Mr. Smeloff. You will remember some years ago we made measurements of the protective ratio of the ground wire in the laboratory on small models. The protective ratios obtained by this method indicated a greater effect from the ground wire than calculations. It thus appeared that ordinary calculations did not take into account all of the factors. This year these tests were repeated in the field with natural lightning and full size towers. One span or antenna arrangement was erected without a ground wire while an adjacent span was erected with a ground wire. Simultaneous measurements on these two spans or antennas during thunder storms showed a protective ratio of approximately the same value as that obtained on the miniature models in the laboratory. Apparently a factor usually neglected in calculations is the corona which effectively increases the size of the conductor. Gradients as high as 85 kv. per foot were measured by these antennas. This value approaches the gradient of 100 kv. per foot which was determined in an entirely different manner.

The papers on lightning arresters already presented at this convention record two important steps forward in lightning arrester development, first, the size of the new arrester has been so reduced that it is now possible to place it near the apparatus that it is intended to protect; second, it is now possible for the first time to predict the performance of the lightning arrester. Reference is here made particularly to Mr. McEachron's paper in which curves are given from which the performance of the arrester can be pre-determined for any condition.

While progress has been made on transmission lines the improvements have been going forward in the factories in making transformers more highly resistant to lightning transient voltage. Perhaps the outstanding achievement in this direction is the non-resonating transformer. During the year 150 or more natural lightning waves were measured on transmission lines. An examination of the oscillograms shows that the waves vary over a considerable range as would be expected. In most cases they were evidently caused by storms some distance away. In this connection it is important to determine from these measured free traveling waves the character of the voltage during the formation of the wave at the point of origin. This is particularly important because the voltage will always be much higher at that point and arc-over is more likely to occur there. Calculations show that for a range of cloud sizes and rates of discharge to be expected, the voltage at the source or origin reaches maximum in from one-half to one-third of the time indicated by the front of the free traveling wave. The effect at the point of origin would thus be that of a wave about twice the steepness and of a higher voltage value than those measured. A study of the measured waves shows that 50 per cent reach crest voltage in less than 3 microseconds and 90 per cent in less than 8 microseconds or, at the point of origin, 50 per cent of these transients would reach maximum voltage in less than 1½ microseconds and 90 per cent in less than 4 microseconds. Insulator spark-over, if it occurs at all, is likely to occur on the rising front of these waves. The laboratory tests show that the spark-over voltage in 4 microseconds on the uniformly rising wave has an impulse ratio of approximately 1.6, and for spark-over in 1½ microseconds, an impulse of 1.9. Thus, from an examination of these waves one would expect 50 per cent of the insulator arc-overs to have an impulse ratio of 1.9 or greater, and 90 per cent of them to have an impulse of 1.6 or greater. Measurements with the surge voltage recorder during the past few years have given approximately these same results. For this reason also it would seem desirable in making laboratory tests to use a wave giving an impulse ratio of 1.9. It so happens that the standard laboratory waves adopted a number of years before these measurements were made give these results.

A further examination of the oscillograms shows that some of the waves have a front of 10 microseconds or more. The front of a wave is usually shorter than the time that it takes for the cloud to discharge. The induced voltage that can occur on a transmission line depends upon the rapidity of the cloud discharge. For a cloud discharging in 10 microseconds and a cloud 1000 ft. long, the maximum voltage that could occur would probably be in the order of 500 kv. with a front of two or three microseconds. This would not be high enough to cause arc-over on a 110-kv. line. From this it appears that induced voltages high enough to cause arc-over on lines 66 kv. and above must have steep wave fronts and thus high impulse ratio. For lines 66 kv. and below it would appear that induced voltages caused by slow discharging clouds might be sufficient to cause arc-over. Accordingly, the average impulse ratio measured for lower voltage lines would probably be less than for higher voltage lines.

An examination of the waves indicates that voltages sufficiently high to cause arc-over on 220-kv. lines could be induced with clouds of a reasonable size and a reasonable rate of discharge. It also appears, however, that the induced voltage would cause a greater number of outages on the low-voltage lines and as the voltage and thus the insulation is increased, the direct stroke would become more and more important as the cause of outages. Voltages caused by direct strokes would be quite steep and thus have high impulse ratio.

During the past year we have made further mathematical and laboratory studies on the possibility of using ground wires to prevent outages from direct strokes as well as induced voltages. The results are very promising. One important factor brought out is the necessity of very low ground resistance at the tower footings for direct stroke protection.

A statistical study of lightning outages on transmission lines in various parts of the country has been completed. Since different voltages, heights of line and insulation obtain, an attempt was made to reduce the data to a common insulation basis by dividing the number of insulator units in the string by the tower height. Since the number of storms varies in different parts of the country, the outages were reduced to a 100 storms per year basis per 100 miles of transmission line. A curve was then plotted between outages per 100 miles per year and insulators per foot height of tower. The curve indicates that outages would be very small with 0.8 of an insulator unit per foot height of tower without ground wires. With ground wires the outages are of the same order with 0.4 of an insulator per foot height of tower. This means that 0.4 × 40 = 16 units for a 40 ft. conductor height should give a very small number of outages, all probably due to direct strokes.

This is the same conclusion reached by our laboratory methods. For example, with a 40 ft. line

$$V = g\,h = 100 \times 40 = 4000 \text{ kv.}$$

With ground wire

$$V = \frac{4000}{2} = 2000 \text{ kv.}$$

which is the maximum induced voltage.

By referring to the lightning spark-over curve (standard ½/5 Pittsfield wave) this corresponds to about 15 units. Thus, 16 units should give negligible outages for induced voltages, which was checked statistically above. It now seems economically feasible to design a line with ground wires so located that direct stroke outages may be greatly reduced or even eliminated.

Mr. Torok has given a very interesting paper on the test results of a certain type of wave. A rectangular wave such as he is attempting to produce, would give results of theoretical interest. It does not appear, however, the wave used really gives this effect or eliminates the variable due to wave-front. In fact, it seems to be a wave with front reaching crest in between 1 and 2 microseconds and decreasing to half value in about 40 microseconds. In making spark-over tests it is desirable to use a wave giving an impulse ratio of about 1.9. Unless this is done, the effects are little different from those caused by the 60-cycle arc-over. The high impulse ratio is a more severe test because it subjects the solid insulation of the insulator to much higher stresses. This follows because of the high arc-over voltage.

It is of interest to find that Mr. Torok now checks the results which we had obtained on the grading shield. His tests show that for the high impulse ratios the arc-over voltage of the string is materially increased, when the grading ring is used. The tests also show that better results are obtained with the hoop type of ring.

A. S. Brookes: In addition to the paper by Messrs. Conwell and Fortescue there are some further points about the surge impedance of grounds which should be considered at this time. Previous investigators have measured the surge impedance of grounds and found that a ground may be considered as pure resistance. One thing they have not considered is that the transmission engineer is not primarily interested in the ground impedance but in the impedance of the ground plus a tower. Consider the action of a tower footing in lowering the voltage of a surge on a ground wire. The tower may be considered as a conductor of appreciable length connecting the footing to the ground wire. When a traveling wave hits the top of the tower, a reduction in voltage takes place and a surge travels down to the ground taking a finite time to reach it. If the footing is of lower surge impedance than the tower, a negative reflection takes place. When this reflected wave strikes the junction, it causes an increase in current and a reduction in voltage. This process of wave reflection continues back and forth until a steady state condition is established some time after the original wave reached the tower.

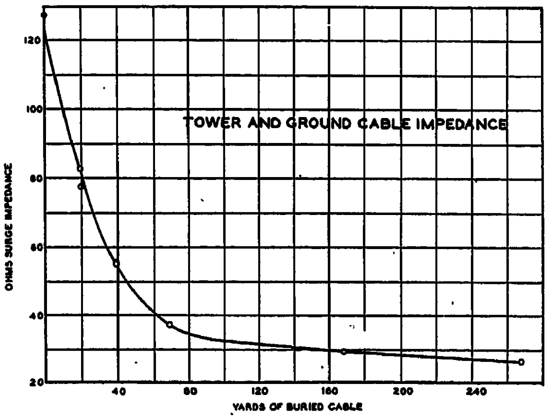


FIG. 4

Tests were made at Stillwater to determine the effective surge impedance of the tower and footing combined. These were made by measuring the reduction in voltage of a surge passing the tower on a conductor of known surge impedance grounded to the tower. Tests were made at different times and as close agreement found as could be expected with the amount of variation present in the moisture content of the soil. The surge impedance seems to be about twice the meggered resistance but the agreement is not close enough to be of much significance.

The method of using a buried footing tie or "counterpoise" is of interest as the action of such a cable is unlike that of a ground concentrated at one point. The cable may be considered for purposes of analysis as connected to the ground through a number of high resistances. When the resistance of such a combination is meggered, a low value is obtained, as might be expected because a steady state condition is reached and all the resistances carry practically equal current. Under surge conditions, a finite time is required for the reflection from each assumed resistance connection to travel back to the top of the tower and it might be expected that only the part of the cable near the tower would be effective, due to the long time required for reaching the steady state condition. This analysis considers the cable only as a path to ground and neglects any "counterpoise" effect. Further tests are really needed to determine if the latter effect is of importance.

Table I in the paper by Messrs. Conwell and Fortescue shows that the cable does reduce the tower surge impedance very greatly. The relatively small effect on tower 41 may be explained by the fact that here the cable ran along the top of rocks for about a hundred feet before entering the ground. Tower 39 showed the impedance with the cable to be about half of that with the cable disconnected. Evidently the cable lying in the earth near the tower footing exerted some effect, even when disconnected, because tower 40 showed an impedance of 115 ohms when the cable was disconnected from the base of the tower and when the entire cable was actually removed from the earth, an impedance of 128 ohms. On this tower the improvement in surge impedance effected by the buried cable was in the order of 6 to 1.

Tests were also run at Stillwater to check the belief that it was only the first part of the cable which was effective. Measurements were made with various lengths of cable and the curve of the surge impedance plotted against length of cable used is shown in Fig. 4. This shows that with surges of the duration used in the tests, the first 80 yards of the cable are by far the most effective portion.

These tests indicate that the use of a cable buried six or eight inches and extending about two hundred feet from a tower footing parallel to the line conductors is a very effective means of improving high resistance grounds, and such a cable is probably cheaper and easier to install than a concentrated ground of equal impedance. Data are insufficient to attempt a complete theoretical analysis of the value of low impedance grounds but a partial analysis and some operating data seem to indicate that a transmission line with low impedance footings does have better ground wire protection and is more free from flashovers.

A. C. Monteith: The beneficial effect of ground wires on reducing outages has been demonstrated on a large number of lines throughout the country. There is some discussion as to how this gain is secured.

If it is accepted that all surges on high-voltage lines producing outages are caused by direct strokes, the line construction employed will be different from that used if surges arising from induced strokes are considered of major importance. Data gathered both in this country and in Germany indicate that the effect of direct strokes should be given first consideration in line and tower construction to build a line which will have a minimum of outages due to lightning. This point of view emphasizes the fact that ground wires should be located with respect to the line wires so as to attract and receive the lightning stroke. It further means that the ground wires will be placed above the line conductors and spread out so as to shield the line adequately. This point of view also stresses the importance of securing the lowest tower and footing resistance economically possible.

Two ground wires will usually give good protection for a twin circuit line with vertical configuration or a single circuit line with horizontal configuration provided the above mentioned factors are recognized.

Work of a theoretical nature has been done by Messrs. Fortescue, Atherton, and Cox and presented in their paper entitled *Theoretical and Field Investigations of Lightning* before this society last year. Based on this work and the results of the work done by Professor Matthias in Germany together with data on probability of frequency of lightning occurrence obtained from klydonograph records, curves have been prepared to show the effect of ground wires and tower footing resistance on the probability of outage of a transmission system equipped with various numbers of insulator units assuming that the line is adequately shielded.

In Fig. 5, Curve *A*, the assumption has been made that the maximum voltage in a direct stroke to the ground wire at the tower will be ten million. If the frequency of occurrence of the ten million volt stroke is taken as unity the probabilities are that N squared times as many surges of $1/N$ times this value will be experienced. Unity may be one or more. Curve *A* intersects the voltage line at ten million with the probability of occurrence as unity. The slope of the curve is such that the probability of a 5-million-volt surge is four times unity and of a one-million-voltage surge 100 times unity.

From Fig. 6 which is Fig. 24 published in the Fortescue, Atherton, and Cox paper, the values of voltages across an insulator were secured due to a direct stroke on the ground wire at the tower for various values of tower and footing resistance in ohm. The Curve *V* indicates the potential on the ground wires, in terms of ground impedance, for a stroke to the ground wire at the tower and in the mid-span. Those of *V* indicate the potentials on the line wire for the same conditions. Those of $V_2 - V_1$ indicate the potentials across the insulator string also for the same conditions. These curves show that next to placing the ground wires so that they receive the direct hit, the value of tower and ground resistance is the most important factor. The percentage reduction accomplished by ground wires in the case of direct strokes, even with reasonably low ground impedance, is very much greater than that in connection with induced strokes.

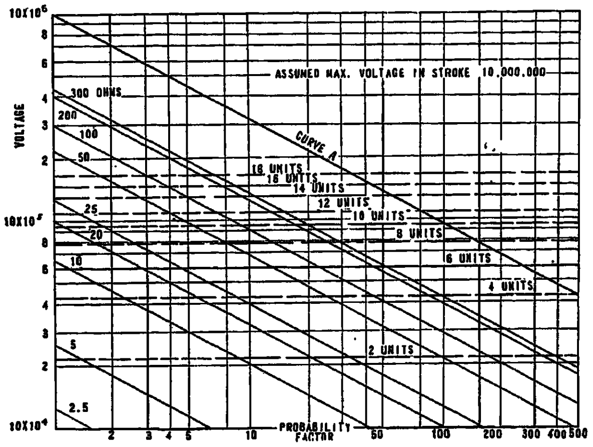


FIG. 5

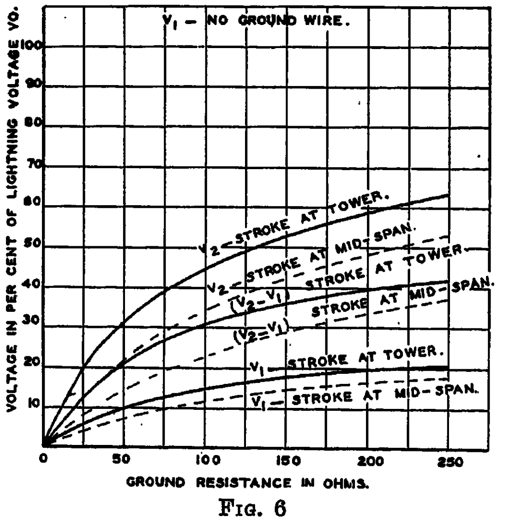


FIG. 6

The family of curves for various values of ground impedance are shown in Fig. 5 and give the relationship existing between crest voltage of the stroke at the tower and the voltage across the insulator string as well as the probability of such voltages occurring. The horizontal lines give the impulse flashover value of the various strings of standard 10-in. insulators at 15 microseconds when subjected to a flat topped wave similar to the Trafford wave.

The curves show that the probability of outage for a line having a 10-insulator string is ten times as great with 200 ohms surge impedance as compared to a line with 25 ohms surge impedance. Also the probability of outage for a line having 6-insulator and 25 ohms surge impedance to ground is 5 times as great as if fourteen units were used. These curves show very strikingly the large gain in utility of insulation made by keeping the tower footing resistance to a minimum. The values of surge impedance referred to are of the tower and footing measured without the interconnecting ground wire in place.

In Fig. 5 the assumption that 10,000,000 volts is the maximum that can be secured from a lightning stroke is open to question as no direct data are available to substantiate this assumption. However, theoretical work together with results, obtained in Germany, of current measurements in a stroke, indicates that this assumption is fairly reasonable. Fig. 7 is identical, in every respect, with Fig. 5 except that the assumption of maximum magnitude of the surge has been increased to 15,000,000.

The data presented in these figures are meant to present the picture of the value of ground wires and low ground impedance from a qualitative standpoint rather than from presenting quantitative results of actual data.

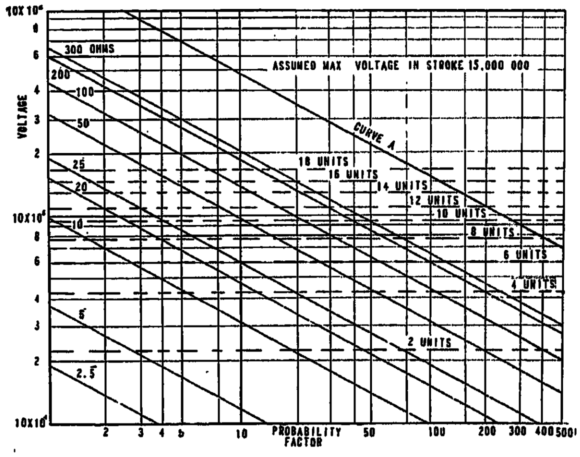


FIG. 7

With the tests already under way in the field and those contemplated for the next lightning season some very interesting data should be obtained to establish definitely the validity of the theories which have been advanced. The direct proof of the theories will have marked influence on the considerations that should be given weight in laying out the types of tower construction that should be used. If the theory of the direct stroke is further substantiated the curves in Figs. 5 and 7 indicate very strongly that extensive work is yet to be done before the designer can predict with certainty a lightning proof line.

Edward Beck: The paper by McEachron, Hemstreet, and Rudge, presents some data on the impulse characteristics of driven grounds, indicating that the apparent resistance of a ground when subjected to impulse voltages is somewhat less than the resistance as measured by the usual means such as by a megger, for instance. The writer has made similar tests on driven grounds and these are in rather good agreement with the results secured by Towne and McEachron, Hemstreet, and Rudge. The writer found in his test that grounds which indicated high resistance when measured by means of instruments showed a much greater reduction in apparent resistance when subjected to surge voltages than grounds which measured low in resistance. The results of these tests have been published in the proceedings of the Signal Section of the American Railway Association for March 1929.

The investigations of lightning phenomena on transmission lines have increased our knowledge of this subject and have already caused us to change some of our ideas, in some instances quite radically. Although it is not among the most important strides towards the solution of the lightning problem, one of the most radical changes in established practise brought about by the lightning research involves the choke coil. For some time past many people who have studied the choke coil theoretically have held that the type of coil in general use is of no benefit as an aid in lightning protection because it is too small and that if it is made sufficiently large to exert an appreciable influence on a lightning transient it is likely to produce secondary effects which may be harmful to apparatus unless taken care of by auxiliary protective equipment. Although this view was held by many, the lack of complete test data restrained the industry from accepting this opinion and from discarding the choke coil as a protective device because there naturally lurked in the background the thought that the theorists might have overlooked something. The lightning investigations have shed light on the general shape of lightning waves to be expected and on the behavior of choke coils. It quickly developed that the type of choke coil usually applied is in general neither beneficial nor harmful, in other words it is not economically justified. Thus a direct economic benefit to the operator has already resulted from the work done by the various investigating groups. In certain specific cases the application of choke coils is sound practise but where they are used they must be designed for the particular installation.

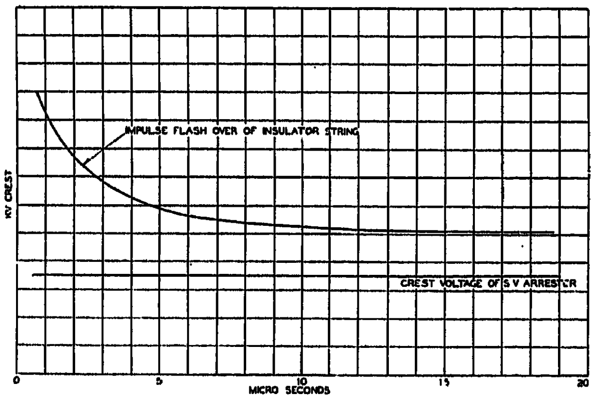


FIG. 8—COMPARISON OF LIGHTNING ARRESTER VOLTAGE AND IMPULSE FLASHOVER OF INSULATOR STRING

The choke coil situation represents one of the changes in ideas which the lightning research has brought about. With data now available on the performance of insulation and protective devices when subjected to impulse voltages and on the general nature of the lightning transients to be expected it appears to me that the time is ripe to reconsider another phase of the lightning problem. During the past few years much thought has been devoted to the proper coordination of line and apparatus insulation purely as an expedient to safeguard apparatus against damage by lightning. It has been proposed that an insulator string be used as a standard for this coordination and for this proposal there is a sound background. However, the impulse data now before us bring out the shortcomings of the insulator string for this office. The Torok paper presents data on the impulse characteristics of insulators. We see from it that the flashover of a string is not a fixed quantity but that it varies with the nature of the applied impulse (Fig. 8). If we say that the flashover or impulse ratio of a string or a gap is thus and so, we must also specify the wave for which this holds, and this gives us no data on the flashover of the string for another wave unless we have before us the entire impulse flashover curve. Thus we have a standard which is a variable and which, by the nature of its high flashover on steep waves may pass on voltages which can be dangerous to apparatus. Also it is known that other insulation does not necessarily parallel the characteristics of a string preserving the same margin; on the contrary, the characteristics of the various types of insulation that enter into system design and must be considered in the coordination may even cross each other. It would be desirable that the standard chosen have a fixed characteristic independent of the wave shape. The value to which modern valve type lightning arresters will limit lightning voltages is practically a constant quantity for the range of waves that occur. This is known from tests on arresters ranging to the largest yet installed. Moreover arresters will limit the voltages to lower values than the proposed insulator strings. The arrester is the most efficient voltage limiting device that is available and the one that must be considered. With standard voltage class insulation in a station and modern valve arresters of that voltage class properly applied it is immaterial to what degree the lines may be overinsulated, the voltage which reaches the station is limited to a definite value by the arrester.

Hence the writer would propose that in the coordination of insulation serious consideration be given to a standard based on the impulse characteristic which has the advantage of being a constant and of being the base, namely that of a lightning arrester as it is used today.

In the projection of a system one of the first things that is determined is the operating voltage. This fixes the ratings of the arrester to be used which in turn fixes the transient voltages that are permitted at arrester locations. The insulation employed on the system can then be based on this.

It may seem somewhat early to suggest the use of lightning arrester characteristics as basis for the coordination of insulation as there still exists some scepticism of their performance and reliability. However, with actual test data before us it appears to the writer that the characteristics of lightning arresters present features which make them more suited as basis than insulators and I am confident that as the lightning research progresses and data continue to be published, the difference between the two will become universally apparent. Moreover the arrester is proposed merely as a basis for coordination. If insulator string or gaps are preferred as protection in some cases there is no reason why they should not be used. This is a matter of economics. However, I would suggest that they be coordinated with the other insulation through the medium of the arrester characteristic.

Harald Norinder: I have read with great interest the paper of J. H. Cox and Edward Beck on the subject of *Cathode Ray Oscillograph Studies on Lightning on Transmission Lines.*

I was particularly interested in seeing whether the records of lightning surges on transmission lines in the United States would show some difference in their general structure from those of surges recorded with similar instruments at the research station of the Royal Board of Water Falls, in Sweden. From theoretical considerations I was unable to discover any reason for a difference in the structure of the surges experienced on transmission lines in Europe and in America. The general physical state of a thunder cloud must be the same in regions not included in the equatorial belt, where the conditions are probably quite peculiar.

However, it does not follow from this that we might be able to use a few records as a basis for a "standard" lightning surge. Physical structures and conditions during thunderstorms are too variable to produce some uniform type of lightning surge.

In Sweden at the research station of the Board of Water Falls during the summer of 1929 two cathode ray oscillographs were connected to transmission lines of either 20 or 80 kv.

A comparison of the records secured at the Upsala station with those presented in the paper by Messrs. Cox and Beck show good agreement. The general duration of the surges and the time to reach their crest values are of the same order.

The absolute crest values in kilovolts are higher in the case of those described by the Cox and Beck paper than our own This difference is brought about by the difference in insulation on the respective lines. We have recorded our surges on moderately insulated transmission lines of 20- and 70-kv. systems and all surges with higher amplitudes have been lost because of the flashovers they produced.

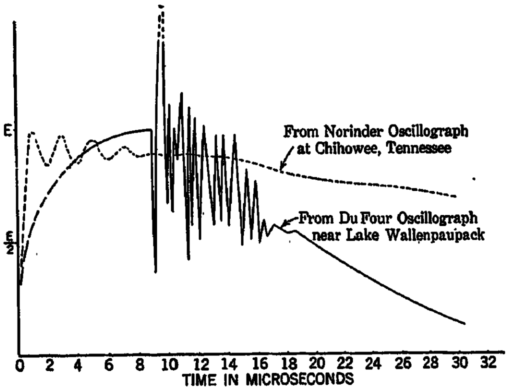


Fig. 9—Lightning Waves

Detailed results of our lightning surge records in Sweden during the season of 1929 will be published in the *I. E. E. Journal*, London, 1930, and the *Bulletin de la Societé Francaise des Electriciens*.

C. L. Fortescue: There are two things we want to avoid in an investigation of this character. First is jumping at conclusions, and second is wishful thinking.

Mr. Peek makes a plea for the short wave, of 20 microseconds on the basis that it is close to the average of the waves observed.

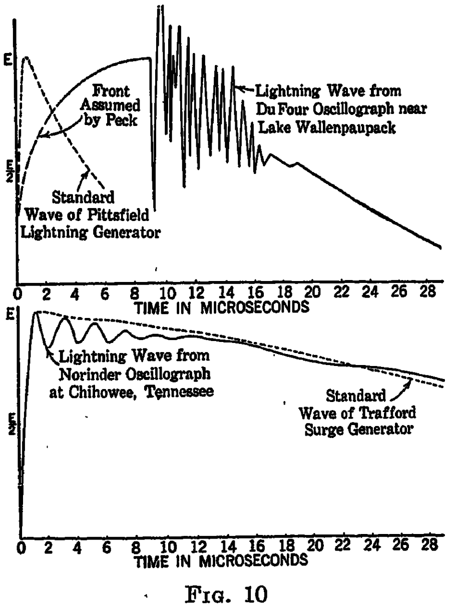


Fig. 10

I might say that if we wanted to design a battleship to withstand gunfire, we wouldn't take the average battery; we would take the largest guns built and would try to design our armor to withstand the impact of those guns. When we are designing transmission lines, what we want to do is to make that transmission line as nearly as possible immune from lightning and therefore we ought to consider the most severe type of wave that is possible due to lightning.

These waves shown in Fig. 9 were taken during the season of 1928. In the first waves the front was indeterminate but you will notice the Tennessee wave here is of considerable length. After 30 microseconds it is only reduced to about 80 per cent of its original value. This other wave, the Wallenpaupack wave, rises to a very high value and then drops off. It is admitted that for that wave there was a flashover of an insulator on an adjoining line. The result of a flashover on an adjoining line is to dissipate part of the surge in this other line. Consequently, the probability is that if it hadn't been for the flashover that wave would have been of about the same character as the Tennessee wave.

Fig. 10 shows the Tennessee wave, the Wallenpaupack wave, the 5 microsecond wave, and the Trafford wave. The wave we are using in our laboratory follows very closely the outline of the Tennessee wave.

Some of the waves we obtained at Stillwater can be seen in Fig. 11. This first wave, obtained on the Wallenpaupack line, went up to an exceedingly high value flashing over the insulator. You will notice there that in spite of that flashover there is so much energy left that the voltage is maintained for a considerable time. This other lower wave you see follows closely the same characteristic, and it is only reduced to 80 per cent of its value. So you can't say that the ordinary lightning wave is a short wave.

I am under the impression from the paper by Mr. Smeloff that a large number of those waves cause flashover some distance before they reach the cathode ray oscillograph. In such cases they are chopped. You can't define them as an actual lightning wave after they are chopped. What we really want is an actual lightning wave as it appears on the line at the point where it happens without any interference due to flashover of insulators or anything else. That is the reason why we seized the opportunity of doing work on the Stillwater line, which is the highest insulated steel tower line in the United States.

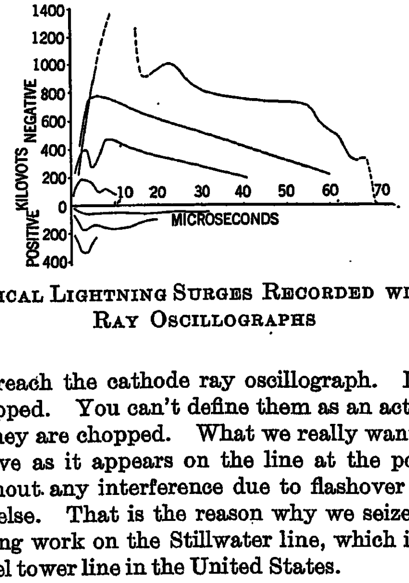


Fig. 11—Typical Lightning Surges Recorded with Cathode Ray Oscillographs

Just a few words about some of the other papers. In Mr. McEachron's paper he mentions the attenuation with ground wires. He says in his conclusions that if that is general, it is an important thing to know that the effect of ground wires is to reduce attenuation, but I don't think that is general. I think that the attenuation will depend a good deal on the surge impedance of the ground.

I drew the conclusion a year or so ago that the effect of the ground wire would be to increase the attenuation. However, the effect of it in the decrease in attenuation, if it is there, is not serious as far as the operation of the line is concerned.

I think the lightning stroke recorder is a good idea. We should use it without jumping to conclusions. I think our friends have jumped to conclusions. I don't think that the method of using it will give the value or even an approximate value of the lightning surge current.

As regards the question of direct stroke versus induced stroke, there is no real controversy there at all, except in this respect, that as far as high-voltage lines are concerned, the group with which I am working has disregarded the induced stroke as a cause of outage, and we are concentrating on trying to find what the maximum value and wave form of direct stroke is. However, there is some evidence in Messrs. Lewis and Foust's paper which seems to show that a considerable number of the voltages producing outages are due to direct stroke, possibly all of them, I don't know. The fact that the transmission line on the storm side gets more outages I think is very positive evidence that the large number of the outages are due to the direct stroke.

F. E. Andrews: I wish to discuss the four papers which had to do with the investigations of actual lightning on transmission lines. The experience which has been obtained on the lines of the Public Service Company of Northern Illinois I believe should add some weight to the information which has already been given in these papers, and in some respects may be considered somewhat more conclusive than some of the data which is given in these papers.

I will refer to three 132-kv. lines originally constructed without ground wires. They are insulated with unshielded strings with a 60-cycle flashover of approximately 570,000 volts r. m. s. These three lines operated for two and three years, depending upon the line concerned, without ground wires, and during that time of operation without ground wires there were 115 outages due to lightning. In other words, with 174 year miles of operation, we had 115 outages, or 66 outages per 100 miles per year. After the installation of the ground wires on these same lines, the record was approximately 1.2 per 100 miles per year; 163 year miles with 2 lightning outages. During the year 1929, for 297 miles of lines operated we had 3 lightning outages, or approximately 1 per 100 miles per year.

The approximate value of lightning surge voltage gradient which we estimate would cause flashover, when the lines were without ground wires, was 20 kv. per foot. That is based on an impulse ratio of 1.8. The estimated voltage which will give flashover with ground wire is about 37 kv. per foot. I therefore believe that we can assume most of the 115 outages due to lightning were caused by lightning surge voltages between approximately 20 and 37 kv. per foot. The performance after the installation of ground wires indicates that the lightning surge voltages exceeding 37 kv. per foot on these lines are of the order of 3 per 100 miles per year.

This brings up the question of the extent to which it pays to go to voltage gradient such as 100 kv. per foot, ordinarily spoken of as the maximum gradient which may be expected. It is a question whether the small number of outages occurring at voltages above the flashover values which I mentioned justifies the expense of insulating, or otherwise designing, to withstand such voltages.

The question of induced strokes versus direct strokes is pertinent. To us it doesn't make very much difference whether the strokes which caused the outages on the line before the ground wires were installed were induced or direct. It appears that voltages were experienced of such a character that they would flashover an insulator string of approximately 570 kv., 60 cycle flashover value. After the ground wire was installed the protective ratio and character of the lightning were such that we did not get flashover except in rare instances. We feel that this experience gives us a reasonably satisfactory basis for the design of our lines.

H. L. Melvin: Referring to the section of the paper by Mr. Torok dealing with wood insulation, I am of the opinion that the dashed portions of the time lag curves should show decreasing values to 8 or 10 microseconds similar to those for other types of insulation and even to a greater degree for wet wood. It is also believed that the failure to obtain breakdown with the longer durations might be accounted for by the limitations of the laboratory lightning generator, or with the longer durations, breakdown may have occurred within the wood section rather than along the surface, thus not being observable. However, if a cathode ray oscillograph was used in making the tests, these results are very interesting.

During the past two years several lines, utilizing the impulse insulation of wood in an experimental way to a greater degree than any lines heretofore constructed, have been under special observation and test. Considerable damage has been done to unprotected sections of poles and crossarms, indicating terrific power in the lightning discharges. These flashovers probably occurred on the front of the wave, in many cases at least, as voltages in excess of 200 kv. per foot of length of wood were recorded a number of times by surge voltage recorders. We are also of the opinion that most of them were due to direct strokes and side flashes. These lines, some of which are insulated as high as an estimated equivalent of 25 to 30 suspension units, do not hesitate to flashover and it is common experience to find 40 to 50 ft. of pole flashed, which, based on the test data, would be the equivalent of more like 50 units or about 20 ft. of air. These values interpreted in times of normal crest voltage to ground are, of course, greatly in excess of the commonly discussed values of 10 to 15 times normal. For example, on a 13-mile section of 66-kv. line in Kansas utilizing the impulse insulation strength of 7 ft. of pole, values of the order of 1800 kv. or 35 times normal were recorded four or five times in 1929.

It is hoped that with another year's experience on these lines and others which have been constructed, that enough will have been learned about wood as insulation and its protection to use it intelligently, though it may not be possible to estimate with any degree of accuracy the relative value of such increased impulse insulation as reflected by improvement to service.

In comparing the data given by Mr. Torok with that in the paper by the speaker, which was presented at the Pacific Coast Convention, it will be noted that spark-over voltage values for time lags of between one and two microseconds compare quite closely with the values obtained on a variety of samples of wood under various conditions of moisture and contamination for tail of wave flashover using the so-called Pittsfield 20-microsecond test wave.

Regarding many of the data contained in the other papers, I wish to enter a plea that lightning voltages be expressed in actual kilovolts rather than times normal. It might also be emphasized that unless the recorder was at the point of origin of the surge and unless spark-over did not occur, the reading obtained is not necessarily a record of the maximum voltage appearing on the line for a given lightning discharge. Whenever insulator spark-over does occur the reading is really a measure of the strength of the insulation to lightning voltages. It therefore follows that the curve between frequency of occurrence and magnitude of lightning voltages cannot be plotted with any degree of confidence on account of the limited ranges of insulation and instruments used in recording the voltages.

H. B. Vincent: We have been hearing a lot about direct strokes. Mr. Fortescue said there was no controversy. So that there will be no confusion in some of our minds I should suggest that various authors using that word give us a definition.

H. K. Sels: In summarizing the results that have been obtained to date at the several field lightning observation stations it might be well to consider the knowledge which is needed to develop the maximum ability of transmission systems to withstand lightning. The three phases of the problem as stated by Messrs. Cox and Beck should be restated to include another important factor, the characteristics of lightning storms. Broadly speaking the three fields which must be covered are:

1. The mechanism of lightning and its influence on a transmission system whether by conduction, induction, or both.
2. The characteristics of the greatest electrical forces which may be imposed on transmission systems and the behavior of such forces on the system.
3. Remedial measures which may be used to protect, relieve, or withstand these forces without interruption to service and damage to the system.

About four years ago there was proposed a research committee between a group of operating companies and manufacturers similar to that now existing in Germany but unfortunately it was not organized. As a result and with some duplication several independent investigations have been initiated and while considerable knowledge regarding the characteristic wave forms and propagation of lightning surges on a few existing lines has been gained, many of the fundamental ideas are purely speculative and theoretical. Actual knowledge therefore appears limited to a relatively few isolated and specific cases and so it is felt that coordination of the investigations has not been of the best. A certain amount of duplication may be excusable to bring out all the facts.

Up until the past year practically nothing had been done to investigate the mechanism of a cloud discharge except for a few simple mechanical observations with rotating or swinging cameras. Even this simple means shows that a cloud discharge may be composed of several lightning flashes occurring for periods up to 0.6 second or more. During the past year greater activity has been evidenced in studying storm movements and characteristics by a systematic recording of weather data in one local area, installation of electrostatic devices for measuring field intensity, and rate of field change and direct stroke recorders. These investigations show that lightning storms are extremely local, that several individual storms may be in progress over a line at one time and that the present practise of using less than 100 microsecond sweep circuits on oscillographs will register only a small part of the phenomenon. Therefore, a great deal more remains to be done to improve the technique and give a complete record. It has been suggested that more attention be paid to both electrostatic and electromagnetic fields, particularly their directions, intensity, and rates of change correlated with the location of the lightning discharges. What happens below the ground plane should not be neglected. A sufficient number of stations should be used to determine the distribution of these various factors giving a complete picture of the magnitude, rate, and extent of a cloud discharge. Practically nothing is known of the sphere of influence of the main stroke and whether lightning is conducted upon the transmission line by the main or a branch stroke or is induced. At present many favor the conduction theory but the data are not conclusive and the deduction may be wrong when applied to all cases. The mechanism of the lightning getting on the line should be determined, particularly when ground wires are used. In this connection a simple device is needed to record flashovers differentiating if possible between surge and power current as the correlating of flashover, surge voltage, and trip-out records has been very difficult and doubtful.

Next to knowing how the lightning gets on the transmission line comes the measuring of its characteristics. The key of this situation seems to be to locate the field observation station so that the lightning can be measured and observed at the point where it strikes the line. After the surge gets on the line comes the ability of the line to hold and propagate the wave. It has been shown that the behavior of a surge on the system can be studied in the laboratory or the field with artificial surges, particularly after the characteristics of the lightning discharges have been determined. Volumes of surge voltage recorder data have been collected and oscillograph records of waves are becoming common but at present the characteristics of the greatest electrical forces that can be imposed upon the system is not known at its source. This is partly due to the location of the measuring instrument but also to the shortcomings of the potentiometer coupling devices and shielding of recorder leads. Mention is made in the papers of questionable recorder operation due to stray fields, corona, or leakage currents so that some part of the data at the higher voltages may be misleading. It is imperative that improved methods and technique be developed to eliminate existing discrepancies as the most accurate instrument in the world is worthless with an uncontrollable multiplying ratio.

Various remedial measures may be applied to the transmission system to improve its performance. The method of using artificial surges in studying the characteristics of different transmission line and apparatus designs has been successfully demonstrated. On this account there has been considerable agitation for the adoption of standard test surges based on the meager data now available. There is also considerable controversy as to whether system insulation fails on the front or tail of a wave. Whatever assumptions are adopted it should be recognized that they should be used as such for uniformity of tests and they may be changed annually to conform with each year's developments. In the last analysis whatever remedial measures such as ground wires, counterpoises, routing of lines around storm centers or paths, additional insulation, arcing and protective devices, improved grounding, line type arresters, etc., are adopted as the result of field or laboratory tests, the actual performance over a period of years should be carefully recorded and considered the final criterion. What the operating engineer ultimately desires in this problem is sufficient design data so that he can survey the storm and physical characteristics of the territory to be covered and design a system to give a predetermined service in the most economical manner.

In conclusion the authors and companies represented deserve every credit for the initiative and thoroughness in conducting this season's investigation which under the program outlined here may take several more years to complete satisfactorily.

A. H. Schirmer: A rapid review of the papers indicates that many of the conclusions of past years will have to be revised in order to bring them in agreement with the more complete data now available.

It may be of interest to outline briefly some of the lightning investigations which have been carried out on telephone lines. In the past, the surge recorders principally used were two-electrode machines with stationary plates and a number of four-electrode machines using a slow moving film. With these recorders we could not be certain after an electrical storm whether the figures were the result of a single stroke of lightning or whether they were the composite effect of several strokes.

In order to overcome these difficulties, a surge recorder was designed early in 1929, which employed a motion picture film which could be made to pass under an electrode at a relatively high speed so that individual figures could be obtained for each lightning flash. In order to permit direct comparison between films from different recorders, all films were started and stopped simultaneously by means of relays operated on a (common) telephone circuit. In order to prevent flashover at the electrode, the dielectric was made considerably wider than the film. The film was driven by 24-volt motor operated by two 12-volt storage batteries.

As originally designed, the film speed was 5 ft. per minute. Later on this speed was reduced to 2 ft. per minute. As the film magazines have a capacity of 200 ft., each surge recorder could be kept in operation for at least an hour and a half.

Six of these new type surge recorders were connected to antennas erected at right angles to a telephone line in Western Maryland. Each antenna was 135 ft. long and approximately 20 ft. above the ground. These antennas were located at intervals of one and a half miles. The purpose of the test was to obtain an indication of the area affected by an individual lightning stroke. The recorders were in service during 14 storms in that location. A number of these storms did not give any figures on the film. The interesting feature, however, is that in no storm did a single stroke of lightning give a figure on more than one surge recorder although several of them showed figures from a single storm.

It is planned to construct several more of these new type surge recorders and to continue the tests on antennas and also to make tests on various types of telephone plant to determine the magnitude and attenuation of lightning surges.

E. W. Boehne: (communicated after adjournment) In several of the papers presented considerable interest was shown in the use of a conductor imbedded in the ground between tower footings. The exact function of such a counterpoise in reducing line flashovers is not known; however, we have two interesting facts on record which, it is believed, will help our understanding of this new protective agent.

We learn from a reliable operating company who preserves an accurate record of their line flashovers, that a section of one of their 66-kv. lines which parallels a railroad has had only a very few flashovers in a period of several years, whereas a section of this same transmission line which is not near the railroad suffered many times as much grief in the same period. On measuring the tower footing resistances it was found that the average resistance of the section near the railroad was 4 ohms in contrast with an average resistance of 25 ohms in the section remote from the rails. The line in question was equipped with ground wires.

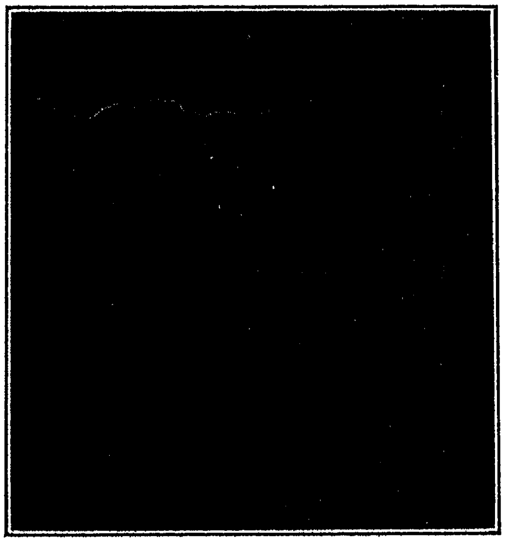


Fig. 12

A similar case is on record in which one of the 220-kv. lines in the East which parallels a large river has never had a flashover.

The action of the steel rails and the river in the above examples is believed to be a counterpoise action. They both reduce the tower footing resistances enabling the ground wires to be closer to ground potential. A second advantage is that they undoubtedly raise the effective ground plane, increasing the capacity of the conductors to ground.

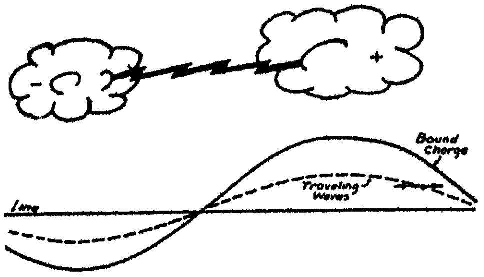


Fig. 13

It is the firm belief of the writer that similar cases are on record which have never been investigated. Additional information would be valuable in building up our knowledge concerning this phenomenon, and the writer wishes to take the initiative in requesting that any data similar to the above mentioned cases if sent to Institute Headquarters will be duly appreciated.

The last lightning season has been particularly fruitful in actual records of lightning surges. On investigating these records one will find many oddly shaped waves, some with reversed polarity, (Figs. 9 and 10 of Mr. Smeloff's paper) others with two unidirectional humps in the same wave.

Fig. 14

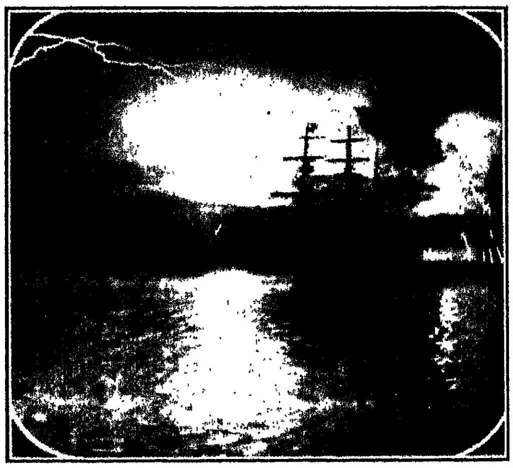


Fig. 15

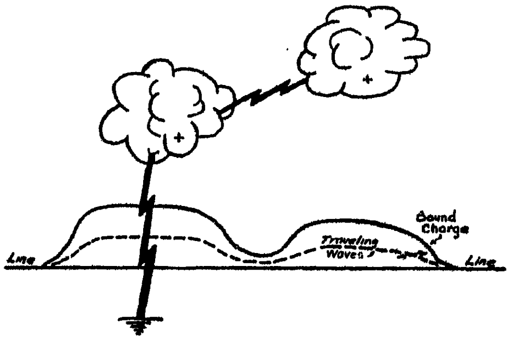


Fig. 16

It is the purpose of this brief discussion to suggest three possible ways to account for these queerly shaped waves.

The first and most probable explanation is that they have their origin, both of magnitude and shape, in the lightning discharge. We have all observed lightning flashes between clouds without observing a simultaneous discharge to ground. Such a discharge, an example of which is shown in Fig. 12, must be due to the presence of both positively and negatively charged clouds in the atmosphere. Consider the case of Fig. 13. Here the presence of the positively and negatively charged clouds produced a bound charge of reversed polarity on the conductor. When the discharge between the clouds takes place a traveling wave of half this magnitude travels in both directions on the line.

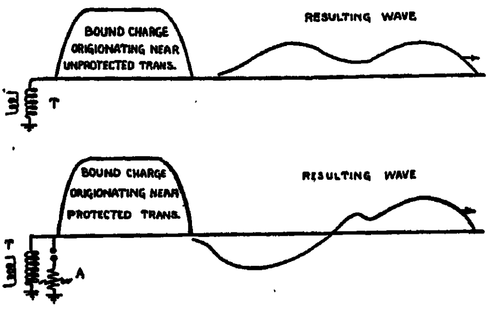

BOUND CHARGE ORIGIONATING NEAR UNPROTECTED TRANS.
RESULTING WAVE
T
BOUND CHARGE ORIGIONATING NEAR PROTECTED TRANS.
RESULTING WAVE
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Fig. 17

Figs. 14 and 15 show two pictures of lightning discharges, the latter being taken in Japan. These show that the lightning phenomenon is not always a simple discharge. Consider, however, the simple case of Fig. 16 in which two positively charged clouds discharge either separately or in series to ground.

In a similar fashion the bound charge is released and in this case gives rise to unidirectional waves having two peaks.

In Fig. 17 is shown a single bound charge which originated near a line terminal. Here, on the release of the charge the waves disburse in both directions. However, the wave traveling to the left is immediately reflected, depending upon the terminal impedance. In this way we can account for all the types of waves recorded.

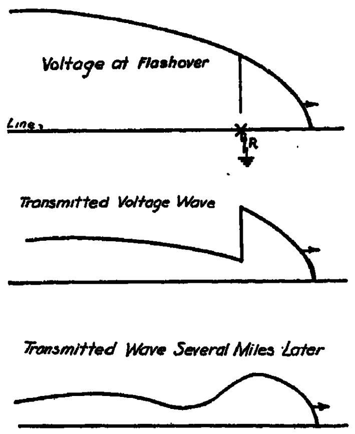

Fig. 18

A third way to account for waves with double peaks is by an insulator flashover. It is demonstrated in Fig. 10 of Mr. Smeloff's paper as well as in another example mentioned by Mr. C. L. Fortescue, that the resistance of such an arc is not always zero. Assuming, therefore, that a large wave originates on the line, Fig. 18, and that a flashover occurs having a definite resistance, this resistance to ground will form a transfer impedance allowing a portion of the remainder of the wave to pass the arc. The evolution of such a wave is shown in Fig. 18. These voltages are undoubtedly strengthened by induced waves from traveling waves on conductors which did not discharge.

J. K. Hodnette: (communicated after adjournment) The field investigation of lightning reported in the symposium on lightning investigations has shown with engineering precision the shape of surges existing on transmission lines, both with respect to the front or rate of rise and the duration or length of tail, and the magnitudes resulting from natural lightning. This information is invaluable in making laboratory tests, since with it the conditions experienced in service can be simulated in the laboratory and the performance of apparatus determined under practical conditions. This reflects immediately in the tests on transmission line insulation as the line and substation structure are most susceptible to lightning strokes and their insulation is relied upon to limit the surge voltage on the system. By assuming the worst conditions for a particular case as indicated by the field tests, the safe performance of insulation can be predetermined from laboratory tests as Mr. Torok has demonstrated for line insulation using a steep front and long tail surge.

Of no less importance is the effect on the study of substation apparatus and particularly transformers which comprise the most valuable substation apparatus subject to line surges. It is essential here to coordinate and correlate field investigations and laboratory tests on transmission line insulation with the tests on the terminal apparatus, because the surges impinging upon the latter are derived from those existing on the line. An investigation of this nature was made last year and reported in a paper before the Institute under the title *The Effect of Surges on Transformer Windings*. In this paper it was predicted that the maximum voltages expected from lightning were 14 to 15 times normal and of short duration and the longest lightning waves were in the order of or less than 60 microseconds to half value. It is a pleasure to have these statements in my paper confirmed as to the magnitude and duration of lightning surges and that, therefore, the magnitude of the voltages resulting from them within the transformer winding will be of the order previously estimated. This further indicates that modern transformer design can progress along sane and logical lines, in step with our increased knowledge of the stresses to be encountered.

E. J. Amberg: One phase which has not been mentioned in either the paper or the discussions is the collection and systematic recording and analysis of statistical data by individual power companies in regard to frequency and general movement of lightning storms and also data in regard to line outages and insulator flashovers due to lightning. As far as the latter are concerned, most companies keep some kind of a record and I am sure that with very little additional work such records could be put in a form in which they will be very valuable later on.

With the progress which has been made in the last two years in lightning investigations, it is quite evident that within a few years concrete recommendations can be made for the construction and operation of lines which will be practically lightning proof.

The important question then will be to what extent can these recommendations be applied economically to new and existing lines, and in order to obtain the greatest benefits for the smallest amount of money it will be necessary to have reliable statistical data as to lightning storms in general and as to performance of existing lines during lightning storms.

As far as new lines are concerned, many of the recommendations for a lightning proof line can be incorporated in the original design at little extra expense; whether such new lines should be made lightning proof for the entire distance will depend entirely on service requirements.

When it comes to improving conditions on existing lines, the problem will be more difficult because the necessary changes will nearly always be expensive. Economic consideration will make it necessary in most cases to confine reconstruction for lightning protection to short sections of the line.

C. A. Jordan: The results of the 1929 lightning investigations have materially advanced understanding of lightning surges and wave propagation on aerial transmission circuits.

In fact, it seems apparent that the progress last year in this branch of study, at least, has been greater than in any previous year. Particularly encouraging is the excellent agreement in the findings of different groups of investigators employing artificial surge generators to study the various factors governing the propagation of traveling surges. While the advance in knowledge of lightning itself has been less satisfactory, the new instruments and technique which have been and are being developed offer promise that in relatively few years enough additional information will have been secured from field studies of natural lightning to make lightning control merely an economic problem.

The measurements at Croton Dam, Michigan, indicate that the transient impedance of tower footings may be of the order of 67 per cent of the steady state impedance. At Stillwater, New Jersey, other investigators using a different method found that transient impedance of towers and footings combined exceeded the steady state impedance by 90 to 187 per cent. The presumption is that this difference in results is due, at least in part, to tower impedance. In view of the fact that most power companies are limited to steady state methods in measuring tower footing impedance, and can make no determination of tower impedance, it is important that the relation between transient and steady state values of footing impedance be established clearly.

While on this subject, it may be well to point out that while tower footing impedance undoubtedly has a very important influence on what happens when towers or ground wires intercept a direct stroke, if the dimensions of the intercepting metallic path are great enough to contain one-half the front of the resulting surge, the metallic path will exhibit characteristic surge impedance to the lightning discharge. In this case, the impedance of the nearest grounding point or tower footing will have no effect on the crest value of potential to ground which will occur. Thus, if on a transmission line equipped with ground wires and having an average span length of 1000 ft., lightning struck a ground wire in the middle of a span and the discharge current rose to its maximum value in one microsecond or less, the potential from ground wire to earth at the point of contact of the stroke would become $0.5\ i\ Z_o$ before the effects of low impedance at the neighboring towers could become apparent.

It is hoped that during 1930 it will be possible for the various investigators to gage more closely the lightning discharge currents which may occur and their rates of change with time. I believe these two factors, together with the coupling between ground wires and line conductors (about which the Stillwater and Michigan investigations furnished some information), contain in large measure the key to the question of whether lightning-proof lines are economically feasible.

I have some difficulty in reconciling the theory on the second ground wire effect outlined in the McEachron, Hemstreet, and Rudge paper with the results of the coupling measurements at Stillwater, New Jersey. This theory is based on the assumption that a traveling wave in one wire will induce a traveling wave of opposite polarity in an adjacent wire, whereas the Stillwater measurements show that the original and induced waves have the same polarity. In measuring surge potentials with sphere-gaps as was done in Michigan, it is of course not possible to determine polarity. However, the accuracy of the numerical results should not be affected. If this is the case, the information obtained in Michigan regarding the effects of ground wires on free traveling waves indicates that some of our previous conceptions were incorrect, and offers considerable food for thought in connection with the installation of discontinuous ground wires adjacent to terminal stations.

While progress in the studies of natural lightning has of necessity been slower, the various groups have been fortunate in obtaining a number of oscillograms of lightning surges. Although, due to the limitations of line insulation, it does not seem probable that the extreme lightning potentials can be recorded on transmission lines of existing types, these oscillograms represent an encouraging start in the investigation of surges which may be imposed on terminal apparatus.

However, all the present records are of surges which originated at some distance from the oscillographs. These surges were at lower voltage levels than are known to have appeared on the respective circuits on which the oscillograms were recorded, and undoubtedly in their travel to the recording stations they were considerably modified in shape. There is every reason to believe, from the evidence at present available, that at the point of surge origin the amplitude and steepness may both be much greater than any recorded to date. I wish, therefore, to emphasize the note of caution expressed in the closing paragraphs of the Cox and Beck paper and in the preceding discussions. Present knowledge of possible lightning surge wave-shapes is too limited to make it advisable at this time to standardize wave shape or shapes for testing insulators and apparatus.

K. B. McEachron: (communicated after adjournment) The paper by Mr. Torok is, I feel, a very valuable contribution to the subject of flashover of insulation under impulse conditions. In approaching this subject, we have to distinguish between two very different conditions. One is the breakdown of insulation on the front of the wave, and the other is the breakdown on the tail of the wave. This wave tail may be nearly horizontal, or it may be sloping, which will, of course, give rise to somewhat different results.

There can be no question that flashovers in practise do occur on the front of the wave, and therefore it seems to me that it is very desirable that a study of flashover of insulators be made on the front of the wave as well as on the tail. Work of this character was done as early as 1915 by Mr. Peek but should be repeated and extended now that the cathode ray oscillograph is available. Such a study was made for two and three suspension units, and for various ratings of pin type insulators, by Wade and Smith, and reported in the August 18 issue of the *Electrical World* for 1928. These tests should be extended for still higher voltages so that the insulator flashover values on the front of the wave may be well established. In general, the agreement between the Wade and Smith results and those as shown by Torok, is quite satisfactory when it is remembered that one is based on the front of the wave breakdown and the other on the tail of the wave.

On the first page of Mr. Torok's paper, he states that the time lag is measured from the point where the voltage first exceeds the 60-cycle crest breakdown value. I do not agree that this is the proper point for measurement, for if breakdown occurs on the front of the wave a certain amount of confusion is likely to arise. For instance, with a rise of voltage equivalent to the 60-cycle rate of rise, the time lag will be zero, whereas with an infinitely steep wave the time lag will also be zero, which gives an unfortunate relationship and does not express very accurately what is meant by the term "time lag." It seems to me that this confusion can be avoided by not using the word "time lag" at all, but instead refer to "time to breakdown," in which case the time is always measured from the beginning of the wave rather than from some arbitrarily established flashover value. Furthermore, the 60-cycle flashover value of insulation is not a fixed quantity. It depends upon a variety of conditions, such as humidity, etc., whereas the impulse flashover is less dependent upon such factors. The 60-cycle flashover value in the case of insulators which have different shaped terminals such as pin type insulators, depends on polarity which introduces a variation in the basis for determining the beginning of the time measurement if the 60-cycle flashover potential is used.

I would like to propose that a method of referring to waves be established, in which the time to reach crest voltage and the time to reach half value, be stated. For instance, a 5-60 wave would be a wave which rises to its crest value in 5 microseconds, and decays to half value in 60 microseconds—in each case time being measured from the beginning of the wave. If the crest voltage is stated in kilovolts, the wave is pretty well defined by such a characterization. This method, of course, assumed a straight line rise and a straight line decay when, as a matter of fact, both of these may be exponential or some other form, but at least it gives an average measure of the front and the tail and the crest value, which are the three factors in which we are most interested. Such a designation would avoid much of the confusion which now exists.

It is gratifying to note that the results obtained by Messrs. Conwell and Fortescue check so well with those we have obtained ourselves in Michigan.

In Fig. 5 of their paper, an oscillogram is shown giving the effect on wave form of the travel of an impulse along the line. I should like to ask the authors if they have any explanation for the change of wave shape as the wave passes along the line. One gets the impression from reading the paper that this wave shape change is due to the action of corona.

In Fig. 11 of the paper by Messrs. Smeloff and Price, is shown an oscillogram which was taken at the time the line was not energized, and shows the flashover of an insulator on the front of the wave in about 36 microseconds. Since this is "front of the wave flashover," the flashover potential should not be far from the 60-cycle flashover value based on the cathode ray oscillograph studies of Wade and Smith. In Table II this surge is shown as having a value of 1360 kv. at or near the point of inception. This represents an impulse ratio of about 1.2 above the 60-cycle flashover value of the line insulation. This oscillogram is very interesting as it does give fair agreement with the laboratory results of insulator flashover on the front of the wave.

In the investigation in the paper by Messrs. Sporn and Lloyd among other data, some information was secured concerning the performance of lightning arresters. The current through one leg of the arrester was measured, and also the voltage across the arrester. In the discussion, the following statement appears, "The data in Table IV show that the lightning arrester, in the case of switching surges has discharged a current of 226 amperes per leg at 450 kv. Since it is known that switching surges do not exceed some 550 kv. on these lines, there are no definite data to show that the arrester has held the voltage down appreciably on switching surges." It is interesting to point out in this connection that whenever a lightning arrester draws current from a transmission line, the voltage is necessarily reduced by the action of the surge impedance of the line. It is true that in arrester operation it is necessary frequently to distinguish between breakdown and $I\,R$ drop through the arrester. However, it is worth while to find out what the voltage reduction would be as a result of drawing a current of 226 amperes. The authors have not stated whether these results come from the end of the line or from an arrester located at a junction point on the line. It is therefore necessary to distinguish the two cases.

The first case is for the arrester at the end of the line. The equation for this condition is:

$$2\,E_1 = E_a + Z\,I_a$$

When E_a is the potential across the arrester, E_1 is the potential of the traveling wave on the line which is half that at the end of the line with no arrester. I_a is the current through the arrester, while Z is the surge impedance of the transmission line. Using 226 amperes for I_a, 500 ohms for Z, and 450 kv. for E_a, $2\,E_1$ becomes 560 kv. This shows a reduction of 113 kv. as the result of arrester operation.

For the second case in which the arrester is tapped to a transmission line, the equation is:

$$2\,E_1 = 2\,E_a + Z\,I_a$$

Using the same figures as in the previous substitution, we arrive at a value of 506 kv. for the voltage of the traveling wave, which voltage has been reduced to 450 kv. by the operation of the arrester. It is interesting to note that 450 kv., which is the voltage measured across the arrester during the discharge, is slightly less than 2.5 times the arrester rating. Referring to the two papers presented earlier at this convention dealing with lightning arresters entitled *Development of the New Auto-Valve Arrester*, by Messrs. Slepian, Tanberg, and Krause, and *Thyrite—A New Material for Lightning Arresters*, by K. B. McEachron, it will be found that a voltage of 2.5 times normal is a very satisfactory voltage for the arrester to go into operation, and is about as good as can be expected at the present time.

A. O. Austin: The fact that lightning strikes near the line without producing a serious disturbance does not necessarily prove that a transmission line will not be subject to flashover due to induced surges. If there is a cloud-to-cloud discharge, the resulting low resistance path may permit a final discharge to ground at relatively low gradients. If this assumption is correct, a lightning discharge could take place near the line without necessarily inducing a high potential on the line. Conversely there are some conditions where the gradient was such that a discharge could be obtained from a metal point held up in the air. On one occasion, the brush over a mountain side has been seen to discharge. Certainly the gradients in these cases were very high and a lightning stroke in the vicinity of the line would produce a high voltage on a transmission line in the field. Apparently there is no question but that a direct stroke is likely to be far more severe than an induced one, particularly where a negative charge is placed on the conductor.

While the general theory of the ground wire in lowering maximum voltage through increase in the effective capacitance of the power conductors has been generally accepted, it would seem that the energy dissipating properties of the ground wire should be given more attention. This is particularly true where it is desired to protect the station equipment and for the application to very long lines where it may be desired to dampen out a disturbance which might otherwise cause an arc to ground over reduced insulation sections. There is probably some ground resistance which will produce the maximum attenuation. In considering the effect of ground resistances, it is essential that the varying height of the conductor be taken into account at the same time. A high ground resistance may not be a disadvantage, but if the effective height of the conductor is increased with the high ground resistance, a higher effective voltage may result. With a high effective ground resistance the potential of the tower may be raised by the charge from the ground wire, causing the ground wire to act as a counter potential wire, thereby permitting the maximum voltage across the insulators.

J. H. Cox: The mechanism of the production of surges by lightning on transmission lines, that is whether produced by induction or direct hits, has been mentioned either directly or indirectly in a number of papers at this meeting. I wish to emphasize the importance of this question as it has a vital bearing on the methods used to protect lines against lightning phenomena. In the past the opinion was quite general that practically all troublesome lightning surges were the result of induction and were more or less amenable to protection, and that the few direct strokes which occurred were so severe that they were entirely beyond control. This opinion is still somewhat prevalent. If this is true the natural conclusion is that any protection accomplished is only relative and that the idea of a lightning-proof line is hopeless since we will always have direct hits with us.

I do not wish to be understood as stating this to be an established fact, since the data are far from conclusive and it is possible that this belief may have to be reversed, but a number of factors brought out by both test and analytical work in investigation of lightning phenomena has led the group with which I have been working to believe that there is no sound foundation for the above opinions and that the only lightning surges which are high enough to give trouble on high-voltage lines are caused by direct strokes. Furthermore, we believe that many methods of protection which are or may be employed at the present time are quite effective in keeping all but the most severe direct strokes within a safe value. Fortunately most of the methods used to protect against direct strokes are also effective against induced strokes if such exist.

I will not take up the space to discuss in detail the factors which have led us to the conclusion that all important surges on high-voltage lines are the result of direct strokes but will merely state the factors as follows: (1) our present knowledge of rate of growth of streamers in relation to the speeds necessary to cause large induced stroke surges; (2) a consideration of the differences between the energies involved in induced and direct strokes; (3) klydonograph data which showed the small number of surges occurring; and (4) observations in field tests where strokes striking the earth very near the line were found to cause no surges. Incidentally Simpson in England and Matthias in Germany working on this phenomenon have independently come to the same conclusions.

Direct stroke surges vary from very weak to the very destructive transients sometimes observed. Therefore, if all important surges are caused by direct hits and since protective measures now being employed protect against the majority of these, protection becomes only a matter of degree and certain measures may need only some modification to make them completely effective. With this conception the problem of a lightning-proof line does not seem hopeless. It should be stated here that actual observations are of great value in the determination of this problem and the operating engineers being in a particularly favorable position to supply this data could contribute materially by reporting such cases as they happen to observe. However, caution should be exercised in reporting observations as it is very easy to draw false conclusions and to appear to see things in lightning phenomena which actually do not happen.

Regarding the wave-fronts as reported in some of the papers, I believe that many of these are misleading as well as some of the conclusions drawn from the data as presented. For instance, several of the papers present tables of wave-fronts varying in time to reach maximum value from as short as zero to one microsecond. It is true these have been indicated as approximate values but I feel that it is misleading to state them at all when they are the result of pure guess work. As a striking example take that in the Sporn and Lloyd paper where the time to 75 per cent is given as 0.2 microsecond and to maximum value as one microsecond. When these values are compared with the oscillograms shown in Fig. 10, which we may assume were the best obtained, it would seem that considerable courage is required to measure times of this order from the blot shown at the beginning of these oscillograms. In this connection the time of wave-front is still considerable of an unknown factor. Complete oscillograms have been obtained and wave-fronts of the order of one microsecond have been obtained; however, it must be admitted that the exact location of these strokes was not known and that it is probable that a surge has not been recorded at the point of inception. It also seems quite probable that wave-fronts of the original surge may be considerably steeper than any of those recorded and the maximum values considerably greater. There is great need for records of surges at the point of inception and on a line of insulation value above the maximum surge voltage.

It should be mentioned here that oscillograms of surges on lines, even though they may be of induced origin, are not measures of cloud field gradients due to the time lag of the stroke and the opportunity for redistribution of the bound charge on the line. This is evident from the great difference in the ease of obtaining oscillograms on antenna and on lines.

Regarding the attenuation of traveling waves on lines we are glad to note that Mr. McEachron's data agree with ours and indicate that attentuation is decreased with a larger number of wires. This also checks theoretical considerations which in this case are based on known factors. It is to be noted that the data given in the Sporn and Lloyd paper indicate the reverse and that attenuation is least rapid on only one conductor. The data in this case were obtained by means of the klydonograph and the curves drawn mean very little as the points given are too scattered. The principal conclusion to be drawn from these latter curves is that the klydonograph is not a sufficiently accurate instrument for this kind of measurement.

In the paper by Messrs. McEachron, Hemstreet, and Rudge data were presented which show that on a particular line equipped with ground wires attenuation was less rapid than on a line without ground wires. In a paper of which I was an author, published in the *Electrical World* about a year ago, from theoretical considerations the opposite conclusion was stated. However the two compensating effects mentioned by Mr. McEachron were pointed out and it is possible that with different configurations the balance might be the reverse from Mr. McEachron's case. In either case the difference is not large and even if it were it is of no great importance in transmission line performance. This is because if a surge is high enough to flash over the line insulation it will do so at the point of origin and after a flashover is produced the only importance of the rate of attenuation is the magnitude of the surge which might be impressed upon terminal apparatus. In this connection it is felt that the last sentence states the important part of Mr. McEachron's conclusion 7, in that the value of additional ground wires near the station is because of the protection they afford against surges originating at that point and I definitely feel that from this standpoint they are more than justified since their cost is low and we are always in danger of a stroke at that point. There can be no question regarding the desirability of continuing the line ground wires into the station structure.

J. H. Cox and Edward Beck: Mr. Sels has presented a comprehensive outline of the lightning problem in general, particularly that part still remaining unsolved, and pointed out the breadth of scope of the program necessary. Mr. Roper has mentioned the value of cooperative and coordinative effort so far as it has been accomplished in his particular lightning problem and has indicated that such collective effort is the shortest path to results. On various occasions in the past we have expressed the desirability of a comprehensive cooperative program and we feel that there is still need for such a program and that it would greatly accelerate the final solution of the problem.

Mr. Silver has pointed out that the recorded waves as reported in this symposium were probably all recorded at some distance from their origin and that this voltage is relatively unimportant since if a flashover occurs as a result of a lightning surge it does so at the point of origin where the voltage is obviously the maximum. This point is well taken and for this reason, as well as the determination of the fundamental nature of lightning, it is desirable to obtain records at the origin. However, apart from their importance as causes of line flashover and their indication as to the nature of lightning surges, records obtained at points distant from the origin are valuable because they give data on the voltages to which station apparatus may be subjected.

Mr. Bewley has presented an interesting discussion of the mechanism of cloud discharge which contains theories which may prove useful. However, we believe that further amplification would be desirable since part of his theory is not quite clear to us. For instance, Mr. Bewley states that during the incipient period of the cloud discharge there is a migration of the charge towards the center of disturbance. Mr. Bewley's idea of the incipient period does not seem clear since there can be no migration of distant charges without a change in field to cause such migration. Such change in field would be caused only by the removal of charge from cloud to earth in the lightning stroke and to be appreciable surely must involve a considerable part of the discharge. All of Mr. Bewley's discussion pertains to induced stroke surges and although according to his calculation such surges may reach appreciable proportions we disagree with the conclusions reached. In the near future Mr. Fortescue will present a mathematical analysis of the induced stroke which we believe will be conclusive proof that high induced voltages are improbable.

In connection with Mr. Peek's comments, tests on models in the laboratory indicate only approximately the protective ratio of ground wires since the difference in size of certain elements cannot be corrected for. Also, potentials measured on antennas in the field are not indicative of the surges which would be induced on transmission lines unless such antennas are connected to ground at their terminals with resistances equal to the antenna surge impedance.

I cannot agree with Mr. Peek's statement that if the insulator flashes over at all it is likely to occur on the front of the wave, since from the point of view of line flashover the minimum wave which just flashes over the insulator is the most important and for such waves the flashover undoubtedly occurs on the tail of the wave. Furthermore, Mr. Peek still mentions impulse ratios for lightning waves of 1.6 to 1.9 and again, except for the magnitude of the chopped wave which enters the station, the important waves are those which just flash over and, as shown conclusively in Mr. Torok's paper, such waves have an impulse ratio of the order of 1.2.

Mr. Boehne discusses the nature of surges induced by cloud to cloud discharges. In this connection the operators of the lightning laboratories in the field have frequently observed intense cloud to cloud lightning activity immediately above the station, both parallel to the line and at right angles to the line, but have never recorded an overvoltage as a result of such activity.

J. J. Torok: Mr. Wade has questioned the results shown in Fig. 17 of my paper. The apparatus used in making this test was the same as that in all the other impulse tests and a very careful calibration was made of all parts. It is believed that this calibration is quite accurate and therefore that the curves show the actual characteristics of the sphere-gaps tested. I do not see that these curves are contrary to expectation since it is known that the characteristics of sphere-gaps change between close spacings at which they are ordinarily used and long spacings of several times their diameter such as used for the upper curves.

Mr. Peek has criticized the wave used in the tests presented in my paper. This wave was as nearly flat-topped as it was possible to obtain with laboratory equipment and without oscillations. It is felt that the deviation from a true rectangular wave is not sufficient to modify the results materially except at the short time lags and these are of the least importance in connection with their application. Mr. Peek states that the wave used should give an impulse ratio of 1.9 but gives no reason and I see none. He also states that the results of a flat-topped wave are little different from those caused by a 60-cycle arc-over. It is quite obvious from the curves presented that this is not the case and that any desired impulse ratio can be obtained by varying the magnitude of the impressed wave. Regarding the results obtained with arcing rings, the tests do not show that the hoop type of ring is necessarily better but that it is sufficiently adequate for the usual line and that the larger, and better, ring, is not economically justified.

In connection with Mr. Melvin's comments, all measurements made in this work were taken with the cathode-ray oscillograph. I do not believe that the shape of the curves for wood insulation were the result of lightning generator limitations as this was not indicated by the oscillograms.

Mr. McEachron comments that tests with the flat-topped wave which give only flashovers on the tail of the wave are incomplete, and that tests giving flashovers on the front of the wave are needed. Such tests would require a definition of the wave and introduce again the third variable and since undoubtedly the flashover involves energy it is believed that flashover on the front of a sloping wave can be readily predicted from curves obtained with the flat-topped wave.

Mr. McEachron also comments on the limitation of these curves when carried to the 60-cycle crest breakdown value and states that the time lag at 60 cycles would be zero. This does not seem correct since the length of the 60-cycle wave is several thousand microseconds and the time during which the voltage slightly exceeds the steady state breakdown is probably over 100 microseconds. Also, I do not feel that the variability of the 60-cycle breakdown is very appreciable since the amount of this variation is not large when considered in connection with the accuracy possible in impulse tests.

C. L. Fortescue: Mr. Rudge questioned the appropriateness of the term "surge impedance" as applied to tower and tower footing resistance, in the paper entitled *The Lightning Laboratory at Stillwater, New Jersey.* In investigations of this character we must consider in what form the results will be most useful. It will be obvious that the actual wave propagation through the tower into the earth is quite a complex phenomenon, involving reflections at the tower footing and change of impedance in the tower from its top to tower footing. Formulas taking all these factors into account will be of very little use in estimating the protective value of ground wires against lightning strokes. The obvious solution therefore will be to obtain an equivalent surge impedance which will apply to the most extreme case, that is to say, to waves having very steep front and long duration. The tests were made with a great deal of care, a resistance type potentiometer which has been verified very carefully in our laboratory, and the steepest wave-fronts and the longest wave which it was possible to obtain by the surge generator available being used. The results, we believe, will be fairly indicative of what may be expected in the case of a lightning stroke.

I should like to state that throughout these tests we exercised the utmost care to verify our results and we found at first that test results using the ordinary type of capacitive potentiometer were inconsistent. We at once changed to the resistance type of potentiometer which, on account of the fact that this line was not energized, could be used. The results then obtained checked up with repeated measurements and were always consistent.

It is pleasing to note that the data obtained by us on attenuation check up very well with those obtained by Mr. McEachron, by entirely different methods.

Referring to Mr. Amberg's remarks on the economics of this situation I wish to assure him that throughout these investigations we have kept our feet on the ground and have not lost sight of the economic significance of the various protective factors we have discussed. Elsewhere in the discussion it is pointed out that a buried counterpoise is only effective for a short percentage of its length. This suggests at once that it may be more effective to run out radial feeders from the tower than to use a long counterpoise extending from tower to tower. Naturally, when driven grounds are effective they are preferable to buried cable or counterpoise and should be used but where it is impossible to drive a ground or where they are not effective I feel sure that some form of artificial ground such as buried cables extending radially from the tower may be made very effective and will prove economical.

Considering now the practical results that will be derived from these lightning investigations I wish to point out that all the measures proposed might be classified as defensive measures. We have three lines of defense. The first line of defense is the proper design and construction of transmission lines which includes such measures as liberal spacing between conductors, ground wires so placed that they will dissipate any stroke of lightning and prevent it from reaching the line wires, adequate spacing between the ground wires and conductors so that whatever surge potential is produced on these ground wires as a result of the lightning stroke the spacing will be sufficient to prevent the stroke from extending down to the line wires, effective grounding of the tower footings so that the full value of the ground wires as a protection against strokes will be realized. By these means properly applied it is hoped that we will be able to obtain transmission lines practically immune to lightning; however, in case an exceptionally severe stroke should cause an outage we still have a second line of defense to fall back on.

Other devices which have been considered are the spacing of lightning arresters at short intervals along the line and the placing of spillover gaps with fuses to interrupt the dynamic arc at frequent intervals along the line. Where flashover takes place on an insulator string not provided with any device to blow out the arc, quick-acting circuit breakers will prevent the outage from resulting in any instability provided the system is properly designed with respect to stability so that the quick-acting circuit breaker may be considered as one of the second lines of defense. If in addition to this we have combined with this quick-acting circuit breaker a quick-acting automatic reclosing device, we shall have gone a long way towards designing a model system.

This second line of defense will consist of such measures as the use of properly designed arcing rings which will prevent flashover, if it takes place, from cascading over the string and fracturing the insulators or puncturing them. Thus even though a flashover should take place the insulator string is still in perfect condition to perform its proper function. In addition we may have other means combined with the arcing ring such as that discussed by one of the previous speakers which, by means of a fuse or other device, will prevent the continuation of the dynamic follow-up arc and thus the line will flash over and come back into service without an outage.

This third line of defense consists in careful design of the system taking all conditions of stability into account. In other words, it is included under the heading of system designing which has been discussed in a previous session. The factors which determine the stability of a transmission system have been pretty well established so that there is little difficulty in determining whether a system will stay in operation after a fault has occurred and has been cleared. The high-speed breaker has been a tremendous step in advance in facilitating the design of system which will be stable under fault conditions. Furthermore, it permits the use of higher levels on existing systems and enables the designer to make use of means of obtaining economical power transmission over long distances, such as the Baum system. The Baum system heretofore has suffered from the fact that with the slow speed breakers in vogue before the advent of the high-speed breakers, the synchronous condensers required to maintain stability under transient conditions were large and costly. The high-speed breaker enables us to use standard designs of condensers which will hold up the voltage under the transient conditions until the circuit is cleared. The combined result obtained by the use of high-speed breakers and the Baum system will permit an economic transmission of power over long distances so that the power may be generated at the points where coal can be obtained cheaply, with the minimum of handling and where there is adequate condensing water, and transmitted to the load centers for distribution. Obviously if we can insure against interruption of service due to lightning we can operate all transmission lines at a much higher power level thus effecting a great economy.

I wish to say that it has been very gratifying to me to see how all the various bodies working on this investigation are gradually approaching each other in their conclusions and it is hoped that this season will see a still closer agreement.

R. H. George: In Mr. Bewley's discussion of our paper, attention was called to the flat place in the waves just beyond the crest, and the question was asked as to whether we could assign any reason for it.

Although this peculiarity of wave shape has received much consideration, no sound reason has been found for its occurence. Checks on the potentiometer and cathode ray oscillograph made by means of the surge generator, described in the paper by Messrs. Hemstreet, McEachron, and Rudge, showed the recording apparatus to be free from distortion. Also the fact that waves coming into the oscillograph station from either the north or the south possessed the same flat characteristic would indicate, since no terminal apparatus was located at Croton, that the wave shape was not due to the contour of the land which the line traversed.

It is very interesting and important that Mr. Bewley's theory of traveling waves apparently offers an explanation for the wave shape of natural lightning surges.

The tests reported in Mr. Schirmer's discussion show that the telephone companies are in a position to obtain very valuable information relative to areas affected by lightning strokes and should be encouraged to make as extensive an investigation as possible.

Mr. Schirmer's results with recorders located one and one-half miles apart, check results obtained at Purdue University last summer. These results indicate that a single stroke does not usually affect a very large area. While taking cathode ray oscillograms of lightning surges on a 140-ft. antenna, a record was kept of the time which elapsed between the lightning flashes and the resultant thunder, where possible, and these data correlated with the surges on the antenna.

Although the results were subject to considerable error they indicated that strokes occurring more than one-half mile away rarely produced appeciable surges on the antenna.

The lightning investigation will be continued at Croton, Michigan during the 1930 season, with the aid of improved apparatus, and it is hoped that surges may be recorded which have their origin in the immediate vicinity of the oscillograph station. Until a number of such surges has been recorded the data must be considered incomplete.

K. B. McEachron: I agree with Mr. Silver that it is very desirable to be able to reconstruct the original impulse at the point of inception from the oscillogram taken at some other point. To acquire the ability to do this is one of the objects of the study which we have conducted with artificial transients on transmission lines.

The picture which Mr. Southgate gives of the action of corona in changing wave shape, in which he suggests an equivalent circuit consisting of a gap with series resistance and capacitor would, it seems to me, be more accurate if the condenser was also shunted by a resistor of proper value. It does not seem likely that much energy is returned to the conductor from the corona formed around it when the voltage begins to decrease; however, it may later be shown that the amount of energy returned to the conductor is greater than now appears possible. I do not believe that very much is known about the formation and disappearance of corona under the transient conditions encountered during the travel of short time impulses along transmission lines. It would be very helpful indeed to have this information and it is to be hoped that the investigations now in progress will give some clew to the laws of corona formation and disappearance under transient conditions.

The results of tests to be presented at the Summer Convention, relating to further studies in Michigan, indicate that there is little measurable change in the form of traveling waves if the potential is kept below the corona voltage. This seems to indicate that the effects which Mr. Ackerman describes as changing surge impedance are dependent, to a considerable degree at least, on the effects of corona. If and when the effect of corona at different voltages on the surge impedance is evaluated by actual test, it will become possible to take these factors into account when making calculations relating to the travel of waves on lines. Tests involving the measurement of current and voltage need to be carefully made not to invalidate the relatively small differences which may result due to corona. Much more evidence under a variety of conditions must be obtained before any conclusion with regard to values of surge impedance can be stated with assurance.

Mr. Fortescue suggests that the resistance of grounds may have considerable effect on the attenuation of surges on conductors protected by ground wires. In this connection it is important to know if the effect of the ground wire on attenuation, found in Michigan, may be applied generally. The effect of the ground wire should be determined both for the case of high resistance grounds and low resistance grounds, so that the effect of the ground wire on the attenuation of surges becomes known for all conditions.

It would seem from Mr. Andrews' experience that quite a high percentage of the outages on the circuits described by him were caused by induced discharges, since the ground wire appeared to reduce very greatly the number of outages. This possible conclusion is based on the proposition that the ground wire as usually installed is in the most efficient location for induced discharges, but is not, in general, in the best location for direct strokes and I assume that the ground wire installation referred to by Mr. Andrews is in the usual location.

Mr. Jordan refers to the fact that if direct strokes occur in mid span that low tower footing resistance will be of little avail if the length of wave front is less than the span length. This again emphasizes the idea that some form of protection other than the usual ground wire is required for protection against direct strokes in mid span. If flashover does not occur in mid span so that power current follows, low tower footing resistance will be helpful in preventing a flashover of the insulator string at the tower.

A. L. Price and E. Bell: Considerable discussion has taken place as to whether induced or direct strokes are responsible for the greater number of service interruptions. We wish to emphasize one bit of information included in the paper by Smeloff and Price relative to this subject, namely the fact that during the 1929 lightning season 24 cloud-to-ground strokes were observed within a one mile radius of the Wallenpaupack Lightning Laboratory.

Mr. Peek has given the dimension of the protected area of a lightning rod as a radius equal to four times the height of the rod. Considering the transmission line conductors as protecting an area of four times their height on either side of the line gives an area protected by the 65 miles of line of about 5.7 square miles. On the basis of 24 strokes per 3.14 square miles and assuming equal storm conditions throughout, we should expect about 43 direct strokes to the Wallenpaupack Tap-Siegfried Roseland Line during the course of the 1929 season. Actually 37 trip-outs occurred during the period when lightning stroke observations were being made. These data would indicate that there is a reasonable expectation of a sufficient number of direct strokes to account for all circuit interruptions.

Fig. 18 (profile of flashed insulators per mile on the Wallenpaupack Tap-Siegfried Roseland Line over a period of years) shows that considerable benefit would accrue if some practical means could be found of measuring storm intensities over proposed transmission line rights of way and thus avoid routing lines through localities where storms are particularly numerous and severe.

While there is considerable evidence that direct strokes are the major cause of line trip-outs on high-voltage lines, induced strokes also cause numerous and often high voltages. During 1929 some surge voltage recorder measurements were made on a wood pole line, and on a telephone line beneath, both lines being supported by the same poles. Voltages exceeding 330 kv. were recorded on the telephone line on several occasions. Of particular interest, however, was the large number of small surges recorded during lightning storms. The telephone line appeared to act as a very long antenna, despite the fact that it was grounded at either end through drainage coils. None of the surges did any damage and their only apparent effect was to make the line noisy.

The average conductor height was 20 ft., hence the voltage gradients measured ranged from zero to about 17.5 kv. per foot maximum. It is therefore apparent that induced strokes do cause very appreciable voltages on aerial conductors, and that on low-voltage lines which are not highly insulated, both direct and induced strokes should share the onus of being responsible for line flashovers and trip-outs.

Philip Sporn: Referring to the question of continuity of service and its relation to the solution of the lightning problem, Mr. George has stated that on a system that he is familiar with 55 per cent of the trouble is due to causes external to the system, such as wind, weather, and other phenomena. On the 132-kv. system of the American Gas & Electric Company subsidiaries the record over four years has indicated that 75 per cent of the outages have been due to lightning. In general where the construction of the line is of a high order, and the line is located in a territory where lightning is prevalent, I believe that a far greater percentage of outages is due to lightning than has been indicated on the system quoted by Mr. George.

Regarding the comments of Mr. Andrews and his experience with some 132-kv. lines before and after ground wires, the reduction in outages obtained on these lines after the installation of ground wire would lead one to conclude, first, that the lines were rather low lines, and that the ground resistance must have been unusually low,—in other words, that the ground wire must have been particularly effective; and second that the intensity of the lightning in that territory is not particularly heavy. On the system with which I am connected our experience before and after the installation of ground wires showed a marked reduction in the number of outages after the ground wire was installed, but the reduction was not nearly as great as indicated by Mr. Andrews. Also from analyses made regarding the relative number of outages of top, middle, and bottom wires we came to the conclusion that voltage gradients as high as 50 to 60 kv. per foot were quite common.

The point discussed by Messrs. Cox and Fortescue with regard to the relative importance of induced and direct lightning strokes on the higher voltage lines is one still open to a wide difference of opinion. There are definite data available to show that both direct and induced strokes are important and one cannot be neglected in favor of the other. I am confident that on this matter a final conclusion can be drawn only after more data are obtained.

Mr. Cox's point with regard to wave-fronts shown in our paper is of course slightly exaggerated in that the data with regard to wave-front steepness are not guesswork but are based on laboratory test of the time taken for the initiating gaps to function and put the oscillograph in operation. While there may be some error in the estimated time of the wave-fronts as given, it is believed that it was better to give approximate values rather than to give nothing at all. The weakness encountered in measuring this portion of the wave has been fully realized and it is expected that in the set-ups used in the coming year's investigation the initiating gap will be speeded up sufficiently to determine more completely the front of the wave.

A Self-Compensating Temperature Indicator

BY I. F. KINNARD[1] and H. T. FAUS[1]

Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The following paper describes a unique adaptation of the temperature sensitive magnetic alloys known as "calmaloy" to the cold junction compensation of thermocouple temperature indicators. It has been known for some time that certain specially prepared copper-nickel alloys possess the property of changing permeability linearly with temperature over very wide ranges. This peculiar function of permeability has been advantageously used for effecting temperature compensation on different devices, particularly watthour meters. The need has recently arisen of providing suitable compensation for millivolt meters used in connection with thermocouples for determining the temperatures of air cooled aircraft motors. It is shown that by employing the restoring torque on the instrument produced by a small vane of calmaloy in addition to the usual restoring torque of the spring, a very satisfactory compensation results. Curves are shown which illustrate the accuracy of compensation obtained and a detailed explanation of just how it is accomplished is also given.

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THE development of air-cooled airplane motors, in which the operative temperatures are rather high as compared with those of water-cooled motors, has made it expedient to find means of measuring these temperatures. For laboratory measurements where the weight and cost of apparatus are not important, and where high accuracy is desirable, the type of pyrometer utilizing a thermocouple and a potentiometer has proved very useful. For permanent installation on an airplane where a direct reading instrument of fairly small size is required, it has been the practise to use a millivoltmeter with an iron-constantan thermocouple, the cold junction being made at the instrument. A mercury thermometer is used to indicate the cold junction temperature. This method has the very serious disadvantage of requiring calculation to determine the temperature of the hot junction.

Various methods of cold junction compensation are successfully used on thermocouple type pyrometers now on the market. In general, they may be divided into two types, electrical and mechanical. The former type uses an e. m. f. derived from a battery and an adjustable portion of this e. m. f. is added to the thermocouple e. m. f. to correct for the change due to the difference between the actual cold junction temperature and that for which the instrument was calibrated. The latter type uses a bi-metallic spring which, for a given change in temperature, will move the pointer by an amount sufficient to compensate for the error caused by the change in cold junction temperature.

The battery method is not considered suitable for aircraft use on account of the weight, and because of the effect of very low temperature on the e. m. f. of the battery. Since the instrument should have a scale range from 100 to 650 deg. fahr., and should give reasonably accurate readings at any cold junction temperature from + 105 to − 40 deg. fahr., it was believed that it would not be practicable to use a bi-metallic spring capable of supplying a correction equal to 27 per cent of the full scale reading of the instrument. This problem is made much more difficult by the small size of the instrument required.

In a previous article, mention was made of a series of temperature sensitive magnetic alloys.[2] These alloys have been manufactured for some years under careful control for use in meters and instruments. Some hitherto unpublished data on these materials are shown in Figs. 1, 2, and 3. Figs. 1 and 2 show that the permeability and point of magnetic transformation are increased with decreasing copper content and that this

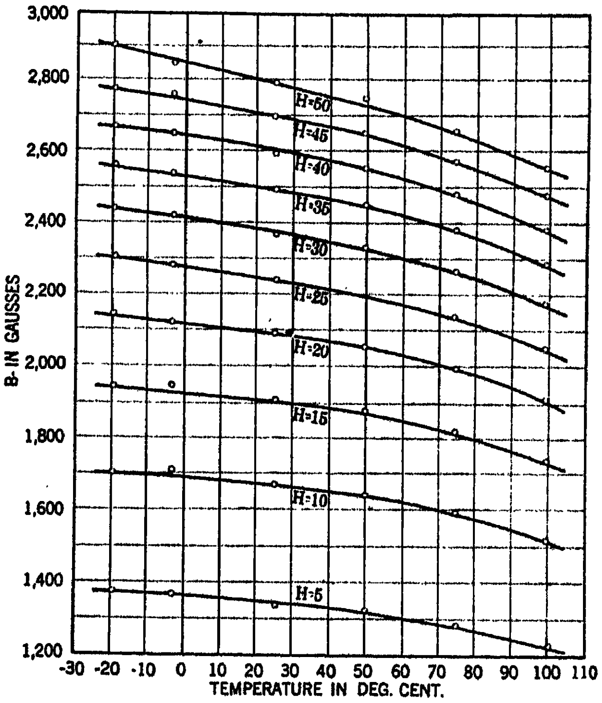

FIG. 1—FLUX TEMPERATURE TESTS ON CALMALOY

Composition: Cu 9.5 per cent, Ni. 88 per cent, Fe. 2.2 per cent
Heat Treatment: Quenched in water

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[1] General Electric Company, West Lynn, Mass.

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[2] *Temperature Errors in Induction Watthour Meters*, A. I. E. E. TRANS., Vol. XLIV, 1925, p. 275. In this paper, the name "thermalloy" was applied to these alloys, but it was subsequently learned that this name had previously been applied to a heat resisting alloy. Hence, the name of this series of temperature sensitive magnetic alloys has been changed to "calmaloy," which is derived from caloric magnetic alloy.

change is accompanied by a decrease in the slopes of the temperature-flux curves. Fig. 3 shows that quenching gives a lower permeability than is obtained when the material is cooled slowly, but that the hysteresis is much less in the quenched material. Since low hysteresis is essential in these materials, the quenching treatment is used. The success which has attended their use in correcting other types of temperature errors led to a consideration of the possibility of using them in correcting for cold junction error.

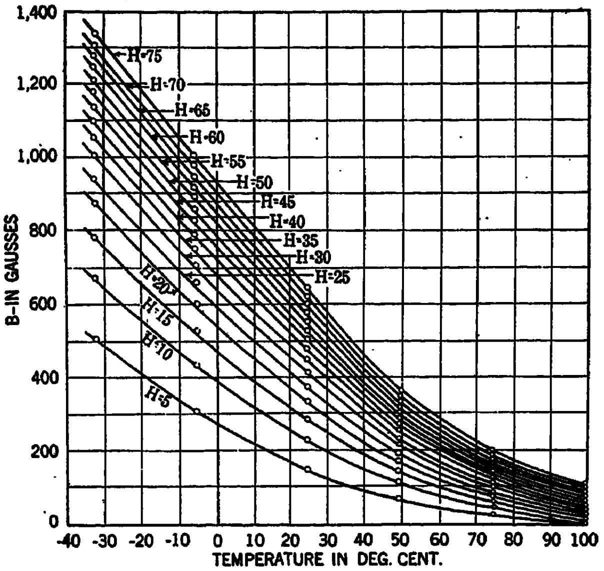

FIG. 2—FLUX TEMPERATURE TESTS ON CALMALOY

Composition: Cu. 30 per cent. Ni. 66.5 per cent, Fe. 2.2 per cent
Heat Treatment: Quenched in water

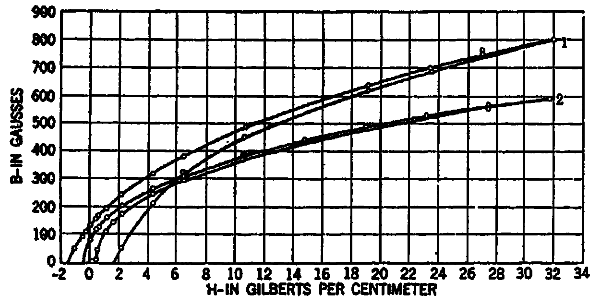

FIG. 3—HYSTERESIS TESTS ON CALMALOY

Composition Cu. 30 per cent, Ni. 66.5 per cent, Fe. 2.2% per cent
1. Cooled slowly in furnace
2. Quenched in water

The first method attempted was the use of a calmaloy shunt in parallel with the air-gap of the instrument. By this method, it was possible to obtain correct compensation at only one point on the instrument scale since the correction supplied by the shunt was proportional to the scale reading, while the cold junction error was the same at all points in the scale. Fig. 4 illustrates the results which would be obtained if an instrument were correctly compensated for cold junction temperature variation at 70 per cent of full scale reading. It is evident that this method of compensation is suitable only for purposes requiring accuracy over a very narrow hot junction temperature range.

Since it is necessary to measure the temperature of the cylinder wall which should not exceed 310 deg. fahr. and the temperature of the cylinder head, which may be as high as 600 deg. fahr., and it is also desirable to use one instrument with two thermocouples and a selector switch for taking both measurements, it is evident that the type of compensation just described is not suitable for use in measuring engine temperatures. What is required is a compensation that will be effective over the entire range of the scale.

The required type of compensation was obtained by mounting a calmaloy vane on the instrument armature in such a relation to an auxiliary pole piece that the magnetic attraction between the vane and the pole piece produces a torque tending to move the instrument pointer toward the lower end of the scale. When the temperature of the instrument is lowered, the permeability of the calmaloy vane is increased, thus causing it to exert a stronger torque; this balances the increase in torque due to the increase in millivolts due to the lowering of the cold junction temperature. Since both effects have a linear variation with temperature, a good compensation may be obtained over a wide range of cold junction temperatures.

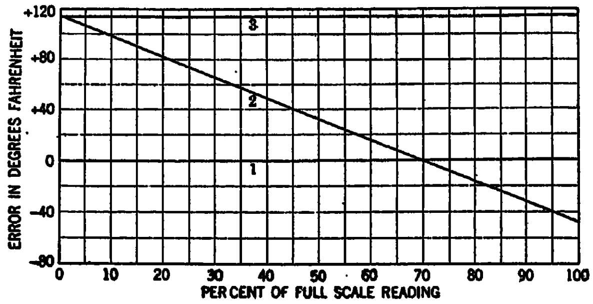

FIG. 4—COLD JUNCTION COMPENSATION OBTAINED BY USE OF A CALMALOY SHUNT

1. Calibration at 75 deg. fahr.
2. Calibration of compensated instrument at − 40 deg. fahr.
3. Calibration of uncompensated instrument at − 40 deg. fahr.

It is the usual practise to connect a sufficient amount of manganin or other resistance wire having a low temperature coefficient in series with the armature of a millivoltmeter to minimize resistance changes due to changes in the temperature of the copper windings. It is evident that this is not a very desirable method to use in a case where only a very small amount of electrical energy is available for the operation of the instrument, since the sensitivity is reduced in the same proportion as is the temperature error.

The temperature error due to the change in resistance of the windings is proportional to the scale readings and hence could be perfectly compensated by a temperature sensitive magnetic shunt similar to that described in connection with the cold junction compensation. Principally on account of the manufacturing difficulties which would be caused by using both methods of compensation in the same instrument, it was decided to use the calmaloy vane to compensate for both the cold junction error and the error due to the change in the resistance of the instrument. This problem may be understood more easily by referring to Fig. 5. It will be noted that the curve representing the total error is the sum of two errors, one increasing with increasing scale deflection, and the other remaining the same for all points on the scale. It is necessary to provide a type of compensation which will increase with increasing deflection in the same manner as shown by the curve representing the total error. This is done by proportioning the magnetic circuit so that the rate of change of flux, and hence the torque, acting on the calmaloy vane is increased with increasing scale reading.

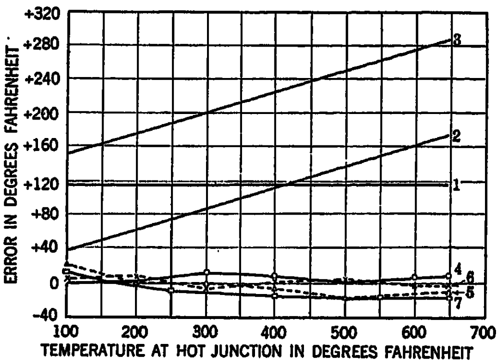

Fig. 5—Results Obtained by the use of a Calmaloy Vane in Compensating for Ambient Temperature Errors in Thermocouple Type Temperature Indicators

1. Error due to cold junction temperature at −40 deg. fahr.
2. Error due to change in instrument resistance at −40 deg. fahr.
3. Total error in uncompensated instruments at −40 deg. fahr.

4-7. Errors in compensated instruments at −40 deg. fahr. All instruments calibrated to read correctly at + 75 deg. fahr.

Fig. 6 illustrates the method of applying this compensation to a standard type d'Arsonval instrument. This instrument has a coil (*e*) of cylindrical section, iron pole pieces (c c^1) having inside surfaces of spherical section and a spherical iron core which, together with the springs, is omitted from the drawing in order to avoid confusion. The compensating pole piece (*b*) which is made of iron is mounted on the pole piece (*c*). The calmaloy vane (*a*) is mounted parallel to the armature shaft. The pole piece (c^1) is cut away as shown so that it will not exert an attraction which will oppose the attraction of the compensating pole piece for the calmaloy vane. The principal path of the flux which produces the compensating effect is indicated by the arrows. It will be noted that the flux, after crossing the air-gap between the compensating pole piece and the calmaloy vane, passes through the coil and across a second air-gap to the core. The length of this second air-gap is not changed when the armature is rotated, but may be varied by raising or lowering the armature by means of the jewel screws. This provides a convenient means for adjusting the compensation.

Fig. 5 shows the results obtained on four instruments compensated by this method. Although it is possible to secure a very accurate compensation, the cost is materially increased if it is attempted to hold an accuracy of better than 20 deg. fahr. for a change in ambient temperature from + 75 to − 40 deg. fahr. This means that the error will be less than one-fifth of the change in ambient temperature from the calibrating temperature of 75 deg. fahr.

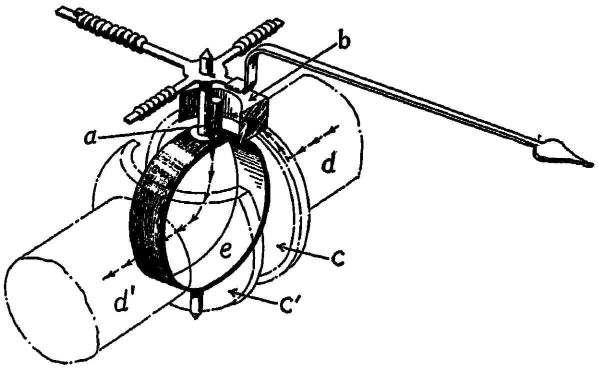

Fig. 6—D'Arsonval Instrument with Calmaloy Compensation

On account of its simplicity, sturdiness, and small size, it is believed that the type of temperature indicator just described will be found useful for many purposes other than that for which it is particularly intended. It also appears that the possible uses of temperature sensitive magnetic alloys have not been exhausted.

Phase Defect Angle of an Air Capacitor

BY W. B. KOUWENHOVEN* and C. L. LEMMON†
Member, A. I. E. E. Non-member

Synopsis.—*This investigation was undertaken in order to determine the conditions of atmosphere under which the air capacitor may be regarded as having zero loss.*

A special guarded and shielded test capacitor was constructed having quartz insulation between the high-voltage and guard plates and low loss molded bakelite insulators between the measuring and guard plates. This capacitor was enclosed in a metal box and subjected to various conditions of temperature and humidity. Readings were made of continuous conduction current and a-c. phase defect angle. The measuring instruments used were the most sensitive available. A D'Arsonval galvanometer having a sensitivity of 1.5 × 10–¹² amperes was used in the continuous current work and a Wien bridge sensitive to three seconds in the a-c. measurements. A gradient of approximately 4000 volts per cm. was used in all tests.

Measurements were made of the effect of humidity at temperatures from 60 deg. fahr. to 100 deg. fahr. and in each run the humidity was varied from 40 to 95 per cent, approximately. Readings were made of continuous conduction current and the phase angle of the test capacitor was compared with that of a standard air capacitor in which the humidity was maintained at a low value. It was found that the conduction was zero and the phase defect angle of an air capacitor was less than three seconds for values of humidity below 90 per cent.

Observations were made also of the effect of the introduction of ions and dust particles into the condenser chamber.

* * * * *

INTRODUCTION

IMPROVEMENTS in impregnated paper cable manufacture have necessitated many experiments in the measurement of phase angle. The a-c. bridge and the dynamometer wattmeter are the two methods commonly employed to measure dielectric loss. Both of these methods use air capacitors and the accuracy of the results is based upon the assumption that the air capacitors are loss free, *i. e.*, the current leads the voltage by exactly 90 deg. The correctness of this assumption was questioned almost simultaneously by both the General Engineering Laboratory of the General Electric Company and by the Electrical Laboratory of the Johns Hopkins University,[1] during the summer of 1927. The trouble with the air capacitors developed during the summer months, when the humidity was high. This investigation was undertaken to determine whether or not the known sources of atmospheric ionization were sufficient to account for the phase defect angle found, and to study the effects of humidity and temperature upon the behavior of air capacitors. The phase defect angle of a capacitor is the angle by which its current fails to lead the alternating voltage by 90 deg.

SOURCES OF ATMOSPHERIC CONDUCTION

It is a well recognized fact that atmospheric air contains charged particles called ions, which will move when exposed to an electric field, thereby causing conduction.[2] The sources[3a] of ionization are radium emanations, soil gases, ultra violet light, cosmic rays, photoelectric effects, the Lenard[4] effect, whirling dust clouds,[5] and the products of combustion.

The normal ionizing agents produce from ten[6] to thirty[7] ions of each sign per cu. cm. per second. These are removed by diffusion and recombination at such a rate that at any one time there are present per cu. cm. of the atmosphere from 700 to 1000 pairs[6] of ions.

The number of ions present in air may be counted by means of the "ion counter."[3b] The number not only varies at different localities, but also from hour to hour throughout the day. In cities the average number of small ions is 800 of both signs, per cu. cm., and of large ions 1800. The number of large ions present at any time depends mainly upon the amount of smoke in the air and may equal 60,000 or more per cu. cm. "Small ions[3c] consist of groups of some ten molecules, more or less grouped around a central charged molecule and forming a fairly stable complex or cluster." Large ions are small ions combined with dust or water particles, or both. The velocity of small ions is from one to two cm. per volt per cm. per second, and the velocity of large ions from 1/100 to 1/1000 that of the small ions. The charge on an ion is usually taken as 1.59×10^{19} coulombs.

ATMOSPHERIC CONDUCTION—PHASE DEFECT ANGLE

If a capacitor is placed in still air, ions will not flow from the outside air into the capacitor and the number of ions available, to cause conduction, will be those generated per unit volume per second by the normal atmospheric ionizing agents. If these produce an average of 20 ions of each sign per cu. cm. per second, and if none of these recombines the current per unit volume of the capacitor will equal 3.2×10^{-18} amperes.

Consider, for example, a high-voltage air capacitor having two measuring plates, electrically connected, one on either side of a high-voltage plate, each plate being 121.9 cm. (4 ft.) by 182.9 cm. (6 ft.). At a spacing of 5.08 cm. (2 in.) the volume of air enclosed by the condenser is 227,000 cu. cm., and assuming that the voltage gradient is sufficient to draw out all of the ions formed, the conduction current amounts to 0.72×10^{-12} amperes. The electrostatic capacity of this capacitor equals 775 $\mu\mu$f. and at 60 cycles potential of 13,200 volts, the charging current is 4 milliamperes. The power factor of the capacitor under these conditions equals 1.8×10^{-8} and this is negligible.

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*Assoc. Professor of Electrical Engg., The Johns Hopkins University, Baltimore, Md.

†Grad. Student in Electrical Engg., The Johns Hopkins University, Baltimore, Md.

[1] For references see Bibliography.

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If the air in the vicinity of the condenser contains 60,000 ions of both signs per cu. cm., and if we assume that the air currents are sufficient to completely change the air in the capacitor once every second, then the conduction current would equal 2.2×10^{-9} amperes, and the power factor, $(\cos \phi)$, 0.55×10^{-6}. This corresponds to a phase defect angle of less than one second of arc and may be neglected.

It is evident, therefore, that the normal ionization of the atmosphere is not sufficient to account for the phase defect angles sometimes reported in air capacitors. Any phase defect that occurs must be the result of either leakage over the insulation, abnormal ionization caused by the flow of highly ionized air into the condenser space, or the presence of a dielectric other than air in the field.

Experimental Work

As stated before, the object of this investigation was to determine the effect of humidity, temperature, and ionization upon the dielectric, atmospheric air. Measurements were made of the conduction current flowing through an air capacitor under high continuous potentials and also of the power factor at corresponding values of 60-cycle voltages. A special test capacitor described below was constructed for this purpose.

The continuous current conduction was measured by a high sensitivity D'Arsonval galvanometer and as a check a-c. measurements were made in a Wien bridge, comparing the test capacitor with a standard capacitor. The effect of temperature was first studied in order to be sure that no thermal e. m. fs. existed in the test capacitor circuit. After proving that thermal e. m. fs. were absent, humidity runs were made, each at a definite temperature ranging from 21 deg. cent. (70 deg. fahr.) to 38 deg. cent. (100 deg. fahr.). In each one of these runs the relative humidity was varied from a low value to the highest attainable. No appreciable continuous conduction current was found and the d-c. and a-c. methods checked within the limits of accuracy of the equipment.

During the work two sources of trouble developed, namely surface leakage over the insulation and conduction due to fibers. Both of these were carefully investigated. Later ionized gases and dust were introduced into the test capacitor and their effect upon the phase defect angle studied.

The Test Capacitor

A flat plate capacitor (Fig. 1) was used as a test specimen, in which the atmospheric conditions could be varied. The high-voltage plate was 51 cm. (20 in.) × 38 cm. (15 in.), the guard ring 6.4 cm. (2.5 in.) wide, and the main or measuring plate 38 cm. (15 in.) × 25.4 cm. (10 in.). The width of the air-gap between guard and main plates was 0.159 cm. (1/16 in.) and all plates were of 0.318 cm. (1/8 in.) brass. The plates were polished and all sharp edges removed. The main plate was suspended from the guard plate by bakelite strips, running the full length of the condenser; brass studs, placed as near the center of these strips as was consistent with rigidity, held the main plate. The main plate lead was a 0.635 cm. (1/4 in.) brass rod soldered by means of leads of equal length, to four symmetrically placed points on the main plate. The main plate was shielded by a brass case soldered and fastened mechanically to the guard ring. A brass tube concentric with the main plate lead was mounted on this shield and served both as the guard plate terminal and as a shield for the lead. The capacitance with 0.254 cm. (0.1 in.) spacing was 300 $\mu\mu$f. The capacitor was completely enclosed in a galvanized iron box. Outlets were provided for circulation of air and observation of conditions. An oil bath was used to control the temperature of the capacitor, which is shown in detail in Fig. 2.

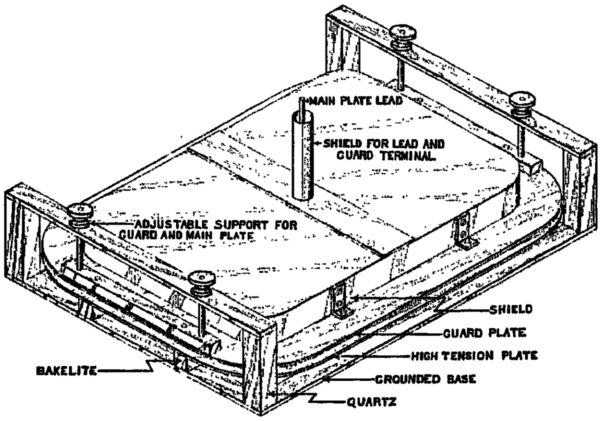
MAIN PLATE LEAD
SHIELD FOR LEAD AND GUARD TERMINAL.
ADJUSTABLE SUPPORT FOR GUARD AND MAIN PLATE
SHIELD
GUARD PLATE
HIGH TENSION PLATE
BAKELITE
GROUNDED BASE
QUARTZ

Fig. 1—Test Capacitor

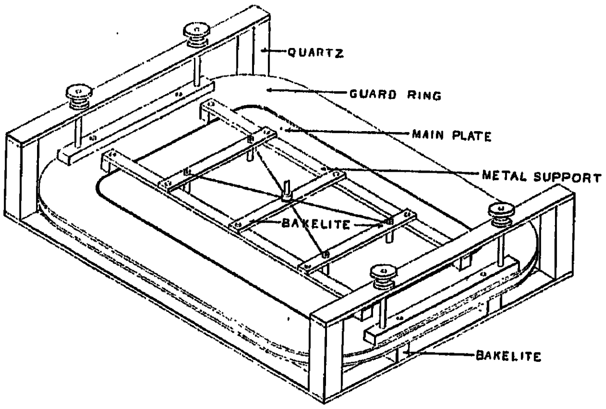

Fig. 2—Test Capacitor Details

All leakage paths were made as long as possible. The leakage path between main and guard plates was 4 in. long and the total cross-section of insulation was 1½ sq. in. Molded bakelite was used to insulate the main plate from the guard. The guard and high voltage plates were both supported from a grounded base. All leakage over insulation from the high-voltage plate was led to ground and did not flow in the guard circuit. The guard ring was supported from the base by four quartz pillars. The best grades of low loss insulation available were used in the capacitor and care was taken to keep it out of the field. To reduce the chance of condensation to a minimum, the insulators were coated with a thin band of ozokerite wax. To conform with present practise brass plates were used, although they probably contained slight traces of radioactive substances.

The temperature of the capacitor was varied by means of an oil bath which heats only the bottom of the capacitor chamber. Heating of the bottom produced a small temperature gradient in the tank and circulating air currents resulted. These currents equalized the atmospheric conditions throughout the chamber.

The overhead suspension of the guard plate permitted variation of capacitance without in any way changing the leakage paths through the capacitor, or the position of the insulating supports with respect to the field. The guard and measuring plates were hung from two horizontal supports by four brass screws and the spacing was varied by adjusting the length of the four supports by thumb nuts. The construction is shown in Fig. 2.

CONTINUOUS CURRENT MEASUREMENTS

A 1500-volt storage battery supplied current for the conductivity measurements. The high-voltage side of this battery was connected to the high-tension plate of the test capacitor. The guard plate of the test capacitor and the other side of the battery were grounded. A high-sensitivity galvanometer was connected between the main plate of the capacitor and ground. By varying the position of the pole pieces the galvanometer sensitivity was adjusted to a value of 1.5×10^{-12} amperes per millimeter deflection at the scale distance used. This value of sensitivity was used throughout the work. A resistance shunt was used to decrease the sensitivity when desired.

The suspension used was a 0.007-in. gold wire rolled flat. This suspension was easily strained and after a large deflection the zero was not the same. In order to eliminate the effects of zero creep, care was taken to keep the deflections small, and at each point two conductivity readings were made with reversed battery polarity.

A-C. MEASUREMENT

The Schering and Wien or Rosa[9] bridges are most frequently used in high-voltage cable work. The Schering bridge has three condensers in the network and the Wien only two; therefore, since we were studying the effect of air conditions upon capacitance, we chose the Wien bridge to check the continuous current results. The diagram of connections is given in Fig. 3. This bridge compares the capacitances and the difference in the phase angles of the two capacitors, C_s and C_t, and does not therefore constitute a direct check upon the continuous current results. It was, however, the most sensitive method available, and by keeping the humidity and temperature in the standard capacitor, C_s, low and constant, while varying that in the test capacitor, C_t, we were able to detect any change in their relative phase defect angles.

In setting up the bridge care was taken to make a symmetrical disposition of the various elements of the network. The resistance in the two sides of the bridge were of the same size and type, symmetrically placed, shielded, and guarded. The leads between corresponding elements were the same length, shielded in an identical manner, and fixed in position. The series resistances r_1' and r_2' were completely shielded and were varied by means of a set of extension dials. The space relations of the parts on the two sides of the bridge were the same with respect to each other and to ground, and by this means the capacity unbalance to ground was brought to a minimum.

The standard capacitor was cylindrical in form and brass tubing was used for the plates. The main plate was entirely enclosed and was completely shielded and guarded. In order to maintain the relative humidity low and constant, a quantity of calcium chloride was placed in a container and mounted in the standard capacitor in a position outside the main field.

A Drysdale and Tinsley vibration galvanometer was used as a detector and as this did not give the desired sensitivity a three-stage impedance coupled amplifier was inserted between the bridge terminals and the galvanometer. The amplifier and bridge were isolated electrostatically by means of a shielded transformer. The power supply to the bridge was taken from a 5-kv-a., 60-cycle, sine wave, a-c. generator manufactured by the General Electric Company. The generator voltage was controlled by varying the field excitation, and the generator was driven by a motor supplied with current from the university storage battery. The speed was held constant.

At 60 cycles, and with the amplifier in circuit, the bridge could be balanced to better than 1/10 ohms in the ratio arms and to within plus or minus 100 ohms in the series resistance, r_1', and r_2', arms. The minimum phase defect angle that we could detect (plus or minus 100 ohms in the series resistance at a frequency of 60 cycles) was three seconds of arc. This was approximately 1/250 of the sensitivity of the continuous current measurements.

METHOD OF VARYING THE HUMIDITY

Ions are not produced by slow evaporation from a water surface.[3d] Therefore trays holding water were placed in the box containing the test condenser and to increase the water surface blotters were allowed to dip into the trays. There was a slight temperature gradient between the top and bottom of the box containing the capacitor and this aided in circulating the air.

A fan was used to stir up the air and this was operated so as to draw air out of the condenser space and discharge it outside of the box. When air carrying drops of moisture passes through a fan the suspended water particles are ionized in accordance with the Lenard effect.

The relative humidity was measured by means of wet and dry bulb thermometers placed at the same level as the capacitor and close to the plates. Runs using this method of humidity control were necessarily long and each test extended over a period of twenty-four hours or more. This was not a disadvantage, however, as it gave sufficient time to obtain all of the desired data.

It is easy to increase the rate of evaporation by heating the water slightly and thereby shortening the time of a run. After a number of attempts to use such schemes they were abandoned because of the danger of ionization and condensation.

OPERATION

In this investigation two sets of data, continuous current conduction and a-c. phase angle, were taken concurrently on the same air capacitor. A complete switching arrangement was installed by means of which the capacitor could be transferred from one circuit to the other. Measurements were made at intervals consistent with the rate of change of the variable under observation.

In order to protect the high-sensitivity galvanometer from transients during the continuous current measurements a definite cycle of operation was adopted. With the galvanometer short circuited, the continuous voltage was applied to the capacitor circuit; after five minutes the galvanometer was placed in circuit, a reading taken, and again short circuited. The circuit was then opened and the capacitor short circuited for five minutes. Then with the galvanometer still short circuited, voltage was applied in the opposite direction, and after five minutes the second or reversed reading was taken. The reversed readings made the accurate determination of the galvanometer zero unnecessary, and also served to eliminate any contact potentials that might develop during a run. This method gave a very good check of conditions in the capacitor, and if the plates were free from dust or fibers and the insulation free from moisture, the difference between the two readings was not greater than half a centimeter.

Three main operations were necessary in balancing the a-c. bridge. The first was to obtain an approximate balance of the main bridge at reduced galvanometer sensitivity. The second was to connect a second galvanometer, not shown in Fig. 3, between the guard and main plates of the standard capacitor and balance by adjusting the resistance r_6. The third step was to connect the second galvanometer between the guard and main plates of the test capacitor and to balance by adjusting resistance r_7. These three steps were repeated until the bridge was in balance for full sensitivity of the galvanometer and full amplification of the amplifier.

Each of the humidity runs was made at a definite temperature and the relative humidity raised slowly from the lowest value obtainable to as near one hundred per cent as possible. Before each run the condenser plates were carefully cleaned. The final cleaning operation consisted of scraping the plates with a safety razor blade to remove the last traces of lint and dust. The capacitor was then assembled and placed in its tank with calcium chloride dryer. After twenty-four hours the dryer was removed and measurements were made with both alternating and continuous current. If the difference between the reversed continuous current galvanometer readings at 1500 volts was less than half a centimeter, it indicated that the condenser was free from lint and the test was continued. If the difference was greater than half a centimeter the condenser was cleaned again. After securing a satisfactory set of readings with dry air, water was placed in the chamber with the condenser, the humidity increased gradually, and observations were made of both phase angle and conduction current.

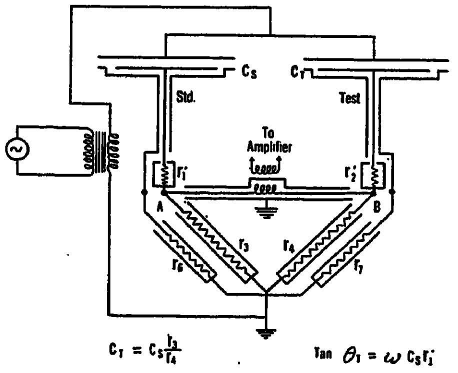

FIG. 3—DIAGRAM CONNECTIONS OF A-C. CIRCUIT

EXPERIMENTAL RESULTS

The results of the temperature test, the four humidity runs, and the effect produced by the introduction of ions into the test condenser, are given below. In addition there are included curves showing the variation of the conduction current with voltage for leakage over both insulation and fibers.

1. *Temperature Effects*: *Table I.* This run was made to determine whether or not a change in temperature would affect the measuring apparatus. The temperature range in this experiment was 7.2 deg. cent. (45 deg. fahr.) to 43.3 deg. cent. (110 deg. fahr.). Both continuous and a-c. measurements were made during the run. In Table I the results of the continuous current measurements are given and also of the capacitance of the test capacitor to the nearest micro-microfarad, as measured in the bridge. The capacitance increased during the run from 295 to 300 micro-microfarads, due to expansion. The power factor

balance of the bridge did not change during the test. The high sensitivity galvanometer remained at practically zero deflection throughout the entire run. The small deflections noted are inherent in a galvanometer with this type of suspension and sensitivity.

It is evident from the results that thermal electromotive forces were not present in the apparatus at any point in the range of temperature covered by the experimental work.

TABLE I
TEMPERATURE VARIATION
The voltage used in this test with both alternating and continuous current was 1500

Time hr. min.	Temp. of test capacitor deg. fahr.	Per cent humidity	Capacitance of test capacitor m. m. f.	Cm. deflec. 1.5×10^{-11} amp./cm.
0:00	45	56	295	0.25
1:00	50	49	295	0.75
2.00	55	53	295	0.75
3:00	60	54	295	0.20
3:45	65	56	296	0.30
4:20	70	56	296	1.00
5:20	75	54	296	0.70
5:55	80	54	296	0.70
6:30	85	49	297	1.50
8:00	90	44	298	0.00
10:00	95	41	298	1.60
10:45	100	35	299	0.00
11:50	105	31	300	0.40
12:35	110	20	300	0.20

2. Humidity Effects at 25.5 deg. cent. (78 deg. fahr.)—Table II. Two runs were made at this temperature. In the first run, which continued for about ten hours, the voltage gradient used was 4000 volts per cm. (10.1 volts per mil), in both the a-c. and d-c. measurements. The relative humidity in the test condenser at the start of the run was 51 per cent and it was gradually increased to 93 per cent. There was no evidence of continuous conduction current until the humidity reached 93 per cent; at that point the current suddenly increased to 18×10^{-11} amperes and the readings became erratic. The test was stopped at this point. The a-c. bridge results also showed that the difference in the phase defect angles of the two capacitors was practically constant up to 93 per cent humidity. Above this point the phase defect angle of the test capacitor was 3 minutes and 44 seconds, as compared to the standard. Readings were also taken at gradients of 2670 and 1350 volts per cm., which checked those at 4000 volts.

The second test at this temperature was made at a higher potential gradient of 5050 volts per cm., and continued for over 32 hr. The data given in Table II were taken during this run. The maximum temperature variation was 1 deg. fahr., and the capacitance remained constant to within one-half of one per cent. The relative humidity ranged from 68 to 91 per cent and the conduction current was zero up to 88 per cent humidity. Above this value it increased slightly, but did not become erratic.

In this test the a-c. measurements were made using a 1:1 ratio bridge and during the run the standard and test capacitors were interchanged a number of times. With the connections as shown in Fig. 3, the bridge measurements indicated that the phase defect angle of the test capacitor was three seconds, as compared to that of the standard. When the capacitors were interchanged the phase angle remained in the same side of the bridge; or in other words, the standard capacitor now had a power factor corresponding to three seconds of arc. Its source, therefore, lay in the bridge circuit, and we were not successful in eliminating it. In Table II the phase defect angle is reported as three seconds, the limit of the sensitivity of the a-c. bridge.

These two runs prove that at 25.5 deg. cent. the conduction current in an air capacitor is zero for values of the relative humidity below 90 per cent, and that if a capacitor has a phase defect angle it remains constant below 90 per cent humidity.

TABLE II
HUMIDITY EFFECT AT 25.5 DEG. CENT. (78 DEG. FAHR.)
Spacing of test condenser 0.18 cm. (70 mils)
Gradient 5050 volts per centimeter (14.3 volts per mil)
Voltage used in both alternating and continuous current tests, 1500

Time hr. min.	Temperature of test cap. deg. fahr.	Per cent humidity	Test cap. capacitance m. m. f.	Phase defect angle	Cm. deflec. 1.5×10^{-11} amp./cm.
0:00	77.5	68.0	463.0	3 sec.	0.50
1:15	Placed water in trays in condenser chamber				
1:25	78.0	75.0	463.0	3 sec.	0.50
2:15	78.0	77.0	463.0	3 sec.	0.50
3:15	77.8	79.0	463.0	3 sec.	0.50
4:15	77.8	79.0	462.5	3 sec.	0.00
5:15	78.0	79.0	462.5	3 sec.	0.25
6:15	78.0	81.5	462.0	3 sec.	0.25
7:15	78.0	82.2	462.0	3 sec.	0.50
8:15	77.8	81.4	462.0	3 sec.	1.50
9:15	77.5	85.0	462.0	3 sec.	0.25
10:15	77.5	85.0	461.5	3 sec.	1.00
11:15	77.2	86.2	461.5	3 sec.	0.50
12:15	77.0	87.0	461.5	3 sec.	0.50
13:15	77.0	87.0	461.0	3 sec.	1.50
20:15	77.0	88.5	461.0	3 sec.	6.75
22:15	77.0	88.5	461.0	3 sec.	3 00
32:15	77.0	91.0	461.0	3 sec.	7.00

3. Humidity Effect at 31.7 deg. cent. (89 deg. fahr.)—Table III. This run extended over a period of thirty-two and one-half hours as the rate of change of water content was very slow. The results are given in Table III. As in the preceding runs readings were taken at several values of potential gradient, namely 4280, 2860, and 1725 volts per cm. The values obtained at the lower gradients check those given in Table III. The humidity increased slowly and in some instances several hours elapsed between successive readings. A careful check of conditions was made throughout the run to prove that singular points did not exist. At the start the humidity was 40 per cent and at the end of the run it had reached 98 per cent. The conduction current through the condenser was zero for the entire run. The maximum deflection obtained at any time was three cm. and in the majority of cases was less than 0.5 cm.

The results of the a-c. measurements in the Wien bridge showed that the phase defect angle of the test capacitor as compared to the standard did not vary by more than three seconds throughout the entire test.

TABLE III
HUMIDITY EFFECTS AT 31.7 DEG. CENT. (89 DEG. FAHR.)
Spacing of test condenser 0.35 cm. (138 mils)
Gradient 4280 volts per cm. (10.9 volts per mil)
Voltage used, 1500

Time hr. min.	Temp. of test cap. deg. fahr.	Per cent humidity	Cm deflection 1.5 × 10⁻¹¹ amp./cm.
0:00	89.5	40	0.00
0:30	Placed water in trays in condenser chamber		
1:00	89.5	46	0.00
1:30	88.2	46	0.00
2:30	88.8	48	0.00
3:30	91.8	51	0.00
4:30	90.2	53	0.70
5:30	89.5	56	0.00
6:30	89.0	61	0.20
7:30	89.0	62.5	1.00
8:30	88.0	62.5	0.75
9:30	87.0	71.0	0.50
10:30	86.8	73.0	0.50
11:30	86.5	79.0	0.50
12:30	87.2	80.0	0.25
13:30	87.4	82.5	0.00
14:30	86.8	84.0	2.00
15:30	87.8	86.0	2.50
17:30	88.2	86.0	1.00
19:30	89.2	88.0	1.00
22:30	88.5	87.0	3.00
25:30	89.8	90.0	0.25
26:30	88.0	90.5	0.00
32:30	88.0	98.0	1.50

This test indicated that at a temperature of approximately 31.6 deg. cent. (89 deg. fahr.) and over a humidity range of 40 to 98 per cent relative humidity there was no change in the apparent phase defect angle of the capacitor and its conduction current remained zero.

4. Humidity Effects at 37.8 deg. cent. (100 deg. fahr.). The run made at this temperature continued for twenty-four and a half hours, and a voltage gradient of 4000 volts per cm. was used. The relative humidity varied from 34 to 90 per cent during the run. The continuous conduction current through the capacitor was zero up to 90 per cent relative humidity, the galvanometer deflections being all one-half cm. or less. At 90 per cent there was a sudden increase of conduction current to 39×10^{-11} amperes and the test was discontinued.

The Wien bridge measurements made at the same potential gradient indicated no change in the relative phase defect angles of the two capacitors up to 90 per cent humidity. At the 90 per cent point the test capacitor apparent phase defect angle was one minute as compared to the standard.

The test showed that below 90 per cent relative humidity the conduction current was zero and the apparent phase defect angle remained constant. Above 90 per cent the two changed in an irregular manner and definite values could not be obtained for them.

EFFECT OF THE INTRODUCTION OF IONS

Ions were introduced into the space between the plates of the capacitor by four methods, (a) Lenard effect, (b) spark discharge, (c) smoke, and (d) dust. Ionization produced by the Lenard effect is caused by the breaking up of drops of water when they strike an obstruction. The fan was set running at 200 rev. per min., driving air having a relative humidity of about 75 per cent, at a velocity of about 10 cm. per second, through the capacitor. Before the fan was started the conduction current was zero, but within less than ten minutes after starting the fan the current had reached a value of 15×10^{-10} amperes. After running the fan for forty minutes it was stopped. About ten minutes later the galvanometer deflection came back on scale and the current gradually decreased. Ions produced by this method are very large and their mobility is low. The gradient in this test was 4000 volts per cm. and the current still persisted for several minutes after the fan had stopped. During the period that the fan was running, a measurement of the phase defect angle gave a value of 19 seconds.

Ions produced by spark discharge are very small and possess such a high mobility that we did not succeed in introducing them into the space between the capacitor plates. Smoke is a more satisfactory method of introducing ions into a capacitor. The size of these ions is intermediate between the electrons set free by the spark discharge and the large ions resulting from the Lenard effect. In Table IV are given data typical of a smoke ionization test. The relative humidity was low and the fan was used to drive tobacco smoke into the condenser. With the guard connected normally the ions were all removed by the guard ring, practically none reaching the main plate of the capacitor, and no appreciable change in either conduction current or phase angle was produced by the smoke. But with the guard connected to the main plate, *i. e.*, operating as an unshielded capacitor, the conduction current rose to 45×10^{-11} amperes, and the phase defect angle to 32 seconds. The ratio of the width of the guard ring to spacing was 25 to 1 in this test. The phase defect angle as calculated from the conduction current does not check with the value found for the a-c. bridge measurement. It was not possible to take simultaneous measurements of loss and conduction current, and the volume of smoke varied.

TABLE IV
SMOKE
Spacing 0.37 cm. (146 mils)
Gradient 4050 volts per cm. (10.25 volts per mil)
Voltage used in alternating and continuous current tests, 1500

Time hr. min.	Temp. of test cap. deg. fahr.	Per cent humidity	Capacitance of test cap. m. m. f.	Phase defect angle	Cm. deflec. 1.5 × 10⁻¹¹ amp/cm.
	Dry air without circulation				
0:00	85.8	47	224	3 sec.	0.00
	Dry air, smoke, filtered through glass wool				
0.30	85.8	47	224	3 sec.	2.25
	Dry air, smoke, blown directly into condenser				
1:00	85.8	47	224	3 sec.	2.25
	Dry air, without smoke, guard, and main plate connected together				
1:30	85.8	47	454	0 sec.	1.00
	Dry air, smoke, guard, and main plate connected together				
2:00	85.8	47	454	32 sec.	30.00

Attempts were made, without success, to introduce sufficient dust particles into the test capacitor to cause conduction. Dust contains a large percentage of carbonaceous matter and a sample was collected from the top of an instrument case about 8 ft. high, and placed on the lower plate of a small capacitor. At a voltage gradient of 4000 volts per cm. a slight haze was noticed and the particles could be seen vibrating between the plates. After a few minutes the action ceased and when the plates were examined both the upper and lower plates were found coated with a closely packed layer of dust. Dust is a possible source of conduction and loss in air capacitors.

It is evident from these results that the use of suitable guard rings will remove the small ions and electrons from air, but the large water vapor ions move so slowly that they can get by a guard ring even where the width is twenty-five times the spacing. It is clear also that if sufficient ions are introduced into the space between capacitor plates, they will cause conduction and produce a loss in an air capacitor.

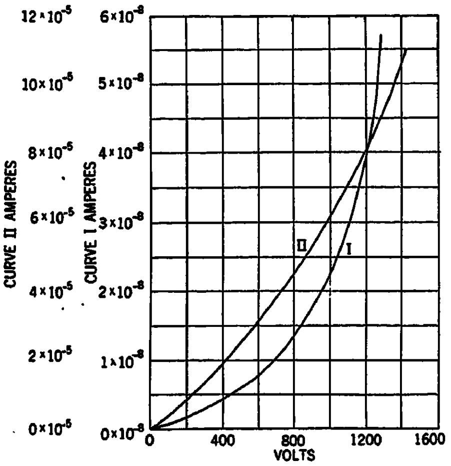

FIG. 4

CURVE I CONDUCTION CURRENT
CURVE II SURFACE LEAKAGE

CONDUCTION DUE TO FIBERS AND CONDENSATION ON INSULATION

Fibers were a constant source of trouble. At the voltage gradients used the fibers would line themselves up with the electrostatic field, and as the plates of our capacitor were close together they would bridge the gap and cause conduction. In Fig. 4 is shown a curve of the variation of the conduction current with applied voltage. In this test there was a large number of cotton fibers between the plates and the relative humidity was 42 per cent. At higher values of humidity the curves were much steeper.

In all our tests condensation on insulation was the limiting factor in the behavior of the air capacitor. The use of ozokerite helped to reduce this trouble, but did not eliminate it. Several tests were therefore made to determine the leakage current that would flow over the surface of a piece of insulation upon which moisture had condensed. In these tests the voltage was varied and a typical curve is reproduced in Fig. 4.

The interesting thing about this leakage current curve and also for the fiber conduction curve, although to a lesser extent, is their similarity to the curve of the ionization current through a gas as it approaches breakdown.

CONCLUSIONS

The following conclusions may be drawn from this investigation:

1. The normal ionization of the atmosphere is not sufficient to produce a measureable phase defect angle in a-c. measurements at commercial frequencies, involving an air capacitor as a standard.
2. The a-c. bridge observations indicate that any apparent phase defect angle in an air capacitor remains constant at values of relative humidity below 90 per cent.
3. The continuous conduction current in a properly shielded and guarded air capacitor in quiet atmosphere is negligible at values of humidity below 90 per cent. Therefore such a capacitor may be considered as an ideal capacitance.
4. In an air capacitor there are present in addition to ionization two sources of conduction, fibers bridging the plates and condensation on insulation. The latter may be reduced by placing a band of ozokerite, or other similar material, around each possible leakage path, and by keeping the humidity low. The leakage path should be made as long as possible.
5. In an air capacitor it is advisable to insert a grounded metal plate in the solid insulation between the guard and high-tension plates. Such an arrangement eliminates practically all leakage current from the guard circuit.
6. In a properly designed air capacitor a guard plate should be provided, but this guard ring cannot be relied upon to remove all of the ions present in the atmosphere.

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Transmission Research and Design with the Field as a Laboratory

BY F. E. ANDREWS* and C. L. STROUP†
Member, A. I. E. E.
Associate, A. I. E. E.

Synopsis.—Overhead transmission lines are usually considered to be a class of equipment subject to certain types of troubles which cannot be avoided by means accepted as ordinary practise. It is the purpose of this paper: (1) to describe improvements in design of wood pole lines of the 33,000-volt class, which improvements have been developed and applied to the system with which the authors have been identified and which, it is believed, will greatly reduce the characteristic troubles, (2) to present an explanation of flashovers on wood structures, (3) to give the facts found in field investigations which form the basis for the improvements adopted, and (4) to describe the method used for field investigations and analysis of troubles.

The studies and investigations referred to deal primarily with the matter of insulation. Considerable information relative to the insulation values of wood in a structure as developed by laboratory tests has been published. Experience is cited in this paper to show the value of wood insulation in the structure developed from field experience on 33,000-volt lines, and there is presented a practical economical design of wood pole structures using wood braces, in place of the usual metal braces, which, it is believed, will be practically immune to lightning troubles. Data from laboratory tests are given to substantiate the conclusions developed in the field investigations. The performance of insulators with respect to mechanical and electrical breakage is shown and the use of insulators of more sturdy designs is discussed.

The most important conclusions are:

1. The lightning performance of wood pole transmission lines of moderate voltage can be greatly improved by the elimination of bonding, or partial bonding between insulator supports such as results from the use of steel crossarms or steel crossarm braces; and by the use of wood or other insulators in guys.

2. The record of wood pole overhead lines with ground wires and the information developed in the paper show that overhead ground wires as usually installed do not materially improve the operation of the lines.

3. Insulator breakage can be greatly reduced by the use of the more sturdy insulators which are available.

4. Field investigations, analysis of records of operation and troubles, and interpretation of results, systematically carried out, can result in great improvement in system reliability, and in better utilization of investment.

* * * * *

INTRODUCTION

OUR present electricity supply systems are characterized by the concentration of power generation in large stations and the supply of loads from transmission systems. The quality of the service is accordingly a function of the dependability of the transmission lines. It is therefore of basic importance that every means be utilized in both the design and the operation of our transmission lines to make them the utmost in reliability.

Within the past few years, remarkable engineering progress has been made with respect to the higher voltage lines; but until recently, lines operating at the lower voltage, *i. e.*, 66,000 volts and below, have received only passing attention, the supposition apparently being that these have been so well standardized in principles of design that they are about as satisfactory in operation as can be reasonably expected. The principal loads are directly dependent upon the lower voltage lines which, from this point of view, constitute the heart of any system. The material in this paper is based upon investigations and experiences with lines of this classification, with which the authors have been identified.

Their work in this connection has been principally with 33,000-volt wood pole lines. However, the principles and methods described are adaptable to lines of other voltages using wood pole construction, and in some respects, to all lines.

DESCRIPTION OF SYSTEM

The system with which the authors have carried out their work covers an area of approximately 6000 square miles in the northeastern part of Illinois. The major portion of the energy is generated in two large stations at 12,000 volts, with a considerable part also furnished from stations of other companies associated with the interconnected system of the Chicago region. The interconnections between principal generating stations and with major distribution centers are at 132,000 volts. The intermediate voltage system, which is the direct transmission supply for most of the loads, operates at 33,000 volts, although a voltage of 12,000 direct from the station busses is used for loads close to the generating stations.

The mileage of each type of line is shown in Table I.

The 132,000-volt lines are of steel tower construction with overhead ground wires. The 33,000-volt and 12,000-volt lines are of wood-pole construction with steel structures used in special situations. The territory traversed by these lines is all of essentially the same character, most of it being flat or slightly rolling, and cultivated. Neutrals of all three systems are solidly grounded, although one portion of the 33-kv. system uses a neutral resistance distributed among several grounding points.

The wood-pole construction for both voltages, except that recently installed, is principally of the conventional type illustrated by Fig. 1. The insulators in general,

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*Public Service Company of Northern Illinois, Chicago, Illinois.

†Jeffrey-DeWitt Insulator Co., Kenova, W. Va.: formerly with the Public Service Co. of Northern Illinois.

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except those recently installed, are of the pin type of so-called 45,000-volt rating, and are multi-part porcelain, two-piece and three-piece. Overhead ground wires, except on a few of the older lines, are not in use on the wood-pole construction.

INVESTIGATIONS OF LINE PERFORMANCE

Detailed investigations and analysis of all troubles and accurate systematic records and summaries of the operation of the 132,000-volt lines have given a basis for the design of these lines that is resulting in a highly satisfactory performance. The results of this work have brought out the necessity of devoting similar attention to the intermediate-voltage system.

Interruptions of 33,000-volt lines more frequently result in interruption to substations, and to customers directly served, than do interruptions of the high-voltage interconnections. Furthermore, the former occur more often. From this point of view, the intermediate voltage lines and substations are considered the heart of the system and warrant the exercise of every reasonable means of improving their performance. Accordingly, investigations of the performance of these lines, carried on over a period of years, have finally culminated in the plan of investigation described and in the records illustrated. This has given the required information upon which to base improvements in design.

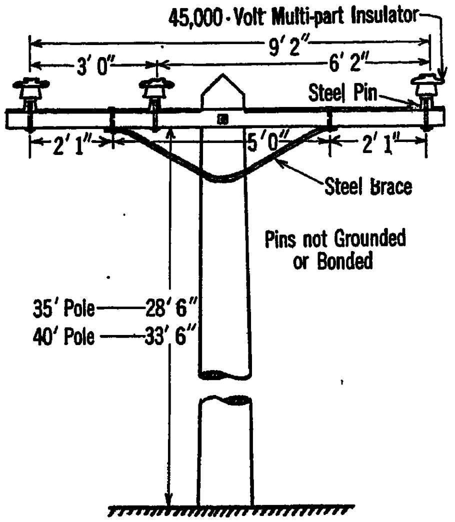


FIG. 1—TYPICAL WOOD POLE CONSTRUCTION 12- AND 33-KV. LINES

When the work of collecting data was started, an attempt was made to derive conclusions from repairmen's reports sent in from the field. The result was unsatisfactory, for while the extent of damage was quite completely described, the information needed for proper analysis of the trouble was not included.

Investigation by an engineer in several cases showed that much information not usually obtained was available with reference to cases of trouble, from which the cause and manner of failure could be determined. Engineers whose first duties were to make field investigation of trouble immediately following its occurrence were accordingly assigned to this work.

In cases in which it was necessary to obtain the facts, poles were climbed and a minute examination made. The following information was reported:

1. Number of insulators flashed (position on structures).
2. Path of arc (phase to phase) (phase to ground).
3. Guys, if any, and location.
4. Overhead ground wire, if any.
5. Earth connection and where connected (to hardware, or overhead ground wire, etc.).
6. A small sketch of the structure and associated equipment.
7. Exact location of all burns on insulators, tie wires, conductors, and hardware.
8. Location of all splinters and burns on crossarms or pole.

All cases of trouble are reported by the repair men who fill out a form provided, giving details of the trouble and repair work. The system operators' daily reports giving the detailed log of all switching operations, are received each morning and show what trouble has occurred during the preceding twenty-four hours. The engineers promptly make field inspections of all situations requiring their attention, to obtain data to supplement that given by the repair reports. In addition to this, a system operator's report is made out for each case of trouble, which shows the switches which opened, and all pertinent data relative to the system as affected by the trouble.

Reports of the bi-weekly line patrols of each line, showing location of broken insulators and other line defects, and the reports of regular line repairs also constitute an important source of information relative to line performance.

A card file by lines is kept, which is used as a working record and follow-up system for investigations in progress.

An insulator-record book is used in which is shown by lines, the insulator replacements reported and the engineer's inspection report on each insulator removed.

Each special investigation of trouble is covered by a report, these reports being filed by lines.

A chronological summary of the operating record for each line is kept, which shows for each interruption the time and duration, the interruption report number, and the cause. This summary, together with the other information described above, forms the basis for the final analysis as shown by Tables I, II, V, and VII.

Insulators are more susceptible to defect than any other single piece of equipment, and they are of paramount importance in maintaining service. Therefore, the practise of inspecting all insulators removed because of defects has been established. By this means, correct and complete information is obtained relative to the performance of various makes and designs of insulators, and a proper basis is set up for their selection and use.

The insulators when removed are marked by cloth tags to show reason for removal and location from which they are taken. An engineer makes an inspection at the earliest opportunity and records the location, type of insulator, cause of removal, and character and extent of damage. The information obtained from the insulator inspection is correlated in cases of electrical trouble with all other information on the case of trouble concerned.

Data collected in this way are of much use as a source for specific items of information needed from time to time with reference to the transmission system, but the most significant and valuable benefits cannot be derived from such information unless special effort is directed toward systematically analyzing and summarizing it. Use for much of the detailed information collected is not foreseen at the time it is recorded, but new questions and problems are continually arising for the answering of which the information is eventually utilized. It has been felt as the result of this experience, that no item of information in connection with the investigation of trouble or line performance is too insignificant to record.

The summary of line performance with respect to lightning and insulator damage is shown in Table I. This shows the comparison of operation of the lines of different voltages. The trip-outs indirectly due to lightning are those caused by lightning disturbances on lines or parts of the system other than the ones on which the trip-outs counted in this classification occurred. The larger proportion of flashovers which were found on plain structures, that is, those without guys or grounds, in comparison with those found at guyed or grounded locations is of significance. It is also of interest to point out that in practically all cases of the former two or three insulators were flashed over.

TABLE I

SYSTEM MILEAGE AND SUMMARY OF LIGHTNING, FLASHOVER, AND INSULATOR PERFORMANCE, 1929

Mileage	132 Kv.	33 Kv.	12 Kv.
Wood pole construction	0	1045	130
Steel tower construction	297	25	10
Operating Record			
Line trip-outs directly due to lightning per 100 miles	1.00	45	15
Line trip-outs indirectly due to lightning per 100 miles		26.5	12
Total cases of insulator flashover located per 100 miles	1.00	36	28
Cases of insulator flashover located on lines with overhead ground wire per 100 miles of this type line	1.00	20	..
Cases of insulator flashover located on grounded structures per 100 miles	..	11.5	6.9
Cases of insulator flashover located on structures with guys but no grounds per 100 miles	..	9.5	7.1
Cases of insulator flashover located on structures not guyed or grounded per 100 miles	..	15	14
Cases of insulator flashover located on pole top switches per 100 switches	..	11	3.3
Cases of insulator flashover located on steel towers per 100 towers	..	7	4.2
Number of insulators changed due to electrical damage per 100 miles	5.15	75.0	58.5
Number of insulators changed due to external breakage per 100 miles	8.1	139.0	204.0

The excess of the number of 33,000-volt direct lightning trip-outs over the number of flashover locations found is explained by the fact that flashovers frequently occur which do not leave sufficient evidence to be visible from the ground to the line patrolman.

Table II shows the classification of interruptions according to causes. It will be noted that the number of interruptions due to lightning was greatly in excess of those due to any of the other causes.

TABLE II

CLASSIFICATION OF INTERRUPTIONS RESULTING FROM AUTOMATIC OPENING OF OIL SWITCHES

12-Kv. and 33-Kv. Lines
Jan. 1—Dec. 31, 1929

Cause	Per cent of total interruptions
Lightning, direct	35.7
Lightning, indirect	22.7
*Failure of insulators other than during lightning storms	1.8
Failure of system equipment external to the line	5.1
System disturbance exclusive of those due to lightning	18.5
Accidents	4.0
Trees	.8
Wind, snow, ice, or other atmospheric conditions	1.8
Animals	.4
Operating errors	.9
From lines interconnecting with other companies	2.0
Unknown	6.3

*Due principally to insulators with cracked porcelain parts which failed during wet weather.

LIGHTNING FLASHOVERS

The best construction of the type generally used for wood-pole lines, and the use of better and higher voltage insulators has been ineffectual in improving the lightning performance. It was accordingly found necessary to consider other means to decrease the susceptibility of the lines to lightning.

Satisfactory explanation of the way in which the lightning flashovers occurred was not available, and a climbing inspection of a number of poles in widely scattered locations on which lightning flashovers had taken place was therefore undertaken for the purpose of finding out just what had happened. The types of structures inspected, the character of burns located, and the arc paths, are illustrated by Figs. 2 and 3. The results of the inspection are classified in Table III.

From this table, it can be seen that out of 47 structures on which flashovers were investigated, 19 structures without guys or grounds showed evidence of line-to-line arcs which did not go to ground. On the remaining 28 structures with guys or grounds, 20 flashovers went to ground, while 8 took place between conductors only. These latter were in locations where there was some separation on the pole between crossarm and guy. This information shows that a considerable number of all flashovers due to lightning occur on unguyed and ungrounded structures.

The following observations are derived from this information:

1. On poles not grounded or guyed, phase-to-phase flashovers occur without arcing to ground, the arc utilizing the steel brace with wood crossarm construction, or the steel crossarm, as part of the short-circuit path.

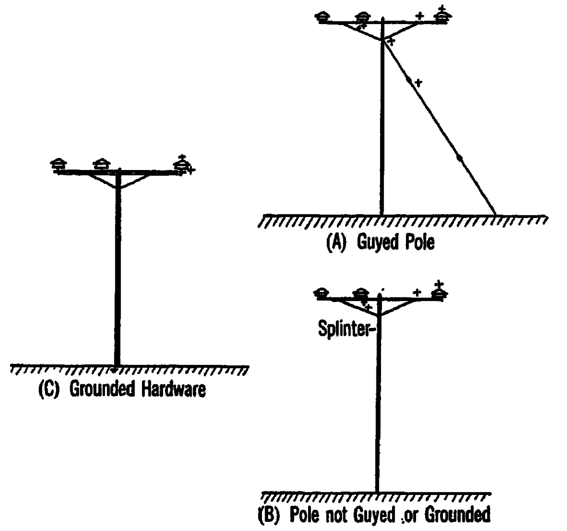
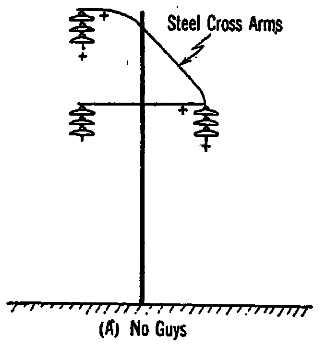
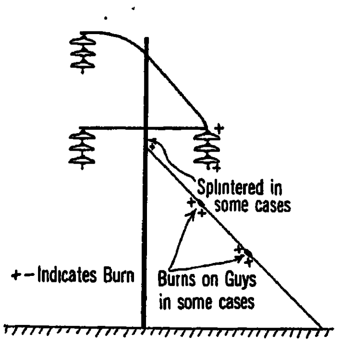


FIG. 2—CONVENTIONAL CONSTRUCTION. + INDICATES BURNS



(A) No Guys



(B) Guyed Structure

FIG. 3—STEEL ARM CONSTRUCTION

2. On grounded structures the flashovers are from line to ground, usually involving more than one phase. The flashovers to ground of the different phases on any one structure, may or may not occur simultaneously.

3. On guyed structures, with the guy separated on the pole from the steel arm or brace, line-to-line flashovers frequently occur without power arcs to ground.

4. Splinters found on poles between crossarms and guys with burns on insulators or conductors and on arms or braces, but with no burns on the guys, indicate that a power arc has taken place between conductors, but that the lightning has taken a path to ground over the guy without a follow-up power arc.

The operating record of the 132-kv. lines in the territory concerned shows that lightning voltages high enough to cause flashovers seldom occur. The lightning flashover value of the insulator strings based upon the wave used by the Pittsfield Laboratory of the General Electric Company and the calculated protective ratio of the overhead ground wire are such that flashover should not occur with induced voltages below 37 kv. per foot. Therefore, the induced lightning voltages on lines carried on wood-pole structures, such as those used for 33,000-volt lines, with the conductors about 33 ft. above the ground, should rarely exceed 1250 kv. The lightning voltage required to flash over the wood-pole structure to ground is estimated to be of the order of 3000 kv. minimum, which is considerably higher than the voltages actually occurring as given above.[1] This reasoning supports the observation made from the inspection of structures that flashovers occur on plain (non-grounded or non-guyed) structures without arcing to ground.

TABLE III
ANALYSIS OF PATH OF FLASHOVERS ON WOOD STRUCTURES

Type of structure	Figure	Number of structures inspected	Number of structures on which no burns were found	Line-to-line burns only	Line-to-ground or guy burns on one or more conductors
Steel brace, wood x-arms, no guys, no grounds	2-B	14		14	
Steel X-arm no guys, no grounds	3-A	79	74	5	
Totals without guys or grounds		93	74	19	
Guyed or grounded structures	2-A 2-C	17		1	16
Guyed structure steel x-arm no ground	3-B	16	5	7*	4
Total, guyed or grounded		33	5	8	20
Total, all structures		126	79	27	20

*Of these seven, four showed pole splinters between steel crossarm and guy.

Consideration of the large number of lightning flashovers which occur on plain structures, in relation to information which has been published on relative frequency of low and high values of lightning surge voltages, also appears to warrant the conclusion that many power arc flashovers must occur at voltages much lower than those required for complete structure spark-over.[2]

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[1] For references see Bibliography.

The evidence and reasoning so far presented lead to the conclusion that flashovers on 33,000-volt wood pole construction occur at lightning surge voltages below those required to spark over the entire structure, and that these flashovers are line-to-line rather than from line to ground.[1,2,3] Inspection of the locations where flashovers have occurred and the information in Tables I and III show that flashovers are reasonably well divided between guyed or grounded structures, and plain structures without guys or grounds. Both classes of trouble have an important bearing on the line performance and must be materially reduced for satisfactory operation. Flashovers on guyed or grounded poles are readily explained by the fact that the lightning spark-over value of this type of structure is only that of the insulator alone or of the insulator plus the small distance over the wood arm from insulator pin to steel brace. This is obviously quite low on 33-kv. lines.

The following explanation of flashovers on plain structures is offered. The voltage distribution between line and ground over the insulators and structure is such that the voltage stress over the insulator, or between line and crossarm brace, is high enough to cause spark-over or incipient sparks between line and brace only, at voltages lower than those required for structure spark-over. The 60-cycle voltage on the line causes a power arc to follow over the simultaneous lightning sparks from more than one conductor to brace, and a short circuit is established from line to line through the brace. In some instances on guyed structures, as indicated by observations four and five referring to Table III, the evidence indicates a line-to-line short circuit without a power arc to ground, but with the lightning taking a path to ground through the guy.

NEW 33,000-VOLT STRUCTURE DESIGN

It was apparent from the foregoing investigations that if material reduction were to be made in the number of lightning flashovers a radical change in design would be required. Experience with insulators of higher voltage rating in certain locations indicated that these alone would not give appreciably better results. More effective use of the inherent impulse insulation value of the wood in insulators was obviously necessary.[1,3,4] The next steps to be considered were the elimination of the conducting crossarm brace and an increase in the separation between conductors on the structures. It was believed that if these objectives could be accomplished, line-to-line short circuits could not occur except at a lightning surge voltage high enough to cause complete structure flashover. The design shown in Fig. 4 was accordingly worked out. It will be noted that this allows approximately 9 ft. of wood between insulator pins. The wood braces are so arranged that the distance on the crossarm from the point of attachment of the brace to the pole is equal to the distance between metal fittings on the brace. The dimensions selected were the result of using a standard crossarm to obtain the maximum practical separation between conductors. Separation less than 9 ft. might be reasonably satisfactory, but the resultant saving would be small and certainly not worth the chance that such lesser separation might not be satisfactory. It is believed that with this construction, incipient sparks forming over the insulators will not develop into short circuits, and that the only flashovers will be those caused by lightning voltages of such character and magnitude as to cause complete structure flashover to ground.

Evidence of the effectiveness of eliminating conducting parts between conductor supports on wood-pole construction in preventing line-to-line flashovers has been furnished by the performance of an H-frame line built with the same insulation as the standard 33-kv. wood-pole lines, but with a 5-ft. separation between insulator pins and with no metal braces short circuiting any portion of the wood crossarms. Investigation of all locations where flashovers had been found showed that the flashovers, with one exception, were all to ground on guyed structures. One case was found where a line-to-line flashover had occurred over the five feet of wood between insulator pins. This indicated that a separation of more than five feet on a wood-structure would generally be sufficient to prevent 33-kv. line-to-line short circuits, except for occasional unusually severe lightning voltages.

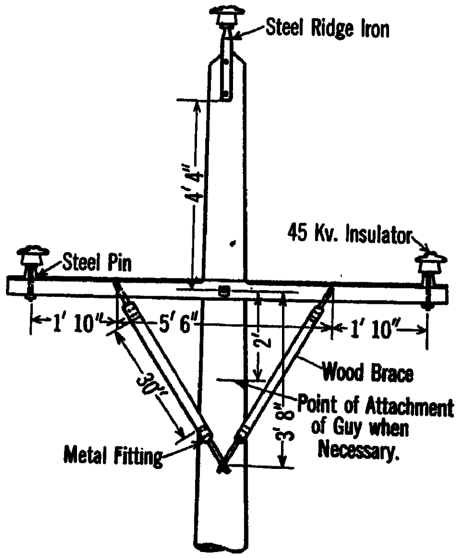

FIG. 4—NEW DESIGN, 33,000-VOLT WOOD POLE CONSTRUCTION

If flashovers of the plain structures are prevented by the new design, the guyed structures become the most susceptible to flashover. The obvious solution is to provide as much insulation as possible with respect to lightning between the line conductors and the grounded portions of the guys. This is accomplished by attaching the guys to the poles as far as possible below the crossarms or insulator attachment to the pole, and by adapting the wood crossarm brace in tension as a guy strain insulator, with two of them used in series on each guy.

The vertical corner construction in use was modified to a design with a six-foot separation between conductors on the pole, wood-guy strain insulators, and the guys attached to the pole midway between line insulator attachments. The line insulation on corners was increased to three insulator units (6.5-inch spacing) in series. Maximum use is thus made of the inherent insulating value of the wood in the structure.

The design of the wood brace and guy insulator, illustrated in Fig. 5, is interesting. A wedge assembly is inserted into a saw-cut at the end of the wood member. The assembly is held together by means of steel plates on the outside of the wood, bolted as shown. The straps which take the load are slotted lengthwise. When tension or compression load is applied to the brace, the load is transferred from the end straps to the wood member by means of the wedge, which places the wood between the wedge and outside plates in compression with the bolts acting in tension. The slots in the end straps prevent shear on the bolts.. The guy strain insulator under test develops a maximum strength in tension of 16,000 lb., the failure occurring in the end straps. The strength developed is not required of the brace, but this design was considered necessary for satisfactory assembly on the structure and for satisfactory operation during the life of the structure.

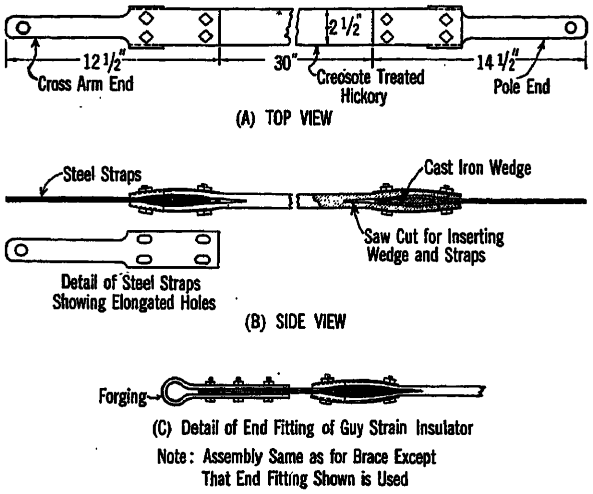
12 1/2"
Cross Arm End
30"
2 1/2"
Creosote Treated Hickory
14 1/2"
Pole End
(A) TOP VIEW
Steel Straps
Cast Iron Wedge
Saw Cut for Inserting Wedge and Straps
Detail of Steel Straps Showing Elongated Holes
(B) SIDE VIEW
Forging
(C) Detail of End Fitting of Guy Strain Insulator
Note: Assembly Same as for Brace Except That End Fitting Shown is Used

FIG. 5—WOOD CROSSARM BRACE AND GUY STRAIN INSULATOR

In order to obtain information as to the expected lightning performance of the new design, tests which gave the following approximate results were made with the cooperation of the General Electric Company at the Pittsfield high-voltage laboratory.

Tests on a guyed structure without guy insulators, using the old design, Fig. 1, and 45-kv. insulators, showed a lightning spark-over value of less than 400 kv., the insulator spark-over by itself occurring very close to the value required for complete structure flashover. On a plain wet structure without guys or grounds, using a 14-ft. effective length of pole and 70-kv. insulators, spark-over of the insulator itself occurred at about 900 kv., whereas the structure spark-over value was 2600 kv. The tests with a wet pole showed that with increasing height, the value at which the insulator spark-over occurred remained very nearly constant, while the structure spark-over value naturally increased with increasing length of pole. With a 45-kv. insulator the incipient spark-over on the insulator alone would have taken place at a value somewhat less than 900 kv. The value of lightning voltage at which this incipient spark takes place is considered to correspond to the value at which a 60-cycle arc would form as a short circuit between conductors with the steel-brace construction.

Tests with dry poles showed that spark-over of both the insulator and the complete structure took place almost simultaneously at nearly the same values of voltage.

The tests accordingly indicate that the line-to-line flashovers which take place on plain structures at low lightning-surge voltages may occur only when the structures are wet or possibly in connection with weathered or dirty structures whose behavior in the field is something like that of wet structures.

Tests of the new design, Fig. 4, showed the lightning spark-over value of the guyed structure without insulators in the guy to be approximately 1040 kv. The lightning generator arrangement used in these tests would not give sufficient voltage to cause structure spark-over with two insulators in the guy, but it is estimated, on the basis of correlated test data, that the two wood insulators to be used in the guy would raise the lightning spark-over value of the structure to approximately 1800 kv. It is believed that this is above the range of the usual maximum lightning-surge voltages which appear on the lines.

Pole-top switches appear to be especially susceptible to lightning flashover. These have heretofore been installed on wood poles by means of a grounded steel supporting frame. It was recognized that, if the trouble on the pole top switches were to be prevented, the same principles worked out for the new line construction would have to be applied to the pole-top switches. The matter has been taken up with the manufacturers, and several have worked out designs using wood-frame mountings with increased separation between phases, with wood-insulator interconnecting units in the operating mechanism and in the operating rod, and with only the operating handle grounded.

Another question which must be considered in connection with the new construction is the relation of line flashover value to that of station equipment flashover value. The change in design on the plain structures does not give a flashover value to ground materially different from that on old construction. However, the use of insulation in the guys will greatly raise the flashover value of all guyed structures. Lines entering substations are usually built with guyed structures close to the line terminal.

These locations in the past have apparently been acting as "safety valves," as their flashover value has usually been below that of the station insulation. A failure in the insulation of station equipment, however, is generally so much more serious that it is important to have the flashover of these "safety valves" always coordinated with the station insulation. Out on the line away from station equipment, the insulation may be as high as the economics of the situation will permit, but adjacent to the station, the insulation should be reduced to a value below that of the station equipment. To prevent service interruptions due to flashover at these points of reduced insulation, lightning arresters may be used. If the lightning arresters function properly, the reduced insulation should not flash over; if the lightning arresters do not function properly, the reduced insulation acting as "safety valves" prevents breakdown of the station equipment. In case it is considered that lightning arresters are not economically feasible because of the small amount of power being transmitted or the relatively small amount of equipment at the station, the use of fused arcing devices or their equivalent may provide one solution to the problem.

The effect of the line insulator with respect to the lightning spark-over of a wood-pole structure not guyed or grounded is relatively small. The lightning spark-over value of a 45-kv. insulator alone is about 260 kv., while that of an average 30-foot pole with crossarm is about 6000 kv.[1] On a guyed structure of the new design with an estimated lightning spark-over value of 1800 kv., the line insulator will still contribute only a small part of the total insulation against lightning. The size of the insulator for lines of this class therefore appears to be relatively unimportant on a structure well-insulated from ground for lightning voltages, and will be determined by the requirements with respect to preventing leakage of power current of normal frequency.

OVERHEAD GROUND WIRES

The effectiveness of overhead ground wires on wood structures in reducing interruptions due to lightning has been an open question.[5] Overhead ground wires were removed from certain lines several years ago. Records before and after removal show no consistent difference in performance. Comparison of performance of the small mileage of lines still equipped with overhead ground wires with other lines in the same territory without ground wires shows the same result. This is illustrated by Table IV, which shows the two-year record of the performance of several lines with overhead ground wires. It is of interest to compare this record with that for the entire 33,000-volt system as given in Table I.

The overhead ground wires are of either ⅜-in. galvanized steel or ¼-in. copper covered steel and are carried either on the pole top or at the end of a four-ft. crossarm, the other end of which carries one conductor. Earth connections are made at intervals of about every 600 to 800 ft. The down leads are fastened directly to the poles for the entire length and are terminated in ten-ft. galvanized ground pipes.

TABLE IV
LIGHTNING PERFORMANCE OF LINES WITH OVERHEAD GROUND WIRES

Line	Length in miles	Number of trip-outs directly due to lightning, 1928, per 100 miles	Number of trip-outs directly due to lightning, 1929, per 100 miles
A	31.13	29	51
B	26.44	33	57
C	30.82	20	10
Total*	88.39	27	36

*Totals figured by adding all T.O. and dividing by total miles.

The difference between the effectiveness of the overhead ground wires as usually installed on wood structures, and on metal towers,[4] is readily explained by consideration of the reduction in flashover value of the wood structure due to the presence of the ground wire. The presence of the ground wire reduces the flashover value of the structure in considerably greater proportion than it reduces the surge voltage on the conductors.

It is also probable that this reduction of induced voltage is in many cases not sufficient for the value of the lightning surge voltage on the conductors to be below the voltage at which incipient insulator spark-over will cause short circuits through the steel brace, as previously discussed.

Overhead ground wires on wood pole lines would undoubtedly be effective in reducing the number of outages due to lightning, if installed with low values of surge impedance and good earth connections and in such a manner as to maintain the lightning insulation value between conductors and ground wires above the values of lightning surge voltage for which a line is designed. The elimination of metal crossarm braces, the installation of the down-lead ground connections insulated by air or otherwise from the pole tops and conductors, and the use of insulated supports for the overhead ground wire, are methods which assist in increasing its effectiveness.

Except where lightning conditions are severe, it is probable that wood pole lines without metal crossarm span braces and with well insulated guys built without overhead ground wires will withstand most lightning surge voltages appearing on the conductors. In situations where splitting of structures from lightning is serious, the overhead ground wire, if installed so that the flashover value of the structure is not too greatly decreased, may be justified. The use of protective gaps to prevent structure damage should, of course, also be considered. The number of cases of lightning damage to wood structures on the system described has not been considered sufficient to warrant the general use of protective measures with respect to pole or crossarm splitting.

INSULATOR PERFORMANCE

The study of damaged insulators removed from the line has yielded valuable information relative to the service obtained from insulators of various types. The total amount of insulator damage on the system from both electrical causes and from external breakage is shown by Figs. 6 and 7. Insulators replaced in connection with the annual line-insulator testing program are not included. The curves, together with the last two items in Table I, indicate that replacements are an item of considerable expense. The external breakage is shown to be more than twice the electrical damage. Fig. 8 shows a portion of a study of one line upon which an unusual number of damaged insulators has been replaced. This study was made with the object of visualizing the effect, if any, of the geographical and physical characteristics of the line upon the insulator performance. The electrical and mechanical insulator damages are shown for each group of ten poles throughout the length of the line.

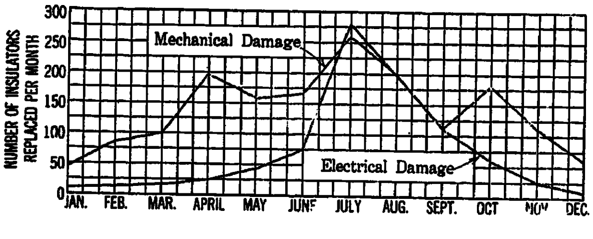

FIG. 6—INSULATOR DAMAGE PER MONTH FOR THE YEAR 1928

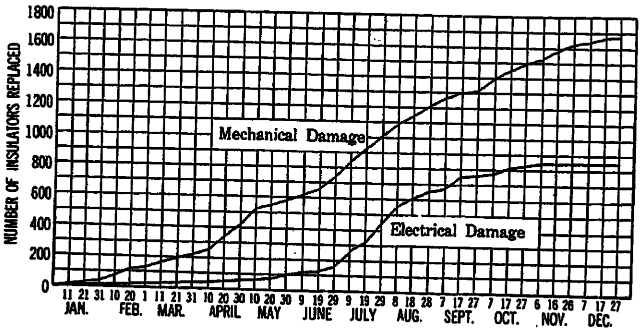

FIG. 7—TOTAL INSULATOR DAMAGE DURING THE YEAR 1928

Table V gives an analysis of damage to insulators replaced on account of defects from electrical causes. This does not include all insulators flashed over during this period, as many arc-overs occur which do not damage the insulator sufficiently to be located. It will be noted that the greater percentage of insulators broken are of the two-piece type. This may be due to the difference in design of the two-piece and three-piece insulators, the latter being generally made up of shorter and thicker skirts of a smaller diameter than the former. A large percentage of the three-piece insulators are of designs now obsolete, which have developed many shell cracks.

It is also seen that a considerable number of both types of insulators is flashed but not broken, while about the same number has only one skirt broken and a much smaller number is shattered. These figures give an indication of the number of insulators which remain on the line in a more or less hazardous condition after a flashover occurs, and until they are discovered by patrol and replaced.

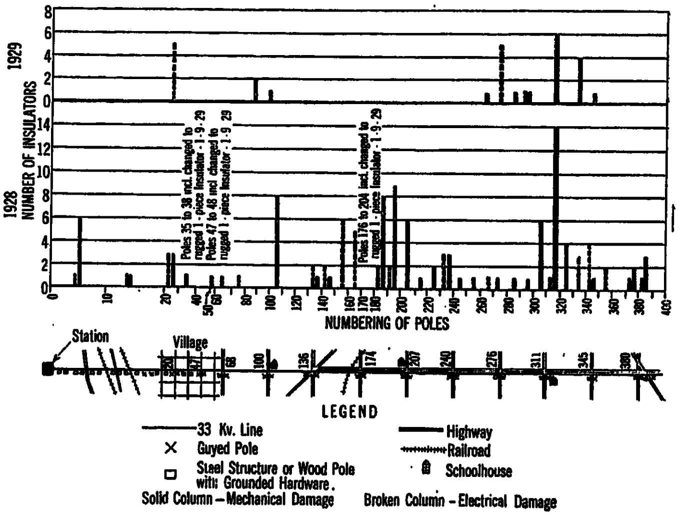

FIG. 8—MAP OF LINE

A study of the records indicates that there is little difference in the susceptibility to lightning flashover between obsolete and new insulators. However, the data show quite conclusively that certain types of the old insulators are much more susceptible to shattering than modern insulators, when flashover occurs. When shattering occurs, the line will often not go back into service, or the wire will drop onto the crossarm causing a structure fire. It appears that the cracks in the top

TABLE V
CLASSIFICATION OF INSULATOR DAMAGE FROM ELECTRICAL CAUSES
Jan. 1—Dec. 31, 1927

Nature and extent of damage	Quantity of insulators damaged		Damaged insulators per 1000 in service of each type	
	2-piece	3-piece	2-piece	3-piece
Small burn (not bad enough to replace)	17	16	0.485	0.188
Flashed but not broken	84	104	2.4	1.22
One skirt broken	82	112	2.34	1.32
More than one skirt broken	15	22	0.429	0.259
Shattered	19	31	0.543	0.264
Broken but unclassified	17	35	0.485	0.412
Total	234	320	6.69	3.76

Total—2-piece insulators in service—35,000
Total—3-piece insulators in service—85,000

shells of the old insulators are an important factor in this kind of trouble.

The performance of the old insulators with shell cracks has been a matter of much concern. A line-insulator testing program which has been under way for several years has resulted in the removal of a large number of the cracked insulators from service. However, visual inspection of insulators in service and of those removed from service for various reasons other than the test program, indicates that because of the progressive cracking which occurs, the lapse in time between test periods, and the difficulty of picking out clean cracks, a considerable number of insulators with cracks is continually in operation. Records are available of two 33,000-volt lines, each with a different type of 45,000-volt insulator. In one case 89 per cent of the insulators had top-shell cracks, while in the other case 45 per cent were so affected. There were no cases of insulator failure (electrical breakdown through the porcelain) on these lines during 1929.

A good many cracks are also found in the center and bottom shells, and inspection of the insulators which have failed, frequently discloses old cracks in more than one shell. There is, accordingly, good ground for the belief that with 45,000-volt insulators on 33,000-volt lines, cracking of only the top shell does not seriously affect the operation of the line, and that the insulator failures which cause interruptions are practically all due to the failure of the porcelain in more than one shell. The study of cases of trouble in which conductors were burned down or in which structures were burned shows that the types of insulators in which cracks were prevalent, largely of the older types, were largely responsible for the conductor or structure failures aside from cases of lightning flashover. This is shown by Table VI.

TABLE VI
NUMBER OF CASES OF CONDUCTOR OR STRUCTURE FAILURES AS INFLUENCED BY INSULATOR CRACKING

	With insulators free from cracking	With insulators subject to cracking
Conductor burned down due to lightning flashovers	22	19
Conductor burned down due to insulator failure (not lightning)	0	19
Total	22	38
Structure burned due to insulator failure (not lightning)	0	103

The damage to insulators from external breakage is shown by Table VII. Damage of this class is due to such causes as blasting where the line is near quarrying operations, stone throwing, shooting, and so on. As was observed with respect to electrical breakage, two-piece insulator damage is relatively greater than the three-piece insulator damage.

It is frequently taken for granted that insulator breakage cannot be materially reduced. However, in an effort to accomplish something in this direction, installations of thick porcelain one-piece insulators were made in locations where an extraordinary amount of breakage was being experienced. The results are shown in Fig. 8. During 1928, with two-piece insulators, breakage was especially severe between poles 176 and 204, in the vicinity of a schoolhouse. One-piece insulators were installed on these poles early in 1929, and since that time no replacements have been required on account of breakage.

TABLE VII
CLASSIFICATION OF INSULATOR DAMAGE FROM EXTERNAL BREAKAGE
Jan. 1—Dec. 31, 1929

Nature and extent of damage	Quantity of insulators damaged		Damaged insulators per 1000 in service of each type	
	2-piece	3-piece	2-piece	3-piece
Small chip	140	365	4.0	4.3
One skirt broken	285	379	8.14	4.46
More than one skirt broken	39	69	1.11	0.812
Shattered	1	8	0.0286	0.0941
Broken but unclassified	119	176	3.4	2.07
Total	584	997	16.7	11.7

Total 2-piece insulators in service—35,000
Total 3-piece insulators in service—85,000

A large number of the one-piece sturdy insulators placed in service during the past year have given an especially satisfactory record of operation. Of the total of 5000 installed during 1929, only five have required replacement because of breakage.

Two types of one-piece insulators are being used, both being selected primarily on account of their ruggedness. Other points, such as dependability of manufacturing processes and freedom from some of the causes which are believed to have contributed to troubles with multi-part insulators, were also considered. One type is similar in shape to the conventional two-piece unit. The other is more cylindrical in outline and has five thick narrow skirts of the same diameter.

The ability of various makes and types of insulators to withstand breakage was investigated in the laboratory. A one-inch steel ball was dropped on the skirt of an insulator by a special testing device from various heights starting at 18 in. and increasing by increments of 6 in. Three tests were made from each height, the tests being continued until the insulator was badly chipped or broken.

The comparison of the insulators tested is shown by Fig. 9. (*A*) is the thick-porcelain one-piece insulator of conventional shape, which is now being installed; *B*, *C*, *D*, *F*, *G*, and *H* are two-piece 45-kv. insulators of different manufactures, the shapes all being quite similar. As one outcome of these tests, suggestions made to one manufacturer resulted in design changes to one insulator which placed it in the class of the most sturdy insulators.

A considerable difference in decrease of flashover value of different types of insulators, when broken, was also found as a result of these tests.

CONDUCTOR FAILURES

A study of the cases of conductor failures due to the arc following lightning flashovers indicated that the susceptibility of wires to burn-down is a function of conductor size. The smaller sizes, No. 0 and smaller, show more cases of this kind of conductor failure than the larger sizes. Satisfactory data are not now available to evaluate the differences in this respect between the several wire sizes.

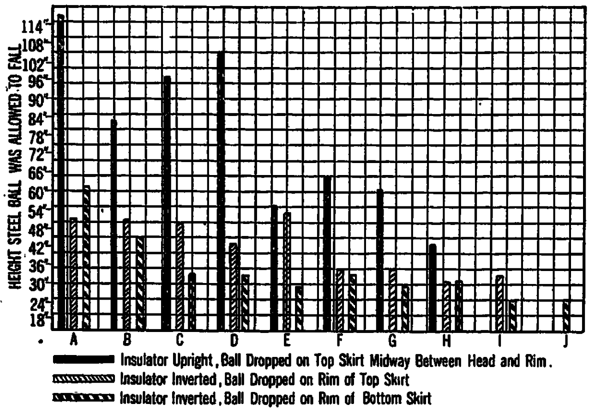

FIG. 9—MECHANICAL SHOCK RESISTANCE TEST OF INSULATORS

The height of the lines, which show the shocks applied in different ways, indicates the points at which large chips or definite breaks occurred. Each group of lines represents one insulator.

CONCLUSION

The high degree of reliability being obtained from transmission lines of the higher order of voltages, 110,000 volts and above, which are insulated to withstand the lightning surge voltages to which they are subjected, indicates that it should be possible to build lower-voltage lines which will give a class of service not obtainable with designs ordinarily in use at present. Wood pole structures of the present ordinary design are subject to flashover from the action of lightning-surge voltages much lower than voltages required to cause structure flashover from line to ground. By revision of the present designs of wood pole structures to make full use of the inherent insulating value of the crossarms and poles against lightning surge voltages, and by elimination of metal braces or other parts tending to make conducting paths between phases, it should be possible to obtain highly satisfactory operation with respect to lightning troubles. Lines are now being constructed with wood crossarm braces and wood guy insulators which, it is believed, will be practically free from lightning troubles.

The performance of lines of the new design will be watched closely, and information relative to this will be available later.

The records of operation of wood-pole lines with overhead ground wires, when considered in connection with the information given relative to lightning flashovers, shows that the overhead ground wires as usually installed do not improve the performance of wood-pole lines. Overhead ground wires, to be effective, should be so installed that as much as possible of the inherent insulating value of the structure is retained.

Electrical breakage of old-style pin-type insulators is serious with respect to number of line failures due to flashover. Breakage of insulators from mechanical causes is greater quantitatively than from electrical causes. Mechanical breakage is greatly reduced by the use of more sturdy insulators which are now available.

Systematic thorough investigation of line troubles, with well-organized records and proper interpretation of the facts thus made available, are necessary if proper application of operating experience is to be made use of in the design of transmission systems. The importance of establishing a systematic plan of periodically summarizing and analyzing the facts available from the records is to be emphasized. The day-to-day records, by themselves, are usually of little value. It is only by regularly gleaning the facts from them by a well defined plan that they become valuable. These facts, properly used, will form a basis for great improvement in system reliability and more effective utilization of investment.

ACKNOWLEDGMENTS

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Discussion

J. T. Kelly: I was particularly interested in one point which the authors brought out which seems rather peculiar at first glance and that is the flashover of an insulator without establishing an arc to ground. On any transmission structure involving wood members we have a series of insulators,—the insulator itself, a short section of crossarm between insulator and metal brace, a short section of pole perhaps, between the metal brace and grounded guy attachment, or other metal attachment. In many structures there is a great number of small insulating elements between conductor and ground. Now, that series of insulators is also something else. It is likewise a series of condensers, since a condenser is a dielectric interposed between two conducting plates, and if one thinks of a transmission structure, particularly a wood pole structure, in those terms, it is apparent that there may be a large series of condensers between line conductor and ground. In any series of condensers, when the weakest member of the series is over-stressed, that is the condenser of lowest capacitance, that condenser will discharge, and that is exactly the phenomenon which takes place and to which Mr. Andrews made reference.

This is a very useful point of view in visualizing some of these phenomena. It is also an extremely useful standpoint when analyzing radio trouble because if one thinks of the setup as a series of condensers, it will help many times in finding the particular condenser causing the radio trouble.

C. L. Stroup: I wish to thank Mr. Kelly for his words. In collecting data it became necessary to set up quite an organization to obtain the information and compile it in a systematic way. This organization is being maintained and will continue to function and it is hoped to obtain information which will be of operating value to the industry.

Arcing Grounds and Effect of Neutral Grounding Impedance

BY J. E. CLEM*
Associate, A. I. E. E.

***Synopsis.**—This paper was written to review and extend the theory of overvoltages due to arcing grounds because of the increasing tendency to use impedances between the neutral point and the ground, thereby losing the advantage of the solidly grounded neutral. The "third class conductor" theory of Steinmetz is touched upon very briefly and is considered as not applying to transmission line conditions. The theory when the phenomenon is controlled by normal frequency arc extinction, as presented by Peters and Slepian, is reviewed and the maximum voltage for this analysis is found to be $3\frac{1}{2}E$, where E is the normal line to neutral voltage. The theory when the phenomenon is controlled by oscillatory frequency arc extinction as originated by Dr. Petersen is given in detail but in a modified and extended form. The maximum voltage for a single-phase circuit when no damping is considered is found to be $6E$.*

The analysis for the three-phase circuit is newly developed for the case in which there is an impedance between the neutral and ground and the maximum voltage is found to be $7\frac{1}{2}E$ when the effect of the damping factors and capacitance between lines is neglected. The method of determining the various reductions or damping factors is outlined.

The effect of a neutral grounding resistor is discussed and it is pointed out that a suprisingly high value of resistance can be used without incurring the possibility of dangerous overvoltages. It is shown that the use of reactance is more liable to result in overvoltages than resistance but that relatively large values of reactance can be used in conjunction with resistance.

The Petersen Coil is usually considered as causing the arc to go out by giving a balance of lagging and leading currents in the arc. It is brought out in this paper that there will be no voltages built up when the Petersen Coil is used whether or not the arc goes out.

The relation of the overvoltages on a non-grounded and an effectively grounded system is outlined, and a criterion for determining whether or not a system is effectively grounded is proposed.

INTRODUCTION

In the operation of power distribution systems overvoltages appear on the transmission lines, whether overhead or cable, from switching, arcing grounds, and lightning. These overvoltages are frequently of sufficient magnitude to cause interruption of service and sometimes to destroy apparatus. The hazard of arcing grounds is completely eliminated by connecting the neutral directly to ground, but the present day tendency is away from the direct ground connection on account of the necessity of keeping down the concentration of power in a short circuit, to reduce the damage to equipment at the location of the short circuit, to increase stability, and to a lesser extent to lower the duty on circuit breakers by inserting an impedance in the neutral. The purpose of this article is to review briefly the various analyses[1] proposed for the explanation of the overvoltages caused by arcing grounds, to extend the theory, and to apply it to grounded neutral circuits.

There are in general three methods of explaining the high voltages produced by arcing grounds. The first two are quite similar and are based on the familiar theory of the oscillatory charge of a capacitance through resistance and inductance. They differ only in the manner in which the arc is considered to go out. In one case the arc is considered to go out at the instant when the normal frequency current passes through zero. When the neutral is grounded through a moderate impedance the phenomenon will most likely be controlled by normal frequency arc extinction but it is useful to know what might happen if the phenomenon was controlled by oscillatory arc extinction. In both the arc is considered to strike again as soon as the faulty line rises to its maximum potential against ground.

The third method is due to Dr. Steinmetz and is based on what he termed a "third class conductor." In ordinary conductors an increase of current requires an increase of voltage and the resistance may be defined as

$$\frac{\Delta E}{\Delta I} = r$$

In an arc the characteristics are such that the ratio of voltage to current is not constant, the resistance tending to rise with decreasing current. An increment of current then requires a decrement of voltage and if the ratio be defined as r'

$$r' = -\frac{\Delta E}{\Delta I}$$

An oscillation dies away because the energy is consumed in resistance and the rate is given by the factor ϵ^{-rt} which occurs in the equations. If the quantity r in the exponent should itself happen to be negative the factor would be ϵ^{rt} and the transient, instead of dying away, would increase beyond all bounds. However, from available data it appears that this characteristic probably does not apply to the arcing ground as it occurs on an overhead line.

SUMMARY

In this paper there is developed the following expression for estimating the maximum voltage that may be

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*Central Station Engg. Dept., General Electric Company, Schenectady, N. Y.

[1] For references see Bibliography.

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produced by an arcing ground on a grounded system, on the basis that the arcing ground voltage is controlled by oscillatory arc extinction.

$$\frac{V}{E} = \frac{3}{2}\,\frac{(2-d)\,(3-2\,b) - \left(1 + 2\,\frac{C}{C_m}\right)(1-d)}{3 - 2\,(1-a)\,(1-d)}$$

V = maximum voltage to which sound lines oscillate

E = normal voltage line to neutral-maximum

$1 - d$ = damping factor of oscillation

$1 - a$ = damping factor of redistribution of bound charge

$1 - b$ = damping factor due to presence of neutral ground impedance

C = average capacitance of single conductor to ground

C_m = average mutual capacitance between pairs of conductors

This equation, as has just been mentioned, is based on the assumption that the phenomenon is controlled by oscillatory frequency, arc extinction, whereas when there is an impedance in the neutral, the phenomenon is very probably controlled by normal frequency arc extinction. Also, the assumption is made that each successive striking of the arc from the faulty line to ground occurs at an increasing voltage. In addition, the ionization of the arc, which may cause it to be maintained, is neglected. These factors tend toward lower voltages.

In any oscillation the voltage will go as far on one side of the average or applied value as it does on the other, unless there is damping. When there is damping the voltage on the first oscillation fails to go to twice the applied and the factor $(1 - d)$ is a measure of this; for instance, if the applied voltage was 100 volts and on the first oscillation the voltage rose to 180 volts, the factor $(1 - d)$ would be 0.9.

In the analysis of the arcing ground condition the fundamental assumption is that a charge is trapped on the sound lines when the arc is extinguished. This bound charge, as it is called, is that required to establish the voltage present on the sound lines at the instant the arc goes out. This charge redistributes throughout the system between the extinguishing of the arc and the restriking of the arc on the next normal frequency cycle and establishes an average potential. However, during this interval a certain part of the charge leaks off to ground and the average potential is reduced proportionately and the factor $(1 - a)$ is the measure of this effect. As an example, if the bound charge was such that it would establish an average potential of 100 volts if no leakage occurred but actually does establish an average potential of 60 volts, the factor $(1 - a)$ is 0.6.

If, on an isolated system, one line becomes grounded the neutral rises to leg voltage above ground, but if the system is grounded through an impedance the shift of the neutral will be less depending upon the relative value of the neutral grounding impedance and the factor $(1 - b)$ makes allowance for this effect. For instance, if the leg voltage is 100 volts and on a fault the neutral rises to 30 volts the factor $(1 - b)$ would be 0.3.

On an ungrounded system the maximum voltage which may be developed by an arcing ground is $3\frac{1}{2}E$ if the phenomenon is controlled by normal frequency arc extinction and $7\frac{1}{2}E$ if controlled by oscillatory frequency arc extinction. The limiting value of $7\frac{1}{2}E$ is obtained when the effect of the mutual capacitance between conductors and all damping factors are neglected. On average lines the ratio of C/C_m ranges from 0.18 to 0.28 so that the effect of mutual capacitance is to reduce the limiting overvoltage from $7\frac{1}{2}E$ to a value between $6.66\,E$ and $6.96\,E$. Fair values of the damping factors $(1 - d)$ and $(1 - a)$ are about 0.95, tending to be slightly higher for $(1 - d)$ and slightly lower for $(1 - a)$, and these reduce the probable maximum voltage for an arcing ground on an isolated system to a value between $5.3\,E$ and $5.7\,E$.

On a grounded system there can be no overvoltage from arcing grounds if the neutral is solidly grounded. In this case there is no neutral shift and the factor $(1 - b)$ becomes zero as also does the factor $(1 - a)$ with the result that no overvoltage appears from the calculations. However, if an impedance is inserted in the neutral the amount of overvoltage that may be calculated on the basis that the phenomenon is controlled by the oscillatory frequency arc extinctions depends upon the amount and kind of such impedance. If the impedance consists of resistance alone a surprisingly high value may be used without incurring any chance of dangerous arcing ground voltages being set up. As the resistance is increased the discharge of the bound charge to ground through the resistance changes from oscillatory to unidirectional and a value of resistance equal to twice the critical value can be used without exceeding a calculated voltage of $2\frac{1}{2}E$. If the impedance consists of reactance the calculated voltages will be higher but values of reactance high enough for practical purposes can be used without danger of serious arcing ground voltages. If the neutral grounding reactance is of such a value as to give a neutral shift of $66\frac{2}{3}$ per cent E on short circuit the arcing ground voltage would calculate from $3.7\,E$ to $4\,E$, the variation in calculating voltage being due to the variation in assumed mutual capacitance between lines—see Table III. When the values of neutral grounding impedance are such that these voltages are not exceeded the system will be effectively grounded.

The following tabulation gives the maximum voltages for the conditions already outlined.

TABLE I
TABULATION OF MAXIMUM VOLTAGES

Isolated system	Single phase	Three phase
Initial arc	$3E$	$2\frac{1}{2}E$
Normal frequency arc extinction	$4E$	$3\frac{1}{2}E$
Oscillatory frequency arc extinction Isolated System		
a—No damping	$6E$	$7\frac{1}{2}E$
b—Capacitance only		$6.7E$—$7E$
c—Capacitance and damping		$5.3E$—$5.7E$
Effectively grounded system (see below)		
a—Through resistance only		$2\frac{1}{2}E$
b—Through reactance only		$3.7E$—$4E$

From this tabulation it is seen that the probable maximum voltage that may be produced on an isolated system by an arcing ground is approximately 5.5 times the normal line to neutral voltage. This is approximately the same as the overvoltages produced by switching surges. The insulation required for both types of overvoltages must be based on the 60-cycle flashover characteristics of the insulation.

It is customary at times to base the insulation of transformers and cables upon a decision as to whether the system can be classified as a grounded or non-grounded system. In the past it has usually been the practise to consider a system as non-grounded unless the neutrals were solidly grounded. For a non-grounded system the apparatus has, in general, insulation based on 3.46 times the rated line to neutral voltage and the maximum arcing ground voltages calculate in the neighborhood of 5.7 times. However, the apparatus gives satisfactory service and so must have some factor of safety. If insulation is used corresponding to a 2.73 test the arcing ground voltage should not calculate more than 4.5 times normal line to neutral voltage for the same factor of safety. This condition is satisfactorily met when the reactance between the neutral and ground is such that the neutral shift is less than two-thirds the normal line to neutral voltage; and is more than met when the resistance between the neutral and ground is such that the lagging component of the fault current is equal to or greater than the leading component. This condition gives a resistance less than the critical resistance previously mentioned. The foregoing statements are based on the assumption that the arcing ground phenomenon is controlled by the oscillatory arc extinction and if the phenomenon is controlled by the normal frequency arc extinction the arcing ground voltages will be less.

On this basis a system may be said to be effectively grounded when:

(a) *Grounded through resistance.* If a system is grounded through resistance only it may be classified as an effectively grounded system when the resistance is of such a value that the lagging component of the fault current is equal to or greater than the leading component of the fault current.

(b) *Grounded through reactance.* If a system is grounded through reactance only it may be classified as an effectively grounded system when the value of reactance is such that the neutral shift on a short circuit is not more than two-thirds the normal line to neutral voltage.

A system may be said to be grounded through resistance when resistance only is used between the neutral and ground. In this case some of the neutrals may be solidly grounded and the others grounded through resistance.

A system may be said to be grounded through reactance when a reactance coil is used between neutral and ground, when only part of the neutrals are grounded, or when grounding transformers are used.

The voltage figures previously given are the voltages that are produced on the sound lines under arcing ground conditions, and it was shown how the insulation might be affected. On an isolated system the voltage on the neutral would be two-thirds of the voltage on the sound lines, so that the maximum voltage which might appear on the neutral of an isolated transformer would be $3.53\,E$ to $3.8\,E$. On the assumption that the line end is insulated for a 3.46 test the neutral end should be insulated for a test of 2.04 to 2.31 times normal line to neutral voltage for the same factor of safety. If the system is effectively grounded as previously defined insulation corresponding to a test of two times leg voltage will be ample and a lower value may be used depending upon the calculated voltages and the neutral shift.

The Petersen Coil is a special case of reactance ground for which the reactance value is such that the lagging component of the short-circuit current is made equal to the leading component of the charging current in the fault. For this case the factor $(1 - a)$ becomes -1 so that the maximum voltage is limited to $1.34\,E$. The adjustment of the Petersen Coil is not critical so that considerable variation from the resonant value may be allowed without giving much increase in voltage. The Petersen Coil will keep the voltages down to harmless values regardless of whether or not it is set at the resonant value to extinguish the arc.

NORMAL FREQUENCY ARC EXTINCTION

In Fig. 1 is shown a single-phase or two-wire circuit with Lines 1 and 2 and the winding of the power source of the circuit. In Fig. 2 are shown the voltages from Line 1 and Line 2 respectively to neutral or ground, and the voltage E_{12}, which is the voltage from Line 1 to Line 2 or the voltage between lines. The neutral or ground potential point is at the midpoint of the winding and the line voltages vary from (+ E) to (− E) in respect to the neutral point.

In Fig. 3 are shown the voltages when an arc strikes from Line 2 to ground. The voltage of Line 2 to ground collapses and the potential of Line 2 becomes zero. This takes place almost instantly after the arc strikes. Line 1 is at an initial potential $E_1 = (+E)$ in respect to ground, but zero potential of Line 2 requires a potential of (+ 2 E) on Line 1. Consequently the potential of Line 1 goes from potential (+ E) to potential (+ 2 E) through an oscillation, the impressed voltage being 2 E. The applied voltage producing the oscillation is (2 E − E) or E, and therefore the amplitude is E, and this, neglecting damping, added to the impressed voltage 2 E gives the maximum value of 3 E. After steady conditions are reached, Line 2 is at ground or zero potential and the line potential E_1, to ground, and voltage E_{12}, between lines, coincide, the arc current I being represented 90 deg. ahead of E_1.

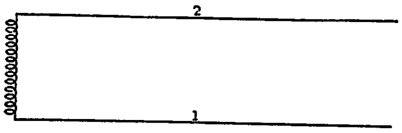

FIG. 1

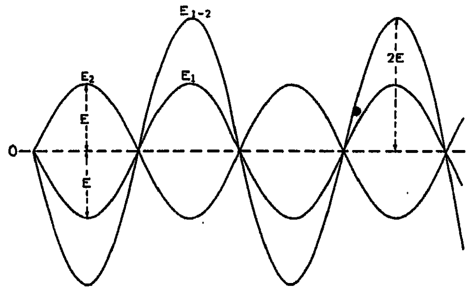

FIG. 2

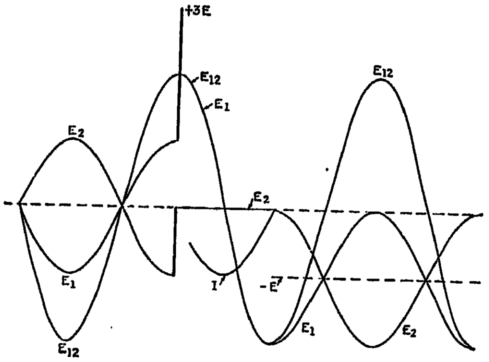

FIG. 3

When the current I passes zero, the arc is assumed to go out. At this instant Line 1 is at a potential (− 2 E) and has a charge sufficient to establish this potential. This charge diffuses throughout the entire system so that the average potential becomes (− E), the line potentials varying harmonically above and below this potential level as shown in the right hand part of Fig. 3, on the assumption that the defective Line 2 does not arc to ground again.

However, the arc is assumed to strike again as in Fig. 4. This time the initial voltage of Line 1 is 0, the impressed voltage is 2 E, and the amplitude of the oscillation is 2 E so that the maximum potential that Line 1 reaches is 4 E. As before, the arc goes out when the arc current passes through zero and Line 1 is at a potential of (− 2 E) and conditions again are exactly as they were before the arc struck.. This process may be repeated with no further increase in voltage until the circuit conditions change in some way.

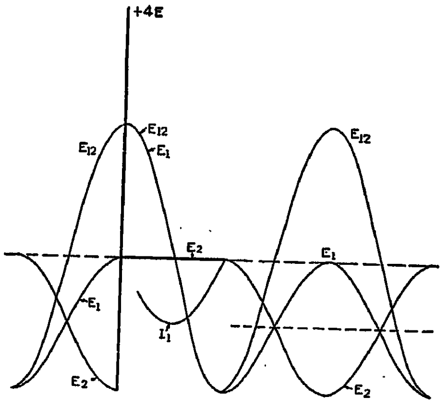

FIG. 4

The analysis for a three-phase circuit is similar to that just given for the single-phase circuit. In Fig. 5 is shown the circuit and in Fig. 6 are shown the voltages. Line 2 is the faulty line and it arcs to ground at the instant of maximum potential and immediately goes to ground or zero potential. Zero potential on Line 2 requires a potential of 1.5 E on both Line 1 and Line 3 but these two lines are at an initial potential $\left(+\frac{1}{2}E\right)$ and pass from the initial condition of $\left(+\frac{1}{2}E\right)$ potential to the ultimate condition of $\left(+1\frac{1}{2}E\right)$ through an oscillation. The impressed voltage producing the oscillation is $\left(+1\frac{1}{2}E\right)$ and the applied voltage

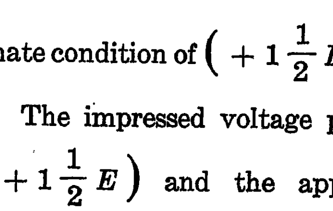

FIG. 5

$\left(1\frac{1}{2}E - \frac{1}{2}E\right) = E$, therefore the amplitude is E, giving $2\frac{1}{2}E$ as the maximum voltage of the first oscillation.

After the oscillation has died out conditions are also shown in Fig. 6. Line 2 is at zero potential and Lines 1 and 3 have the potential E_{12} and E_{32} as shown. The arc current is made up of I_3 and I_1 which are shown 90 deg. ahead of their respective voltages. The arc will go out at the instant the arc current $(I_3 + I_1)$ is zero. At this instant Lines 1 and 3 are both at a potential of $1\frac{1}{2}E$ and the charge on the conductors necessary to produce this potential diffuses throughout the system and establishes an average system potential of $(-E)$, about which the potentials of the three lines vary.

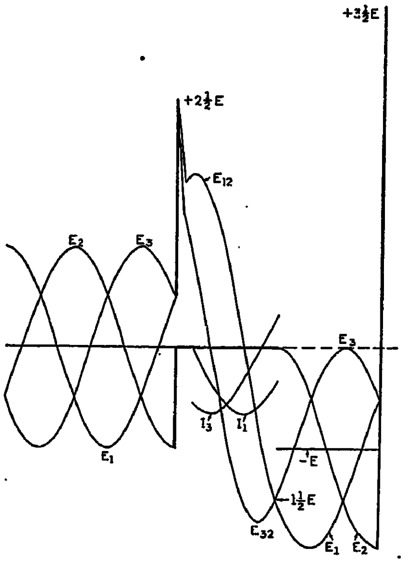

FIG. 6

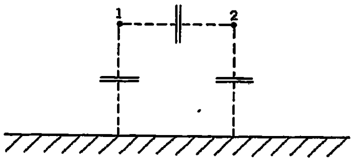

FIG. 7

When the faulty Line 2 reaches its maximum potential of $(-2E)$ the arc again strikes and at this instant Lines 1 and 3 are at an initial potential $\left(-\frac{1}{2}E\right)$. Line 2 then goes to zero potential which condition requires a potential of $\left(+1\frac{1}{2}E\right)$ on both Lines 1 and 3. However, Lines 1 and 3 are at a potential $\left(-\frac{1}{2}E\right)$ and an oscillation results, the applied voltege is $2E$, therefore the amplitude is $2E$ which gives a maximum voltage on Lines 1 and 3 of $3\frac{1}{2}E$. As before the arc will go out when the arc current passes through zero at which time the potentials of Lines 1 and 3 will again be at $\left(-1\frac{1}{2}E\right)$ and the sequence of events will be repeated with no further increase in voltage.

OSCILLATORY FREQUENCY ARC EXTINCTION

Single-Phase Circuit. The analysis for the single-phase circuit will be given complete and in detail to illustrate the method of analysis. In Fig. 7 are represented the two wires of a single-phase circuit, together with the respective capacitances. The effective value of these capacitances at any instant depends upon the potentials of Line 1 and Line 2, as is brought out in the Appendix. In Fig. 2 is shown the voltage wave of the two wires in respect to ground. The maximum voltage to ground of either line wire is E and the voltage between wires is $(E_2 - E_1)$ or $(E_1 - E_2)$ with a maximum of $2E$.

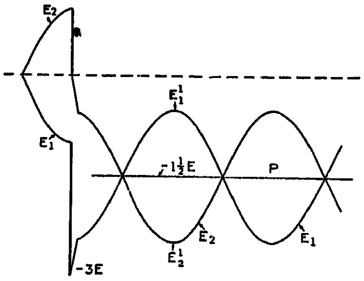

FIG. 8

The arc is assumed to strike from Line 2 to ground, thereby reducing the potential of Line 2 to zero and short circuiting the capacitance between Line 2 and ground. The initial arc is considered to occur when Line 2 is at its maximum potential to ground $(E_2 = +E)$. This condition of zero potential of Line 2 requires at this instant for steady state conditions a potential of $(-2E)$ on Line 1, which is equivalent to impressing a voltage of $(-2E)$ on Line 1. At the instant the arc strikes the potential of Line 1 is $(-E)$ and the transition from $(-E)$ to $(-2E)$ takes place as an oscillation (see Fig. 14A). It is assumed that the arc goes out the first time the oscillatory arc current passes through zero which is practically simultaneous with the instant of maximum potential of Line 1.

In Fig. 8 are shown the conditions immediately after the arc has struck and the conditions at the instant the arc has gone out. At the instant the arc goes out the voltage to ground on Line 1 is $(-3E)$, which necessitates a corresponding charge $(-Q)$ on the sound line. After the arc goes out this charge distributes itself over the system passing through the apparatus at the end of the line and establishes the average potential to ground of the system as $\left(-1\frac{1}{2}E\right)$, corresponding to a charge of $\left(-\frac{Q}{2}\right)$ on each of the two lines. This redistribution of charge takes place through an oscillation which is not shown in the diagrams.

After steady conditions are attained, conditions are as indicated on the right of Fig. 8. However, this steady state cannot be maintained as the faulty Line 2 will again arc to ground and it is assumed that this will occur when its potential to ground becomes a maximum. This is shown in Fig. 9 in which the second arc is shown as striking when the voltage of the faulty Line 2 has reached a value of $\left(-2\frac{1}{2}E\right)$. At this instant the potential of Line 2 immediately goes to zero. For the resulting oscillation the initial potential of Line 1 is $\left(-\frac{1}{2}E\right)$ and the impressed voltage is $(+2E)$, as required by the condition of zero potential on Line 2. The transition from a potential of $\left(-\frac{1}{2}E\right)$ to $(+2E)$ takes place through an oscillation which has an applied voltage of $\left\{+2E-\left(-\frac{1}{2}E\right)\right\}$ or $\left(+2\frac{1}{2}E\right)$. The maximum potential to which Line 1 oscillates during the oscillation is $\left(2\frac{1}{2}E+2E\right)$ or $\left(+4\frac{1}{2}E\right)$, at which instant the arc goes out, leaving a charge corresponding to a potential $\left(+4\frac{1}{2}E\right)$ on Line 1. This charge then diffuses itself throughout the entire system establishing an average potential of $\left(+2\frac{1}{4}E\right)$ through the entire system.

As before, the potentials of Line 1 and Line 2 vary about this average potential and when the potential of Line 2 reaches its maximum value of $\left(+3\frac{1}{4}E\right)$ the arc strikes again. At this instant the potential of Line 1 is $\left(+1\frac{1}{4}E\right)$ so that the resulting oscillation has an impressed voltage of $(-2E)$ and an applied voltage and amplitude of $\left(-2E-1\frac{1}{4}E\right)=\left(-3\frac{1}{4}E\right)$. The potential of Line 1 goes to a maximum of $\left(-5\frac{1}{4}E\right)$ at which instant the arc goes out. A charge corresponding to a potential of $\left(-5\frac{1}{4}E\right)$ is left on Line 1 and this charge diffuses as before throughout the entire system, establishing an average potential over the two lines of $\left(-2\frac{5}{8}E\right)$. This process repeats itself until the maximum voltage is attained or something happens to remove the system voltage.

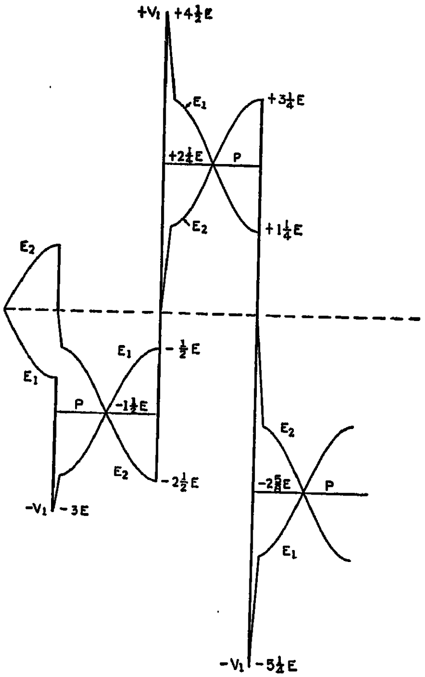

FIG. 9

The maximum possible voltage can be determined from the following considerations and reference to Table II. Let the maximum potential of a negative oscillation be denoted by $(-V_1)$, (see Fig. 9). Immediately following the extinction of the arc the charge associated with the voltage $(-V_1)$ will distribute itself throughout the system and establish a potential P. To determine P it is necessary to know the relationship existing between the various capacitances involved; these are developed in the Appendix. When Line 2 arcs to ground it goes to zero potential and the potential $(-V_1)$ to which Line 1 oscillates is established over the capacitance C_0, requiring a charge

$$Q_1 = -V_1 C_0 \qquad (11)$$

The capacitance C_0 is defined as the capacitance of one line wire to ground when the other line wire is grounded, *i. e.*, at zero potential.

TABLE II

DERIVATION OF MAXIMUM VOLTAGE

Arc No.		1	2	3	4	5	N	$N+1$	$N+2$
Line 2	E_2	+1	$-2\frac{1}{2}$	$+3\frac{1}{4}$	$-3\frac{5}{8}$	$+3\frac{13}{16}$		−4	+4
Line 1	E_1	−1	$-\frac{1}{2}$	$+1\frac{1}{4}$	$-1\frac{5}{8}$	$+1\frac{13}{16}$		−2	+2
Line V_1	E_1	−1	$-\frac{1}{2}$	$+1\frac{1}{4}$	$-1\frac{5}{8}$	$+1\frac{13}{16}$		−2	+2
Impressed V	E_o	−2	+2	−2	+2	−2		+2	−2
Amplitude	A	−1	$+2\frac{1}{2}$	$-3\frac{1}{4}$	$+3\frac{5}{8}$	$-3\frac{13}{16}$		+4	−4
Max. V	V_1'	−3	$+4\frac{1}{2}$	$-5\frac{1}{4}$	$+5\frac{5}{8}$	$-5\frac{13}{16}$	−6	+6	−6
Average V	P	$-1\frac{1}{2}$	$+2\frac{1}{4}$	$+2\frac{5}{8}$	$+2\frac{13}{16}$	$-2\frac{29}{32}$	−3	+3	−3
Line 1	E_1'	$-\frac{1}{2}$	$+1\frac{1}{4}$	$-1\frac{5}{8}$	$+1\frac{13}{16}$	$-1\frac{29}{32}$	−2	+2	−2
Line 2	E_2'	$-2\frac{1}{2}$	$+3\frac{1}{4}$	$-3\frac{5}{8}$	$+3\frac{13}{16}$	$-3\frac{29}{32}$	−4	+4	−4

Voltages given in the table are in terms of the line voltage E. All reducing factors have been neglected.

Since Line 2 is assumed to arc to ground the impressed voltage is $2 \cdot E$ with the sign taken as opposite to that of E_2 and the initial voltage is the voltage of Line 1.

$A = E_0 - E_1$ taking proper sign of E_0 from sign of E_2

$V_1' = E_a + A \; E_1' = P - E$

$P = \frac{V_1'}{2} \qquad E_2' = P + E$

The values for the first three arcs can be taken from or checked against Fig. 9 and the rest of the values follow obviously

The arc goes out when the oscillatory arc current passes through zero and this charge Q_1 now diffuses throughout the system and establishes the average potential P over the capacitance of the two line wires in parallel. If C_a be defined as the capacitance of one of two wires to ground when both are at the same potential, the capacitance of the two wires together will be $2 \cdot C_a$ and we can write

$$P = -\frac{1}{2} V_1 \frac{C_g}{C_a} (1 - a) \tag{12}$$

The factor $(1 - a)$ is added to make allowance for any charge that may leak off and for the charge that may be taken up by the terminal apparatus. It should be noted that the resistance in the circuit does not have any effect upon the final average voltage P except to influence the time required to reach the steady state conditions.

After a half cycle the voltages are

$$E_1 = -\frac{1}{2} V_1 \frac{C_g}{C_a} (1 - a) + E \tag{13}$$

$$E_2 = -\frac{1}{2} V_1 \frac{C_g}{C_a} (1 - a) - E \tag{14}$$

and when the faulty Line 2 reaches this potential the arc will strike again. These expressions for E_1 and E_2 have been obtained directly from (12) by adding or subtracting E. These expressions can be obtained by considering that the charges on the two lines must total to the value given in (11) and using the expressions for Q_1 and Q_2 given in the appendix.

When the arc strikes, the potential of Line 2 drops to zero losing its charge, and the charge Q_1' which is associated with the voltage E_1, must now establish another voltage E_1' over the capacitances C_g, of Line 1 to ground, when Line 2 is at ground potential. This involves a change in the electrostatic field only and takes place very rapidly.

The charge associated with the potential E_1 according to the equations in the Appendix is

$$Q_1' = C \frac{E_1 - E_2 \dfrac{C}{C_m}}{1 - \dfrac{C^2}{C_m^2}} \tag{15}$$

and upon substituting the values of E_1 and E_2 from (13) and (14) there results

$$Q_1' = C_g \left\{ E \left(1 + \frac{C}{C_m} \right) - \frac{1}{2} V_1 (1 - a) \right\} \tag{16}$$

and the potential established by this charge is

$$E_1' = E \left(1 + \frac{C}{C_m} \right) - \frac{1}{2} V_1 (1 - a) \tag{17}$$

The fact that Line 2 is at zero or ground potential requires that Line 1 be at a potential $(+ 2 E)$ but instead it has the potential E_1' and this difference between impressed and initial potential produces an oscillation having an amplitude of

$$A = \{ 2 E - E_1' \} (1 - d) \tag{18}$$

and giving a maximum voltage of

$$V_1' = 2 E + A \tag{19}$$

The limiting value will occur when the maximum potential of any oscillation is the same as that of the immediately preceding oscillation, that is when $V_1' = V_1$ and then

$$V_1 = 2 E \frac{2 + \left(1 - \dfrac{C}{C_m} \right) (1 - d)}{2 - (1 - a)(1 - d)} \tag{20}$$

This expression gives the maximum voltage that can be set up by an arcing ground on a single-phase circuit with the analysis based on the extinction of the arc when the oscillatory current first passes through zero.

The maximum possible value is six times normal line to neutral voltage as compared with four times as obtained from the preceding analysis. Table II further illustrates the building up of the voltage on the sound line and also fixes six times as the limit. By inspection of Table II and Fig. 9 it can be seen that the values of V_1 and V_1' have the following relation between any two consecutive values,

$$V_1' = V_1 + \frac{1}{2} (6 - V_1)$$

which indicates that V_1 can never be greater than 6. Writing in the values 6 for V_1 in the column for the Nth arc it is seen that the $(N + 1)$th and $(N + 2)$th arcs give no higher values, thereby establishing the limit as 6 times line to neutral voltage, for the voltage produced by an arcing ground in a single-phase system.

THREE-PHASE CIRCUIT—IMPEDANCE IN THE NEUTRAL

Since the present day tendency is toward limiting fault currents by placing an impedance in the neutral ground connection, the analysis for the three-phase circuit will be made for that condition. As mentioned before it is likely that the phenomenon will be controlled by normal frequency arc extinction when the neutral is grounded but it is useful to determine limiting values of neutral grounding resistance or reactance on the basis that the voltages are controlled by oscillatory frequency arc extinction. This method will give conservative values. Let the circuit be as shown in Fig. 10 with r_n and L_n as the neutral grounding impedance. The capacitances involved are indicated in Fig. 10A and the voltages are as shown on the left in Fig. 11.

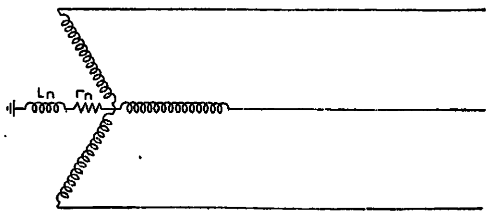

FIG. 10

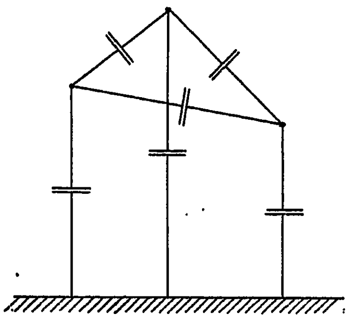

FIG. 10A

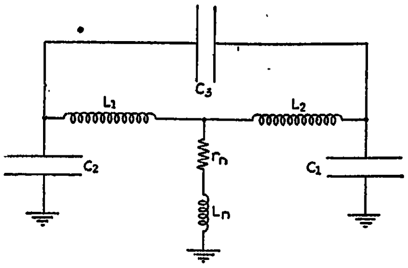

FIG. 10B

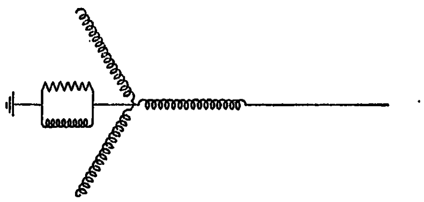

FIG. 10C

When the faulty Line 2 first arcs to ground its potential drops at once to zero and cuts out the capacitance between Line 2 and ground. Lines 1 and 3 are at a potential $\left(+ \frac{E}{2} \right)$ while the condition of zero potential on Line 2 requires a potential of $\left(+ 1 \frac{1}{2} E \right)$ on Lines 1 and 3, thus impressing this voltage on the two sound lines. The sound lines have an initial voltage of $\left(+ \frac{1}{2} E \right)$ so that the voltage applied is $\left(1 \frac{1}{2} E - \frac{1}{2} E \right) = E$ which initiates an oscillation with an amplitude of E. In the oscillation the maximum voltage attained by the sound lines is $\left(+ 2 \frac{1}{2} E \right)$ (neglecting all damping or reducing factors) at which instant the arc is assumed to go out. The charge required to produce a potential of $V_1 = \left(+ 2 \frac{1}{2} E \right)$ on the two sound lines now diffuses throughout the system and establishes an average potential over the three lines of $P = \frac{2}{3} V_1$. As soon as the faulty Line 2 reaches its new maximum $E_2 = P + E$, it again arcs to ground and the sound lines oscillate to a new maximum and then the arc goes out. The bound charge on the two lines establishes a new average potential, the faulty line arcs to ground at the next instant of its maximum potential, and the process is repeated.

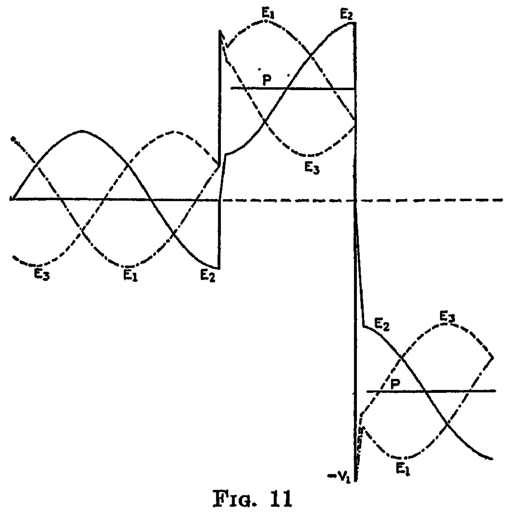

FIG. 11

The limiting voltage is found as follows. Let $(- V_1)$, Fig. 11, be the maximum potential which the sound lines reach on any oscillation. This potential on the two sound lines requires a charge dependent upon the capacitance to ground of the two sound lines when the third line is grounded, it is

$$Q_1 = -2\,C_g\,V_1 \tag{21}$$

This charge establishes an average potential P over the capacitances of the three wires in parallel, or $3\,C_a$. If there was no connection to ground from the neutral no charge would be lost and the potential would be in proportion to the capacitances involved. When there is a connection between the neutral and ground part of the charge passes to ground and the potential P is reduced proportionately. When V_1 is the potential of the sound lines at the instant the arc goes out we can write for P

$$P = -\frac{2\,C_g}{3\,C_a}\,V_1\,(1-a) \tag{22}$$

The factor $(1 - a)$ is introduced as before to take into account the reduction in potential due to part of the charge passing off to ground through the ground connection.

When the faulty line arcs to ground, and goes to ground or zero potential, this condition requires on the sound lines a potential dependent upon the impedance in the neutral ground connection. The potential of the neutral is affected and does not shift to full leg potential above ground. The neutral shift can be represented by

$$E_n = E\,(1-b) \tag{23}$$

If there were zero impedance in the neutral, *i. e.*, a solid ground, b would be equal to 1 and there would be no shift of the neutral. If the ground connection were open b would be equal to zero and the neutral shift would equal the leg voltage.

At the instant the arc strikes the potential conditions are

$$E_1 = -\frac{2\,C_g}{3\,C_a}\,V_1\,(1-a) + \frac{1}{2}\,E \tag{24}$$

$$E_2 = -\frac{2\,C_g}{3\,C_a}\,V_1\,(1-a) - E \tag{25}$$

$$E_3 = -\frac{2\,C_g}{3\,C_a}\,V_1\,(1-a) + \frac{1}{2}\,E \tag{26}$$

The arc now strikes and Line 2 drops to zero or ground potential which condition requires a potential of approximately $\left(E\,(1-b) + \frac{1}{2}\,E\right)$ on the sound lines so that the impressed voltage E_o is

$$E_o = E\,(1-b) + \frac{1}{2}\,E \tag{27}$$

At the instant just before the arc struck the sound Lines 1 and 3 had charges dependent upon the voltages E_1, E_2, and E_3 at that instant. Substituting Equations (24), (25), and (26) in the equations developed in the Appendix the charge Q_1' on the sound line is found to be

$$Q_1' = C_g\left\{\frac{1}{2}\,E\left(1+\frac{2\,C}{C_m}\right) - \frac{2}{3}\,V_1\,(1-a)\right\} \tag{28}$$

This charge together with an equal one on Line 3 establishes a potential E_1' over the capacitance $2\,C_g$ of the two sound wires to ground

$$E_1' = \frac{1}{2}\,E\left(1+\frac{2\,C}{C_m}\right) - \frac{2}{3}\,V_1\,(1-a) \tag{29}$$

The amplitude of the oscillation is then

$$A = (E_o - E_1')\,(1-d) \tag{30}$$

and the maximum voltage

$$V_1 = E_o + A \tag{31}$$

The limiting value will occur when the maximum voltage of any oscillation is equal to the maximum voltage of the immediately preceding oscillation. It is

$$V_1 = \frac{3}{2}\,E\,\frac{(2-d)\,(3-2\,b) - \left\{1+\frac{2\,C}{C_m}\right\}(1-d)}{3-2\,(1-a)\,(1-d)} \tag{32}$$

DETERMINATION OF REDUCTION FACTORS

$(1-a)$ The factor $(1 - a)$ which allows for the charge that passes to ground can be estimated in the following manner. The equivalent circuit is as shown in Fig. 10B. C_2 represents the capacitance of the two sound lines and L_2 the inductance; C_1 and L_1 represent the capacitance and inductance of the faulty line; C_3 represents the effect of the direct capacitance between the sound and faulty lines; r_n and L_n represent the neutral grounding impedance. A rigorous analysis of this circuit leads to a fourth degree differential equation which renders a general solution out of question.

It is noticed in the diagram, Fig. 10B, that the C_2 can discharge directly into C_1 through C_3 and that there is no appreciable inductance nor resistance in this circuit. On this account it can be assumed that an average potential P is established very quickly and then the three lines begin to discharge to ground through the neutral impedance. The discharge circuit will then be as shown in Fig. 10B. The equation for condenser discharge when the resistance is low is

$$e_c = e\,\epsilon^{-\alpha t}\left\{\operatorname{Cos}\,\omega\,t + \frac{r}{q}\,\operatorname{Sin}\,\omega\,t\right\} \tag{4}$$

r_3 = phase resistance of circuit plus three times resistance of grounding impedance, *i. e.*, $r_3 = r_1 + 3\,r_n$

x_3 = phase reactance of circuit plus three times the reactance of the grounding impedance, *i. e.*, $x_3 = x_1 + 3\,x_n$

L = phase inductance of circuit plus three times inductance of grounding impedance

C = capacitance of one line wire to ground when all three have same potential.

If x_3 is the inductive reactance of one phase plus three times the reactance of the grounding impedance, then

$$L = \frac{x_3}{\omega_o} \tag{33}$$

If x_a is the capacitive reactance of one line when all three are at the same potential then

$$C = C_a = \frac{1}{\omega_o x_a} \tag{34}$$

The arc on the faulty line restrikes at the end of one-half cycle of normal system frequency and the time t is

$$t = \frac{\pi}{\omega_o} \tag{35}$$

Substitute these three relations in Equation (4) just given and there results

$$\frac{e_c}{e} = \epsilon^{-\pi n_3} \left\{ \operatorname{Cos} \pi\, \theta + \frac{n_3}{\theta} \operatorname{Sin} \pi\, \theta \right\} \tag{36}$$

$$\theta = \sqrt{\frac{x_a}{x_3} - n_3^2} \quad n_3 = \frac{r_3}{2\, x_3} \tag{37}$$

On making the same substitution in the equation for condenser discharge when the resistance is high there results

$$\frac{e_c}{e} = \epsilon^{-(n_3 - \phi)\pi} \frac{1 + \dfrac{n_3}{\phi}}{2} + \epsilon^{-(n_3 + \phi)\pi} \frac{1 - \dfrac{n_3}{\phi}}{2} \tag{38}$$

$$\phi = \sqrt{n_n^2 - \frac{x_a}{x_3}} \qquad n_3 = \frac{r_3}{2\, x_3} \tag{39}$$

The factor $(1 - a)$ depends upon the charge which leaks off the ground and the charge which remains is proportional to the voltage left on the capacitance which is given as e_c in the preceding equation. If no charge leaked off $(1 - a)$ would equal 1 and also $e_c/e = 1$; if all the charge leaked off $(1 - a)$ would be equal to zero and also $e_c/e = 0$. Since by definition the factor $(1 - a)$ is a measure of the charge that leaks off to ground, it is obvious that

$$(1 - a) = \frac{e_c}{e} \tag{40}$$

The expression for $(1 - a)$ in the oscillatory case appears to indicate that with a solidly grounded neutral, $r = 0$, no damping would result, the average potential P being then the same as if the neutral were isolated. This would be true if there were no resistance at all in the circuit, the only difference then would be that the solidly grounded case would be oscillatory and the isolated case non-oscillatory. However, there is a certain amount of resistance in the rest of the circuit which will cause damping. Also, this equation apparently indicates that infinite ohms in the neutral give 100 per cent damping, but it should be remembered that increasing resistance gradually changes the discharge over into a non-oscillatory condition and it can then be shown that infinite ohms give zero damping. Such very high values of ohms would not be considered for grounding purposes so this condition is only of passing interest.

$\underline{(1 - b)}$ The factor $(1 - b)$ which allows for the reduced neutral shift may be estimated as follows. When the faulty line arcs to ground a short circuit is established and the current flowing through the neutral grounding impedance produced a voltage drop. The neutral shift is equal to the voltage drop across the neutral grounding impedance, and if this is designated by E_n and normal line to neutral voltage as E there results

$$(1 - b) = \frac{E_n}{E} \tag{41}$$

$\underline{(1 - d)}$ The factor $(1 - d)$ is simple of determination. Since the arc is assumed to go out at the first time the oscillatory frequency arc current passes through zero this corresponds to the instant of maximum voltage on the two sound lines. At this instant $\operatorname{Cos} \omega\, t = -1$ and we have from Equation (1) appendix

$$\frac{e_c}{e} = 1 + \epsilon^{-at} \tag{42}$$

$$\epsilon^{-at} = (1 - d) \tag{43}$$

In this case the resistance involved r'' is the resistance of one phase plus the two times the parallel resistance of one phase and the grounding impedance, the inductance x'' is the inductance of one phase plus two times the parallel inductance of one phase and the grounding impedance and the capacitance is the capacitance C_g of one of the two sound lines to ground when the third line is at ground potential. Making the proper substitutions we get

$$(1 - d) = \epsilon^{-b} \tag{44}$$

$$b = \frac{\pi\, n_2}{\theta_g} \quad x_g = \frac{1}{2\, \pi\, f\, C_g} \theta_g$$

$$\theta_g = \sqrt{\frac{x_g}{x''} - n_2^2} \quad n_2 = \frac{r''}{2\, x''} \tag{45}$$

The factor $(1 - d)$ can be taken as unity without serious error.

EFFECT OF GROUNDING IMPEDANCE

Resistance Ground. When it becomes desirable to put an impedance in the neutral ground connection the value chosen should be such that the possibility of arcing ground conditions is remote when the analysis is based on the assumption that the phenomenon is controlled by oscillatory frequency arc extinction. This method of analysis gives higher voltages than the analysis based on normal frequency arc extinction, and accordingly if limits are set by the pessimistic analysis the values chosen will surely be conservative and safe. When resistance alone is used it is surprising how high a value may be used. The limiting value can be determined by calculations on the circuit involved. There are some general considerations that apply and these will be discussed briefly.

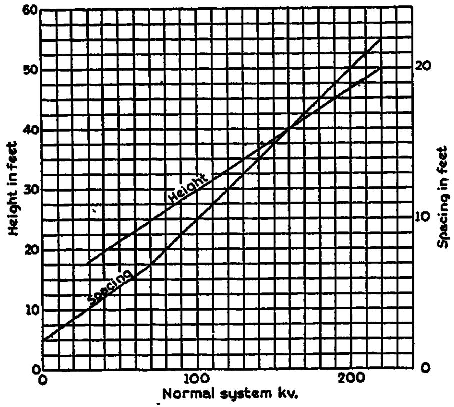

FIG. 12

When the arc goes out the bound charge redistributes itself over the system and drains off to ground through an oscillation when the resistance in the neutral is lower than the critical value. As long as the resistance is below the critical value the bound charge will be entirely drained off at the end of half a 60-cycle period and no arcing ground voltages can be built up. For this condition θ of Equation (37) must be real and therefore

$$\frac{r_3}{2\,x_3} = n_3 < \sqrt{\frac{x_a}{x_3}} \qquad (46)$$

In this expression $r_3 = 3\,r_n$; $x_3 = x_1$, (x_1 is the circuit reactance) and $x_a = \frac{1}{\omega\,C_a}$ so that

$$r_n < \frac{2}{3}\sqrt{x_a\,x_1} \qquad (47)$$

As long as the grounding resistance r_n is less than the critical resistance $\frac{2}{3}\sqrt{x_a\,x_1}$ no arcing ground voltage will be built up.

To calculate the limiting resistance as based upon this criterion it is first necessary to find out what the value of x_a might be. In Fig. 12 are two curves drawn to show the average spacing and height of conductors as dependent upon the nominal system voltage. These curves represent average conditions and are of course approximate. They are based upon data published by the *Electrical World* some years ago. In Table III are the results of capacitance calculations made for circuits of these proportions and various voltage ratings. An examination of this tabulation shows that the value

TABLE III
CAPACITANCE CALCULATIONS—AVERAGE CIRCUIT PROPORTIONS

Kv	33	44	66	110	132	154	220
Cond. spacing	4	5	7	11	13	15	22
Ave. spacing	5.0	6.3	8.8	13.9	16.4	18.9	27.7
Horizontal spacing							
Height—h	18	20	24	32	35	39	50
Ave. image R	36.4	40.6	49.0	65.9	72.4	81.0	104.8
C—0.250	0.010960	0.010810	0.010580	0.010225	0.010101	0.009970	0.009705
C—0.375	0.011503	0.011365	0.011104	0.010695	0.010595	0.010453	0.010150
C—0.500	0.011990	0.011710	0.011502	0.011070	0.010960	0.010815	0.010495
C_m—(mutual)	0.04498	0.04800	0.05220	0.05720	0.06000	0.06140	0.06550
C_a—0.375	0.00761	0.00771	0.00779	0.00778	0.00782	0.00780	0.00775
C/C_m 0.375	0.256	0.237	0.213	0.187	0.177	0.170	0.155
I	16.44	22.1	33.7	56.0	67.5	78.3	111.4
Vertical spacing							
Height—h	22	25	31	43	48	54	72
Ave. image R	44.0	50.0	61.7	85.5	95.4	107.3	143.3
C—0.250	0.010708	0.010548	0.010298	0.009900	0.009788	0.009678	0.009383
C—0.375	0.011255	0.010780	0.010797	0.010391	0.010265	0.010121	0.009789
C—0.500	0.011680	0.011487	0.011185	0.010745	0.010613	0.010464	0.010119
C_m—(mutual)	0.0412	0.0432	0.0458	0.0493	0.0508	0.0515	0.0542
C_a—0.375	0.00728	0.00733	0.00733	0.00731	0.00731	0.00726	0.00719
C/C_m—0.375	0.273	0.256	0.236	0.211	0.202	0.197	0.181
I ave	15.7	21.1	31.7	52.6	63.1	72.9	103.4

Conductor Radius in Inches
$I = 3\,E\,(377)\,C_a$ for 100 miles.

of C varies only slightly over the entire voltage range, and that C_a is practically constant for either vertical or horizontal spacings. The difference in value of C_a for horizontal and vertical spacing is small so that it appears reasonable to use the average value throughout the entire voltage range for both spacings.

Information has been published[8] that the charging current flowing in the ground, I_f of Fig. 13C and D averages 1.5 I_n, where I_n is the normal three-phase changing current, and that I_f averages 2 I_n. These figures were based on tests on low-voltage transmission systems, and would be exact if the ratio $\frac{C}{C_m}$ were 0.25. The average of this ratio for 33 kv., 44 kv., and 66 kv. is 0.245 with a variation less than that which might be encountered in such tests so that it was quite reasonable to form such a conclusion from the tests.

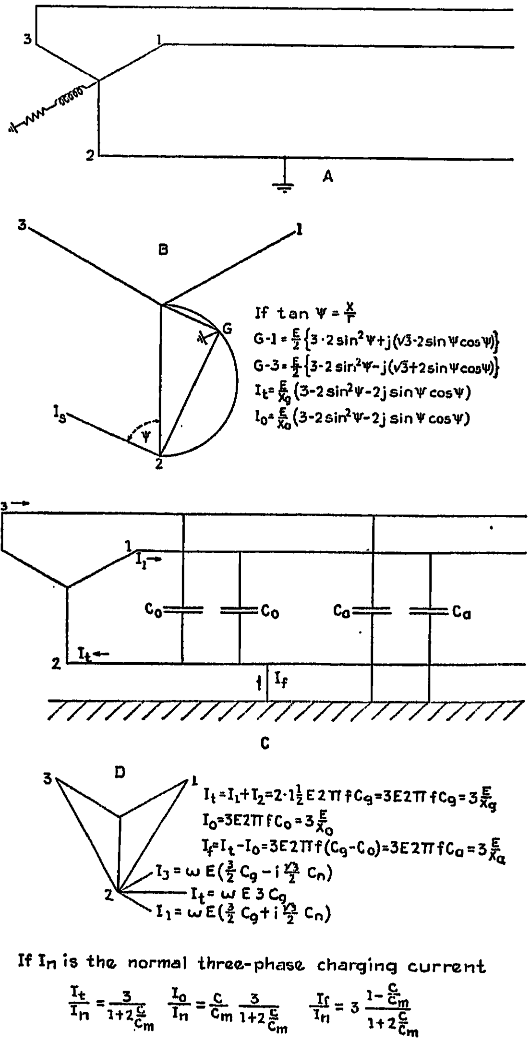
3
1
2
A
3
B
1
G
I_s
ψ
2
If $\tan\psi = \frac{X}{r}$
$G\text{-}1 = \frac{E}{2}\{3\cdot 2\sin^2\psi + j(\sqrt{3} - 2\sin\psi\cos\psi)\}$
$G\text{-}3 = \frac{E}{2}\{3 - 2\sin^2\psi - j(\sqrt{3} + 2\sin\psi\cos\psi)\}$
$I_t = \frac{E}{X_g}(3 - 2\sin^2\psi - 2j\sin\psi\cos\psi)$
$I_0 = \frac{E}{X_a}(3 - 2\sin^2\psi - 2j\sin\psi\cos\psi)$
$I_3 \rightarrow$
3
1
$I_1 \rightarrow$
C_0
C_0
C_0
C_a
2
$I_t \leftarrow$
I_f
C
3
D
1
$I_t = I_1 + I_2 = 2\cdot 1\frac{1}{2} E 2\pi f C_g = 3E2\pi f C_g = 3\frac{E}{X_g}$
$I_0 = 3E2\pi f C_0 = 3\frac{E}{X_0}$
$I_f = I_t - I_0 = 3E2\pi f(C_g - C_0) = 3E2\pi f C_a = 3\frac{E}{X_a}$
$I_3 = \omega E(\frac{3}{2}C_g - i\frac{\sqrt{3}}{2}C_n)$
$I_t = \omega E\,3C_g$
$I_1 = \omega E(\frac{3}{2}C_g + i\frac{\sqrt{3}}{2}C_n)$
2
If I_n is the normal three-phase charging current
$\frac{I_t}{I_n} = \frac{3}{1+2\frac{C}{C_m}}$ $\frac{I_0}{I_n} = \frac{C}{C_m}\frac{3}{1+2\frac{C}{C_m}}$ $\frac{I_f}{I_n} = 3\frac{1-\frac{C}{C_m}}{1+2\frac{C}{C_m}}$

FIG. 13

Using the average figure of 0.00752 μ f. for C_a, x_a in ohms can be written, if L is length of line in miles.

$$x_a = \frac{.3525\ 10^6}{L} \tag{48}$$

If x_1 is expressed in per cent then x_1 ohm is given by

$$x_1 \text{ ohms} = \frac{x_1\sqrt{\text{kv.}^2}\ 10}{\text{kv-a.}} \tag{49}$$

Inserting these values in Equation (47) we find

$$r_n < 1.25\sqrt{\frac{x_1\,10^6}{L.\ \text{kv-a.}}} \tag{50}$$

The critical resistance obtained from (50) for various overhead circuits has been tabulated in Table IV. If the resistance in the neutral is less than the values in this table no arcing ground troubles should be experienced.

TABLE IV
CRITICAL RESISTANCE FOR OVERHEAD SYSTEM

$$z = 1.25\text{ kv.}\sqrt{\frac{X_1\,.\,10_0}{L\,.\,\text{kv-a.}}}\quad X_1 \text{ assumed 25 per cent}$$

$$z = \frac{2}{3}\sqrt{x_a\,x_1}$$

Kv.	Kv-a. L	5,000	10,000	25,000	50,000	100,000	200,000
33	25	584	413	261	....	....	....
	50	413	292	185	....	....	....
	100	292	207	131	....	....	....
44	25	777	550	347	....	....	....
	50	550	388	246	....	....	....
	100	388	275	174	....	....	....
66	25	1166	825	521	370	....	....
	50	825	583	370	261	....	....
	100	583	413	261	185	....	....
110	50	....	980	616	435	308	....
	100	....	687	435	308	218	....
	200	....	486	308	218	154	....
132	50	....	1166	739	522	370	....
	100	....	825	522	370	261	....
	200	....	582	370	261	185	....
154	50	....	....	862	610	431	308
	100	....	....	610	431	308	215
	200	....	....	431	308	215	154
220	50	....	....	1231	871	616	435
	100	....	....	871	616	435	308
	200	....	....	616	435	308	218

For cable circuits the quantity x_a is quite variable and so the critical resistance must be calculated from the expression

$$z = \frac{2}{3}\sqrt{x_1\,x_a} \tag{47a}$$

For a cable circuit using three-phase cables the critical resistance is shown in Table V.

It is possible to use values much higher than those determined by the preceding analysis and still have the arcing ground voltages within safe limits. When the resistance is in excess of the critical resistance fixed by (47) the discharge to ground is no longer oscillatory and the factor $(1 - a)$ must be determined from Equation (38). If we let z be the critical or limiting value

TABLE V

CRITICAL RESISTANCE FOR CABLE SYSTEMS

(Three-Phase Cables)

Rated voltage	X_1 100,000-kv-a.	X_a 100 miles		Z	
	10%	Isolated	Grounded	Isolated	Grounded
7..5	0.05625	104.3	88.2	1.616	1.485
12..5	0.15625	119.3	103.8	2.876	2.683
15	0.225	126.5	111.2	3.556	3.333
20	0.4	140.0	126.0	4.99	4.742
25	0.625	155.0	142.2	6.566	6.288
33	1.089	171.0	158.0	9.10	8.75

$$Z = \frac{2}{3} \sqrt{x_1 x_a}$$

Z will vary inversely as $\sqrt{\text{Kv-a.}}$
Z will vary inversely as $\sqrt{\text{Length of line}}$
Z will vary directly as $\sqrt{\text{Per cent reactance}}$
Thus at 7.5 kv. for 25,000 kv-a., 25 miles, and 15 per cent reactance Z = 7.92 ohms

as determined from (47) and $r_n = k\,z$ the value used we can write

$$r = \frac{2}{3} k \sqrt{x_a x_1} \tag{51}$$

$$\phi = \sqrt{\frac{x_a}{x_1}}\,(k^2 - 1)^{\frac{1}{2}}\, n_3 = k \sqrt{\frac{x_a}{x_1}} \tag{52}$$

$$1 - a = \frac{k + \sqrt{k^2 - 1}}{2\sqrt{k^2 - 1}} \quad \frac{\epsilon^{\pi}\sqrt{\frac{x_a}{x_1}}\,\sqrt{k^2 - 1}}{\epsilon^{\pi}\sqrt{\frac{x_a}{x_1}}\,k} \tag{38a}$$

If a resistance 50 per cent greater than the value determined from (47) were used, then k in these last two expressions would be 1.5.

The quantity x_a is the capacitive reactance of one line when all three are at the same potential and this is very approximately twice the customary line to neutral capacitive reactance. If we consider that the normal charging current will never be more than 100 per cent, then x_a will never be less than 200 per cent and for a fair average value of x_1 25 per cent the factor $\sqrt{\frac{x_a}{x_1}}$ will not be less than 2.83.

Using this relation in (38a) and then in (32) the following is obtained for increasing values of r_n. In the calculation $(1 - d)$ and $(1 - b)$ are taken as unity and $\frac{C}{C_m}$ as 0.15.

TABLE VI

k	$(1 - a)$	V
1.1	0.0056	2.35 E
1.2	0.0119	2.37 E
1.5	0.0357	2.41 E
2.0	0.0962	2.51 E
3.0	0.227	2.77 E

It is quite obvious from this tabulation that the resistance in the neutral can easily be three times the critical resistances obtained from Equation (47) without running into any danger of trouble from arcing grounds.

It is also possible to fix a limit[2] for r_n such that the lagging component of the current in the arc is equal to or greater than the leading component. The leading component of the arc current is that part of the charging current of the two sound lines in parallel that goes to ground. When Line 2 goes to ground or zero potential the two sound lines take a potential as indicated in Fig. 13. The voltages $1 - 2$ and $3 - 2$ are 60 deg. apart as are the charging currents I_1 and I_3. The charging currents for this condition are dependent upon the capacitance C_0, and it is obvious then that the total charging current is

$$I_t = I_1 + I_3 = (2)\left(1\frac{1}{2}\right)\left(\frac{E}{x_0}\right) = \frac{3E}{x_0} \tag{53}$$

Part of this current returns through the ground and part directly through the faulty Line 2. The part that returns through the faulty line depends upon the capacitance C_o, if this is I_o

$$I_o = (2)\left(1\frac{1}{2}\right)\left(\frac{E}{x_o}\right) = \frac{3E}{x_o} \tag{54}$$

The current I_f in the arc or fault is the difference

$$I_f = I_t - I_o = 3E\left(\frac{1}{x_0} - \frac{1}{x_o}\right) \tag{55}$$

$$I_f = \frac{3E}{x_a} \tag{55A}$$

It is noticed that the charging current in the fault has been calculated as though the neutral were not grounded. This is done because with the maximum resistance so determined such conditions are very closely approximated.

If the short circuit current in the fault is $I_s = \frac{E}{X_s}$ then the reactive current I_x is

$$I_x = I_s \frac{x_s}{\sqrt{r_n^2 + x_s^2}} = \frac{E_x}{r_n^2 + x_s^2} \tag{56}$$

and since I_x is set equal to $I_f = \frac{3E}{x_a}$

$$r_o < \frac{1}{\sqrt{3}} \sqrt{x_a x_s} \sqrt{1 - \frac{3\ x_s}{x_a}} \tag{57}$$

The limiting value as determined from (57) is less than that as determined from (47).

Reactance Ground. When reactance is used for grounding the neutral the following equation, which is obtained from Equation (32) by letting the factor

$(1 - d)$ be unity, may be used to estimate the arcing ground voltage.

$$V_1 = \frac{3}{2} E \frac{5 - 4b - 2\dfrac{C}{C_m}}{3 - 2(1 - a)} \tag{58}$$

The neutral grounding reactor affects the factor (b) and also the factor $(1 - a)$. If I_s is the short-circuit current flowing in the fault and x_s the system reactance such that

$$I_s = \frac{E}{x_s} \tag{59}$$

then

$$1 - b = \frac{x_n}{x_s} \tag{60}$$

and

$$b = 1 - \frac{x_n}{x_s} \tag{61}$$

The factor $(1 - a)$ in the form previously given contains an exponential part and a trigonometric part. When the neutral is grounded through reactance, the resistance will usually be low and the inductance high so that the exponential part can be taken as unity without appreciable error. With this in mind we obtain from Equations (**40**) and (**37**)

$$(1 - a) = \text{Cos}\, n\, \theta \tag{62}$$

$$\theta = \sqrt{\frac{x_a}{x_3}} = \sqrt{\frac{x_a}{x_1} \frac{1}{1 + 3\dfrac{x_a}{x_1}}} \tag{63}$$

$$x_c = \frac{x_a - x_1}{3} \text{ for } \theta = 1 \tag{63a}$$

The value x_c might be called the critical reactance.

A brief inspection of Equation (**50**) shows that the lowest value of the arcing ground voltage V_1 will be obtained when the factor $(1 - a)$ is equal to -1. Therefore, the value of neutral grounding reactance x_n should be chosen so that θ as calculated from Equation (**63**) is an odd integer. Because of necessary assumptions made in the derivation of the formula and uncertainty in the exact determination of the line constants, the use of these expressions should be limited to values of θ less than 4.

For the condition that $\theta = 1$ as in Equation (**63a**) this gives a value of neutral grounding reactance about the same as the Petersen Coil but it should be noted that the method here given is different from the ordinary method of calculating Petersen Coils. The value of θ is not critical in the neighborhood of $\theta = 1$. This is brought out in Table VII, in which are shown calculated voltages for various values of neutral grounding reactance above and below the critical value. It is quite apparent from this table that considerable variation from the critical value can be permitted without any high voltages resulting.

TABLE VII

Tabulation shows variation of calculated arcing ground voltages for variation of neutral grounding reactance above and below the critical value

$\frac{x_n}{x_c}$	θ	$\frac{V_1}{E}$	$\frac{x_z}{x_c}$	θ	$\frac{V_1}{E}$
1.0	1.0	1.36	1.0	1.0	1.36
1.1	0.913	1.38	0.9	1.105	1.39
1.2	0.84	1.43	0.8	1.234	1.52
1.3	0.778	1.92	0.7	1.4	2.60

Petersen Coil. The Petersen Coil is a special type of neutral grounding reactor in which the lagging current which the coil permits to flow when one line becomes grounded is equal to the charging current of the two sound lines. For this condition the lagging component of the fault current is practically equal to the total short circuit current so that

$$I_s = \frac{E}{x_s} = I_f = \frac{3E}{x_a} \tag{64}$$

But $x_s = x_n + x$ in which x_n is the reactance of the Petersen Coil and x is one-third the sum of the zero, positive, and negative sequence reactances of the circuit. And then

$$x_n = \frac{x_a}{3} - x \tag{65}$$

$$x_n = \frac{x_a - (x_o + x_1 + x_2)}{3} \tag{65a}$$

The value of θ corresponding to the value of x_n given for the Petersen Coil by Equation (**65**) is

$$\theta = \sqrt{\frac{x_a}{x_a - x_o - x_2}} \tag{66}$$

The reactance of the Petersen Coil as given by (**65a**) will be less than the critical value given by (**63a**) by an amount depending upon the relative values of the zero and negative phase sequence reactances. The value of θ as given by (**66**) for the Petersen Coil is very nearly unity.

The principle of operation of the Petersen Coil is that of balance between the short-circuit current and charging current in the fault. When this balance exists the arc is supposed to go out from lack of current to maintain it. When the Petersen Coil is out of tune there is no longer a balance of lagging and leading currents and the arc persists. However, this is not serious because no harmful overvoltages will result unless the unbalance is considerable, as reference to Table VII will show.

Series Resistance and Reactance. For a neutral grounding impedance consisting of resistance and reactance the damping factor $(1 - a)$ should be determined from (40) in combination with (38) or (36) depending upon the values of the circuit constants.

Parallel Resistance and Inductance. It has been found cheaper to obtain a given required value of ohms by the use of an inductance than by use of resistance. However, an inductance in the neutral ground connection causes the arcing ground voltages to calculate higher than resistance. This can be overcome by the use of a high resistance in parallel with the inductance, and this resistance need not be expensive because it need not have a high current carrying capacity.

An exact analysis of this circuit leads to a third degree differential equation which does not readily permit a general solution. With the circuit as shown in Fig. 10C the equation for the current is

$$i = \frac{E_o}{L} \frac{\left(\frac{r_n}{L_n} p + p^2\right)}{F(p)}$$

$$F(p) = p^3 + \left(\frac{r_n}{L_n} + \frac{r}{L} + \frac{r_n}{L}\right) p^2 + \left(\frac{1}{L\,C} + \frac{r\,r_n}{L\,L_n}\right) p + \frac{r_n}{L\,L_n\,C} = (p+c)\,(p+b)\,(p+a)$$

In this equation (i) is the discharge current to ground from the charges trapped on the two sound lines. What we are interested in is the voltage left on the lines at the end of one-half of a normal frequency cycle or the voltage on the condenser part of the equivalent circuit, which is

$$E_c = E_o - \frac{i}{p\,C}$$

$$E_c = E_o - \frac{E_o}{LC} \left\{ \frac{\left(\frac{r_n}{L_n} - c\right)}{(c-a)\,(c-b)\,(p+c)} + \frac{\left(\frac{r_n}{L_n} - b\right)}{(b-a)\,(b-c)\,(p+b)} + \frac{\left(\frac{r_n}{L_n} - a\right)}{(a-b)\,(a-c)\,(p+a)} \right\}$$

$$= E_o - \frac{E_o}{LC} \left\{ \frac{\left(\frac{r_n}{L_n} - c\right)(1-\epsilon^{-ct})}{c\,(c-a)\,(c-b)} - \frac{\left(\frac{r_n}{L_n} - b\right)(1-\epsilon^{-bt})}{b\,(b-a)\,(c-b)} + \frac{\left(\frac{r_n}{L_n} - a\right)(1-\epsilon^{-at})}{a\,(b-a)\,(c-a)} \right\}$$

In the solution the numerical values of the roots are substituted at this point.

Let $b = n - j\,m \qquad K = c - n$

$a = n + j\,m \qquad K' = \frac{r_n}{L_n} - n$

$$\phi = \frac{n\,(K\,K' + m^2) - m^2\,(K - K')}{n^2 + m^2}$$

$$\theta = \frac{(K\,K' + m^2) + n\,(K - K')}{n^2 + m^2}$$

$$E_c = E_o - E_o \left\{ \frac{\frac{r_n}{L_n} - c}{L\,C\,(K^2 + m^2)\,c} (1 - \epsilon^{-ct}) + \frac{\theta}{L\,C\,(K^2 + m^2)} - \frac{\theta}{L\,C\,(K^2 + m^2)} \epsilon^{-nt}\,\text{Cesmt} + \frac{\phi}{L\,C\,(K^2 + m^2)m} \epsilon^{-nt}\,\text{Sinmt} \right\}$$

As an example, consider a 66-kv. line, 50 miles long, carrying 25,000-kv-a. and assume 25 per cent system reactance at point of fault, and also assume that the neutral is grounded through 50 per cent reactance in parallel with 1000 ohms resistance. Then

	Actual	Effective
r_n	1000 ohms	1000 ohms
L_n	0.2306 H	0.2306 H
L	0.1153 H	0:0384 H
r	12 ohms	4 ohms
C_a	0.375 μ f.	1.125 μ f.

The solution of the differential operator gives

$c = 29760$

$b = 305 - j\,1859 = n - j\,m$

$a = 305 + j\,1859 = n + j\,m$

From which

$K = 29.41\ 10^3 \qquad K' = 4.035\ 10^3$

$\phi = -\ 14.52\ 10^3$

$\theta = 36.6$

$$\frac{1}{LC} = 24.2\ 10^6$$

$$K^2 + m^2 = 869\ 10^6$$

$$\frac{1}{L\,C\,(K^2 + m^2)} = 0.02785$$

$$E_c = E_o - E_o\,\{\,0.0245\,(1 - \epsilon^{-29760t}) + 1.018 - 1.018\,\epsilon^{-305t}\,\text{Cos}\,1859\,t - .2175\,\epsilon^{-305t}\,\text{Sin}\,1859\,t\}$$

When $t = 0$ this equation gives the condenser voltage as the initial voltage and when t is very large the condenser voltage is practically 0. The fact that it is not 0 is probably due to some slight inaccuracy in the solution for the roots. The factor $(1 - a)$ is equal to ϵ^{-305t} and at the end of one-half a normal frequency cycle is approximately 0.08, so that there is no chance for arcing ground voltages.

As a second example assume that the grounding device consisted of a 500 per cent reactance in parallel with 1210-ohm resistance and then $r_n = 1210$ ohms and $L_n = 2.306\,H$. The roots of the differential operator are ($c = 31370$), ($b = 380 - j\,1037$), ($a = 380 + j\,1037$) so that the damping factor is $(1 - a) = \epsilon^{-380t}$ and at the end of half a normal frequency cycle $(1 - a)$ is about 0.043.

It can be said in general that it is possible to ground the neutral through a very high relative value of reactance shunted by a resistance and still be entirely free from danger of arcing ground voltages.

Appendix

BASIS OF ANALYSIS

The analysis of arcing ground phenomena is based on the familiar theory of condenser and discharge charge through a resistance and inductance with the constants considered concentrated as shown in Fig. 14. When a constant voltage e is suddenly applied the condenser voltage e_c is given by

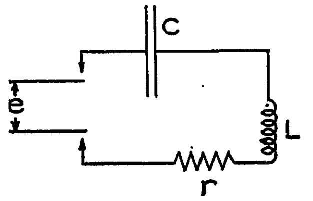

FIG. 14

$$\frac{e_c}{e} = 1 - \epsilon^{-at}\left\{ \operatorname{Cos} w\,t + \frac{r}{q}\operatorname{Sin} w\,t \right\} \qquad (1)$$

$$\frac{e_c}{e} = 1 - \epsilon^{-at}\left\{ 1 + \alpha\,t \right\} \qquad (2)$$

$$\frac{e_c}{e} = 1 - \epsilon^{-at}\left\{ \operatorname{Cosh} \mathfrak{B}\,t + \frac{r}{s}\operatorname{Sinh} \mathfrak{B}\,t \right\} \qquad (3)$$

$$= 1 - \frac{1}{2}\,\epsilon^{-at}\left\{ (r+s)\,\epsilon^{-\frac{s}{2L}t}\,\frac{s}{2L}t - (r-s)\,\epsilon^{-\frac{s}{2L}t} \right\} \qquad (4)$$

$$a = \frac{r}{2L} \quad \omega = \sqrt{\frac{1}{LC} - \frac{r^2}{4L^2}} \qquad q = \sqrt{4\frac{L}{C} - r^2} \qquad (5)$$

$$\mathfrak{B} = \sqrt{\frac{r^2}{4L} - \frac{1}{L\,C}} \qquad s = \sqrt{r^2 - 4\frac{L}{C}} \qquad (6)$$

e_c = voltage across capacitor at time t
e = applied voltage for charge-initial voltage for discharge
r = resistance in ohms
L = inductance in henrys
C = capacitance in farads

Equations (1), (2), and (3) are for the oscillatory, critical, and non-oscillatory cases respectively. For discharge the expressions are the same except that the figure 1 is left out, leaving on the right side only the term affected by ϵ^{-at}.

It is easy to see from Equation (1) that the maximum value of condenser voltage occurs when $\omega\,t = \pi$ and is

$$e_c = e\,(1 + \epsilon^{-at}) \qquad (7)$$

The limiting maximum occurs when $r = o$ and is $2\,e$. The amplitude A of the oscillation is equal to the applied voltage. Suppose the condenser already had a voltage e_1 at the time a voltage e_o was impressed, then the voltage applied to the condenser would be the difference between the impressed and the initial voltage. The following relations would hold

Initial voltage $= e_1$
Impressed voltage $= e_o$
Applied voltage $e = e_o - e_1$ (8)
Amplitude $A = e_o - e_1$ (9)
Maximum voltage $V = e_o + A$ (10)

These relationships are illustrated in Fig. 14A.

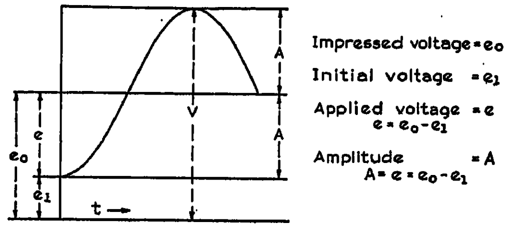

FIG. 14A

CAPACITANCE RELATIONS—OVERHEAD LINES

The capacitance relations in a three-phase or single-phase circuit can be obtained easily from the standard potential equations. These equations give the potential of a conductor due to its own charge and to all the charges on nearby conductors. These equations are

$$V_1 = Q_1 A_1 + Q_2 B_{12} + Q_3 B_{13}$$
$$V_2 = Q_1 B_{21} + Q_2 A_2 + Q_3 B_{23}$$
$$V_3 = Q_1 B_{31} + Q_2 B_{32} + Q_3 A_3$$

In this equation the voltages are above zero potential. For overhead lines the surface of the earth is usually taken as the neutral plane and the voltage V_1, etc., are then voltage to ground.

Capacitance is defined as ratio of charge to potential and therefore

$$C_{11} = \frac{Q_1}{V_1} \qquad C_{22} = \frac{Q_2}{V_2} \qquad C_{33} = \frac{Q_3}{V_3}$$

C_{11} = Effective capacitance of conductor to ground

$$Q_1 = \frac{D_3'}{D_3} \qquad Q_2 = \frac{D_3''}{D_3} \qquad Q_3 = \frac{D'''}{D_3}$$

$$D_3 = \begin{vmatrix} A_1 & B_{12} & B_{13} \\ B_{21} & A_2 & B_{23} \\ B_{31} & B_{32} & A_3 \end{vmatrix} \qquad D_3'' = \begin{vmatrix} A_1 & V_1 & B_{13} \\ B_{21} & V_2 & B_{23} \\ B_{31} & V_3 & A_3 \end{vmatrix}$$

$$D_3' = \begin{vmatrix} V_1 & B_{12} & B_{13} \\ V_2 & A_2 & B_{23} \\ V_3 & B_{32} & A_3 \end{vmatrix} \qquad D_3''' = \begin{vmatrix} A_1 & B_{12} & V_1 \\ B_{21} & A_2 & V_2 \\ B_{31} & B_{32} & V_3 \end{vmatrix}$$

If we write

$$K_1 = A_2 A_3 - B_{23}^2 \qquad K_{12} = A_3\ B_{12} - B_{23} B_{31}$$
$$K_2 = A_3 A_1 - B_{31}^2 \qquad K_{23} = A_1\ B_{23} - B_{31} B_{12}$$
$$K_3 = A_1 A_2 - B_{12}^2 \qquad K_{31} = A_2\ B_{31} - B_{12} B_{23}$$
$$K = A_1 K_1 - B_{12} K_{12} - B_{31} K_{31}$$

then

$$Q_1 = V_1 \frac{K_1}{K} - V_2 \frac{K_{12}}{K} - V_3 \frac{K_{31}}{K}$$

$$Q_2 = -V_1 \frac{K_{12}}{K} + V_2 \frac{K_2}{K} - V_3 \frac{K_{23}}{K}$$

$$Q_3 = -V_1 \frac{K_{31}}{K} - V_2 \frac{K_{23}}{K} + V_3 \frac{K_3}{K}$$

$$I_1 = 2\pi t \left(V_1 \frac{K_1}{K} - V_2 \frac{K_{12}}{K} - V_3 \frac{K_{31}}{K} \right)$$

I_1 = charging current line 1, others in same manner.

$I_o = I_1 + I_2 + I_3$

I_o = residual charging current in the ground.

In the equations for I, etc., the voltages V_1, V_2, and V_3 are to be taken as vectors.

In this system of equations the units used are the farad and volt. The factor A is the capacitive coefficient of self induction and B is the capacitive coefficient of neutral induction.

$$A = \frac{1}{C} \qquad C = \frac{0.03883}{\operatorname{Log} \frac{2h}{r}} \text{ in mcf./mi.}$$

$$B = \frac{1}{C_m} \qquad C_m = \frac{0.03883}{\operatorname{Log} \frac{R}{S}} \text{ in mcf/mi.}$$

Log's to be taken to base 10.

C = self capacitance of a conductor to ground.
C_m = neutral capacitance between pairs of conductors.
h = height of conductor above ground.
r = radius of conductor.
R = distance from a conductor to image of the other.
S = spacing between conductors.

Single-phase Circuit, Fig. 15.

For two-wire circuit the foregoing method gives

$$C_{11} = C_1 \frac{1 - \frac{V_2}{V_1}\frac{C_2}{C_{12}}}{1 - \frac{C_1 C_2}{C_{12}^2}} \qquad C_{22} = C_2 \frac{1 - \frac{V_1}{V_2}\frac{C_1}{C_{12}}}{1 - \frac{C_1 C_2}{C_{12}^2}}$$

C_{11} = effective capacitance to ground of first wire
C_{22} = effective capacitance to ground of second wire.

For a single-phase circuit there results

$$C_a = C_1 \frac{1 - \frac{C_2}{C_{12}}}{1 - \frac{C_1 C_2}{C_{12}}} \qquad C_o = C_1 \frac{\frac{C_2}{C_{12}}}{1 - \frac{C_1 C_2}{C_{12}^2}}$$

$$C_g = C_1 \frac{1}{1 - \frac{C_1 C_2}{C_{12}^2}} \qquad C_n = C_1 \frac{2}{1 + \frac{C_1}{C_2} - 2\frac{C_1}{C_{12}}}$$

C_a = capacitance to ground of one of two wires, when connected in multiple
C_g = capacitance to ground and second wire, when second wire is grounded
C_o = capacitance to second wire only, when second wire is grounded
C_n = capacitance of one wire to neutral when part of single-phase circuit.

If the conductors are transposed to obtain balanced condition to ground there results

$$C = C_1 = C_2 \quad C_m = C_{12}$$
$$h = \sqrt{h_1 h_2} \qquad r = \sqrt{r_1 r_2}$$
$$C_a = \frac{C}{1 + \frac{C}{C_m}} = \frac{1}{(A + B)}$$

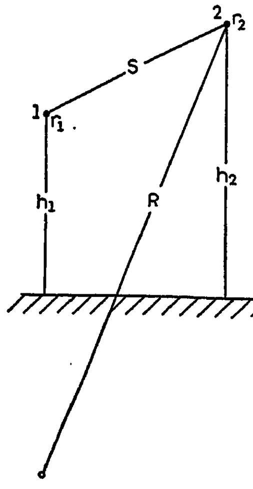

FIG. 15

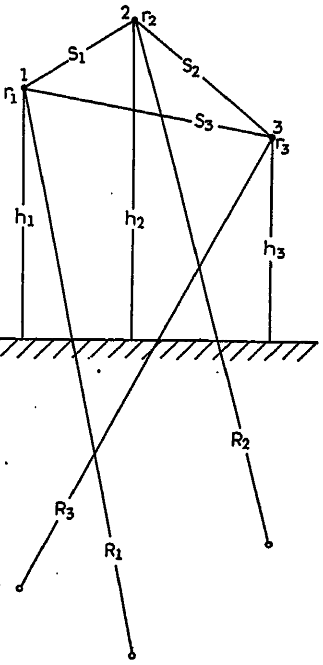

FIG. 16

$$C_g = \frac{C}{1 - \frac{C^2}{C_m^2}} = \frac{A}{(A - B)(A + B)}$$

$$C_o = C \frac{\frac{C}{C_m}}{1 - \frac{C^2}{C_m^2}} = \frac{B}{(A - B)(A + B)}$$

$$C_h = \frac{C}{1 - \frac{C}{C_m}} = \frac{1}{(A - B)}$$

$$C_g = C_a + C_o$$

Three-Phase Circuit, Fig. 16.

For the three-phase circuit only the transposed condition will be considered, and then:

$$C = C_1 = C_2 = C_3 \qquad C_m = C_{12} = C_{23} = C_{31}$$
$$H = \sqrt[3]{h_1 h_2 h_3} \qquad r = \sqrt[3]{r_1 r_2 r_3}$$
$$R = \sqrt[3]{R_1 R_2 R_3} \qquad S = \sqrt[3]{S_1 S_2 S_3}$$

$$C_a = \frac{C}{1 + 2\frac{C}{C_m}} = \frac{1}{A + 2B}$$

$$C_g = \frac{C}{\left(1 - \frac{C}{C_m}\right)\left(1 + 2\frac{C}{C_m}\right)} = \frac{A}{(A - B)(A + 2B)}$$

$$C_o = \frac{C}{C_m}\,\frac{C}{\left(1 - \frac{C}{C_m}\right)\left(1 + 2\frac{C}{C_m}\right)} = \frac{B}{(A - B)(A + 2B)}$$

$$C_n = \frac{C}{1 - \frac{C}{C_m}} = \frac{1}{(A - B)}$$

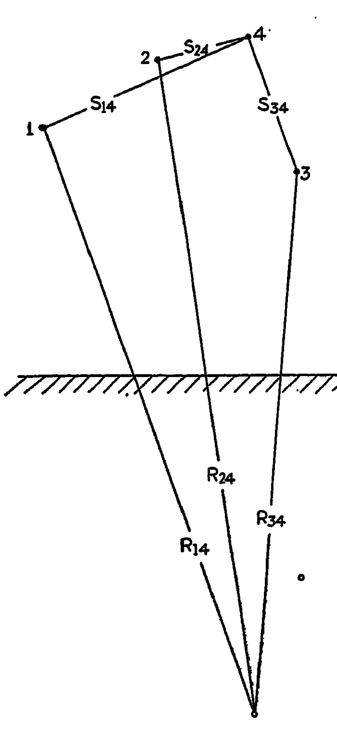

FIG. 17

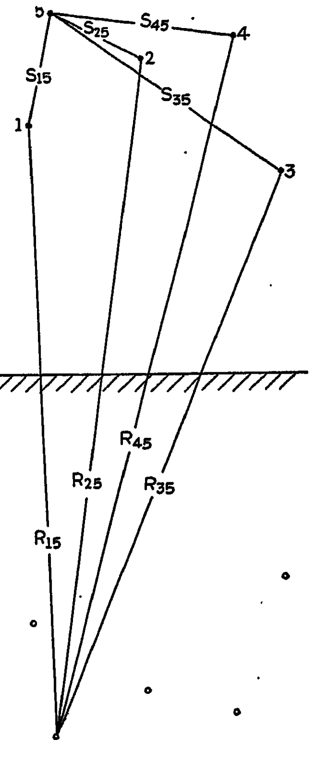

FIG. 18

C_a = capacitance to ground of each of three conductors when all three are in multiple

C_g = capacitance to ground and third conductor, when third conductor is grounded, of each of two conductors in multiple

C_o = capacitance to grounded wire of each conductor for conditions as for C_g

C_n = capacitance to neutral of each conductor of a three-phase circuit.

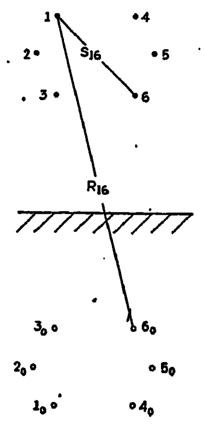

FIG. 19

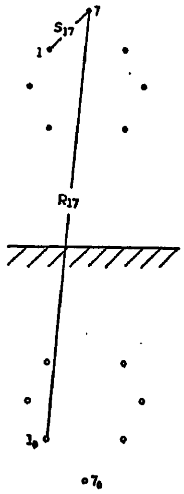

FIG. 20

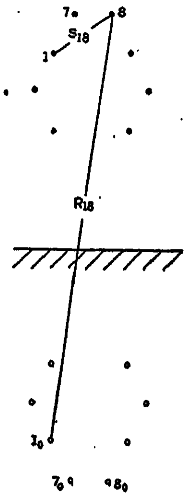

FIG. 21

Three-Phase Circuit—One Ground Wire. Fig. 17.
h, r, R, S, same as Fig. 16.

$$S_4 = \sqrt[3]{S_{14} S_{24} S_{34}}$$
$$R_4 = \sqrt[3]{R_{14} R_{24} R_{34}}$$

The formulas given for the three-phase circuit apply if

$$A - \frac{B_4^2}{A_4} \text{ is used in place of } A$$

$$B - \frac{B_4^2}{A_4} \text{ is used in place of } B$$

Three-Phase Circuit—2 Ground Wires. Fig. 18.
h, r, R, S, R₄, S₄, same as Fig. 16 and Fig. 17.

$$S_5 = \sqrt[3]{S_{15} S_{25} S_{35}}$$
$$R_5 = \sqrt[3]{R_{15} R_{25} R_{35}}$$

The formulas given for the three-phase circuit apply if

$$A - \left(\frac{A_5 B_4 - B_5 B_{45}}{A_4 A_5 - B_{45}^2}\right) B_4 - \left(\frac{A_4 B_5 - B_4 B_{45}}{A_4 A_5 - B_{45}^2}\right) B_5$$

is used for A

$$B - \left(\frac{A_5 B_4 - B_5 B_{45}}{A_4 A_5 - B_{45}^2}\right) B_4 - \left(\frac{A_4 B_5 - B_4 B_{45}}{A_4 A_5 - B_{45}^2}\right) B_5$$

is used for B

Three-Phase Double Circuit Line. Fig. 19.

$$S' = \sqrt[3]{S_{14} S_{25} S_{36}}$$
$$R' = \sqrt[3]{R_{14} R_{25} R_{36}}$$
$$S'' = \sqrt[3]{S_{24} S_{26} S_{16}}$$
$$R'' = \sqrt[3]{R_{24} R_{26} R_{16}}$$

The formulas given for the single three-phase circuit apply if

$$A + B' \text{ is used for } A.$$
$$B + B'' \text{ is used for B.}$$

Double Circuit Line with One Ground Wire. Fig. 20.

$$S_7 = \sqrt[3]{S_{17}\, S_{27}\, S_{37}}$$
$$R_7 = \sqrt[3]{R_{17}\, R_{27}\, R_{37}}$$

The equations for the single three-phase circuit apply if

$$A + B' - \frac{B_7^2}{A_7} \text{ is used for A}$$

$$B + B'' - \frac{B_7^2}{A_7} \text{ is used for B}$$

B' and B'' obtained as from Fig. 19.

Double Circuit Line—Two Ground Wires. Fig. 21.

$$S_8 = \sqrt[3]{S_{18}\, S_{28}\, S_{38}} = S_7$$
$$R_8 = \sqrt[3]{R_{18}\, R_{28}\, R_{38}} = R_7$$

The equations given for a single circuit line apply if

$$A + B' - \frac{2\,B_7^2}{A_7 + B_{78}} \text{ is used for } A.$$

$$B + B'' - \frac{2\,B_7^2}{A_7 + B_{78}} \text{ is used for B.}$$

Bibliography

1. *Voltages Induced by Arcing Grounds*, by Peters and Slepian, A. I. E. E. Trans., Vol. 42, 1923.

Frequency Conversion by Third Class Conductor and Mechanism of the Arcing Ground and Other Cumulative Surges, by C. P. Steinmetz, A. I. E. E. Trans., Vol. 42, 1923.

"The Intermittent Grounding Effect," by W. Petersen, *Elektrotech. Zeitsch.*, Nov. 22 and 29, 1917.

2. "Grounding the Neutral through Resistance or Reactance," by W. W. Lewis, *General Electric Rev.*, June 1929.

3. "Inductive Interference Report," Railroad Commission of California, April 1, 1919. See also Reference No. 2.

Discussion

L. V. Bewley: There are two generally accepted theories of arcing grounds, which have been called the normal frequency arc control, and the oscillatory frequency arc control, corresponding respectively to the way in which the arc is assumed to go out. Hitherto these theories have been developed from the following assumptions and premises:

(1) A system of lumped constants.
(2) Line-to-line capacitance neglected.
(3) Line-to-ground capacitances equal for each line.
(4) Line reactance neglected.
(5) Transformer reactance neglected.
(6) Transformer saturation neglected.
(7) Load connections neglected.
(8) Load at instant of fault neglected.
(9) Line losses neglected.
(10) Neutral impedances neglected.
(11) Gap ionization and characteristics neglected.
(12) Traveling waves non-existent.
(13) Location of fault on the line not taken into account.
(14) High-frequency oscillation assumed to overshoot 10 per cent.
(15) Arc waits each time for a successively increasing normal system frequency crest before striking.
(16) The possibility of a superimposed subsequent switching surge not taken into account.
(17) Neutral voltage not considered.

In his paper Mr. Clem has contributed the following three refinements to the oscillatory frequency arc extinction theory:

(a) Effect of line-to-line capacitance.
(b) Effect of impedances in the neutral.
(c) Approximate expressions for damping.

He has not made a definite decision, however, as to which theory is nearer the truth, but has favored the one which gives the most pessimistic results.

It seems to me that the principal defects of the theory as it now stands, lie in assumptions (12), (15), and (16). Certainly the presence of traveling waves is going to change the character of the phenomena, and how these traveling waves behave will depend decidedly upon items (4), (5), (6), (7), (8), (9), (10), (11), (13), and (16). Some of these effects are optimistic and some pessimistic, but in what direction their resultant points, I do not know. It does not appear reasonable to suppose that a gap which breaks down initially at the crest of the normal voltage wave, will require 2½ times as much potential to break it down the second time, 3¼ times as much the third time, and so on. In fact, just the reverse would be expected, due to gap ionization, and partial disintegration of the insulation.

In a series of laboratory arcing ground tests on a circuit of lumped constants representing a three-phase transmission line, taken at Pittsfield, the high-frequency oscillations were confined entirely to the neutral voltage, and occurred there only for very high neutral impedances. There was no evidence of a high-frequency oscillation in either the free or faulty line-to-ground voltages. The high-frequency oscillation in the neutral did not overshoot more than 75 per cent, yet the normal frequency voltage of the free line built up, in the case of an isolated neutral, to 314 per cent normal leg voltage. In these tests the gap became ionized a cycle or two after the initial arc, and thereafter reignited at insignificant voltages. It was a very short gap, however, and the long arc of a transmission line would no doubt behave quite differently.

A severe arcing ground may cause a circuit breaker to open, so that there is a possibility of a switching surge being superimposed on the arcing ground surges. Perhaps the resulting rise in potential will exceed that due to either type of surge by itself.

In my opinion, this question of arcing grounds will not be settled until exhaustive tests have been made in the fields on actual transmission systems, including both overhead lines and cable systems.

The present theories and laboratory tests of arcing grounds are hedged in by so many assumptions and neglected factors that results obtained with them can hardly be regarded as conclusive.

As an interesting little side light on Table II of Mr. Clem's paper, it will be noticed that the maximum voltages of successive arcs are related to each other as

$$V_k = 3 + \frac{V_{k-1}}{2}$$

Substituting repeatedly into the right hand member until $k = 2$, and summing the resulting finite series, there is

$$V_k = 3 + \frac{V_{k-1}}{2} = 3 + \frac{1}{2}\left(3 + \frac{V_{k-2}}{2}\right)$$
$$= \dots = 6 - \frac{3}{2^{k-1}}$$

Thus the ultimate maximum voltage approaches $V = 6$ as a limit.

H. M. Trueblood: Where operating conditions in power systems are such as to permit it, the use of neutral impedance as a means of limiting short-circuit currents is a valuable and important inductive coordination measure. The figures Mr. Clem has arrived at for overhead systems, as shown in his Table IV, are therefore very encouraging. They indicate that from the standpoint of arcing grounds, at least, neutral resistances in overhead systems may have values sufficiently large to afford very definite relief in the average case of inductive exposure. On the other hand, the figures in Mr. Clem's Table V are not so encouraging, that is to say, the magnitudes of neutral impedance shown in the right-hand column of the table would in most cases be too small to be of much value as a measure of inductive coordination where the problem involves voltages induced in communication circuits at times of faults to ground on power systems.

This leads to the thought that a more careful examination should be made of the effects of various factors which Mr. Clem has found it necessary to ignore or merely to approximate in his analysis of this very complicated question. For example, the assumption is made that extinction of the arc occurs on the oscillatory frequency basis, although it is stated that the phenomenon is probably controlled by normal frequency arc extinction. Other assumptions are also made in the direction of conservatism in estimating the upper limiting values of neutral impedance which may safely be used. In the absence of information, it is natural and indeed advisable to do this, as the two examples cited by Mr. Clem in his presentation indicate. Incidentally, it is interesting that in both examples, neutral reactance and not resistance was involved. I think it is important to evaluate these unknown factors more closely, and while I have no specific suggestions to offer at this time as to how we might go about doing this, I sympathize strongly with the view expressed by Mr. Bewley that the real way to get an answer to questions of this kind is by tests.

In reading the paper I found myself somewhat confused by the considerable variety of capacitance coefficients employed. There seems to be a distinct need of standardization of terminology in this field. It seems to me that it would help a good deal if the various kinds of capacitance coefficients could all be made to depend upon the fundamental idea of direct capacitance.

Mr. Clem's conclusion from Table II, that the maximum voltage is six times normal line-to-neutral voltage, can be obtained rather simply as follows: Considering absolute values in the line of figures "Max. V" in Table II, any term can, of course, be obtained by adding the difference between it and the next preceding term to that preceding term. Examination of the sequence with this in mind shows that the absolute value of the nth term is

$$3\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2} n - 1\right)$$

The limit of the sequence in parentheses is 2, so that the ratio of the maximum voltage to normal voltage is 3 times 2, or 6.

H. H. Spencer: Expression No. 47 of Mr. Clem's paper shows that the limiting value of ground resistance is two-thirds of the geometrical mean of the reactance due to capacitance and the system reactance. I should like to ask Mr. Clem to designate that for us, so we can be sure what system reactance to use: whether it is positive, negative, or zero phase sequence or some weighted average value. I should think the arithmetical mean of the three would be the best value since the ground short circuit current with a single-phase fault is equal to three over the sum of the per unit reactances.

J. E. Clem: Mr. Bewley called attention to the fact that several assumptions were made in working out the theory of my paper. This is perfectly true and it is in line with the general practise used in solving problems which involve many variables. This is done because it is expected that the assumptions will be made in such a way as will not seriously impair the usefulness of the final result. In connection with this it is interesting to note that Mr. Bewley presented an equation for ground wire protection in his discussion of Mr. Dwight's paper which he said he had worked up by considering practically everything he could think of. That equation is identical with one which I worked up independently on the simple assumption of static conditions. The decision as to what to neglect and what to consider depends to a great extent upon the purpose of the analysis. If, as in this case, the purpose is the determination of the limiting values then only the major items need be brought in. Frequently the attempt to include a lot of minor factors in an analysis merely increases the work without improving the engineering value of the result.

However, it appears that I have included more factors than Mr. Bewley realizes in the analysis with impedance in the neutral. For that case his items 4, 5, 7, 9, 13, and 17 are included, either directly or indirectly, in addition to the items he mentioned. That leaves 6, 8, 11, 12, 15, and 17. The important items are 11, 13, and 16. If the gap is relatively short, ionization will probably tend to cause the arc to maintain itself and to prevent the building up of the arcing ground phenomenon. For item 15 it is hard to visualize just how the arc would wait for successively increasing voltages before restriking but the fact that overvoltages have been built up due to an arcing ground indicates that something akin to this does take place. In regard to item 16 if a switching surge were to be superposed upon the arcing ground phenomenon then voltages higher than 5½ to 6 times normal line to neutral voltage should be observed. However, during the surge voltage investigation of the past few years no voltages higher than these have been observed as a result of the switching surge on the system of the Consumers Power Company. This system operates with an isolated neutral and therefore should be subject to the higher arcing ground voltages.

I am heartily in agreement with Mr. Bewley's ideas in regard to tests on the transmission line. Tests in the field under operating conditions are the ultimate proof of any analysis and I should like to venture the prophecy that the tests will not show as high voltages as are indicated in the analysis. However, the design of insulation must be based on extreme conditions and an analysis which gives limiting values which are not entirely impossible of attainment has a value.

Dr. Trueblood calls attention to the fact that the limiting values of resistance in Table V are relatively less than those in Table IV. Table V refers to cable systems and the relatively larger capacitance of cable systems reduces the critical resistance. This need not cause undue apprehension because cable circuits do not cause as much telephone interference as overhead lines.

There is no need for anyone to be confused by the capacitance coefficients since they all depend upon two fundamental values of direct capacitance. These are the capacitance of a conductor to ground and the mutual capacitance between pairs of conductors. All capacitance values of involved configuration of conductors can be expressed by means of these two fundamental values.

The reactance x_1 used in Equation (47) should be taken as the positive sequence reactance.

Fused Grading Shields

BY H. A. FREY[1]
Associate, A. I. E. E.

and

E. M. SKIPPER[1]
Non-member

***Synopsis:**—The fused grading shield was developed primarily to reduce the number of outages due to insulator flashovers on transmission lines. Recent practise of reducing the insulation adjacent to terminals has increased the number of flashovers in these sections. The fused grading shield while not always economically feasible for entire lines provides a means of obtaining the reduced insulation without increased outages.*

Laboratory and field data are presented showing the characteristics of the device as well as the results of several years of operating experience.

THE use of fuses in conjunction with grading shields and arcing horns was first tried on two circuits of the Columbia-Terminal Station line of the Union Gas and Electric Company in the spring of 1927. The device as first installed has been very completely described by Mr. Philip Stewart of the Union Gas and Electric Company in previous papers.[2] It consisted of the usual grading shield and arcing horn combination except that the grading shield was insulated from the line by two standard suspension units. These two units were then short circuited by means of a high-voltage fuse of the carbon tetrachloride type. In the event of a flashover of the insulator string the arc formed between the grading shield and the top arcing horns, forcing the ground current to flow through the fuse. The operation of the fuse extinguished the arc by disconnecting the grading shield from the line. This operation was so rapid that no line outage occurred.

This device operated so successfully during the 1927 lightning season that in 1928 the company decided to equip the remaining two circuits between Columbia and Terminal Station with similar equipments. The original fused grading shields, requiring as they did two additional insulator units per string, were not particularly economical. The redesigned assemblies eliminated this feature. The grading shield was placed in its usual position at the bottom of the string with fuses in place of arcing horns at the ground end. Expulsion type fuses replaced the carbon tetrachloride fuses and with this set-up it was found possible to use two fuses per assembly in place of one. Tests showed that except in rare cases no arc would strike to both fuses at once. This, of course, meant that after one operation there would still be one fuse available in the event of further trouble at the same point.

The 1928 operating experience on this line was again very favorable and has been completely reported by Mr. Stewart.[3]

The adoption of the double ended expulsion type of fuse which is now standard on fused grading shield assemblies was the outcome of a large number of short circuit tests using various shield and fuse combinations. For various line voltages the proper voltage rating of the fuse is as follows:

Line voltage	Fuse
22,000	15 kv.
33,000	25 "
66,000	37 "
110,000	73 "
132,000	73 "

Coincident with these tests a number of impulse tests was also run to determine the correct setting of the grading shield and the fuses in order to assure the formation of the arc between the shield and the tips of the fuses. The tests which were made for this purpose were conducted using an impulse of very steep wave-front and with a tail falling to one-half crest value in about five microseconds. This type of wave, having a very steep front, is more severe from the standpoint of cascading than the usual wave encountered in practise. Obviously if the arc cascaded the insulator string and failed to strike to the fuse, the entire object of the device would be defeated.

It was found that if the arcing distance between fuse and shields did not exceed 80 per cent of the striking distance across the unshielded string, satisfactory arc clearing characteristics could be expected.

The foregoing tests, of course, all fall under the general heading of Development Tests but a number of tests has also been made to determine the operating characteristics of the device. These tests have been conducted both in the high-current testing laboratory of the General Electric Company and in the field. On all of the short circuit tests the arc was originated by means of a piece of fine fuse wire strung between the fuse tips and the shield. Table I gives a summary of these tests.

It will be noted that out of 21 tests there were 2 arcs which failed to clear or, ignoring the laboratory tests, 2 out of 15. On both of these tests the available current was very small. The time required for any fuse to operate and clear the circuit becomes greater as the short circuit current decreases. With currents of the order of 100 amperes the time required for the fuse to clear is so great that even moderate winds are likely to blow the arc away from the fuse tips and onto the tower hardware in which position, of course, it is beyond

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[1] Locke Insulator Corp., Baltimore, Md.

[2] Report to Overhead Systems Committee of N. E. L. A. presented in Chicago in October 1927.

[3] *Fused Arcing Horns and Grading Rings*, by P. Stewart, A. I. E. E. TRANS., Vol. 48, July, 1929, p. 891.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

TABLE I—LABORATORY AND FIELD TESTS

Test No.	Date	Nature of test	Line voltage Kv.	Kv. to ground	Frequency cycles per sec.	Interrupted current	Time to clear sec.
1	Apr. 1928	Laboratory		37	25	572	0.06
2	" "	"		37	25	630	0.06
3	" "	"		37	25	770	0.025
4	" "	"		37	25	1482	0.04
5	" "	"		37	25	1720	0.04
6	" "	"		37	25	2340	0.02
7	May 1928	Field	66	38	60	100	0.60 Failed to clear
8	" "	"	66	38	60	115	1.0 Failed to clear
9	" "	"	66	38	60	250	0.075
10	" "	"	66	38	60	250	0.067
11	" "	"	66	38	60	400	0.042
12	" "	"	66	38	60	2500	0.0125
13	June 1929	"	110	63.5	60	314	0.0675
14	" "	"	110	63.5	60	388	0.05
15	" "	"	110	63.5	60	630	0.146
16	" "	"	110	63.5	60	1112	0.0416
17	" "	"	110	63.5	60	1955	0.046
18	" "	"	110	63.5	60	2878	0.025
19	" "	"	110	63.5	60	3454	0.021
20	" "	"	110	63.5	60	4002	0.021
21	" "	"	110	63.5	60	5120	0.021

the control of the fuse. This is what happened in the two cases in question. Two hundred amperes appear to be the minimum current limit for satisfactory operation of the fused grading shield. The upper limit of short circuit current has not as yet been determined but tests have been made on short circuit currents as high as 5120 amperes with perfect success.

Figs. 1, 2, and 3, are respectively the oscillograms taken during tests 17, 19, and 21. It should be noted that the maximum recovery voltage shown on these oscillograms is 1.225 times normal.

In addition to the tests listed in this tabulation, a series of tests was conducted by the Carolina Power and Light Company at the Swannanoa Substation near Asheville, North Carolina. Unfortunately no oscillograph was available to record accurately the current and duration of the arc. It is estimated, however, that the short circuit current of the system at this point is around 250 amperes. The system voltage is 66 kv.

The four fuse grading assembly of the type shown in Fig. 4 was used for this test. Seven arcs were started, all of which cleared successfully without operation of the line relays. During one short circuit two fuses were blown. In this case the wind carried the arc from the tip of the fuse from which it had started to the tip of an adjacent fuse. Attempt was made to repeat this on succeeding tests by starting the arc on the windward side. Despite this no further example of more than a single fuse operating occurred.

Fig. 5 shows the fused grading shield during another field test clearing a ground current of 2878 amperes. The flame is quite clearly being ejected from both ends of the tube. It will also be noted that the original arc between the tips of the fuse and grading shield is remaining well clear of the insulator string.

Fig. 6 shows a grading shield from which six arcs had been initiated prior to the taking of the photograph. While the shield was slightly burned, it is still perfectly serviceable. Indications are that with the rapid clearing of the arc the life of the grading shield will be practically indefinite.

Fused grading shields were first designed for use on all the insulator strings of a complete transmission line but it is unlikely that this type of installation is to be recommended except under special conditions where line outages assume extremely serious proportions. Where the expenditure can be justified by conditions there is little doubt that almost perfect protection will result.

The fused grading shield, however, has another very definite use which is certainly economical and practical. This use is as relief gaps on those insulator strings immediately adjacent to terminals.

The modern tendency in transmission line design is largely toward the overinsulation of the line with a view to reducing the number of outages caused by insulator flashover. This practise has resulted in improved line service but where the path of the storm is close to a line terminal it has in many cases merely shifted the burden to the substation apparatus. It is relatively easy to add a few units to an insulator string and for the higher voltages at least relatively inexpensive. On the other hand it is very difficult and expensive to attempt to raise the insulation of the substation apparatus to the level of the increased line insulation. It has therefore become customary to reduce the line insulation immediately adjacent to the terminal to a value which will balance with the terminal equipment insulation.

Due to the rapidity with which lightning surges on transmission lines are attenuated most disturbances originating several miles out on the line will probably be reduced below dangerous proportions before they reach the reduced insulation. Nevertheless, there will naturally be an increased number of flashovers adjacent to the terminals resulting in a corresponding increase in number of line outages. There is also the added danger of damaging the insulator units at this point, thus causing a prolonged outage while repairs are being made. The use of a standard grading shield on the shortened strings will prevent the insulator damage but cannot prevent the line trip-out when flashover occurs.

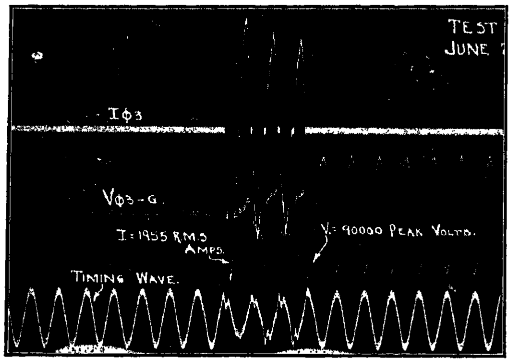

Fig. 1

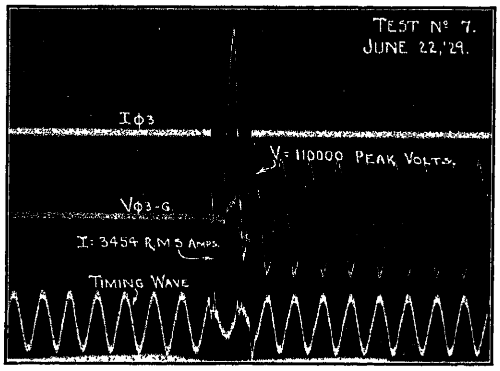

Fig. 2

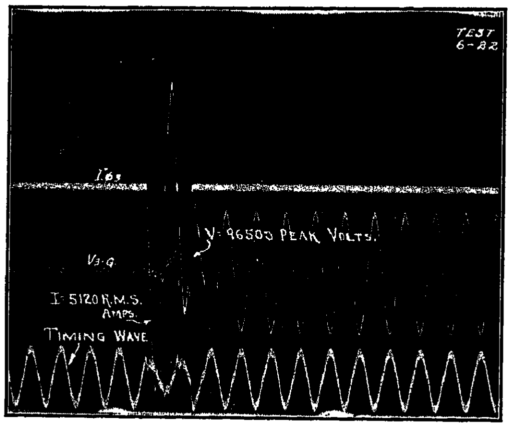

Fig. 3

Figs. 1, 2, and 3—Oscillograms Obtained during Field Tests

Showing short-circuit current and time to clear

If the fused grading shield is applied to the reduced insulation at the terminals and the gap between the fuse tip and shield adjusted to give the desired flashover voltage, then the excess voltage on the line will be relieved without an outage since the power current to ground is interrupted by the fuse without operation of the line relays. Unlike the lightning arrester the fused grading shield can be expected to care for only those surges whose magnitude is sufficient to arc the gap

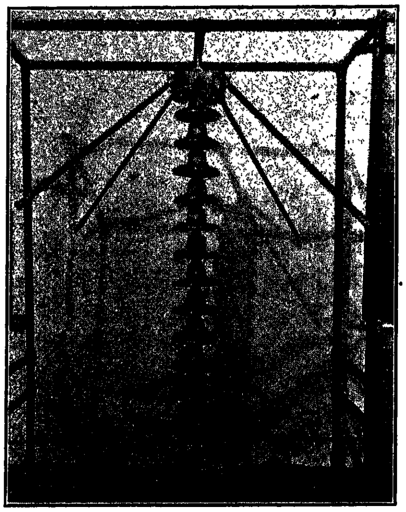

Fig. 4—Four-Fuse Type Assembly

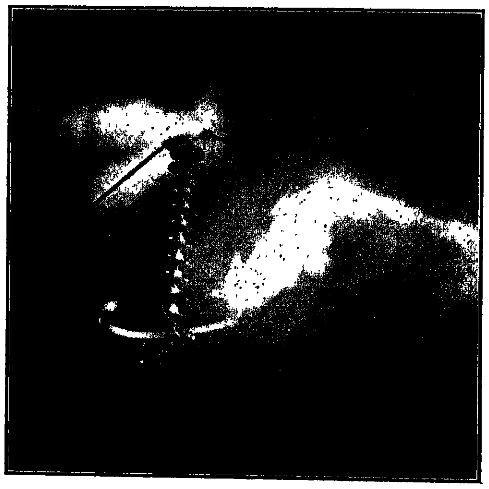

Fig. 5—Fused Grading Shield in Operation during One of Field Tests

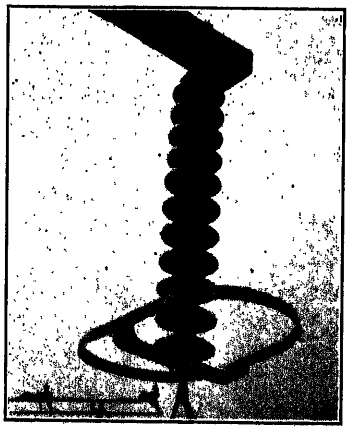

Fig. 6—Grading Shield after Six Tests at High Current

The lightning arrester would protect the station equipment from the hammer blows of smaller transients.

For this class of service a fused grading shield equipped with four fuses, so that each assembly is capable of clearing four arcs without fuse renewal, has been developed. Two and three fuse assemblies have also been applied to this service to fit special conditions and each type of installation has been attended with marked success.

These reduced gaps should be installed on each line for a distance of approximately one-fourth mile out from the terminal. A number of installations has been made equipped with only a single assembly on each line wire. So far, no difficulties have arisen as a result of the use of this single gap but as an additional safety factor it is certainly advantageous to have more than one gap on each conductor.

It is important to remember that the last gap adjacent to the terminal should be placed as close as possible to the apparatus to be protected, otherwise it is quite possible that reflections will so build up within the substation as to result in breakdown or arc-over of the apparatus without affecting the fuse grading shield.

In Table II is listed the proper gap setting of a fused grading shield to balance with modern substation equipment. Here again it must be remembered that much of the older equipment was designed with lower insulation than that which is now being used. Hence in some cases it may be necessary to use a still smaller gap in order to balance with this old substation insulation. In many cases the switch insulators or bus supports in the substation constitute the weak link. When such is the case it may be necessary to set the reduced gap below the tabulated values in order to prevent flashover on disconnect switches and bus supports.

TABLE II—GAP SETTINGS TO BALANCE WITH STANDARD STATION EQUIPMENT

Line voltage in kv.	Fused grading shield gap in in.	Approximate impulse FO of gap	Approximate 60-cycle flashover of gap
33	10 in.	310	103
66	21 in.	605	206
88	31 in	900	290
110	37 in.	1060	345
132	41 in.	1170	380

Figs. 4, 7, and 8, show the 4, 3, and 2, fuse assemblies respectively. The 3 and 4 fuse types are most commonly used for substation protection where it is probable that more than one arc may occur on an assembly during the course of a storm. The two fuse assembly has been applied to substations but is primarily applicable to line service.

The double ended expulsion fuse which was chosen for use with fused grading shields presents several unique features.

The fusible link of this fuse is placed at the upper end of a weatherproof fiber tube. To the lower end of the link is fastened a long copper pigtail which runs the entire length of the tube. The pigtail is held taut by a compression spring at its lower end. The tube is capped at either end. The cap on the lower end is held in place by friction but the cap at the upper end is sweated in. When the fusible link is melted the compressed spring is released. The combination of forces of the released spring and of the gases generated in the tube blow out the cap and vent the tube, extinguishing the arc.

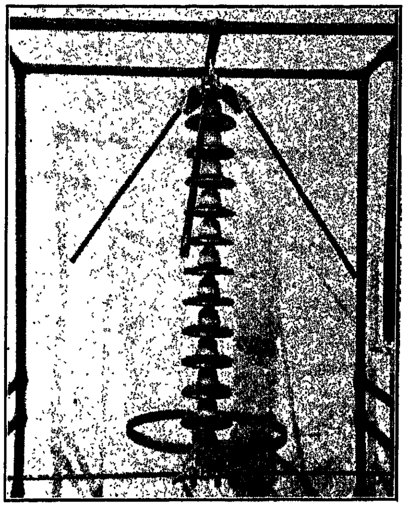

Fig. 7

With the expulsion type of fuse it is desirable to build up the internal gas pressure promptly in order to obtain quick clearing. With the double end fuse only the lower cap opens on low-current operation so that there is no loss of pressure from the upper end of the fuse. The spring also aids in obtaining quick clearing under low currents since it hurls the copper pigtail free of the fuse tube as soon as the fuse element melts. The maximum current rupturing capacity of an expulsion

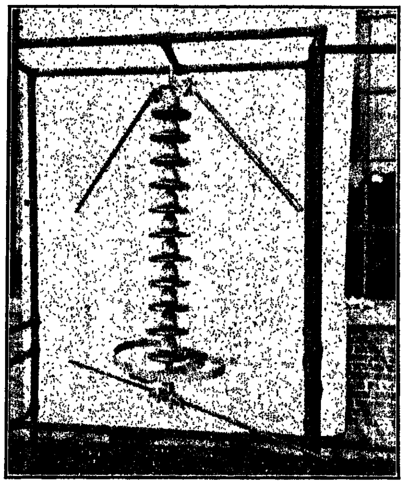

Fig. 8

Figs. 7 and 8—Three- and Two-Fuse Type Assemblies fuse may be the point at which the fuse tube bursts due to excessive pressure. When the internal pressure becomes excessive in a double ended fuse the upper cap is blown out and the tube is vented from both ends. The double ended action is particularly desirable for fuses used on Fused Grading Shields since it reduces the unbalanced reaction. In rupturing high currents this unbalanced reaction might otherwise become sufficiently severe to give a violent kick to the supporting insulators.

Some fears have been expressed that the spring and copper pigtail which are expelled from the fuse might be sufficiently hot to start fires in grass or other inflammable material on which they might fall. During the tests it was found that in most cases the spring and pigtail were cool enough to be picked up by hand immediately after striking the ground. After a few of the very high current tests, notably No. 20 and No. 21, the elements were quite hot. In no case, however, was there any evidence of actual burning of the metal. It is this continued combustion of metal after falling from the line which has been responsible for grass fires in the past.

During the high-current tests the expelled elements were thrown about 30 ft. from the insulator string.

Further tests were made to determine the point at which corona formation would start about the copper pigtail in the fuse tube. It was found that with a standard 110-kv. fused grading shield of the two fuse type mounted on an eleven-unit string, the corona point was 82 kv. when the fuses were parallel to the conductor. With the fuse at right angles to the conductor, the corona point was 89 kv. The normal corona point on the standard grading shield assembly would be in the vicinity of 90 to 95 kv.

OPERATING EXPERIENCE

Since the installation of the first fused grading shield in actual service an attempt has been made to keep an accurate record of its performance.

Table III represents a summary of the operating characteristics of those fused grading shields which have been installed during the past three years. This table shows the approximate short circuit currents to ground, the number of failures to clear, and the number of cases of damage of substation apparatus protected by the fused grading shield and damage to the insulators on which the fused grading shield is mounted. Four trip-outs which may have been caused by arcs on the fused grading shield are also listed. Only the operations on the Union Gas and Electric Company's system which have not been previously reported by Mr. Stewart are included in this table. Lightning arresters were not used on any of the installations listed in Table III.

Table IV gives a list of those fused grading shield installations on which there have been no operations reported to date. All of the installations represented in this table are made as substation relief gaps except those on the Union Gas and Electric Company's system.

The only case of flashover or damage to the substation equipment to be protected is that reported by the Pennsylvania Water and Power Company, where there have been four bushing flashovers since the fused grading shields were installed. Two of the bushing flashovers occurred simultaneously, one at each end of the line, apparently as a result of the same surge. In an effort to prevent a repetition of this the gap was reduced somewhat. After the reduction there were two more bushing flashovers. These occurred during an exceptionally bad lightning season during which a large number of fuses was blown on this same system. There is little doubt that had no fused grading shields been installed the number of substation flashovers would have been considerably greater.

On this system each end of the line wire is provided with 15 reduced gaps. This probably accounts for the large number of fuse operations shown on the operating records. The bulk of these fuse operations occurred during the second season.

The installation of fused grading shields on the Memphis Power and Light Company's North Memphis,

TABLE III—OPERATING DATA

Operating company	Line or substation	System voltage	Ground current amperes	Connected capacity kv-a.	No. of gaps on each line wire	Distance nearest gap to apparatus to be protected	Date put in operation	No. of operations	No. of failures to clear	No of cases of insulator damage	No. of case of F. O. of substation apparatus	No. of trip-outs of line relay
Memphis Pr. & Lt.	N. Memphis Ripley & Covington Stations	66 kv.	500 to 800		1	100-400 ft.	June or July 1929	2	None	None	None	2
Minn. Pr & Lt.	Hibbing Substa.	110 kv.	415 to 920	72,000 kv-a.	3	100 ft.	May 27 1929	1	"	"	"	0
Minn. Pr. & Lt	Nashwauk Substa.	110 kv.	422–1033	72,000 kv-a.	3	200 ft.	May 27, 1929	1	"	"	"	1
Union Gas & Elec. Co.	Hartwell Substa.	66	1500–2000	285,000 kv-a.	On all towers	200 ft.	May 1928	4	"	"	"	0
Penna. Water & Pr. Co.		70 kv	300–600	16,000	15	40–75 ft.	Spring 1928	95	1	1	4	1
Carolina Pr. & Lt. Co.	Swannanoa Substa.	66 kv.	250–300	90,000	1	70	April 1929	8	None	None	None	0
Carolina Pr. & Lt. Co.	Brisco Substa.	110	About 500	108,000	1	2500–3000 ft.	Sept. 1929	2	"	"	"	0

TABLE IV—FUSED GRADING SHIELD INSTALLATIONS FROM WHICH NO OPERATIONS HAVE BEEN REPORTED

Operating company	Station or line	System voltage	Approx. ground current	Connected capacity kv-a.	No. of gaps on each line wire	Distance of nearest gap to apparatus	Date put in operation	Remarks
Alabama Pr. Co.	Leeds Substa.	110 kv.	4460	300,000	3	900 ft.	About Aug. 1929	
Alabama Pr. Co.	Anniston Substa.	110 kv.	2700	300,000	3	1000 ft.	About Aug. 1929	
Carolina Pr. & Lt. Company	Spruce Pines Substa.	66 kv.	350	90,000	1	30 ft.	Oct. 1929	
Carolina Pr. & Lt. Company	Cape Fear Substa.	110 kv.	1500	108,000	1	about 1 mile	Sept. 1929	
Louisiana Pr. & Lt. Company	Sterlington Sta.	110 kv.	1500–2000	100,000–175,000	1	500–1000 ft.	July 1929	
Union Gas & Elec. Co.	Armco Line	66 kv.	1000–1500 amperes	24,000	Used on every tower		June 1928	No outages on line

Ripley and Covington Substations are of the four fuse type. The distance from North Memphis to Covington is 29 mi. and from Covington to Ripley 14 mi. There have been two cases of fuse operation, each accompanied by a line trip-out. The first operation resulted in the blowing of all fuses on all line wires at North Memphis, a total of 12 fuses. The second operation resulted in the blowing of one fuse on one line wire at Covington and one fuse on each of two line wires at Ripley. Apparently these operations were due to the same surge.

Both of these surges must have been very severe in order to have arced so many gaps. The line trip-outs were undoubtedly caused by the flashover of other insulator strings out on the line which were not protected by fused grading shields.

On the installation at the Minnesota Power and Light Company's Nashwauk Substation there has been one fuse operation and a simultaneous line trip-out. This line trip-out is believed to have been due to the arc at the fused grading shield but it is possible that it may have been due to another flashover out on the line on an unprotected insulator string.

It is interesting to note that none of the operating companies has reported any trouble arising from the recovery voltage after a fuse operation. In addition to the operations listed in Table III Mr. Stewart has already reported 7 interruptions and 12 interruptions prevented by the fused grading shields on the Union Gas and Electric System. Some of the fuses on this system are of the carbon tetrachloride type and some of the single ended expulsion type. None of the double ended fuses are in use on the Union Gas and Electric system.

So far no installations have been made using relief gaps on the station side of the choke coils. There is some doubt as to whether such an installation would be successful since it is possible that disturbances set up by the operation of the fuse would cause oscillation between the choke coils and the transformer. A relatively small recovery voltage might be magnified by such reflections to dangerous proportions.

The successful operation of the fused grading shield on any system is dependent not only upon the characteristics of the device itself but also upon the operating characteristics of the system. Before being installed on any system a careful check should be made on the relay settings to see that they are not too fast for the operation of the fuses. Obviously if the relays operate before the fuse clears the line will be tripped out even though the fuse might be quite successful in clearing the arc.

The use of fused grading shields on delta connected systems should be given very careful preliminary consideration since on such systems the only current available for operating the fuse is the very small charging current of one line to ground, and any unbalanced currents which may be present on the system. There is some reason to believe that when used on delta systems flashover on one conductor would result in a corresponding flashover on another conductor. If this is the case there are really two fuses and two arcs in series with the line voltage.

Where difficulty is encountered in installing the fused grading shield relief gaps close to the substations, several rather interesting types of construction have been developed. In one case, the Biscoe substation of the Carolina Power and Light Company, it was impossible to get the relief gap on a suspension string close to the station due to the presence of dead end towers at this point. There was not sufficient mechanical clearance in the jumper loop to place the fused grading shield on insulator strings supporting the jumper. The problem was solved by placing a vertical insulator string between the jumper and the tower crossarm and then suspending the insulators which support the fused grading shield on the bottom of this string with the fuse at the live side and the grading shield connected electrically to ground by means of vertical tie down. This installation at the Biscoe station has had two satisfactory operations so far.

SUMMARY

1. The minimum short circuit current which the fused grading shield, using a double ended expulsion fuse, can be expected to clear satisfactorily is about 200 amperes.

2. The maximum short circuit which can be cleared has not been determined but it is in excess of 5100 amperes.

3. Out of a total of 28 tests made in the laboratory and in the field there have been only two cases of failure to clear. Both of these were with currents under 200 amperes.

4. Operating records show that out of a total of 113 operations on modern assemblies in service there has been only one failure to clear with consequent damage to line insulators and four cases of flashover within the substation. There have been four cases in which the line has tripped out simultaneously with a fused grading shield operation. Of the four, two on the Memphis Power and Light Company system were probably due to other flashovers on unprotected strings. This leaves two cases of line trip-out for which the fused grading shield is probably responsible.

Discussion

F. E. Andrews: In regard to Mr. Frey's paper, it is interesting to point out that in connection with the new construction described in *Transmission Research and Design with the Field as a Laboratory* (Andrews and Stroup, see p. 959), there is quite a considerable problem, particularly on the pole top switches. We have old style pole top switches with steel crossarms and grounded structures. If we wish to improve the performance of a line we change it to the improved construction, with a new pole-top switch of corresponding design. It is a pretty expensive thing. So it seems to me that the fused grading shield using two fuses for each assembly may be the answer and we are making a number of such installations with the expectation that these installations will take care of the flashovers which occur very frequently on these pole top switches without causing us any line outage.

There are two or three questions I should like to ask Mr. Frey. What methods are recommended in regard to the inspection and replacement of fuses on these grading shields? Can the work be done without taking the lines out of service or should they be inspected after every lightning storm? Also, what size fuse is used and what would be the ampere rating of the fuses of this type, and how does the fuse rating compare with the current required for rupturing.

J. E. Clem: Messrs. Frey & Skipper in their paper state that the arcing distance between the fuse tip and grading ring must not exceed 80 per cent of the unshielded length of the insulator string to insure that the arc will always strike to the fuse. This requirement is correct and means either that the flashover voltage of the insulator string must be reduced by the application of the fused grading shield assembly, or that the insulator string must be increased in length if it is desired to maintain the lightning flashover voltage. Part of the increased length required is due to the ring itself, but by far the greater part is due to the space required by the fuses. The rating of the fuses for various line voltages as given in the paper was selected on the basis of using a fuse having a voltage rating approximately equivalent to the normal line to neutral voltage of the system. This gives fuses that sometimes are quite long relative to the length of the insulator string.

The use of lightning arresters is considered to be the most satisfactory method of protection, but in order that the best results be secured, it is desirable that the insulation level adjacent to the station be kept at the proper value for the rated circuit voltage, either by reducing the number of line insulators near the station, or by the installation of a limiting gap. When the fused grading shield assembly is used to establish the required reference insulation level, it should be borne in mind that it is necessary that the reference insulation level be definitely and positively established. This requires an assurance that there are more fuses available than will be used up before the fuses can be replaced. This requirement probably holds when the fuse grading shield assembly is applied on the line for a short distance outside the station. However, when only one installation is made it is advisable to install at this point a plain gap having the proper setting and then to supplement this gap with the fused grading shield assembly set at a lower value. It seems that the use of the limiting gaps at one point is preferable to the use of several on the line outside the station because the lightning arrester has a much better chance of protecting the one gap closely adjacent to it than it has of protecting several gaps at varying distances up to a quarter or half a mile out from the station.

O. R. Schurig: In referring to the application of expulsion fuses to fused grading shield assemblies, the authors have pointed out that at certain low currents, say 100 amperes, an occasional failure of the fuses has been observed, that is, the arc whipped to ground before the fuse cleared the circuit. It must be pointed out, however, that similar fuses in the same series of tests interrupted satisfactorily at all higher currents up to 2500 amperes. In subsequent tests short-circuit currents up to 5000 amperes were cleared with the fuses in fused grading shield assemblies.

With the above points in mind attention is now called to the authors' data in Table III showing that the range of currents over which grading shield fuses have to operate is relatively small, something like 2 or 3 to one. On the basis of Table III, therefore, a grading ring fuse required to interrupt several thousand amperes would not be called upon to interrupt currents of 100 or 200 amperes and vice versa.

On systems requiring grading shield fuse operation at low interrupting currents of the order of 100 or 200 amperes, fuses suitable for the lower current range should be used.

I should like to ask the authors if it is necessary to have fuses to clear at currents below 200 amperes.

The second point to which I wish to call attention is the application of expulsion fuses to the protection of transformers in substations and branch circuits. In this application, the fuses are mounted with both ends connected to metal conductors on insulators. In other words, we do not begin by drawing an arc in the open air as in the gradual shield fuse. Hence the tendency to arc to ground is eliminated at all currents both low and high within the interrupting rating and fuses will interrupt satisfactorily over a wide range of currents.

W. H. Colburn: This paper presents another method of protection against those surges which may result in flashover of insulators or damage to equipment and is itself a testimony to the unsatisfactory conditions which exist in regard to such protection. Among operating engineers there is no unanimity of opinion as to the best method of affording such protection.

What is most needed now is full and accurate data on operating experience and maintenance costs on the various methods, lightning arresters, ground wires, plain and fused grading shields, etc., over a long period, as well as capital outlay therefor, to enable us to arrive at conclusions regarding the best method of surge protection to adopt.

H. A. Frey: Replying to Mr. Andrews' questions concerning the determination of whether or not a fuse is blown and what steps would need to be taken to keep the fuses in proper order on a transmission line,—these fuses have all been equipped with white caps in the lower end which are expelled when the fuse blows. The open end of the fuse can then be seen from the ground. Blown fuses can be replaced while the line is alive. Originally 10 ampere fuses were used but more recently 5 ampere fuses have been used with success. The 5 ampere fuse is the present standard.

Mr. Clem has pointed out that in many cases it may be desirable to use a single fused grading shield gap set for somewhat lower value than would be otherwise necessary and used in conjunction with a plain grading shield gap set for the value just necessary to balance the station insulation. That certainly is a very good idea where there will be less than the usual number of fuses used on each line wire. Ordinarily enough fuses are placed on each line wire so that there is very little possibility of all fuses blowing during one storm before they can be replaced.

Mr. Schurig asks if there is any necessity for a fuse which would operate below 200 amperes. Present experience leads us to believe there is no necessity for such a fuse. As a matter of fact, the fuses which are being used at present seem to fulfill all the requirements very well. Such need may arise when fuses are to be applied to delta connected systems.

In closing I should like to point out again that the tests which are included in the paper were made in three different locations. The first set was made in the high-current laboratory of the General Electric Company at Schenectady, the second set was made at the Hartwell Substation of the Union Gas and Electric Company, and the last set in the Spring of 1929 at the Bessemer Field Laboratory of the Alabama Power & Light Company.

Reduction of Eddy Current Losses by the Inverted Turn Transposition and the Twisted Lead Transposition

BY JOHN M. LYONS*
Associate, A. I. E. E.

Synopsis.—The first section of the paper contains a brief review of the general theory underlying the determination of extra copper losses in armature windings carrying alternating currents. Simplified methods of calculation are then described.

The second part extends the study of the inverted turn transposition to the special case of three-phase windings having two phases connected in series to form a single-phase winding and the third phase left idle. Formulas and tables are presented to show what reductions in copper losses may be expected for this connection, and to aid in deciding what type of transposition is best for a given winding.

The final section of the paper deals with the twisted lead transposition, and presents formulas and tables showing reductions in extra losses to be gained from this method of inversion and connection. Details of the derivations of the formulas and examples of the use of the twisted lead transposition are contained in appendices at the end of the paper.

* * * * *

I. INTRODUCTION

THE problem of determining the extra copper losses in rectangular conductors composing the armature windings of large alternators has been covered in detail in the well-known papers listed in the references. From these and many other investigations several methods have been adopted commercially for reducing the parasitic eddy current losses and the extra heating which they produce. The methods now in use group themselves into two general classes. One class includes transpositions of the conductor laminations in the slot, and the second includes inversions or twists of whole conductors or leads in the end portions. This paper will be concerned only with the second class.

The "inverted turn transposition" has been used to great advantage on large multi-turn coils. In a previous Institute paper by Mr. I. H. Summers[1] the four arrangements of inverted turns as shown in Figs. 2-7 inclusive were described, and the corresponding reductions in copper losses were calculated and tabulated. These reduction factors can be applied to three-phase, quarter-phase, and single-phase windings in general.

There is, however, one special type of single-phase winding which is not completely covered in that paper. Reference is made to the standard three-phase winding in which two phases are connected in series to form a single-phase winding, and the third phase is left idle. This connection has been applied to several large alternators in the last few years, and will, no doubt, find further use in the future. When high untransposed losses are encountered in this kind of winding, any one of the four types of inverted turn transposition can be utilized just as in standard windings, although the reduction factors will in general have different values. In the present paper the formulas and tables of the previous paper are extended to include this special single-phase winding.

Curves are presented in Figs. 8 to 12 inclusive which will decrease the work involved in making routine numerical calculations. These curves apply not only to windings with integral numbers of slots per pole per phase, but also to the general case of windings with fractional numbers of slots per pole per phase.

The second purpose of the paper is to describe the so-called "Twisted Lead Transposition." As the name suggests, this transposition consists of a 180 deg. twist in one lead of each armature coil. To make the inversion effective, the conductor laminations are insulated throughout each phase belt. Formulas are derived and tables showing reductions in extra copper losses are presented in the second part of the paper. Various types of windings and coil groupings are treated, including the special case of the three-phase winding connected single-phase with one leg idle.

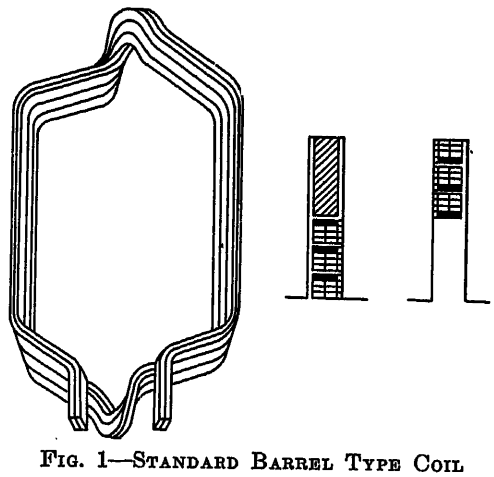

FIG. 1—STANDARD BARREL TYPE COIL
Showing successive positions in slot of a single strand—Type I, Table I

II. GENERAL THEORY

A brief review of the fundamental theory underlying the determination of extra copper losses in rectangular conductors is essential to a complete discussion of the problem. The basic cause of extra copper losses in slot-wound conductors carrying alternating current is the uneven distribution of the current over the cross-section. The flux linkages, and hence the reactance, are greatest at the bottom of the conductor, and least at the top. The current is in effect forced toward the top of the conductor, since the same voltage drop occurs along all the elements of the conductor between points where the elements are solidly joined. This means that the copper losses at the top are greatly in excess of the average, and while the losses at the bottom are less than the average, the total losses in the conductor are greater than occur with the same total uniform current. Likewise, in the several turns in the slot, the extra loss is greatest in the top conductor and least in the bottom one.

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*Turbine Generator Engg. Dept., General Electric Co., West Lynn, Mass.

[1] *Reduction of Armature Copper Losses*, by I. H. Summers, A. I. E. E. TRANS., Vol. 46, 1927, p. 101.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

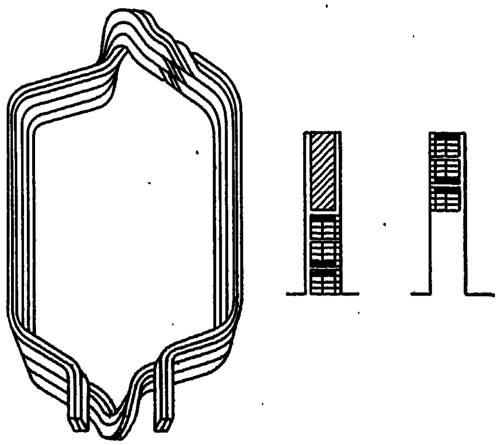

Fig. 2—Barrel Type Coil with Odd Number of Turns

All inverted at the end opposite connections—Type II, Table I

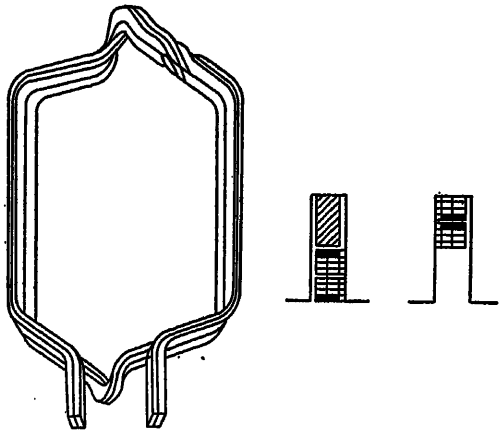

Fig. 3—Barrel Type Coil with Even Number of Turns

All inverted at the end opposite connections—Type II, Table I

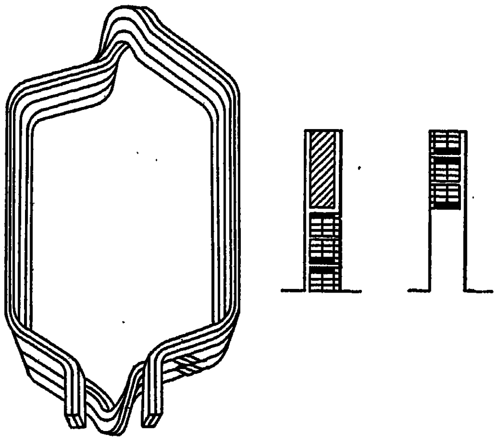

Fig. 4—Barrel Type Coil with Odd Number of Turns

All inverted at the connection end—Type III, Table I

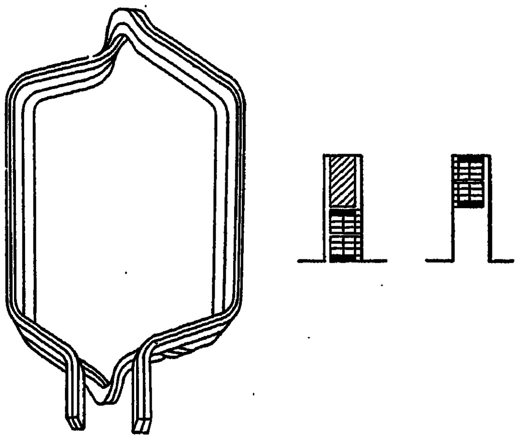

Fig. 5—Barrel Type Coil with Even Number of Turns

All inverted at the connection end—Type III, Table I

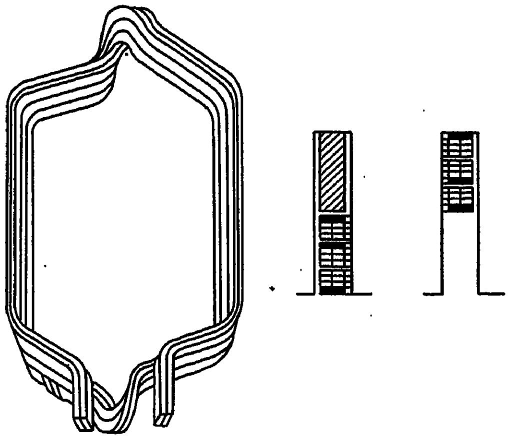

Fig. 6—Barrel Type Coil with Even or Odd Number of Turns

Top half turn in upper coil side only inverted at connection end—Type IV, Table I

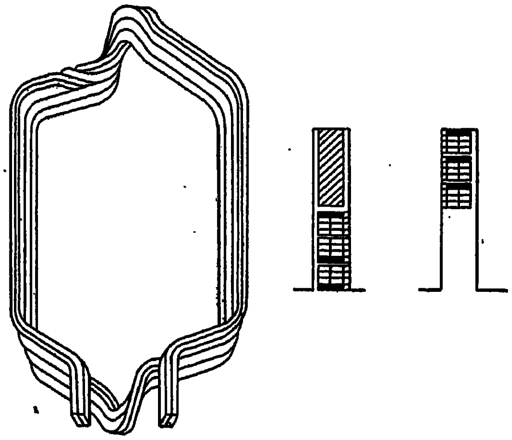

Fig. 7—Barrel Type Coil with Even or Odd Number of Turns

Bottom half turn in upper coil side only inverted at end opposite connections—Type V, Table I

Theories of this action have been worked out by various authors[2, 3], and equations for the variation in current density over a conductor have been developed. For practical use, formulas have been derived for calculating the ratio of a-c. copper loss to uniform current loss for solid conductors, infinitely laminated conductors, and laminations of finite thickness. The reader is referred to these papers for the complete derivations of the formulas, and for explanations of the simplifying assumptions on which they are based. It is to be noted that four factors enter into all formulas and equations for current density and copper loss. These are: (1) frequency; (2) ratio of total copper width to slot width; (3) resistivity of copper; (4) total depth of conductor. These variables are generally collected into one factor D according to the equation:

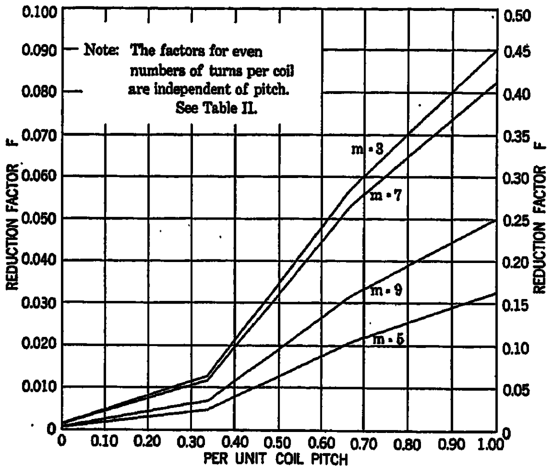

FIG. 8—REDUCTION FACTORS FOR TYPE II TRANSPOSITION SINGLE-PHASE CONNECTION

Use scale on right for $M = 3$ and $M = 5$
Use scale on left for $M = 7$ and $M = 9$

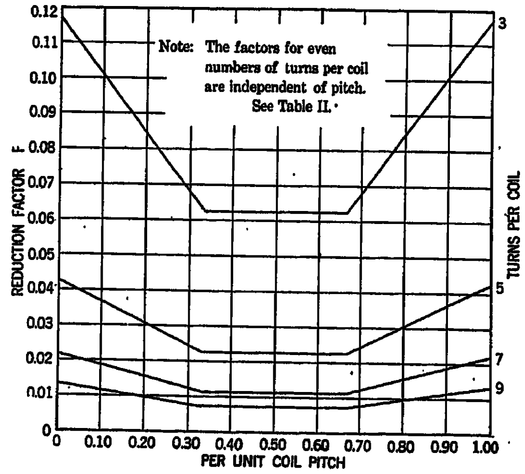

FIG. 9—REDUCTION FACTORS FOR TYPE III TRANSPOSITION SINGLE-PHASE CONNECTION

$$D = 1.19\left[\frac{f}{60/\text{sec.}}\right]^2 [b\, r\, c\, (n\, d/\text{in.})^2]^2 \qquad (1)$$

which holds good for copper at 75 deg. cent. The

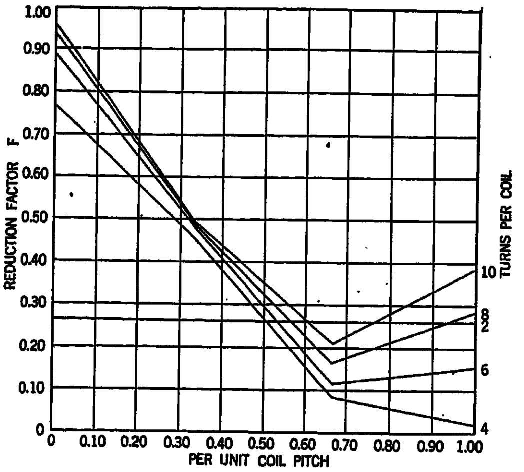

FIG. 10—REDUCTION FACTORS FOR TYPE IV TRANSPOSITION SINGLE-PHASE CONNECTION

Even number of turns per coil

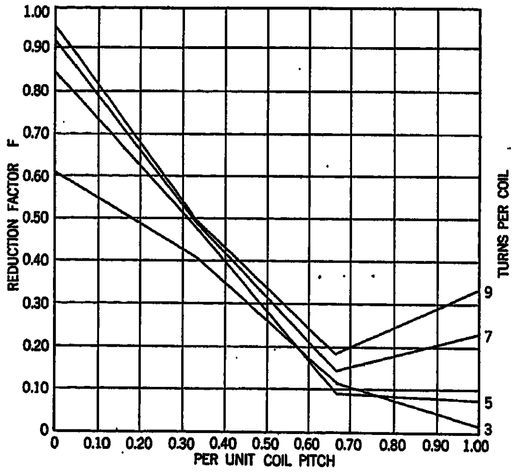

FIG. 11—REDUCTION FACTORS FOR TYPE IV TRANSPOSITION SINGLE-PHASE CONNECTION

Odd numbers of turns per coil

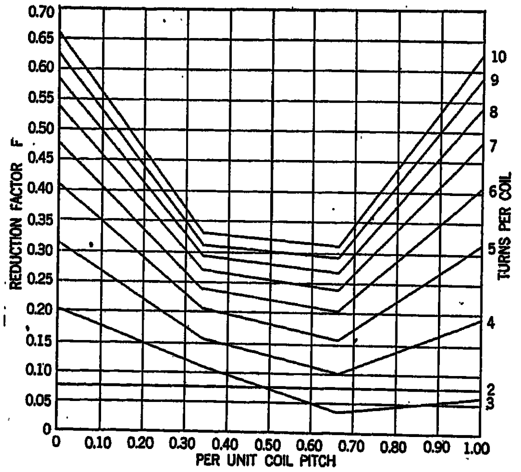

FIG. 12—REDUCTION FACTORS FOR TYPE V TRANSPOSITION SINGLE-PHASE CONNECTION

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[2] *Heat Losses in the Conductors of Alternating-Current Machines*, Waldo V. Lyon, A. I. E. E. TRANS., Vol. 40, p. 1361.

[3] Bibliography given in Reference 1.

meanings of the symbols are to be found in the table at the end of the paper. The ratio b takes account of the fact that the uneven current distribution is due to flux linkages in the core portions of the coil, while the laminations are insulated throughout the whole length of each turn.

The copper loss in any conductor due to circulating currents depends not only on the current in that conductor, but also on the total current below it in the slot. Following the rules derived by Professor W. V. Lyon,[2] in order to find the average loss in a coil it is necessary to find a vector quantity $\mathbf{I}_o$ as follows. Let $\mathbf{I}_b$ be the vector total current in the slot below the given conductor, and let $\mathbf{I}_1$ be the vector current in the conductor. Then, for all conductors having laminations in a direct order (arbitrarily chosen), and for all solid conductors:

$$\mathbf{I}_o = \mathbf{I}_b \qquad (2)$$

For conductors having laminations in the reverse order

$$\mathbf{I}_o = -\mathbf{I}_1 - \mathbf{I}_b \qquad (3)$$

The average value of $\mathbf{I}_o$ over all the conductors in a coil must then be found and used in the equation

$$L = \frac{\mathbf{I}_o \cdot \mathbf{I}_o + \mathbf{I}_o \cdot \mathbf{I}_1}{\mathbf{I}_1 \cdot \mathbf{I}_1} \qquad (4)$$

where the dot product stands for product of the magnitudes times the cosine of the angle between the vectors.

The formula for L has not been presented before in the simple vector form of Equation (4). The vector form lends itself most readily to simplified derivations of the equations for L, and is to be generally preferred to scalar forms previously used.

The general formula for the ratio of a-c. copper loss to uniform current loss is

$$K = M_r + L\,N_r \qquad (5)$$

M_r and N_r are the real parts of complex hyperbolic expressions, and relate to the copper loss due to current in a conductor, and to current below the conductor respectively. Using only the first terms in the series for the hyperbolic quantities, and subtracting the constant uniform current loss from the series for M_r, the following formula is derived for the ratio of extra loss to constant loss

$$k = (4 + 15\,L)\,D \qquad (6)$$

This formula will be used to calculate all extra copper losses in this paper.

III. INVERTED TURN TRANSPOSITIONS IN SINGLE-PHASE WINDINGS

The quantity L, as defined in Equation (4), depends on the number of turns per coil and on the phase angle between the currents in the upper and lower coil sides. In the special single-phase winding with one phase idle there are three possibilities as regards the phase angle:

1. The current in the lower coil side is in phase with that in the upper coil side.

$$\mathbf{I}_2 = m\,\mathbf{I}_1$$

TABLE I
VALUES OF "L" FOR POLYPHASE AND SPECIAL SINGLE-PHASE WINDINGS

Type	Description of winding	Turns per coil	Value of L for polyphase winding	Value of L for single-phase winding		
				$\mathbf{I}_2 = m\,\mathbf{I}_1$	$\mathbf{I}_2 = -m\,\mathbf{I}_1$	$\mathbf{I}_2 = 0$
I	Multi-turn coil. Two coil sides per slot. Involute at each end. (Standard coil)	Either Even or Odd	$\frac{m^2-1}{4}$	$\frac{m^2-1}{4}$	$\frac{m^2-1}{4}$	$-1/4$
II	All turns inverted on end opposite connections	Even	$-1/4$	$-1/4$	$-1/4$	$-1/4$
		Odd	$3/4 - \sin^2\left(\frac{\theta}{2}\right)$	$3/4$	$-1/4$	0
III	All turns inverted on connection end.	Even	0	0	0	0
		Odd	0	0	0	$-1/4$
IV	Top half turn in upper coil side only inverted on connection end	Either Even or Odd	$\left[\frac{(m-1)(m-2)}{m}\right]\left[\left(\frac{m^2-5m+2}{4m}\right)+2\sin^2\left(\frac{\theta}{2}\right)\right]$	$\left[\frac{(m-1)(m-2)}{m}\right]\left[\frac{m^2-5m+2}{4m}\right]$	$\left[\frac{(m-1)(m-2)}{m}\right]\left[\frac{(m+1)(m+2)}{4m}\right]$	$\frac{(3m-2)(m-2)}{4m^2}$
V	Bottom half turn in upper coil side only inverted on end opposite connections	Either Even or Odd	$\left[\frac{m^4-4m^3+m^2+4m+1}{4m^2}+\frac{m-2}{m}\sin^2\left(\frac{\theta}{2}\right)\right]$	$\left(\frac{m^4-4m^3}{4m^2}\right)+\left(\frac{m^2+4m+1}{4m^2}\right)$	$\left(\frac{m^4-4m^3}{4m^2}\right)+\left(\frac{5m^2-4m+1}{4m^2}\right)$	$\frac{1-m^2}{4m^2}$

Note: Top always refers to open end of slot.

2. The current in the lower coil side is 180 deg. out of phase with that in the upper coil side.

$$\mathbf{I}_2 = -m\,\mathbf{I}_1$$

3. The current in the lower coil side is zero.

$$\mathbf{I}_2 = 0$$

Table I gives all the possible values of L in terms of turns per coil for standard coils, and for the four types of inverted turn transposition. Derivations of L for a few typical cases are shown in Appendix A.

Referring to the table, the value of L for standard multi-turn coils is

$$L = \frac{m^2 - 1}{4} \qquad (7)$$

Substituting this value in Equation (6)

$$k = \frac{(15\,m^2 + 1)}{4}\,D \qquad (8)$$

In a coil with any other value of L the ratio of extra circulating current loss to the untransposed loss is, therefore

$$F = \frac{16 + 60\,L}{15\,m^2 + 1} \qquad (9)$$

This formula was used in calculating the reduction factors of Table II and the supplementary curves of Figs. 8-12. The table and curves cover the four types of transposition listed in Table I and shown in Figs. 2-7. They cover all ordinary numbers of turns per coil and all coil pitches.

It is important to note that Equation (9) is used to compare the extra losses in transposed single-phase windings with the extra losses in normal untransposed three-phase double-layer windings having the same coil specifications. In other words the reduction factors are applied to the losses in standard windings of Type I, as calculated by the approximate formula given by Summers,[1] or by the exact equations derived by Professor Lyon.[2] The factors are not applied to the losses in the actual single-phase connection of the normal three-phase winding. The first method leads to simpler computations of extra losses, and at the same time gives the same results when comparing the loss reductions offered by the different transpositions.

Several significant facts can be seen from an inspection of the table and curves. In particular, Type II transpositions are most effective for coils with even numbers of turns. Type III transpositions are to be preferred for coils with odd numbers of turns, especially when the pitch lies between 33⅓ per cent and 66⅔ per cent. Types IV and V are not as effective in general as Types II and III, and are to be used only when space prohibits the use of Types II and III in coils with many turns.

The special case of single-layer windings with standard coils of Type I can easily be treated by referring to Table I. For all coils of this type of winding $\mathbf{I}_2 = 0$, and hence for standard coils without transposition

$$L = -1/4$$

For this value of L the extra losses in a given winding with barrel-type coils are a minimum. No reductions can be made in the losses by any of the transpositions treated in this paper. Single-layer one-turn coils generally require reduction in extra losses, and such coils should be made up of half-turn bars with laminations transposed in the slot portion by well-known methods.

IV. STRAND LOSS IN SPECIAL SINGLE-PHASE WINDING

The approximate formula for extra loss in the strands themselves, arising from the non-uniform distribution of current over each strand cross-section is

$$k_s = 6.0\,b\,\psi\,a\,v\left[\frac{f}{60/\text{sec.}}\right]^2 [r\,n\,(d/\text{in.})^2]^2 \qquad (10)$$

This is the same equation as given in Reference (1). The variation of $\psi\,a\,v$ with coil pitch for the special single-phase winding is shown in Fig. 13.

TABLE II

REDUCTION FACTORS FOR CIRCULATING CURRENT LOSSES OBTAINED WITH INVERTED TURN TRANSPOSITIONS

Three-Phase Windings Connected Single-Phase with One Phase Idle

Turns per coil	Type of Transposition															
	II				III				IV				V			
	Pitch				Pitch				Pitch				Pitch			
	0	1/3	2/3	3/3	0	1/3	2/3	3/3	0	1/3	2/3	3/3	0	1/3	2/3	3/3
2	0.0164	0.0164	0.0164	0.0164	0.262	0.262	0.262	0.262	0.262	0.262	0.262	0.262	0.0778	0.0778	0.0778	0.0778
3	0.00736	0.0625	0.2833	0.449	0.1176	0.0625	0.0625	0.1176	0.607	0.405	0.114	0.0197	0.2035	0.112	0.0380	0.0564
4	0.00415	0.00415	0.00415	0.00415	0.0664	0.0664	0.0664	0.0664	0.768	0.456	0.0820	0.0197	0.319	0.1635	0.1015	0.195
5	0.00266	0.0226	0.1023	0.1620	0.0425	0.0226	0.0226	0.0425	0.845	0.475	0.0929	0.0809	0.411	0.208	0.160	0.315
6	0.00185	0.00185	0.00185	0.00185	0.0295	0.0295	0.0295	0.0295	0.891	0.485	0.1157	0.1525	0.482	0.242	0.205	0.408
7	0.00136	0.0115	0.0523	0.0828	0.0217	0.0115	0.0115	0.0217	0.920	0.491	0.1417	0.222	0.540	0.271	0.242	0.482
8	0.00104	0.00104	0.00104	0.00104	0.0166	0.0166	0.0166	0.0166	0.939	0.494	0.1659	0.283	0.586	0.294	0.270	0.539
9	0.00082	0.00695	0.0316	0.0501	0.01307	0.00695	0.00695	0.01307	0.950	0.495	0.1884	0.337	0.625	0.313	0.2935	0.586
10	0.00067	0.00067	0.00067	0.00067	0.01063	0.01063	0.01063	0.01063	0.960	0.497	0.2080	0.383	0.655	0.328	0.3125	0.624

V. Reduction of Extra Circulating Current Losses by the Twisted Lead Transposition

The circulating currents in any winding can be reduced by increasing the length of path which they must follow. For this reason, a coil with strands insulated from each other throughout the whole length of the coil suffers a lower extra copper loss than a coil with solid conductors, or with laminations solidly joined at the end of each half turn. If coils of the first type are clipped together so that the strands are insulated from each other throughout a phase belt, the extra loss is still further reduced. The reduction can be made comparable to the inverted turn transposition by making a 180 deg. twist in one lead of each coil, as illustrated in Fig. 14. The effect of the twist is to reverse the order of the laminations in corresponding half turns of successive coils, and thus partly to equalize the unbalanced voltages induced in the strands by the leakage fluxes.

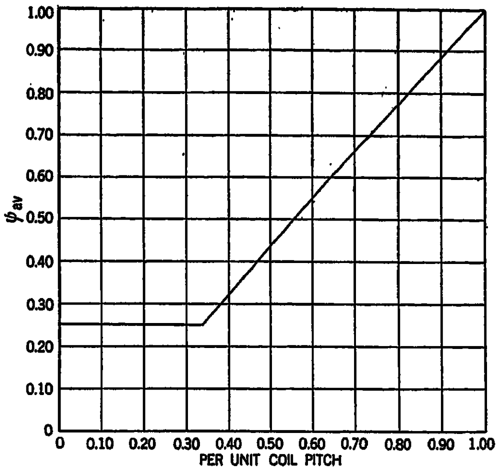

Fig. 13—The Factor ψ_{av} Used in Strand Loss Calclations

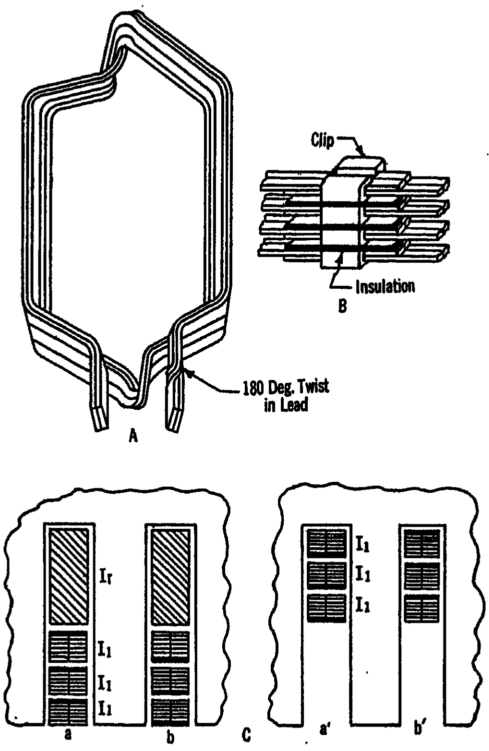

Fig. 14—Twisted Lead Transposition of Barrel Type Coils

A. Standard multi-turn coil with twisted lead

B. Simplified detail of insulated clip connections

C. Conventional slot cross-sections of two consecutive coils $a\ a^1$ and $b\ b^1$ in a phase belt

The twisted lead transposition, described above, was suggested by Mr. M. A. Savage, one of the author's associates. It has been used successfully on large alternators, where space prohibited using the inverted turn transposition. Fig. 15 shows the application of this transposition to a synchronous condenser rated ATI-12-30,000-kv-a., 600-rev. per min., 11,000-volt, which was built in 1923. Approximately 15 machines have been built since then with armature windings transposed by this method. Although this is a relatively small number compared to the number using the inverted turn transposition, it should be noted that in machines where the twisted lead method is applied it is indispensable in reducing extra losses. An added impetus has been given to its use by the development of methods of calculating the ratio of the circulating current loss in such a winding to the loss in the same winding untransposed. The formula which expresses this ratio is as follows:

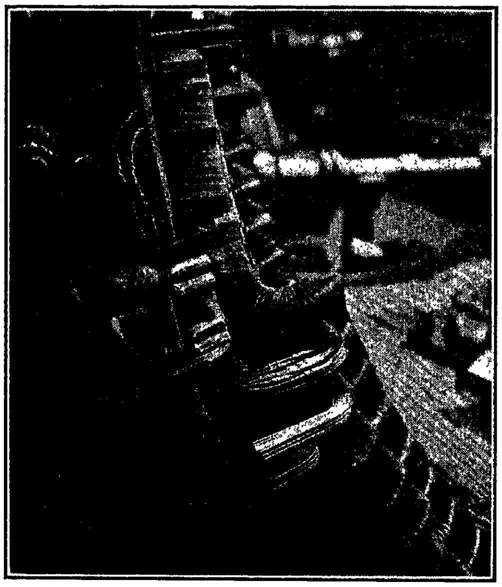

Fig. 15—Stator Coil Connections, Twisted Lead Transposition Showing Insulated Groups of Laminations Clipped Together

$$F = \frac{15\,u\,(m/t)^2 + 1}{15\,m^2 + 1} \tag{11}$$

This is a remarkably simple formula considering the complexity of the problem. The development of this equation is detailed in Appendix C. In the formula, m is the number of turns per coil, t is the number of coils in the phase belt considered, and u is a factor depending on the way the phase belt is split up as regards phase angles between upper and lower coils sides. A complete mathematical definition of u is given in Appendix C after the derivation of Equation (**11**). This factor can have any one of three values, namely, 0, 1, 2.

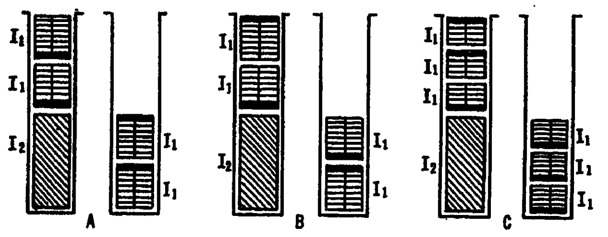

FIG. 16—TYPICAL SLOT CROSS-SECTIONS INVERTED TURN TRANSPOSITIONS

A. Standard coil—Type I
B. Type II transposition—Even number of turns per coil
C. Type V transposition—Even or odd number of turns per coil

Table III gives a summary of the possible groupings of the coils in single-phase, quarter-phase, and three-phase windings, with the corresponding values of u.

The reduction factors to be expected from the twisted lead transposition are shown in Table IV. The table covers only the usual range of turns per coil and coils per phase belt. The reduction factors for other conditions can easily be calculated from Table III and Equation (**11**). The procedure for finding reductions in circulating current losses in the twisted lead transposition reduces, therefore, to checking what part of Table III describes the winding or phase belt, noting the value of u, and going to Table IV or Equation (**11**) for the reduction factor corresponding to the given values of m and t. In the general case of a winding with a fractional number of coils per pole per phase, it is necessary to first lay out the winding in order to see how many coils belong to each phase belt under each pole, and how the belt is split up. A reduction factor can then be found for each phase belt, and all these factors averaged to give the factor for the whole winding.

It is necessary to point out again that Equation (**11**), like Equation (**9**), gives the ratio between the extra losses after transposition in either a polyphase or a single-phase winding and the extra losses in a standard double-layer polyphase winding with the same coils. This fact should be borne in mind when using Table IV in the case of the single-phase connection with one phase idle, and the untransposed losses for this winding should be figured as though all coils were active.

Comparing Table IV with Table II, in the first part of this paper, and Table II in reference (1), it is apparent that in most cases the twisted lead transposition gives lower losses than Types IV and V of the inverted turn transposition. For a large number of coils per phase belt the twisted lead transposition also compares favorably with Types II and III. In general, there is little if any sacrifice involved in using the twisted lead transposition when space or other reasons prohibit the use of the inverted turn transposition.

TABLE III
VALUES OF "u" FOR TWISTED LEAD TRANSPOSITION

Phase belt divided into two parts, each having a different phase angle between currents in top and bottom coil sides.					All coils in phase belt have same phase angle between currents in top and bottom coil sides.		
Both parts have even number of coils. Any number of phases.	Both parts have odd number of coils		One part has even number of coils. Other part has odd number.		Total number of coils per phase belt is odd.		Total number of coils per phase belt is even.
	Three-phase and special single-phase	Two-phase	Any number of phases except single-phase with the odd number of coils having $I_2 = 0$	Single-phase with the odd number of coils having $I_2 = 0$	Any number of phases except single-phase having $I_2 = 0$ in whole belt.	Single-phase with $I_2 = 0$ over whole belt.	
$u = 0$	$u = 1$	$u = 2$	$u = 1$	$u = 0$	$u = 1$	$u = 0$	$u = 0$

TABLE IV
REDUCTION FACTORS TO BE APPLIED TO EXTRA CIRCULATING CURRENT LOSSES WHEN TWISTED LEAD TRANSPOSITION IS EMPLOYED

Turns per coil		$u = 1$												$u = 2$		
		Coils in phase belt												Coils in phase belt		
	$u = 0$	1	2	3	4	5	6	7	8	9	10	11	12	2	6	10
2	0.0164	1.00	0.262	0.126	0.078	0.0557	0.0438	0.0365	0.0318	0.0285	0.0252	0.0245	0.0232	0.508	0.071	0.0361
3	0.00735	1.00	0.256	0.118	0.0694	0.0470	0.0349	0.0276	0.0229	0.0196	0.0173	0.0155	0.0143	0.504	0.0625	0.0272
4	0.00415	1.00	0.253	0.115	0.0664	0.0440	0.0318	0.0245	0.0197	0.0164	0.0141	0.0124	0.0111	0.502	0.0595	0.0241
5	0.00266	1.00	0.252	0.1135	0.0649	0.0426	0.0303	0.0230	0.0182	0.0150	0.0126	0.0109	0.0096	0.5015	0.0582	0.0226
6	0.00185	1.00	0.251	0.113	0.0641	0.0417	0.0296	0.0222	0.0174	0.0142	0.0118	0.0101	0.0088	0.501	0.0572	0.0218
7	0.00136	1.00	0.251	0.112	0.0638	0.0413	0.0291	0.0217	0.0170	0.0137	0.0113	0.0096	0.0083	0.5005	0.0568	0.0213
8	0.00104	1.00	0.251	0.112	0.0635	0.0410	0.0288	0.0214	0.0167	0.0134	0.0110	0.0093	0.0080	0.500	0.0565	0.0210
9	0.00082	1.00	0.251	0.112	0.0633	0.0408	0.0286	0.0212	0.0164	0.0132	0.0109	0.0091	0.0078	0.500	0.0564	0.0208
10	0.00067	1.00	0.251	0.112	0.0631	0.0406	0.0284	0.0210	0.0162	0.0130	0.0107	0.0089	0.0076	0.500	0.0564	0.0206

The objection of greater cost may be raised against this transposition because of the extra labor involved in clipping the coils together so that the laminations are insulated throughout a phase belt. However, the extra cost of connecting deep conductors with many strands can be reduced appreciably by bunching the laminations in groups of 2 and 3. Corresponding groups are then clipped together as illustrated in Fig. 15, which shows three center laminations grouped. Not more than three or four strand levels should be joined in this manner, since they are joined solidly at the ends of each coil, and an extra circulating current loss is thus developed. In Appendix F an approximate formula is worked out for this extra loss. The formula is:

$$k = \left\{ 4 + \frac{15}{4} \left[(m + a)(m + a - 2) - 4\,m\,(a - 1) \sin^2\left(\frac{\theta}{2}\right) \right] \right\} D \qquad (12)$$

and the symbols are defined in the Table of Symbols. The factor D, as defined by Equation (1), varies as the fourth power of the number of strands in a group. Hence, the grouping should be a compromise between cost of connection and extra loss to be tolerated. The strand loss in a twisted lead winding is found from Equation (10) in the same manner as for untransposed windings.

VI. CONCLUSION

The study of the inverted turn transposition, begun in the previous paper referred to in the Bibliography, has been extended to cover the special single-phase case of two phases in series and one leg idle. The twisted lead transposition has been described and analyzed to show its effect on extra losses, with the result that its use can now be definitely recommended in cases where other transpositions are impracticable.

ACKNOWLEDGMENT

The author wishes to acknowledge with appreciation the many important contributions to this paper by Mr. I. H. Summers, especially in the development of the formula for reduction in extra loss by the twisted lead transposition. The author also wishes to express his appreciation of the many constructive suggestions and criticisms offered by Mr. P. L. Alger.

Appendix A

Derivation of Formulas for L.

Type I. Standard Coil. Applying the general rules for finding the average value of $\mathbf{I}_o$ over a coil side, in the upper coil side

$$\mathbf{I}_o = -\frac{1}{m} \sum_{p=1}^{m} \left[(p-1)\,\mathbf{I}_1 + \mathbf{I}_1 + \mathbf{I}_2 \right] \qquad (13)$$

where p represents the number of the turn from the bottom of the coil side. In the lower coil side

$$\mathbf{I}_o = \frac{1}{m} \sum_{p=1}^{m} (p - 1)\,\mathbf{I}_1 \qquad (14)$$

Averaging over both coil sides

$$\mathbf{I}_o = -\frac{1}{2\,m} \sum_{p=1}^{m} (\mathbf{I}_1 + \mathbf{I}_2) = -\left(\frac{\mathbf{I}_1 + \mathbf{I}_2}{2}\right) \qquad (15)$$

Equation (4) can also be written in this form

$$L = \frac{\mathbf{I}_o \,.\, (\mathbf{I}_o + \mathbf{I}_1)}{\mathbf{I}_1 \,.\, \mathbf{I}_1} \qquad (16)$$

Substituting for $\mathbf{I}_o$ from Equation (15), the following result is obtained:

$$L = -\frac{(\mathbf{I}_1 + \mathbf{I}_2) \,.\, (\mathbf{I}_1 - \mathbf{I}_2)}{4\,\mathbf{I}_1 \,.\, \mathbf{I}_1} \qquad (17)$$

which reduces to

$$L = \frac{I_2^2 - I_1^2}{4\,I_1^2} \qquad (18)$$

But $I_2 = m\,I_1$, and therefore

$$L = \frac{m^2 - 1}{4} \qquad (19)$$

If $\mathbf{I}_2 = 0$, as in the special single-phase winding

$$\mathbf{I}_o = -\frac{\mathbf{I}_1}{2} \qquad (20)$$

and

$$L = \frac{I_1^2 - 2\,I_1^2}{4\,I_1^2} \qquad (21)$$

$$L = -\frac{1}{4} \qquad (22)$$

Type II. Even Number of Turns per Coil. (See Table I and Fig. 8.) In the upper coil side

$$\mathbf{I}_o = \frac{1}{m} \left\{ \sum_{\substack{s=1 \\ p = 2s}}^{m/2} \left[(p - 1)\,\mathbf{I}_1 + \mathbf{I}_2 \right] - \sum_{\substack{s=1 \\ p = 2s-1}}^{m/2} \left[\mathbf{I}_1 + (p - 1)\,\mathbf{I}_1 + \mathbf{I}_2 \right] \right\} \qquad (23)$$

where s = any integer except 0.

In the lower coil side

$$\mathbf{I}_o = \frac{1}{m} \left\{ -\sum_{\substack{s=1 \\ p = 2s}}^{m/2} \left[\mathbf{I}_1 + (p - 1)\,\mathbf{I}_1 \right] + \sum_{\substack{s=1 \\ p = 2s-1}}^{m/2} (p - 1)\,\mathbf{I}_1 \right\} \qquad (24)$$

Substituting for p, and averaging over both coil sides

$$\mathbf{I}_o = \frac{1}{2m}\sum_{s=1}^{m/2}[(2s-1)\mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_1 - (2s-2)\mathbf{I}_1$$

$$- \mathbf{I}_2 - \mathbf{I}_1 - (2s-1)\mathbf{I}_1 + (2s-2)\mathbf{I}_1] \quad (25)$$

$$\mathbf{I}_o = \frac{1}{2m}\sum_{s=1}^{m/2} - 2\mathbf{I}_1 \quad (26)$$

$$\mathbf{I}_o = -\frac{\mathbf{I}_1}{2} \quad (27)$$

Substituting in the general equation for L

$$L = \frac{I_1^2 - 2I_1^2}{4I_1^2} \quad (28)$$

$$L = -\frac{1}{4} \quad (29)$$

This value of L is independent of $\mathbf{I}_2$.

Type V. Even or Odd Number of Turns. In the upper coil side

$$\mathbf{I}_o = \frac{1}{m}\left[-(\mathbf{I}_1 + \mathbf{I}_2) + \sum_{p=2}^{m}\{(p-1)\mathbf{I}_1 + \mathbf{I}_2\}\right] \quad (30)$$

In the lower coil side

$$\mathbf{I}_o = -\frac{1}{m}\sum_{p=1}^{m}[\mathbf{I}_1 + (p-1)\mathbf{I}_1] \quad (31)$$

Averaging over both coil sides

$$\mathbf{I}_o = \frac{1}{2m}\left\{\sum_{p=2}^{m}[(p-1)\mathbf{I}_1] + (m-1)\mathbf{I}_2 - m\mathbf{I}_1\right.$$

$$\left. - \sum_{p=1}^{m}[(p-1)\mathbf{I}_1] - \mathbf{I}_1 - \mathbf{I}_2\right\} \quad (32)$$

$$\therefore \mathbf{I}_o = \frac{(m-2)\mathbf{I}_2 - (m+1)\mathbf{I}_1}{2m} \quad (33)$$

$$\mathbf{I}_o + \mathbf{I}_1 = \frac{(m-2)\mathbf{I}_2 + (m-1)\mathbf{I}_1}{2m} \quad (34)$$

Substituting in the equation for L

$$L = \frac{(m-2)^2 I_2^2 - (m^2-1)I_1^2 - 2(m-2)\mathbf{I}_1 \cdot \mathbf{I}_2}{4m^2 I_1^2} \quad (35)$$

which finally reduces to the form

$$L = \frac{m^4 - 4m^3 + m^2 + 4m + 1}{4m^2} + \frac{(m-2)}{m}\sin^2(\theta/2) \quad (36)$$

If $\mathbf{I}_2 = m\,\mathbf{I}_1$ $\quad (\theta = 0)$

$$L = \frac{m^4 - 4m^3 + m^2 + 4m + 1}{4m^2} \quad (37)$$

If $\mathbf{I}_2 = -m\,\mathbf{I}_1$ $\quad (\theta = 180\text{ deg.})$

$$L = \frac{m^4 - 4m^3 + m^2 + 4m + 1 + 4m^2 - 8m}{4m^2} \quad (38)$$

$$= \frac{m^4 - 4m^3 + 5m^2 - 4m + 1}{4m^2} \quad (39)$$

If $\quad \mathbf{I}_2 = 0\ , \quad \mathbf{I}_o = -\frac{(1+m)}{2m}\mathbf{I}_1 \quad (40)$

$$\mathbf{I}_o + \mathbf{I}_1 = -\frac{(1-m)}{2m}\mathbf{I}_1 \quad (41)$$

$$L = \frac{1-m^2}{4m^2} \quad (42)$$

Appendix B

SAMPLE CALCULATIONS OF REDUCTION FACTORS FOR INVERTED TURN TRANSPOSITION OF SINGLE-PHASE WINDINGS

A. Assume Type II transposition, three-turn coils, two-thirds pitch, one pole being shown in Fig. 17.

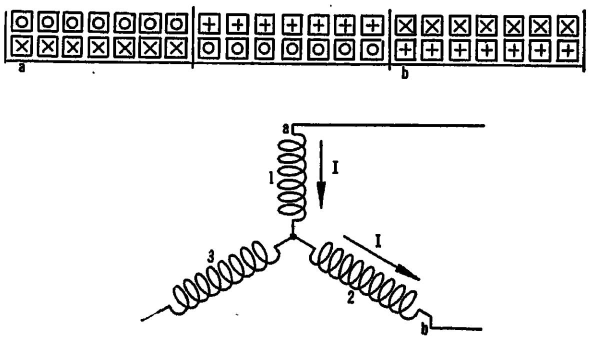

FIG. 17—CROSS-SECTION, DOUBLE-LAYER SINGLE-PHASE WINDING

×—Coils of phase 1
+—Coils of phase 2
○—Coils of phase 3 (idle phase)

Considering the two active phase belts, the following tabulation can be constructed by the use of Table II:

Number of coils	$\mathbf{I}_2$	L	m	F
7	$m\,\mathbf{I}_1$	3/4	3	0.449
7	0	0	3	0.1176

$\therefore F_{av} = 0.2833$

Since there are an integral number of slots per pole per phase this average ratio applies to the whole winding.

B. Assume Type II transposition, three-turn coils, 42 slots, 6 poles, 7/3 slots per pole per phase, four-sevenths pitch. See Fig. 18 where three poles are shown.

Considering the two active phases and the three poles, the following tabulation can be made by means of Equation (9):

Number of coils	$\mathbf{I}_2$	L	m	F
5	$m\,\mathbf{I}_1$	3/4	3	0.449
2	$-m\,\mathbf{I}_1$	$-1/4$	3	0.00736
7	0	0	3	0.1176

$$\mathrm{F}_{av} = 5/14\ (0.449) + 2/14\ (0.00736) + 1/2\ (0.1176) = 0.220$$

Since the winding repeats itself under the other three poles this ratio holds for the complete winding. The value of F_{av} will be found to check the curve Fig. 8 for Type II, three turns per coil, 57.2 per cent pitch. These illustrations show that the factors in Table II hold for any type of winding.

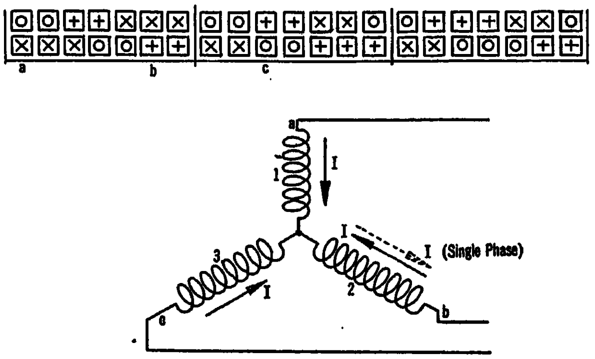

FIG. 18—CROSS-SECTION, DOUBLE-LAYER WINDING

Fractional number slots per pole per phase

×—Coils of phase 1
+—Coils of phase 2
⊙—Coils of phase 3 (idle for single phase)

Appendix C

DERIVATION OF FORMULA FOR REDUCTIONS IN EDDY CURRENT LOSSES IN TWISTED LEAD WINDING

Referring to Fig. 14 in the body of the paper, each strand in a twisted lead winding is theoretically insulated throughout the length of the m turns per coil and also throughout the t coils in each phase belt. The order of the strands in corresponding conductors reverses from coil to coil, so that it is necessary to determine $\mathbf{I}_o$ separately for the even numbered coils and for the odd numbered coils. $\mathbf{I}_o$ is determined for each half turn in the usual way as described in Part II of the paper, and these values are then averaged over all the coils in the phase belt.

The determination of $\mathbf{I}_o$ in a phase belt where t is even is as follows. Considering the odd numbered coils, which are assumed to have laminations in the direct order in the bottom coil side:

$$\mathbf{I}_e = \frac{1}{m\,t} \sum_{\substack{s=1 \\ r=2s-1}}^{t/2} \left\{ \sum_{p=1}^{m} (p-1)\,\mathbf{I}_1 - \sum_{p=1}^{m} [(p-1)\,\mathbf{I}_1 + \mathbf{I}_r + \mathbf{I}_1] \right\} \qquad (43)$$

Considering the even numbered coils, which will have the laminations in the reverse order:

$$\mathbf{I}_o = \frac{1}{m\,t} \sum_{\substack{s=1 \\ r=2s}}^{t/2} \left\{ \sum_{p=1}^{m} [(p-1)\,\mathbf{I}_1 + \mathbf{I}_r] - \sum_{p=1}^{m} [(p-1)\,\mathbf{I}_1 + \mathbf{I}_1] \right\} \qquad (44)$$

In the above equation $\mathbf{I}_r$ is the vector total current in the lower coil side in the rth slot of the phase belt being considered. Let

$$\mathbf{I}_r = m\,I_1\,\mathbf{v}_r \qquad (45)$$

where $\mathbf{v}_r$ is a unit vector displaced from $\mathbf{I}_1$ by the phase angle between the currents in the upper and lower coil sides in the rth slot. Substituting this relation in Equations (43) and (44), and averaging $\mathbf{I}_o$ over all coil sides in the phase belt:

$$\mathbf{I}_o = I_1 \frac{m}{2\,t} \left[\sum_{\substack{s=1 \\ r=2s}}^{t/2} \mathbf{v}_r - \sum_{\substack{s=1 \\ r=2s-1}}^{t/2} \mathbf{v}_r \right] - \frac{1}{2}\,\mathbf{I}_1 \qquad (46)$$

In the same way it can be shown that when t is odd

$$\mathbf{I}_o = I_1 \frac{m}{2\,t} \left[\sum_{\substack{s=1 \\ r=2s}}^{(t-1)/2} \mathbf{v}_r - \sum_{\substack{s=1 \\ r=2s-1}}^{(t+1)/2} \mathbf{v}_r \right] - \frac{1}{2}\,\mathbf{I}_1 \qquad (47)$$

Let

$$\sum_{r=2s} \mathbf{v}_r = \mathbf{v}_e \qquad (48)$$

$$\sum_{r=2s-1} \mathbf{v}_r = \mathbf{v}_o \qquad (49)$$

In words, $\mathbf{v}_e$ is the vector summation of the unit vectors $\mathbf{v}_r$ for all even numbered coils, and $\mathbf{v}_o$ is the vector summation of the unit vectors $\mathbf{v}_r$ for all the odd numbered coils.

Then Equations (46) and (47) can be reduced to one general form as follows:

$$\mathbf{I}_o = \frac{m}{2\,t}\,I_1\,(\mathbf{v}_e - \mathbf{v}_o) - \frac{1}{2}\,\mathbf{I}_1 \qquad (50)$$

For simplicity let

$$\mathbf{v}_e - \mathbf{v}_o = \mathbf{v} \qquad (51)$$

Referring to Equation (16)

$$L = \frac{\mathbf{I}_o \cdot (\mathbf{I}_o + \mathbf{I}_1)}{\mathbf{I}_1 \cdot \mathbf{I}_1}$$

Substituting the value of $\mathbf{I}_o$ from (50) and (51)

$$L = \frac{[\,(m/2\,t)\,I_1\,\mathbf{v} - 1/2\,\mathbf{I}_1\,]\,.\,[\,(m/2\,t)\,I_1\,\mathbf{v} + 1/2\,\mathbf{I}_1\,]}{I_1^2} \qquad (52)$$

$$= \frac{(m/t)^2\,\mathbf{v}\,.\,\mathbf{v} - 1}{4} \qquad (53)$$

Let $\mathbf{v}\,.\,\mathbf{v} = u$ (a scalar quantity) (54)

Then $$L = \frac{u\,(m/t)^2 - 1}{4} \qquad (55)$$

But the first order of eddy currents is proportional to

$$4 + 15\,L = \frac{15\,u\,(m/t)^2 + 1}{4} \qquad (56)$$

For the standard untransposed winding

$$4 + 15\,L = \frac{15\,m^2 + 1}{4} \qquad (57)$$

The reduction factor due to the twisted lead transposition is

$$F = \frac{15\,u\,(m/t)^2 + 1}{15\,m^2 + 1} \qquad (58)$$

It is now necessary to define the factor u which enters into all calculations of reductions in eddy current losses in twisted lead windings. A mathematical definition is all that can be given, as the factor has no apparent physical meaning. In brief, u is the square of the magnitude of the vector difference between two vectors, one representing the sum of as many unit vectors as there are even numbered coils in a phase belt, and the other representing the sum of as many unit vectors as there are odd numbered coils in the belt. The angular displacement of each unit vector from the origin is equal to the phase angle between currents in the upper and lower coil sides in the slot to which it applies.

Appendix D

EXAMPLE OF CALCULATION OF REDUCTION FACTOR FOR TWISTED LEAD TRANSPOSITION IN THREE-PHASE WINDING

Assume the winding of Fig. 18 to be operating as a normal three-phase winding with the twisted lead transposition in the coils. Considering the coils of Phase 1 in each of the phase belts shown, and noting the directions of current flow in the three phases, the following tabulation can readily be made:

Phase belt	$\mathbf{v}_e$	$\mathbf{v}_o$	$\mathbf{v}$	u
1	$1\underline{/60°}$	$1\underline{/60°} + 1\underline{/120°}$	$-\,1\underline{/120°}$	1
2	$1\underline{/60°}$	$1\underline{/60°}$	0	0
3	$1\underline{/60°}$	$1\underline{/120°}$	$1\underline{/0°}$	1

The two phase belts in which $u = 1$ will have the following reduction factors:

(a) Phase belt No. 1 (3 coils)

$$F = \frac{15\,(3/3)^2 + 1}{15\,(3)^2 + 1} = 0.1176$$

(b) Phase belt No. 3 (2 coils)

$$F = \frac{15\,(3/2)^2 + 1}{15\,(3)^2 + 1} = 0.255$$

In the phase belt in which $u = 0$

$$F = \frac{1}{15\,(3)^2 + 1} = 0.00736$$

The average reduction factor for the whole winding is

$$F_{av} = \frac{1}{3}\,(.1176 + .255 + .00736)$$

$$= 0.127$$

Appendix E

EXAMPLES OF CALCULATION OF REDUCTION FACTORS IN SINGLE-PHASE WINDING WITH TWISTED LEAD TRANSPOSITION

For those slots having zero current in the lower coil sides, the vector current $\mathbf{I}_r$ in Equations (43) and (44), Appendix C, is zero, and hence for the active coils so located, the equations reduce to the following.

$$\mathbf{I}_o = -\,\frac{1}{2}\,\mathbf{I}_1$$

$$L = -\,\frac{1}{4}$$

Comparing this with Equation (55) it is evident that for any group of coil sides having $\mathbf{I}_r = 0$

$$u = 0$$

$$\mathbf{v} = \mathbf{v}_e - \mathbf{v}_o = 0$$

This result simplifies the calculations of loss reduction factors in the various phase belts of single-phase windings having the twisted lead transposition. In those phase belts having some of the lower coil sides carrying no current it is only necessary to consider the slots with active lower coil sides in forming the vector difference $\mathbf{v}$, and determining the factor u for the phase belt. If all the lower coil sides in the phase belt carry zero current, the factor u is zero for that group.

As an example, consider the single-phase winding of Fig. 17 with the twisted lead transposition, each coil having three turns. In the phase group lying over the coils of the idle phase

$$u = 0$$

In the other active phase belt all slots have the same phase angle between currents in upper and lower coil sides. Since the phase group has an odd number of coils, there is one more of the odd numbered coils than of the even numbered coils, and hence

$$\mathbf{v}_e - \mathbf{v}_o = -\,1$$

$$u = 1$$

With the aid of Table IV the following tabulation can be made:

Phase	u	m	t	F
2	0	3	7	0.00735
1	1	3	7	0.0276

$$F_{av} = 0.0175$$

Since the winding repeats under the other pole this factor holds for the whole armature.

As a second example, consider the single-phase connection of the winding shown in Fig. 18, and assume three turn coils. Considering only the phase belts of phase 1, and noting the directions of current flow, the following tabulation can be made with the aid of Table IV:

Phase belt	v_e	v_o	v	u	m	t	F
1	1	1	0	0	3	3	0.00735
2	1	1	0	0	3	2	0.0164
3	1	0	1	1	3	2	0.256

The average reduction factor for phase 1 is

$$F_{av} = 0.0784$$

Considering next the phase belts of phase 2, another tabulation can be made as follows:

Phase belt	v_e	v_o	v	u	m	t	F
1	0	1	−1	1	3	2	0.256
2	..	..	..	0	3	2	0.00735
3	0	1	−1	1	3	3	0.118

The average reduction factor for phase 2 is

$$F_{av} = 0.1258$$

The resultant factor for determining the extra loss in the whole winding is

$$F = 0.1021$$

It is interesting to note the possibility that the effective resistance and extra loss in one phase of a single-phase winding with the twisted lead transposition can be greater than in the other phase.

The preceding calculation is typical of that to be followed in the general case of fractional slots per pole per phase. The values of u in Table III were found in this way.

Appendix F

EXTRA CIRCULATING CURRENT LOSS IN GROUP OF LAMINATIONS JOINED AT COIL ENDS IN TWISTED LEAD WINDING

Assume uniform current density over each conductor. This assumption is justifiable where the twisted lead transposition is used, since it approximately produces this effect. Also, let

n = number of laminations vertically per conductor.

n_c = number of laminations vertically in group.

n_b = number of laminations vertically from bottom of conductor to bottom of group in one coil side.

Then, because of inversion of the conductors

$n - (n_b + n_c)$ = number of laminations vertically from bottom of conductor to bottom of group in other coil side.

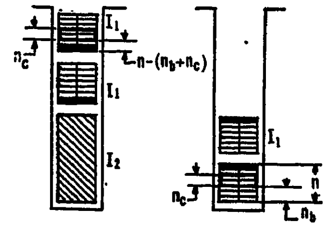

FIG. 19—SLOT CROSS-SECTIONS SHOWING LAMINATION GROUPING—TWISTED LEAD TRANSPOSITION

Referring to Fig. 19, in the upper coil side

$$\mathbf{I}_o = -\frac{1}{m}\sum_{p=1}^{m}\Big[\Big(\frac{n_c}{n}\Big)\mathbf{I}_1 + \Big(\frac{n - n_b - n_c}{n}\Big)\mathbf{I}_1 + (p-1)\,\mathbf{I}_1 + \mathbf{I}_2\Big] \tag{59}$$

In the lower coil side

$$\mathbf{I}_o \frac{1}{m}\sum_{p=1}^{m}\Big[\Big(\frac{n_b}{n}\Big)\mathbf{I}_1 + (p-1)\,\mathbf{I}_1\Big] \tag{60}$$

Averaging over the whole coil

$$\mathbf{I}_o = \frac{1}{2\,m}\sum_{p=1}^{m}\Big[\Big(\frac{2\,n_b}{n}\Big)\mathbf{I}_1 - \mathbf{I}_1 - \mathbf{I}_2\Big] \tag{61}$$

$$= -\frac{\mathbf{I}_1}{2}\Big(\frac{n - 2\,n_b}{n}\Big) - \frac{\mathbf{I}_2}{2} \tag{62}$$

Let

$$\Big(\frac{n - 2\,n_b}{n}\Big) = a \tag{63}$$

Then

$$\mathbf{I}_o = -a\,\frac{\mathbf{I}_1}{2} - \frac{\mathbf{I}_2}{2} \tag{64}$$

$$\mathbf{I}_o + \mathbf{I}_1 = -\frac{(a-2)}{2}\,\mathbf{I}_1 - \frac{\mathbf{I}_2}{2} \tag{65}$$

$$L = \frac{a\,(a-2)\,I_1^2 + m^2\,I_1^2 + 2\,(a-1)\,\mathbf{I}_1\,.\,\mathbf{I}_2}{4\,I_1^2} \tag{66}$$

which can be reduced to the form

$$L = \frac{(m+a)\,(m+a-2) - 4\,m\,(a-1)\sin^2(\theta/2)}{4} \tag{67}$$

The value of L so found can then be put into the usual formula

$$k = (4 + 15\,L)\,D \tag{6}$$

in order to find the extra loss produced in the group being considered. The value of D to be used for the lamination group is

$$D = 1.19 \left[\frac{f}{60/\text{sec.}} \right]^2 [b\, r\, c\, (n_c\ d/\text{in.})^2\,]^2 \quad (68)$$

If $n_b = 0$, or if all laminations are solidly joined at each coil end as in a standard coil,

$$a = 1$$

and

$$L = \frac{m^2 - 1}{4} \text{ as before.}$$

Table of Symbols

D	A numeric depending on the slot and conductor dimensions and assembly, and on the frequency.
F	Reduction factor representing the ratio of the extra circulating current loss in any transposed coil, group of coils, or winding to the loss in the same coil, group of coils, or winding untransposed.
$\mathbf{I}_1$	The vector current in any conductor.
$\mathbf{I}_2$	The total vector current in the bottom coil side.
$\mathbf{I}_b$	The resultant vector current in the slot below any conductor.
$\mathbf{I}_o$	A vector current defined by Equations (2) and (3).
$\mathbf{I}_r$	The vector total current in the lower coil side in the rth slot of a phase belt.
K	Ratio of total heat loss in a winding with alternating current to loss with uniform current.
L	A numeric defined by Equation (4).
a	The ratio $(n - 2\, n_b)/n$
b	A ratio $= \left(\frac{2 \times \text{core length}}{\text{mean length of turn}} \right)$
c	$\left[\frac{\text{gross depth of strand}}{\text{net copper depth of strand}} \right]^2$
d	Net depth of copper in a strand.
f	Frequency of alternating current.
k	Ratio of extra circulating current loss to uniform current loss.
k_s	Ratio of extra loss due to uneven current distribution in strands to uniform current loss.
m	Turns per coil.
n	Number of strands depthwise per conductor.
n_b	Number of strands depthwise below a group of laminations in a twisted lead transposition.
n_c	Number of strands depthwise in the group of laminations.
p	The order of a conductor, numbered from the bottom of a coil side. For the first conductor $p = 1$.
r	The ratio of total copper width to slot width.
r	(As a subscript) The order of a coil in a phase belt in a twisted lead winding, the numbering beginning at the start of the phase belt with $r = 1$.
s	Any integer, except 0.
t	Number of coils in a phase belt throughout which the conductor laminations are insulated from each other depthwise.
u	A scalar quantity used in twisted lead transposition formulas. It depends on the way the t coils in a phase belt are split up as regards phase angles between upper and lower coil sides.
$\mathbf{v}$	The vector difference $\mathbf{v}_e - \mathbf{v}_o$.
$\mathbf{v}_e$	The vector sum of as many unit vectors $\mathbf{v}_r$ as there are even numbered coils in a phase belt in a twisted lead winding.
$\mathbf{v}_o$	The vector sum of all the unit vectors $\mathbf{v}_r$ in the odd numbered coils in the phase belt.
$\mathbf{v}_r$	A unit vector in phase with $\mathbf{I}_r$.
θ	Phase angle between currents in upper and lower sides in a slot.
ψ_{av}	The average of the quantity $[1 - 3/4 \sin^2 (\theta/2)]$ over all coils in a winding.

Discussion

I. H. Summers: I should like to point out that the formulas, tables, and equations which Mr. Lyons has presented are not merely of theoretical interest. In Schenectady for the past four or five years in practically all the large water wheel generators built the inverted turn transposition has been used with the exception of those having a large number of turns which do not need it and those using the twisted lead transposition, which is also covered in Mr. Lyons' paper. These latter have been built over a period of approximately the last two years, and there have been about 15 machines built with this transposition. As a result of this practise no large water wheel alternators are now built in Schenectady having a parasitic eddy current loss of greater than about 15 per cent of the legitimate copper loss.

J. M. Lyons: I wish to thank Mr. Summers for pointing out the extent to which the transpositions described in this paper have been applied on large water wheel generators. The inverted turn and twisted lead transpositions naturally find their greatest use in such machines having multi-turn coils with fairly deep conductors. The reduction of extra copper loss to 15 per cent of the ohmic loss obviously represents a very worth while saving.

The problem of predicting loss reductions due to the twisted lead transposition having been simplified by the development of Equation (11) and the corresponding tables, it would seem that this transposition should find a relatively larger number of applications than in the past.

The scheme of transposition described by Mr. Palueff as being used on heavy transformer coils is apparently very effective in securing a uniform current density and low extra copper loss. This method is essentially different from those described in this paper in that the strands are transposed relative to each other at regular intervals around each turn instead of being twisted all together at one point. Such a transposition would be very difficult to apply on multi-turn coils of generator windings, while for large one-turn generator coils there is available a more effective method. I am referring to the Roebel Transposition for which each coil is made up of two half-turn bars. The laminations are offset and bent in the slot portion so that in the assembled bar every strand passes twice through every level in the bar. In this way the current density over the bar cross-section becomes practically uniform and the extra loss is reduced to a negligible value.

There is seldom any need of transposing generator armature coils in machines smaller than 5000 kw. because of the relatively shallow conductors characteristic of these machines. For all larger ratings the Roebel or one of the twisted conductor transpositions will meet the call for reduction of extra copper losses in any winding.

Effect of Armature Resistance Upon Hunting
of Synchronous Machines

BY C. F. WAGNER*
Member, A. I. E. E.

Synopsis.—The ability of systems to withstand shocks, such as faults or switching operations, is much greater for small operating angles. The present paper has to do with system stability for small oscillations as contrasted with the more usual concept of stability as being its ability to withstand shocks.

The analysis shows that synchronous machines without amortisseur windings and with no resistance in the armature are inherently stable for small oscillations in prime mover input or load output. With resistance in the armature, stability for such conditions is dependent upon load.

A new proof is given for the relation previously derived by Nickle and Pierce which states that the limiting angle at which a machine becomes stable is $\tan^{-1}\dfrac{r}{x_q}$.

Arguments are presented which show that any standard type of damper which may have been installed in generators for other incidental reasons, possesses the property of preventing spontaneous hunting except for the most abnormal conditions met in practise.

* * * * *

FROM time to time cases of spontaneous hunting arise, the cause of which can be traced to an excessive proportion of resistance in the armature circuit, namely in the transmission line. This phenomenon was particularly evident in the early days of rotary converters. A rough working rule in use at that time was to limit the line resistance to 25 per cent of the reactance. In 1911 Dreyfus[1] in Germany showed that the tendency toward hunting decreased with increasing load and that the presence of damper windings tended to stabilize the conditions. In 1924, reporting stability tests made in the works of the Westinghouse Company, Evans and Bergvall[2] likewise demonstrated this same effect using salient pole generators and condensers. More recently Wennerberg[3] in Sweden and Nickle and Pierce[4] in this country, have extended the analysis, developing the results into more usable form. The writer has been working on this same problem and has used a somewhat different method of attack which he believes gives a somewhat clearer conception of the mechanism involved.

I. QUALITATIVE ANALYSIS

To get a picture of what occurs when a machine becomes unstable, and begins to hunt because of the presence of an excessive amount of resistance in its armature circuit, consider a synchronous generator without damper windings and having constant exciter voltage, and connected to an infinite bus through a transmission line. The analysis of transients of synchronous machines may be expedited by the use of the so-called "transient reactance" (x_d'), and the "transient internal voltage" (e_d'). The transient reactance is the reactance to be used in connection with the determination of current flow for sudden changes. It includes not only the armature leakage reactance but also the effects due to the field leakage. The subscript d signifies that the reactions are taken with reference to the direct axis. The voltage e_d' for zero power factor will be defined as the arithmetic sum of the terminal voltage and the transient reactance drop, and for any other power factor, as the arithmetic sum of the component of the terminal voltage in the direct axis and the transient reactance drop due to the component of armature current in the direct axis. This is the fictitious voltage corresponding to flux linkages in the field and for any sudden change in circuit condition the flux linkages and this voltage remain fixed. The voltage e_d will be defined in a similar manner except that the synchronous reactance x_d must be used. It follows, therefore, that e_d and e_d' for steady state conditions differ only by the magnitude of the two drops and that

$$e_d - e_d' = x_d i_d - x_d' i_d$$

or

$$e_d' = e_d - (x_d - x_d') i_d \tag{1}$$

in which i_d is the component of armature current in the direct axis, that is, it is the demagnetizing component of current for i_d positive. The quantity $(x_d - x_d')$ takes something of the nature of a reactance corresponding to armature reaction. It is clear then from the above equation and also from the physical conception of the quantities, that for constant exciter voltage, which is equivalent to constant e_d, that the voltage e_d' which is proportional to flux linkages in the field winding should decrease as the demagnetizing component of current, i_d, increases.

In Fig. 1a is shown the familiar power circle diagram showing the output characteristics of the machine for constant internal voltage assuming no resistance in the armature. This diagram is not strictly correct as it does not introduce the distortion due to the saliency of the poles. It is, however, sufficiently correct for this preliminary analysis. For any operating point such as p the abscissa gives the power output of the machine and the ordinate, the demagnetizing kv-a. The angle θ is the angle between the generated voltage and the voltage of the infinite bus at the receiving end. The particular circle drawn assumed constant voltage e_d so that the circle represents the steady state characteristics.

Now assuming that the generator is operating at a

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*Transmission Engineer, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

1. For reference see Bibliography.

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mean angle θ and that in some manner, the exact nature of which is, at present, of no importance, the rotor is made to oscillate sinusoidally about θ with an amplitude $\Delta \theta_m$ and a frequency f. The amplitude of these oscillations is indicated in Fig. 1A, from which it will be observed that for $\Delta \theta$ positive the demagnetizing kv-a. or the demagnetizing current i_d increases. But from Equation (1) it was shown that if i_d increases e_d' decreases. It follows, therefore, that for positive increments in θ that the increment of e_d' is negative and for negative increments of θ the increment of e_d' is positive. This result is shown graphically by the straight line in Fig. 1b. However, this relation merely shows how $\Delta e_d'$ would vary if θ were increased an amount $\Delta \theta$ and kept there or the manner in which $\Delta e_d'$ would vary for very slow oscillations. Due to the inductance associated with the field circuit e_d' cannot change instantly so that the variations of $\Delta e_d'$ instead of oscillating along the straight line will form an elliptical loop (Lissajous figure) of the form shown. The direction of rotation of the generating point around this loop is extremely important. It will be observed that if $\Delta \theta$ be increasing positively from its value $-\Delta \theta_m$, the operating point a in generating the loop must lag, that is, it must pass through zero at a later time than when $\Delta \theta$ passes through zero. This determines the direction of rotation as being clockwise. With very low frequency the loop degenerates into the straight line $b\,c$ and for extremely high frequencies degenerates into a straight line along the horizontal axis.

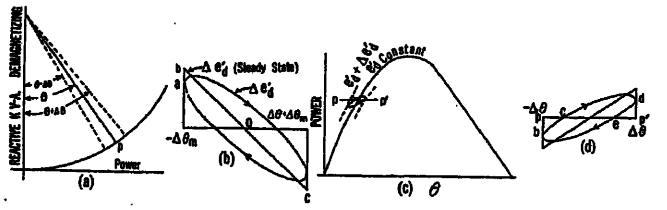

FIG. 1—PURE REACTANCE LINE

a. Demagnetizing current increases with $+\Delta \theta$, so that $\Delta e_d'$ tends to decrease
b. Log of instantaneous value of $\Delta e_d'$ behind value which it would attain for steady state conditions
c. Power loop obtained from instantaneous values of $\Delta e_d'$. System stable for clockwise loops
d. Enlargement of power loop

The power-angle diagram for the machine including the effect of saliency is shown in Fig. 1c. This curve was drawn for a constant value of e_d'. For small positive increments in e_d' the resultant curve will, in general, for almost all cases met in practise, lie above the one for e_d' and for negative increments of e_d' will lie below the curve for e_d'. The loop of Fig. 1B gives the instantaneous values of $\Delta e_d'$ as a function of the angle so that using these instantaneous values it can be seen that a similar power loop is obtained on the power angle diagram and following the point-by-point substitution it will be observed that the rotation of this movement is also clockwise. Assuming constant input into the prime mover of the value corresponding to the generator power output for the mean value of θ, the instantaneous acceleration is determined by the distance between the curves of instantaneous power and the line $p\,p'$. Starting at the point b of maximum negative travel (see Fig. 1d) where the relative velocity between the rotor of the machine and the voltage of the infinite bus is zero, the velocity of oscillation under the assumed conditions of constant input increases until the point c is reached, at which point the acceleration is zero. At this point the rotor has accumulated an amount of stored kinetic energy proportional to the area of the triangle $b\,p\,c$, so that the rotor overshoots this point. The relative velocity is again brought to zero at the point d, but it will be observed that the area representing the energy available to retard this velocity—area $d\,p'\,c$—is greater than the area $b\,p\,c$. The significance of this observation is that, left free to oscillate from point b with constant prime mover input, the generating point will not move all the way to p', but will be damped; or if the generating point does move actually along the locus $b\,c\,d$, then the assumption of constant prime mover is incompatible with the results, the prime mover input represented by the line $p\,p'$ must be altered so that the two areas are equal. In moving from b to d, the excess energy tending to produce damping is proportional to the difference of the areas $d\,p'\,c$ and $b\,p\,c$; but since area $b\,p\,c$ is equal to area $d\,p'\,e$, this difference is equal to the area $c\,d\,e$. Similarly in moving from d to b, the excess energy tending to produce damping is proportional to the area $e\,b\,c$, or during one complete cycle, to the area of the loop.

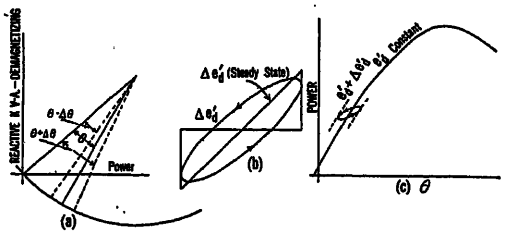

FIG. 2—LINE WITH RESISTANCE

a. Demagnetizing current decreases with $+\Delta \theta$, so that $\Delta e_d'$ tends to increase
b. Log of instantaneous value of $\Delta e_d'$ behind value which it would attain for steady state conditions
c. Power loop obtained from instantaneous value of $\Delta e_d'$. System unstable for counter-clockwise loop

The system may be said to be stable. It will be observed that this condition holds for all values of θ when the armature resistance is zero.

An entirely different condition may prevail when the armature and line resistance is other than zero. Consider the conditions when such a system is operating at the angle θ shown in Fig. 2a on the power-reactive power circle diagram. Due to the presence of the resistance, the center of the circle is shifted from the axis of reactive kv-a. to the position shown. For positive increments in θ, the demagnetizing current decreases, thus increasing the value $\Delta e_d'$ attains for steady state increments of $\Delta \theta$. This relation is shown in Fig. 2b. During system oscillations $\Delta e_d'$ cannot attain the value corresponding to the steady state value, but lags forming the loop shown, which is generated in a counter-clockwise sense. The voltage loop can be converted to the power loop in Fig. 2c, which is also generated in a counter-clockwise sense. Pursuing the same argument as used previously, the area of this loop is proportional to the energy given out,—the system is unstable.

This condition follows from the fact that for small angles the demagnetizing current decreases with increasing angle, but it will be noted that for larger angles this same relation no longer obtains. As the load increases a point is finally reached beyond which the system becomes stable. The qualitative treatment thus shows the mechanism of the phenomena involved and why a machine without damper windings becomes more stable, from the standpoint of hunting, with increasing load.

This phenomenon should not be confused with energy absorbed thermally by the series resistance, which is not effective in damping oscillations between machines. The truth of this statement may be verified by assuming that the oscillations are so fast or that the time constant of the field is so large as to justify the assumption that e_d' is constant. Any effect of the series resistance should still be present. In this case the operating point in the power-angle diagram generates the same locus with increasing angles as with decreasing angles. The area of the loop and the decrement of the oscillations must therefore be zero. For constant prime mover input and constant load output, the energy absorbed by the series resistance is supplied by the stored kinetic energy of rotation, resulting in a decrease of mean system frequency.

II. SALIENT POLE MACHINE WITHOUT AMORTISSEUR WINDING

In the mathematical treatment which follows, the case of a salient pole machine without amortisseur windings will be discussed. A number of simplifying assumptions has been made which agree with those used by Nickle and Pierce.[4]

It has been shown by a number of investigators that transients in synchronous machines involving symmetrical armature conditions can be resolved into a unidirectional component in the field winding (and associated with it a transient of synchronous velocity in the armature) and a unidirectional component in the armature (and associated with it a transient of synchronous velocity in the field winding). These components have very little effect upon each other so that small error is introduced into the calculations effecting the unidirectional component of the field by neglecting entirely the unidirectional component of the armature. For the case under consideration the machine is assumed to be connected to an infinite bus, that is, to a source of e. m. f., E, which will not change in phase position or magnitude by any alteration in the machine or the circuit. Small angular oscillations of the rotor of frequency f are superposed upon the operating angle. These oscillations produce a superposed oscillation of field current of frequency f and a modulation of the armature current of the same frequency. In addition a current of frequency f appears in the armature and a fundamental frequency current, modulated at frequency f, appears in the field circuit. Arguing from the above considerations affecting transients it should be clear that these latter have very little influence upon the magnitude of the former. Further, while the former pair of currents produces variations in torque and power of the first order (since the modulated alternating current in the armature reacts with the average value of the field), the second pair of currents produces variations of the second order. In this analysis the assumption will be made that all currents in the armature but the modulated component of fundamental frequency will be neglected. Thus the instantaneous torque and power will be determined by inserting the instantaneous value of field current in the usual vector diagrams of the machine, depicting the steady state characteristics.

Saturation is neglected to the extent that the impedances of the machine are assumed to be independent of the current flow, that is, the constants may be altered somewhat to take saturation into effect.

The per unit system of notation will be used, by which is meant that units will be used as bases instead of 100 per cent. For example, terminal voltage will be expressed as a fraction of rated terminal voltage, field current will be expressed as a fraction of the field current required to produce rated terminal voltage at no load, and other quantities similarly.

Assume a salient pole machine without amortisseur windings and the absence of other paths which might permit circulating currents in the quadrature axis and thus prevent the change of flux in that axis. Let such a machine be connected to an infinite bus. The plan of attack shall be to first obtain an expression for the flux linkages with the field winding and the time rate of change of these flux linkages in terms of the constants of the machine. A small sinusoidal variation of angle will then be superposed upon the operating angle and the resultant variation of flux linkages and internal voltages determined. These incremental values may be converted to power increments and the area of the loop obtained.

Following the method of Blondel the reactions within the machine will be resolved into the two components, one in phase and one in quadrature to the axis of the main field structure. The nomenclature used will be the same, with some few exceptions, as that used by Doherty and Nickle in their excellent papers on *Synchronous Machines*.

TRANSIENT REACTANCE AND TRANSIENT VOLTAGE

In Fig. 3 let e_{dl} be the no load excitation curve with abscissa representing the field current. In the "per unit system" unit field current will be defined as that current required to produce unit terminal voltage at no load. Also, defining unit air-gap flux as that required to produce unit voltage, e_{dl} will also represent air-gap flux. The component of armature current in phase with the field structure, i_d, is directly demagnetizing. Its effect in field terms is $x_a i_d$, indicated by the line $p - n$, in which x_a converts the armature current to its field equivalent. The total field current required then to produce the air-gap voltage e_{dl} is $e_{dl} + x_a i_d$. But this is numerically equal to e_d, the nominal voltage due to excitation in the direct axis. Therefore

$$e_d = e_{dl} + x_a i_d \tag{2}$$

In addition to the air-gap flux, the field windings are also linked with a field leakage flux* which is directly proportional to the field current e_d. Defining the field leakage, f_l, as a fraction of the air gap flux at no load, the field leakage will be $f_l e_d$. This is indicated in the figure by the line $n - q$. The total flux linking the field winding is then

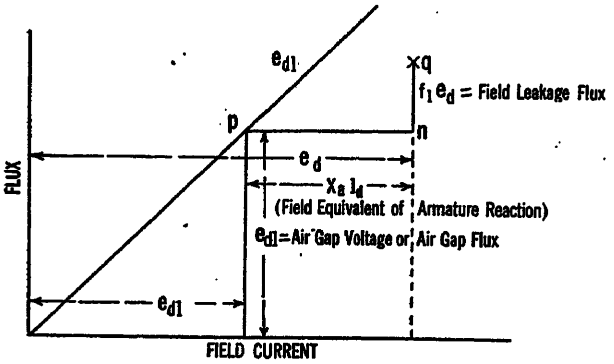
FLUX
FIELD CURRENT
e_{dl}
p
q
n
$f_l e_d$ = Field Leakage Flux
e_d
$X_a I_d$
(Field Equivalent of Armature Reaction)
e_{dl}=Air Gap Voltage or Air Gap Flux
e_{dl}

FIG. 3—FLUX RELATIONS IN DIRECT AXIS

$$e_{dl} + f_l e_d$$

Now defining unit flux linkages as the flux linkages at no load normal voltage, and since the flux linking the field winding at no load normal voltage is $(1 + f_l)$, the flux linkages ψ_d in per unit values are

$$\psi_d = \frac{e_{dl} + f_l e_d}{1 + f_l} \tag{3}$$

Inserting e_{dl} from (2), ψ_d is also

$$\psi_d = e_d - \frac{x_a}{1 + f_l} i_d \tag{4}$$

The "transient reactance" of a synchronous machine is that reactance which when divided into the normal voltage gives the maximum r. m. s. symmetrical current (neglecting the additional component having a very rapid decrement) at the occurrence of a three-phase short circuit at the terminals of the machine which had been operating at no load. Its value can be obtained by the aid of Fig. 3 as follows.

For unit armature current with a three-phase short circuit at the terminals, the air-gap voltage is equal to the armature leakage reactance x_l and the armature reaction expressed in terms of field current is x_a. To maintain this flux and armature current the total field current required is $x_l + x_a$, which is equal to the synchronous reactance x_d. The field leakage corresponding to this current is $f_l x_d$. The total field linkages per unit armature current are then, in per unit terms, $\frac{x_l + f_l x_d}{1 + f_l}$.

For any sudden change in circuit conditions, the flux linkages with the field winding at the first instant remain constant. Since, by definition, the flux linkages with the field winding at normal no load voltage is unity, for any transition from this condition the flux linkages at the first instant for the new condition must remain unity. Knowing then the linkages per unit armature current and the total linkages, the armature current is equal to $\frac{1 + f_l}{x_l + f_l x_d}$.

From the previous definition of the transient reactance, x_d', it may be seen that x_d' is the reciprocal of this quantity.

$$x_d' = \frac{x_l + f_l x_d}{1 + f_l} \tag{5}$$

Remembering that $x_d = x_l + x_a$, other forms of this expression are

$$x_d' = x_l + \frac{f_l}{1 + f_l} x_a \tag{6}$$

$$x_d' = x_d - \frac{1}{1 + f_l} x_a \tag{7}$$

Expression (6) is interesting in that it gives a better picture of the nature of x_d' as being the armature leakage reactance plus a term proportional to the field leakage and the relative strengths of field and armature ampere turns.

From (7)

$$\frac{x_a}{1 + f_l} = x_d - x_d' \tag{8}$$

Inserting in (4)

$$\psi_d = e_d - (x_d - x_d') i_d \tag{9}$$

or

$$e_d - \psi_d = (x_d - x_d') i_d \tag{10}$$

From Equation (10) it will be observed that the difference between e_d and ψ_d in the per unit system is equal to the difference in synchronous reactance and transient reactance drops. It follows, therefore, that ψ_d has all the characteristics of a fictitious voltage, which shall be designated the "transient" voltage, e_d', and which is obtained in the same manner as e_d for steady state conditions, except that x_d is replaced by x_d'. The flux linkages ψ_d or e_d' can be obtained during transients by the same kind of formula as that used to determine e_d for steady state conditions.

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*Leakage flux turns weighted so that they may be replaced by a leakage flux which links all of the turns.

TIME RATE OF CHANGE OF e_d'

In per unit terms, the unit of instantaneous value of exciter voltage e_x will be defined as the voltage required to circulate unit current in the field circuit. The value of e_d has already been defined as the nominal voltage in the direct axis, that is, the internal voltage of the machine obtained by using the synchronous reactance x_d, and is also numerically equal to the instantaneous value of field current or instantaneous resistance drop in the field circuit. The voltage applied to the field must at any instant be equal to the resistance drop plus the inductive drop, which latter is proportional to the rate of change of flux linkages. In per unit terms the voltage equation may, therefore, be written

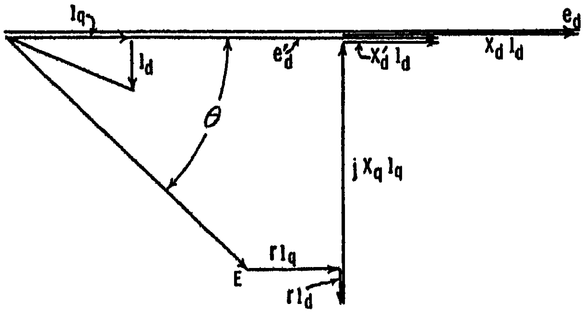

FIG. 4—VECTOR DIAGRAM OF SALIENT POLE SYNCHRONOUS MACHINE WITHOUT AMORTISSEUR WINDING

$$e_x = e_d + T_{do} \frac{d\,\psi_d}{d\,t} \tag{11}$$

The coefficient, T_{do}, is the no load time constant of the machine and, since the units of e_x, e_d, and ψ_d have already been defined this coefficient is introduced to convert the time rate of change of linkages in "per unit" values to "per unit" values of e. m. f. in the field circuit.

Equation (11) may also be written

$$e_x = e_d + T_{do} \frac{d\,e_d'}{d\,t} \tag{12}$$

In what follows, incremental values will be designated by the prefix Δ, and when this prefix is not used the mean operating values will be understood unless specifically stated otherwise. Since constant exciter voltage is assumed, Equation (12) may be written

$$0 = \Delta\, e_d + T_{do} \frac{d\,\Delta\, e_d'}{d\,t} \tag{13}$$

In Fig. 4, the vector diagram of the machine is shown in which

E = Voltage of the infinite bus
e_d = Nominal voltage in direct axis
e_d' = Transient voltage in direct axis
θ = Angle between direct axis of machine and voltage E
r = Resistance in armature circuit
x_q = Synchronous reactance in quadrature axis.

Equating reals and imaginaries and using x_d' and e_d'

$$E \sin\theta + r\, i_d - x_q\, i_q = 0 \tag{14}$$

$$E \cos\theta + r\, i_q + x_d'\, i_d = e_d' \tag{15}$$

Solving for i_d

$$i_d = \frac{1}{r^2 + x_d'\, x_q} \left[x_q\, e_d' - x_q\, E \cos\theta - r\, E \sin\theta\right] \tag{16}$$

As indicated previously and shown graphically in Fig. 4

$$e_d = e_d' + i_d\,(x_d - x_d') \tag{17}$$

Substituting (16)

$$e_d = \frac{1}{r^2 + x_d'\, x_q} \left[(r^2 + x_d\, x_q)\, e_d' - (x_d - x_d')\, E\, (x_q \cos\theta + r \sin\theta)\right] \tag{18}$$

and

$$\Delta\, e_d = \frac{1}{r^2 + x_d'\, x_q} \left[(r^2 + x_d\, x_q)\, e_d' + (x_d - x_d')\, E\, (x_q \sin\theta - r \cos\theta)\right] \tag{19}$$

Now substituting (19) in (13),

$$(r^2 + x_d\, x_q)\, \Delta\, e_d' + (r^2 + x_d'\, x_q)\, T_{do} \frac{d\,\Delta\, e_d'}{d\,t} = (x_d - x_d')\, E\, (-\, x_q \sin\theta + r \cos\theta)\, \Delta\,\theta \tag{20}$$

Dividing through by $(r^2 + x_d\, x_q)$ and letting

$$T_d = \frac{r^2 + x_d'\, x_q}{r^2 + x_d\, x_q}\, T_{do} \tag{21}$$

Equation (20) becomes

$$\Delta\, e_d' + T_d \frac{d\,\Delta\, e_d'}{d\,t} = \frac{x_d - x_d'}{r^2 + x_d\, x_q}\, E(-x_q \sin\theta + r \cos\theta)\, \Delta\,\theta \tag{22}$$

It may be observed in passing that T_d is the time constant of the field winding for the particular circuit condition. Equation (21) is similar to the expression derived by Park and Robertson[6] showing how the time constant of the unidirectional transient in the field varies with the reactance in the short circuited armature, except that Equation (21) goes one step further and includes the effect of the resistance. When r, x_d', x_d, and x_q, refer specifically to machine constants, the effect of external constants R and X may be included as follows:

$$T_d = \frac{(r + R)^2 + (x_d' + X)\,(x_q + X)}{(r + R)^2 + (x_d + X)\,(x_q + X)} \tag{23}$$

VARIATION OF $\Delta e_d'$ FOR SMALL SINUSOIDAL VARIATIONS OF ANGLE

Equation (22) enables one to determine the time rate of change of $\Delta e_d'$ for instantaneous values of $\Delta e_d'$ and $\Delta \theta$ for the condition of constant exciter voltage. Now let small sinusoidal variations of θ of frequency f and magnitude $\Delta \theta_m$ be superposed upon the main operating angle,—the mechanism whereby this may be accomplished is immaterial at this point. Then

$$\Delta \theta = \Delta \theta_m \sin 2 \pi f t \qquad (24)$$

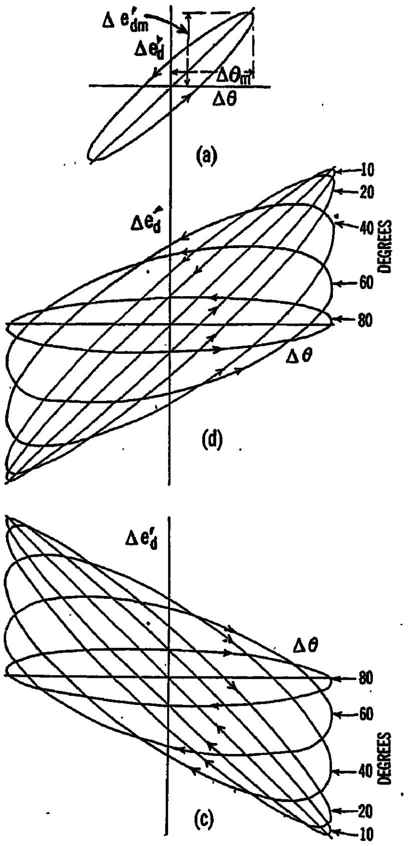

FIG. 5—VOLTAGE LOOPS SHOWING EFFECT OF LAG UPON SHAPE AND DIRECTION OF GENERATION

Since these oscillations are assumed to be sustained, e_d' will also attain some form of sinusoidal oscillation of the same frequency. Let the value of $\Delta e_d'$ be

$$\Delta e_d' = \Delta e_{dm}' \sin (2 \pi f t + \alpha) \qquad (25)$$

in which $\Delta e_{dm}'$ and α are to be determined.

$$\frac{d \, \Delta e_d'}{d t} = 2 \pi f \Delta e_{dm}' \cos (2 \pi f t + \alpha) \qquad (26)$$

Inserting (24), (25), and (26) in (22)

$$\Delta e_{dm}' \sin (2 \pi f t + \alpha) + 2 \pi f T_d \Delta e_{dm}' \cos (2 \pi f t + \alpha) = \frac{x_d - x_d'}{r^2 + x_d x_q} E(-x_q \sin \theta + r \cos \theta) \, \Delta \theta_m \sin 2 \pi f t \qquad (27)$$

Since the above relations must be satisfied for all value of t, $\Delta e_{dm}'$, and α can be determined by letting $2 \pi f t = 0$ and $\pi/2$, producing the two following equations:

$$\Delta e_{dm}' \sin \alpha + 2 \pi f T_d \Delta e_{dm}' \cos \alpha = 0 \qquad (28)$$

$$\Delta e_{dm}' \cos \alpha - 2 \pi f T_d e_{dm}' \sin \alpha = \frac{x_d - x_d'}{r^2 + x_d x_q} E(-x_q \sin \theta + r \cos \theta) \, \Delta \theta_m \qquad (29)$$

As will be shown later, it will not be necessary to determine α and $\Delta e_{dm}'$, but rather the quantity $\Delta e_{dm}' \sin \alpha$. This quantity can be obtained from (28) and (29)

$$\Delta e_{dm}' \sin \alpha = -\frac{2 \pi f T_d}{1 + (2 f T_d)^2} \, \frac{x_d - x_d'}{(r^2 + x_d x_q)} E(-x_q \sin \theta + r \cos \theta) \, \Delta \theta_m \qquad (30)$$

By analogy with the ordinary a-c. circuit it is possible to see immediately from Equation (22) that $\Delta e_d'$ lags behind $\Delta \theta$ by an angle

$$\alpha = \tan^{-1} (2 \pi f T_d)$$

and that $\Delta e_{dm}'$ is equal to the right hand member divided by $\sqrt{1 + (2 \pi f T_d)^2}$.

Plotting $\Delta e_d'$ as a function of $\Delta \theta$ results in the familiar elliptical Lissajous figure as shown in Fig. 5a. When the right hand member of Equation (22) is positive, loops of the form shown in Fig. 5b are obtained and when negative those shown in Fig. 5c are obtained. Further, it will be observed that the point generating the loops rotates counterclockwise and clockwise in Figs. 5b and 5c, respectively. For $T_d = 0$ and ∞ the loops become a diagonal straight line and a horizontal straight line, respectively.

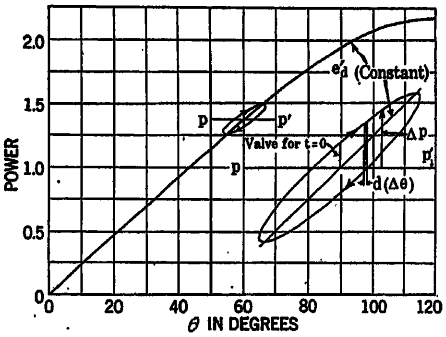

FIG. 6—POWER-ANGLE DIAGRAM WITH ANGULAR OSCILLATION SUPERPOSED

POWER LOOP

Equation (107) in Appendix I determines the total power output of the generator in terms of e_d, e_q, and θ. Letting e_d equal to e_d', x_q equal to x_q', and e_q equal to zero, the power output as a function of θ for a constant value of e_d' may be plotted as shown in Fig. 6.

Having obtained the variation in $\Delta e_d'$ with respect to $\Delta \theta$, the corresponding increment in power output above the value corresponding to e_d' constant is, for small values of $\Delta\ \theta$,

$$\Delta P = \frac{d P}{d e_d'} \Delta e_d' \tag{31}$$

This differential has been derived in Appendix I and is given by Equation (**116**). Now since the variation of $\Delta e_d'$ with $\Delta\ \theta$ forms an elliptical loop, the increment in ΔP will likewise form an elliptical loop about the curve of power output for e_d' constant. When $\frac{d P}{d e_d'}$ is positive, the direction of generation is the same as the voltage loop.

As shown in the qualitative treatment, the energy per cycle is proportional to the area of the power loop and the system is stable for clockwise generation of the loop. The quantitative evaluation of the energy absorbed per cycle follows. ΔP varies in time phase relation with $\Delta e_d'$ so that

$$\Delta P = \frac{d P}{d e_d'} \Delta e_{dm}' \sin (2 \pi f t + \alpha) \tag{32}$$

The area of the loop is equal to the integration around one complete cycle, say from $t = 0$ to $t = \frac{1}{f}$, of the incremental area of height ΔP and width $d (\Delta \theta)$. (See Fig. 6)

$$d (\Delta \theta) = d (\Delta \theta_m \sin 2 \pi f t) = \Delta \theta_m 2 \pi f \cos 2 \pi f t . d t \tag{33}$$

$$W/\!\sim\ = \text{Area} = \int_0^{\frac{1}{f}} \frac{d P}{d e_d'} \Delta e_{dm}' \sin (2 \pi f t + \alpha) \Delta \theta_m 2 \pi f \cos 2 \pi f t . d t$$

$$= \pi f \Delta e_{dm}' \Delta \theta_m \frac{d P}{d e_d'} \left\{ \int_0^{\frac{1}{f}} \sin (4 \pi f t + \alpha) d t + \int_0^{\frac{1}{f}} \sin \alpha\, d t \right. = \pi \Delta \theta_m \frac{d P}{d e_d'} \Delta e_{dm}' \sin \alpha \tag{34}$$

Insert Equation (30)

$$W/\!\sim\ = - \frac{2 \pi f T_d}{[1 + (2 \pi f T_d)^2]} \frac{\pi (x_d - x_d')}{(r^2 + x_d x_q)} E (- x_q \sin \theta + r \cos \theta) \frac{d P}{d e_d'} (\Delta \theta_m)^2 \tag{35}$$

This expression gives the excess of power output per cycle of oscillation, $\frac{d P}{d e_d'}$ being defined by Equation (116) in the appendix and T_d by Equation (21), for constant prime mover input. The expression thus represents the energy that is absorbed per cycle of oscillation,—a positive value indicating a stable condition and negative value an unstable operating condition; or in the event of cyclic variations in prime mover input it represents the energy per cycle that can be absorbed without producing hunting.

The differential $\frac{d P}{d e_d'}$ for practically all conditions met in practise is positive for generators as also is $(x_d - x_d')$.

It follows, therefore, that the sign of the quantity is dependent upon the expression $(- x_q \sin \theta + r \cos \theta)$. Therefore, for

$$\theta > \tan^{-1} \frac{r}{x_q} \tag{36}$$

the energy per cycle is positive and the machine is stable. This relation was first pointed out by Nickle and Pierce.

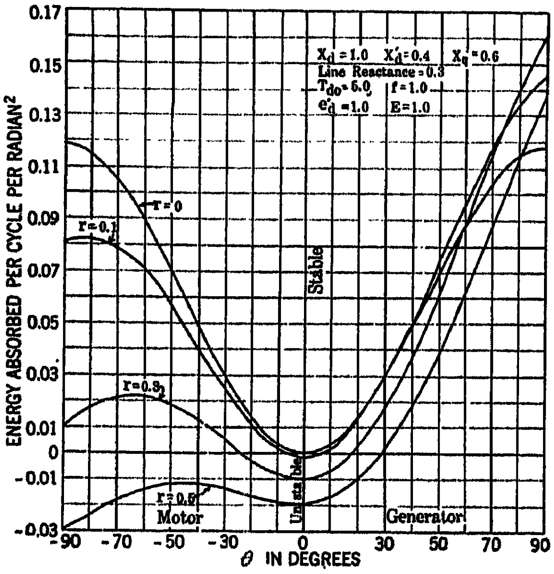

FIG. 7—ENERGY ABSORBED PER CYCLE OF SYSTEM OSCILLATION

No amortissour windings

$$\text{Energy per cycle} = - \frac{2 \pi f T_d}{[1 + (2 \pi f T_d)^2]} \frac{\pi (X_d - X_d')}{(r^2 + X_d X_q)} E (- X_q \sin \theta + r \cos \theta) \frac{d P}{d e_d'} (\Delta \theta_m)^2$$

in which

$$T_d = \frac{r^2 + X_d' X_q}{r^2 + X_d X_q} T_{do}$$

$$\frac{d P}{d e_d'} = \frac{1}{(r^2 + X_d' X_q)^2} \left\{ 2 r (r^2 + X_q^2) e_d' + [r^2 (2 X_d' - X_q) + X_d' X_q^2] E \sin \theta - r [r^2 + X_q (2 X_q - X_d')] E \cos \theta \right\}$$

To form an idea of the relative importance of the operating angle and armature resistance, the curves in Fig. 7 have been calculated for the particular machine constants designated in the figure. The constants were chosen for a typical machine with an external reactance of 0.3. Positive angles indicate a generator action and negative angles motor action. As will be seen, the angle for which $W/\!\sim$ is equal to zero increases with increasing angle following the relation expressed by Equation (**36**). Observe also that the machine is more stable for a given e_d' (flux linkages with the field winding) when operating as a generator than when operating as a motor. This difference becomes very pronounced for the larger values of armature resistance.

It was stated that $\frac{d P}{d\, e_d'}$ is usually positive for generators. Actually the sign of this quantity is dependent upon the excitation. It is difficult to conceive a case in which the coefficient of sin θ of Equation (**116**) in Appendix I is negative, so that if the quantity within the brackets is positive for $\theta = 0$ it will be positive for all positive values of θ. Now to show that the function is positive for $\theta = 0$ it is only necessary to show that

$$2\,(x_q^2 + r^2)\, e_d' > [x_q\,(2\,x_q - x_d') + r^2]\, E \qquad (\mathbf{37})$$

or substituting the particular values from Fig. 7, that

$$\frac{e_d'}{E} > \frac{.99 + r^2}{2\,(.81 + r^2)}$$

that is, for this particular case, the expression will be positive for $\frac{e_d'}{E}$ greater than the values shown below.

r	very small	0.1	0.2	0.3
$\frac{e_d'}{E}$	0.611	0.617	0.606	0.60

Generators will always be working well beyond these limits, except when charging an unloaded line requiring a large charging kv-a. It is a very rare case, however, for which even this requirement decreases the value of e_d' below those given in the table.

For values of $\frac{e_d'}{E}$ below those indicated in the table it is possible that $\frac{d P}{d\, e_d'}$ may be negative but, due to the sine term, $\frac{d P}{d\, e_d'}$ again turns positive for larger angles.

To get an idea of the relative importance of the various terms, the constants given in Fig. 7 were substituted for $r = 0.1$ with the following result

$$\frac{d P}{d\, e_d'} = 0.164\, e_d' + 0.572\, E \sin\theta - 0.1\, E \cos\theta$$

The effect of excitation upon spontaneous hunting was verified by an experiment described in the Evans and Bergvall[2] paper referred to previously. A 425-kv-a., 2300-volt condenser (without damper winding) was connected to the 2300-volt shop circuit through an impedance of 6.66 + j 0 ohms (0.53 + j 0 per unit). This is equivalent to a generator operating at practically $\theta = 0$. For low excitations the system was stable. However, when the excitation was increased to four amperes (7 amperes = N. L. excitation) the system became unstable.

III. SALIENT POLE MACHINE WITH AMORTISSEUR WINDINGS

The effect of the amortisseur winding in the quadrature axis is to prevent the change of flux linked therewith. The rate of change of the cross flux with respect to angle is greater for small operating angles, that is, for light loads, so that under small sustained angular oscillations the currents induced in the quadrature axis should be greater for small operating angles. One should expect, therefore, that the energy absorbed per cycle should be greater for the small operating angles.

The problem is further complicated by the presence of two windings in the direct axis, namely, the main field winding itself and the amortisseur. Several simplifying assumptions suggest themselves with respect to the windings in the direct axis.

a. The presence of the amortisseur winding in the direct axis may be neglected, the currents in the main field only being taken into consideration.

b. The resistance of the main field may be assumed to be zero so that the flux linkages with it remain constant, with the effect of the amortisseur winding taken into consideration.

c. The presence of the amortisseur winding in the direct axis may be neglected and the main field resistance assumed to be equal to zero.

Any of these sets of assumptions render the problem readily susceptible to analysis, the complete derivation of the expressions giving the energy absorbed per cycle of system oscillation being given in the Appendix. In Fig. 8 are given two sets of curves for the above assumptions for a typical machine equipped with copper amortisseur (upper set) and with brass amortisseurs (lower set). The constants of the machine and line utilized are also given in the figure. The line resistance for this case is zero. It will be observed that for both the copper and the brass amortisseurs the three curves comprising a set coincide for $\theta = 0$ and then diverge slowly reaching a maximum divergence for $\theta = \pm\, 90$ deg. The coincidence for $\theta = 0$ becomes clear when one considers that for small angular oscillations about $\theta = 0$, the demagnetizing component of armature current in the direct axis is not altered, so that the flux in the direct axis has no tendency to change. The phenomenon must of necessity, therefore, be independent of the assumptions as to the characteristics of the windings in the direct axis, the entire energy loss being due to current flow in the quadrature axis. It will be observed that for large angles the energy absorbed is, for copper, larger under assumption (b) than assumption (a) but that for brass, larger under assumption (a) than assumption (b). The best assumption to employ is therefore dependent upon the character of the amortisseur in question. As a matter of fact, all of the simplifying assumptions stated introduce considerable error at large angles, the correct value of energy absorbed being very near to the value for assumption (c) plus the difference for assumptions (a) and (c) plus the difference for assumptions (a) and (b). The justification for this statement is that the currents in the field winding and in the amortisseur winding in the direct axis, because of the large difference in their time constants, are very nearly in time quadrature and consequently do not react upon each other to any considerable extent. This hypothesis has been checked by analysis and calculation for the copper amortisseur and found to be substantiated. It therefore follows that one cannot say without giving consideration to the particular type of amortisseur which assumption will give the best results. Fortunately, however, as will be seen later, this question is usually not of much importance. In most cases the problem is to determine whether the machine is stable or unstable, so that unless other periodic power impulses, such as prime mover pulses, enter the problem and require a quantitative evaluation of the energy absorbed per cycle of system oscillation, it is adequate to determine only the sign of $W/\sim$.

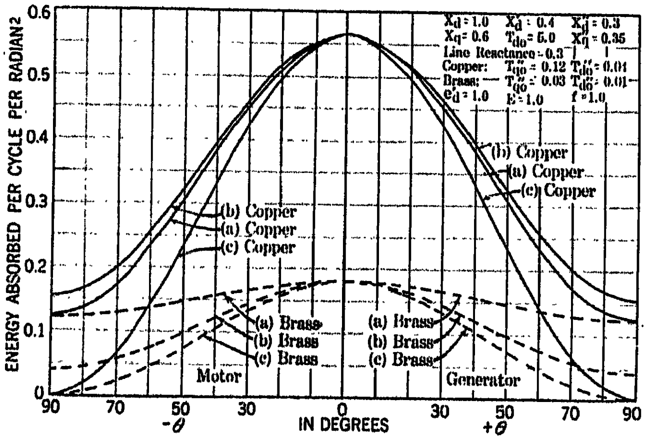

Fig. 8—Energy Absorbed per Cycle of System Oscillation with Armature Resistance = 0 for the Machine Equipped with Copper and Brass Amortisseurs for the Following Assumptions:

Assumption	Amortisseur		Resistance of main field
	In quad. axis	In direct axis	
a	Yes	No	Normal
b	Yes	Yes	Zero
c	Yes	No	Zero

Effect of Armature Resistance

To determine the effect of armature resistance, Figs. 9 and 10 were prepared, the former showing the energy absorbed per cycle for copper amortisseurs using assumption (b) and the latter for brass amortisseurs using assumption (a). It will be observed that the machine, equipped either with brass or copper amortisseurs, should be stable for any normal operating angle or line resistance. The machine is somewhat more stable operating as a generator than as a motor. While resistance decreases the stability of the machine, a very considerable amount will be required to make it unstable within the normal operating range.

The dotted curves in Figs. 9 and 10 were added to further demonstrate the effect of the particular assumptions chosen with respect to the windings in the direct axis. It will be observed that the assumption chosen matters little for the motoring range but becomes of increasing importance, in a quantitative sense, as the angle is increased in the generating range. It is clear, however, that for either copper or brass amortisseurs, the true curve would result in a positive value of $W/\sim$. For smaller machines in which T_{do} is smaller, the assumption of neglecting the amortisseur in the direct axis becomes more justifiable.

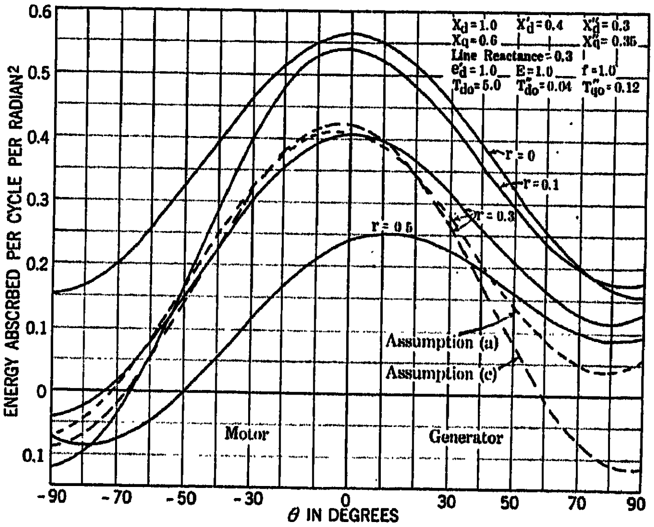

Fig. 9—Effect of Armature Resistance with Copper Amortisseurs

Using assumption (*b*) of Fig. 8. Dotted curves using assumptions (*a*) and (*c*) of Fig. 8

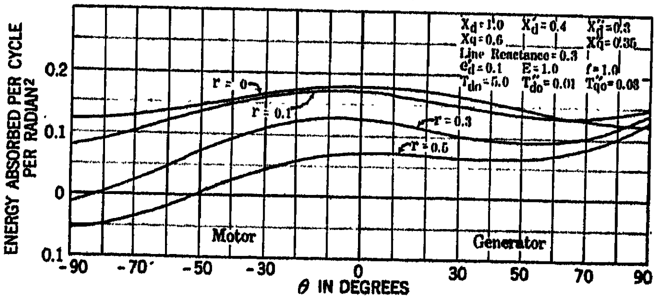

Fig. 10—Effect of Armature Resistance with Brass Amortisseurs

Using assumption (*a*) of Fig. 8. Dotted curves used assumptions (*b*) and (*c*) of Fig. 8

General Discussion

In general, as previously pointed out by Nickle and Pierce, for practical machines, if a generator is stable at zero load it will be stable for all loads. Of course, theoretical machines may be imagined for which this is not true; machines in which the resistance of field winding is very small and in which the amortisseur winding is connected for the quadrature axis only, resulting in characteristic curves such as that for similar assumptions in Figs. 9 and 10 in which the lower curve passes through zero near 60 deg. The plausibility of the statement may be made apparent by arguing from the type of curve obtained in Fig. 7 in the generator range, which shows that a machine without amortisseurs becomes less unstable and then more stable as the angle is continuously increased. Now the general effect of amortisseurs in the curves shown is a maximum near $\theta = 0$ so that one would expect that, combining the effect of the two when the effect of amortisseur is small, should elevate the curve uniformly near zero but when the effect of the amortisseur is much greater than that of the field winding, the type of curve will be governed largely by the characteristics of the curve obtained for the amortisseur.

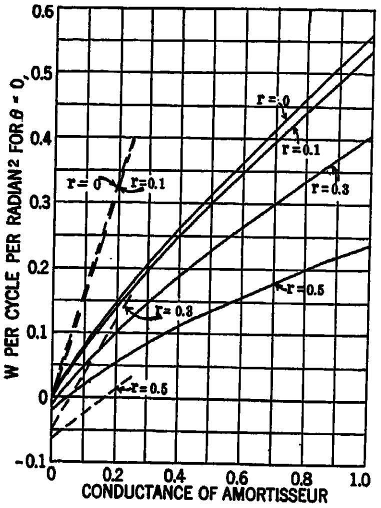

Fig. 11—Energy Absorbed per Cycle as a Function of the Conductance of the Amortisseur for Different Values of Line Resistance and Reactance. Conductance of Copper Winding used as Base = 1.0. Line Reactance for Full Lines = 0.3

Line reactance for dotted lines = 0. Same constants used as for Figs. 9 and 10

Accepting then this statement, the effect of the conductivity of the amortisseur is illustrated in Fig. 11, which shows the energy loss for $\theta = 0$ plotted as a function of the conductance of the amortisseur, using the copper amortisseur as unity. The variation in conductance may be obtained either by changing the quantity or quality of the material employed. Brass dampers for the same section will have about 20 to 30 per cent of the conductance of copper. The curves are plotted for different values of armature resistance and two values of line reactance, the full lines for 0.3 and the dotted for 0. It would appear from these curves that even for such a large value of armature resistance as 0.5, so long as the reactance external to the machine is 0.3, an amortisseur of 4 per cent of the conductance of a copper amortisseur is all that is necessary to prevent spontaneous hunting. Almost any kind of amortisseur installed for the purpose for which it is normally installed will incidentally be of sufficiently low conductance to take care of all but the most abnormal cases giving rise to hunting of this nature. As the series reactance in the armature circuit decreases, the curves show that the need for the amortisseurs to stabilize the system, increases, that is, a higher conductance for the amortisseurs is required, but even this value is still quite small.

The analysis of the machine without amortisseurs assumed an ideal machine with respect to the quadrature axis, that is, the absence of any path that might permit flow of current in that axis, and showed that any such machine connected to an infinite bus through resistance in the armature is unstable at no load. An ideal water wheel generator of this character (and there is always some resistance in the armature circuit) cannot be connected to a transmission line without hunting. As a matter of fact, no trouble of this character is experienced. The answer may be found in either other sources of positive damper or the current paths in the quadrature axis furnished by pole face rivets or the natural paths for eddy currents. Considering both the rivet area in the pole face and its resistivity, the conductance of the rivets may be of the order of 5 per cent of the conductance of the copper damper and since they are surrounded by a complete iron magnetic circuit their time constant is still greater. Of course to offset this effect the subtransient reactance will be greater. In general, however, the stable operation of generators without amortisseurs at no load may well be attributed to the currents in the rivets.

CONCLUSIONS

Theoretically, an ideal machine without amortisseurs should hunt, when operating at light loads. The lack of such hunting in modern transmission systems is explained by the presence of current paths in the quadrature axis afforded by rivets and parts of the field structure. Except for very abnormal conditions any normal design of amortisseur, that is installed for other purposes, will be found adequate incidentally to prevent spontaneous hunting.

Appendix I

POWER EQUATION

The vector diagram for a salient pole machine with the excitation e_d and e_q in the direct and quadrature axes, respectively, is shown in Fig. 12. The reactances in the two axes are x_d and x_q.

Equating the reals and imaginaries

$$E \sin \theta + r\, i_d - x_q\, i_q + e_q = 0 \qquad (101)$$

$$E \cos\theta + r\, i_q + x_d\, i_d - e_d = 0 \qquad (102)$$

From (1)

$$i_q = \frac{1}{x_q}\left(e_q + E \sin\theta + r\, i_d\right) \qquad (103)$$

Substituting in (102)

$$E\cos\theta + \frac{r}{x_q}\, e_q + \frac{r}{x_q}\, E\sin\theta + \frac{r^2}{x_q}\, i_d + x_d i_d - e_d = 0$$

From which

$$i_d = \frac{1}{r^2 + x_d\, x_q}\left[x_q\, e_d - r\, e_q - r\, E\sin\theta - x_q\, E\cos\theta\right] \qquad (104)$$

and substituting in (103)

$$i_q = \frac{1}{r^2 + x_d\, x_q}\left[r\, e_d + x_d\, e_q + x_d E\sin\theta - r\, E\cos\theta\right] \qquad (105)$$

The power output, that is, the power at E is equal to the sum of the products of the in phase components of E and I. To get the total power, including the loss in the resistance, it is necessary to use the sum of the products of the in phase components of I and e, the latter including the resistance drop.

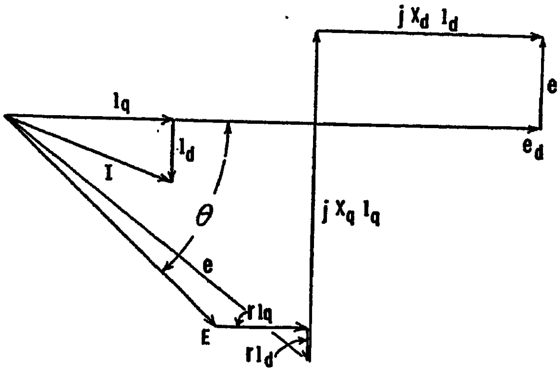

Fig. 12—Vector Diagram of Salient Pole Synchronous Machine with Excitation in Both the Direct and Quadrature Axes

$$P = (e_d - x_d\, i_d)\, i_q + (-\, e_q + x_q\, i_q)\, i_d$$
$$= e_d\, i_q - e_q\, i_d + (x_q - x_d)\, i_d\, i_q \qquad (106)$$

Substituting (104) and (105)

$$P = \frac{1}{(r^2 + x_d\, x_q)^2}\, \{ e_d\, (r\, e_d + x_d\, e_q$$
$$+ x_d E\sin\theta - r\, E\cos\theta)\, (r^2 + x_d\, x_q)$$
$$- e_q(x_q\, e_d - r\, e_q - r\, E\sin\theta - x_q\, E\cos\theta)(r^2 + x_d\, x_q)$$
$$+ (x_q\, e_d - r\, e_q - r\, E\sin\theta - x_q\, E\cos\theta)(r\, e_d + x_d\, e_q$$
$$+ x_d E\sin\theta - r\, E\cos\theta)\, (x_q - x_d) \} \qquad (107)$$

The increment in power for small increments in e_d and e_q is

$$P = \frac{\partial P}{\partial e_d}\, \Delta e_d + \frac{\partial P}{\partial e_q}\, \Delta e_q \qquad (108)$$

But

$$\frac{\partial P}{\partial e_d} = \frac{1}{(r^2 + x_d\, x_q)^2}\, \{ (2\, r\, e_d + x_d\, e_q$$
$$+ x_d E\sin\theta - r\, E\cos\theta)(r^2 + x_d\, x_q) - x_q\, e_q(r^2 + x_d\, x_q)$$
$$+ x_q(r\, e_d + x_d\, e_q + x_d\, E\sin\theta - r\, E\cos\theta)(x_q - x_d)$$
$$+ r(x_q\, e_d - r\, e_q - r\, E\sin\theta - x_q\, E\cos\theta)(x_q - x_d) \}$$

which, after collecting terms becomes

$$\frac{\partial P}{\partial e_d} = \frac{1}{(r^2 + x_d\, x_q)^2}\, \{ 2\, r(r^2 + x_q^2)\, e_d - 2\, r^2\, (x_q - x_d)\, e_q$$
$$+ [r^2\, (2\, x_d - x_q) + x_d\, x_q^2]\, E\sin\theta - r\, [r^2$$
$$+ x_q\, (2\, x_q - x_d)]\, E\cos\theta \} \qquad (109)$$

Similarly,

$$\frac{\partial P}{\partial e_q} = \frac{1}{(r^2 + x_d\, x_q)^2}\, \{ x_d\, e_d\, (r^2$$
$$+ x_d\, x_q) - (x_q\, e_d - 2\, r\, e_q - r\, E\sin\theta - x_q\, E\cos\theta)(r^2$$
$$+ x_d\, x_q) - r(r\, e_d + x_d\, e_q + x_d\, E\sin\theta - r\, E\cos\theta)(x_q - x_d)$$
$$+ x_d(x_q\, e_d - r\, e_q - r\, E\sin\theta - x_q\, E\cos\theta)\, (x_q - x_d) \}$$

and collecting terms

$$\frac{\partial P}{\partial e_q} = \frac{1}{(r^2 + x_d\, x_q)}\, \{ -\, 2\, r^2\, (x_q - x_d)\, e_d$$
$$+ 2\, r\, (r^2 + x_d^2)\, e_q + r\, [r^2 - x_d\, (2\, x_d - x_q)]\, E\sin\theta$$
$$+ [r^2\, (2\, x_q - x_d) + x_q\, x_d^2\,]\, E\cos\theta \} \qquad (110)$$

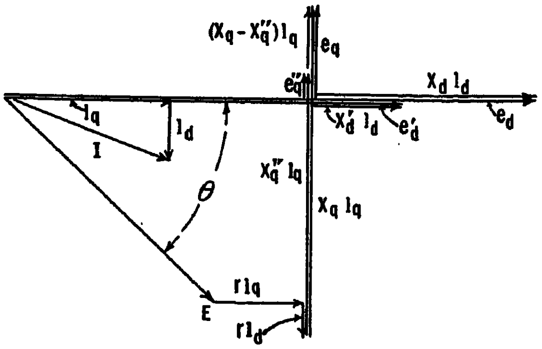

Fig. 13—Vector Diagram of Salient Pole Synchronous Machine showing the Relation between e_d and e_d' and e_q and e_q''

The rate of change of power with respect to e_d' and e_q'' may be obtained by replacing

e_d by e_d'
e_q by e_q''
x_d by x_d'
and x_q by x_q''

giving

$$\frac{\partial P}{\partial e_d'} = \frac{1}{(r^2 + x_d'\, x_q'')^2}\, \{ 2\, r(r^2 + x_q''^2)e_d' - 2\, r^2(x_q'' - x_d')e_q''$$
$$+ [r^2\, (2\, x_d' - x_q'') + x_d'\, x_q''^2\,]\, E\sin\theta$$
$$-\, r\, [r^2 + x_q''\, (2\, x_q'' - x_d')\,]\, E\cos\theta \} \qquad (111)$$

$$\frac{\partial P}{\partial e_q''} = \frac{1}{(r^2 + x_d'\;\; x_q'')^2}\, \{ -2r^2(x_q'' - x_d')e_d' + 2\, r(r^2 + x_d'^2)e_q''$$
$$+ r\, [r^2 - x_d'\, (2\, x_d' - x_q'')\,]\, E\sin\theta$$
$$+ [r^2\, (2\, x_q'' - x_d') + x_q''\, x_d'^2\,]\, E\cos\theta \} \qquad (112)$$

in which e_q'' is given below by Equation (115) which is derived as follows. From Fig. 13 which shows a more complete vector diagram of the machine, it can be seen that

$$e_q'' = e_q - (x_q - x_q'')\, i_q \tag{113}$$

From Equation (**105**), replacing

x_d by x_d'
e_d by e_d'
and e_q by 0

one obtains

$$i_q = \frac{1}{r^2 + x_d'\, x_q}\, [r\, e_d' + x_d' E \sin\theta - r\, E \cos\theta] \tag{114}$$

which when substituted in (**113**) gives

$$e_q'' = -\frac{x_q - x_q''}{r^2 + x_d'\, x_q}\, [r\, e_d' + x_d' E \sin\theta - r\, E \cos\theta] \tag{115}$$

In considering the machine without amortisseur windings e_q'' can change instantaneously so that the characteristics of the machine with respect to the quadrature axis can be determined by the constants which are used for determining the steady state performance. Therefore replacing in Equation (**109**)

e_d by e_d'
e_q by 0
and x_d by x_d'

$$\begin{aligned}\frac{\partial P}{\partial e_d'} = \frac{1}{(r^2 + x_d'\, x_q)^2}\, \{\, 2\, r\, (r^2 + x_q^2)\, e_d' \\ + [r^2\, (2\, x_d' - x_q) + x_d'\, x_q^2\,]\, E \sin\theta \\ - r\, [r^2 + x_q\, (2\, x_q - x_d')\,]\, E \cos\theta\, \}\end{aligned} \tag{116}$$

For the case in which the main field resistance is assumed equal to zero and the effect of the amortisseur in the direct axis is to be included, it is necessary to introduce a third voltage, namely, e_d'', which is proportional to the flux linkages with the amortisseur in the direct axis. The "subtransient reactance" in the direct axis, x_d'', must be used with e_d''. Equations (**111**) and (**112**) for this condition become

$$\begin{aligned}\frac{\partial P}{\partial e_d''} = \frac{1}{(r^2 + x_d''\, x_q'')^2}\{2\, r\, (r^2 + x_q'')\, e_d'' - 2\, r^2 (x_q'' - x_d'')\, e_q'' \\ + [r^2\, (2\, x_d'' - x_q'') + x_d''\, x_q''^2\,]\, E \sin\theta \\ - r\, [r^2 + x_q''\, (2\, x_q'' - x_d'')\,]\, E \cos\theta\end{aligned} \tag{117}$$

$$\begin{aligned}\frac{\partial P}{\partial e_q''} = \frac{1}{(r^2 + x_d''\, x_q'')^2}\{-2\, r^2 (x_q'' - x_d'') e_d'' + 2 r (r^2 + x_d''^2) e_q'' \\ + r\, [r^2 - x_d''\, (2\, x_d'' - x_q'')\,]\, E \sin\theta \\ + [r^2\, (2\, x_q'' - x_d'') + x_q''\, x_d''^2\,]\, E \cos\theta\, \}\end{aligned} \tag{118}$$

in which e_q'' is determined by Equation (**115**) and e_d'' by the following Equation (**121**) which is derived as follows.

$$e_d'' = e_d' - (x_d' - x_d'')\, i_d \tag{119}$$

But i_d from Equation (**104**) inserting the appropriate constants is

$$i_d = \frac{1}{r^2 + x_d'\, x_q}\, [x_q\, e_d' - r\, E \sin\theta - x_q E \cos\theta] \tag{120}$$

Substituting in (**119**)

$$\begin{aligned}e_d'' = \frac{1}{r^2 + x_d'\, x_q}\, [(r^2 + x_d''\, x_q)\, e_d' \\ + (x_d' - x_d'')\, (r \sin\theta + x_q \cos\theta)\, E\,]\end{aligned} \tag{121}$$

Appendix II

ENERGY PER CYCLE INCLUDING EFFECT OF WINDINGS IN BOTH AXES

1. Analysis will first be made of a salient pole machine with an amortisseur winding in the quadrature axis only.

Equation (**17**) gives the instantaneous value of e_d and substituting i_d from Equation (**104**) of the appendix in terms of e_d', e_q'', x_d', and x_q'', there results

$$\begin{aligned}e_d = \frac{1}{r^2 + x_d'\, x_q''}\, [(r^2 + x_d\, x_q'')\, e_d' - r\, (x_d - x_d')\, e_q'' \\ - (x_d - x_d')\, r\, E \sin\theta - (x_d - x_d')\, x_q''\, E \cos\theta]\end{aligned} \tag{122}$$

Obtaining $\Delta\, e_d$ from this equation and substituting in Equation (**13**)

$$\begin{aligned}(r^2 + x_d\, x_q'')\, \Delta\, e_d' - r\, (x_d - x_d')\, \Delta\, e_q'' \\ + (x_d - x_d')\, (x_q'' \sin\theta - r \cos\theta)\, E\, \Delta\, \theta \\ + (r^2 + x_d'\, x_q'')\, T_{do}\, \frac{d\, \Delta\, e_d'}{d\, t} = 0\end{aligned}$$

or

$$\begin{aligned}\Delta\, e_d' + T_d\, \frac{d\, (\Delta\, e_d')}{d\, t} - \frac{r\, (x_d - x_d')}{r^2 + x_d\, x_q''}\, \Delta\, e_q'' \\ = -\frac{(x_d - x_d')}{r^2 + x_d\, x_q''}\, (x_q'' \sin\theta - r \cos\theta)\, E\, \Delta\, \theta\end{aligned} \tag{123}$$

in which T_d is defined by

$$T_d = \frac{r^2 + x_d'\, x_q''}{r^2 + x_d\, x_q''}\, T_{do} \tag{124}$$

In Fig. 13 is shown the vector diagram including the nominal quadrature voltage, e_q. The nominal voltage is numerically equal to the instantaneous value of current in the quadrature axis; the positive sense being taken for values leading e_d'. The voltage e_q'' is the subtransient voltage in the quadrature axis and is called so because it corresponds to flux linkages associated with the damper winding, the double prime being used to correspond with subtransient reactances x_d'' and x_q''. For steady state conditions $e_q = 0$ and e_q'' is negative when the machine is operating as a generator.

Since there is no source of e. m. f. in the amortisseur winding, the voltage equation may be written

$$0 = e_q + T_{qo}''\, \frac{d\, \psi_q}{d\, t} \tag{125}$$

But since ψ_q may be shown to be equal to e_q'' in a manner analogous to relation $e_d' = \psi_d$ for the direct axis,

$$0 = e_q + T_{qo}'' \frac{d\, e_q''}{d\, t}$$

or

$$0 = e_q + T_{qo}'' \frac{d\, \Delta\, e_q''}{d\, t} \qquad (126)$$

But from Fig. 13 it may be seen that

$$e_q = e_q'' + (x_q - x_q'')\, i_q \qquad (127)$$

Substituting i_q from Equation (**105**) of the appendix, in terms of e_d', e_q'', and x_q'',

$$e_q = \frac{1}{r^2 + x_d'\, x_q''} [r\, (x_q - x_q'')\, e_d' + (r^2 + x_q\, x_d')\, e_q''$$
$$+ (x_q - x_q'')\, E\, (x_d' \sin\theta - r\cos\theta)\,] \qquad (128)$$

Obtaining $\Delta\, e_q$ from this equation and substituting in Equation (**126**)

$$(r^2 + x_q\, x_d')\, \Delta\, e_q'' + r\, (x_q - x_q'')\, \Delta\, e_d'$$
$$+ (x_q - x_q'')\, (x_d' \cos\theta + r\sin\theta)\, E\, \Delta\,\theta$$
$$+ (r^2 + x_d'\, x_q'')\, T_{qo}'' \frac{d\, \Delta\, e_q''}{d\, t} = 0$$

or

$$\Delta\, e_q'' + T_q'' \frac{d\, \Delta\, e_q''}{d\, t} + \frac{r\, (x_q - x_q'')}{r^2 + x_q\, x_d'}\, \Delta\, e_d'$$
$$= -\frac{(x_q - x_q'')}{r^2 + x_q\, x_d'}\, (x_d' \cos\theta + r\sin\theta)\, E\, \Delta\,\theta \qquad (129)$$

in which

$$T_q'' = \frac{r^2 + x_q''\, x_d'}{r^2 + x_q\, x_d'} \qquad (130)$$

Now if $\Delta\,\theta$ varies sinusoidally, $\Delta\, e_d'$ and $\Delta\, e_q''$ will also vary sinusoidally. Representing these quantities vectorially as $\underline{\Delta\,\theta}$, $\underline{\Delta\, e_d'}$, and $\underline{\Delta\, e_q''}$, and substituting in (**123**) and (**129**), there results

$$(1 + j\, 2\, \pi\, f\, T_d')\, \underline{\Delta e_d'} - \frac{r\, (x_d - x_d')}{r^2 + x_d\, x_q''}\, \underline{\Delta\, e_q''}$$
$$= -\frac{(x_d - x_d')}{r^2 + x_d\, x_q''}\, (x_q'' \sin\theta - r\cos\theta)\, \underline{E\, \Delta\,\theta} \qquad (131)$$

and

$$(1 + j\, 2\, \pi\, f\, T_q'')\, \underline{\Delta\, e_q''} + \frac{r\, (x_q - x_q'')}{r^2 + x_q\, x_d'}\, \underline{\Delta\, e_d'}$$
$$= -\frac{(x_q - x_q'')}{r^2 + x_q\, x_d'}\, (x_d' \cos\theta + r\sin\theta)\, \underline{E\, \Delta\,\theta} \qquad (132)$$

It will be observed that for $r = 0$, the second terms on the left hand numbers are zero so that $\underline{\Delta\, e_d'}$ and $\underline{\Delta\, e_q''}$ are independent of each other and may be solved separately. For other values of r, $\Delta\, e_d'$ and $\Delta\, e_q''$ will be dependent upon each other. Let the solution of the simultaneous Equations (**13**) and (**132**), be

$$\underline{\Delta\, e_d'} = (a + j\, b)\, \underline{\Delta\,\theta} \qquad (133)$$

and

$$\underline{\Delta\, e_q''} = (c + j\, d)\, \underline{\Delta\,\theta} \qquad (134)$$

Now
$$\Delta\, P = \frac{\partial\, P}{\partial\, e_d'}\, \Delta\, e_d' + \frac{\partial\, P}{\partial\, e_q''}\, \Delta\, e_q'' \qquad (135)$$

in which $\frac{\partial\, P}{\partial\, e_d'}$ and $\frac{\partial\, P}{\partial\, e_q''}$ are given by Equations (**111**) and (**112**), respectively.

The energy absorbed per cycle is obtained by inserting the instantaneous values of $\Delta\, e_d'$ and $\Delta\, e_q''$ and integrating the quantity $\Delta\, P \frac{d\, (\Delta\,\theta)}{d\, t}$ around one complete cycle. This has been done before and it was seen (Equation (**34**)) to be proportional to $\Delta\, e_{dm}' \sin\alpha$. In the case at hand this quantity is equal to be $\Delta\,\theta_m$ for $\Delta\, e_d'$ and $d\, \Delta\,\theta_m$ for $\Delta\, e_q''$. The energy per cycle thus becomes

$$W/\!\sim\; = \pi \left[b \frac{\partial\, P}{\partial\, e_d'} + d \frac{\partial\, P}{\partial\, e_q''} \right] (\Delta\,\theta_m)^2 \qquad (136)$$

in which b and d are defined by Equations (**133**) and (**134**) and determined by solving (**131**) and (**132**) and $\frac{\partial\, P}{\partial\, e_d'}$ and $\frac{\partial\, P}{\partial\, e_q''}$ are obtained by Equations (**111**) and (**112**).

2. The analysis for the case in which the amortisseurs in both axes are included and the resistance of the main field is assumed equal to zero will be discussed next. With zero resistance field, the flux linkages associated with the field winding, must remain constant. This is equivalent to assuming e_d' constant. The voltage corresponding to the flux linkages with the amortisseur winding in the direct axis will be designated e_d'' and the associated reactance x_d''. It follows, therefore, that the analysis will follow the same lines as for the case of the amortisseur winding in the quadrature axis only and finite field resistance, except that e_d' will be replaced by e_d'', x_d' by x_d'', and x_d by x_d'.

The equations determining $\underline{\Delta\, e_d''}$ and $\underline{\Delta\, e_q''}$ become, from (**131**) and (**132**)

$$(1 + j\, 2\, \pi\, f\, T_d'')\, \underline{\Delta\, e_d''} - \frac{r\, (x_d' - x_d'')}{r^2 + x_d'\, x_q''}\, \underline{\Delta\, e_q''}$$
$$= -\frac{(x_d' - x_d'')}{r^2 + x_d'\, x_q''}\, (x_q'' \sin\theta - r\cos\theta)\, \underline{E\, \Delta\,\theta} \qquad (137)$$

$$(1 + j\, 2\, \pi\, f\, T_q)\, \underline{\Delta\, e_q''} + \frac{r\, (x_q - x_q'')}{r^2 + x_q\, x_d''}\, \underline{\Delta\, e_d''}$$

$$= -\frac{(x_q - x_q'')}{r^2 + x_q x_d''}(x_d'' \cos\theta + r\sin\theta)\underline{E\,\Delta\,\theta} \qquad (138)$$

and

$$T_q'' = \frac{r^2 + x_d'' x_q''}{r^2 + x_d'' x_q} T_{qo}'' \qquad (139)$$

$$T_d'' = \frac{r^2 + x_q'' x_d''}{r^2 + x_q'' x_d'} T_{do}'' \qquad (140)$$

Now if $\underline{\Delta e_d''} = (g + j\,h)\,\underline{\Delta\,\theta}$ (141)

and $\underline{\Delta e_q''} = (k + j\,m)\,\underline{\Delta\,\theta}$ (142)

the energy absorbed per cycle is

$$W/\sim\, = \pi\left[h\frac{\partial P}{\partial e_d''} + m\frac{\partial P}{\partial e_q''}\right](\Delta\,\theta_m)^2 \qquad (143)$$

in which $\frac{\partial P}{\partial e_d''}$ and $\frac{\partial P}{\partial e_q''}$ are defined by Equations (**117**) and (**118**).

Appendix III

MACHINE WITH FIELD RESISTANCE = 0 AND AMORTISSEUR WINDING IN QUADRATURE AXIS ONLY

With a field winding having a resistance equal to zero the flux linking the winding in the direct axis must remain constant. The flux in the quadrature axis is still permitted to vary, the amount of which can be obtained from Equation (**132**) by letting $\underline{\Delta e_d'} = 0$, giving,

$$\Delta e_q'' = -\frac{1 - j\,2\,\pi f T_q''}{1 + (2\,\pi f T_q'')^2}\,\frac{(x_q - x_q'')}{r^2 + x_q x_d'}(x_d' \cos\theta + r\sin\theta)\,E\,\Delta\,\theta \qquad (144)$$

Since b in Equation (**136**) is zero and d is equal to the imaginary part of Equation (**135**), then

$$W/\sim\, = \frac{2\,\pi f T_q''}{1 + (2\,\pi f T_q'')^2}\,\frac{(x_q - x_q'')}{(r^2 + x_q x_d')}(x_d' \cos\theta + r\sin\theta)\,E\frac{\partial P}{\partial e_q''}(\Delta\,\theta_m)^2 \qquad (145)$$

in which $\frac{\partial P}{\partial e_q''}$ is determined by Equation (**112**).

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Discussion

H. E. Edgerton: The person who has never had experience with, or heard of self-sustained oscillation of synchronous machines is at first disturbingly puzzled by the idea. Why should the currents that are induced in the stator and rotor by momentary departures from synchronism tend to drive the machine further from synchronism? It appears to be a contradiction to Lenz's law which tells us that the induced currents set up a force that tends to stop the motion producing them. The induced currents in the rotor of an induction motor obey Lenz's law and tend to bring the rotor to synchronous speed and we would think that the induced currents in the field of the synchronous machine should be of the same nature.

There is however an important difference between the rotor of an induction motor and the field of a synchronous machine which, I believe, is the cause for self-sustained oscillations. The induction motor has a *polyphase* winding on the rotor while the field winding of the synchronous machine appears as a *single-phase* winding to the stator. An amortisseur winding tends to introduce a winding in the quadrature axis, thus allowing the rotor winding to approach a polyphase combination. In this discussion I wish to show how the theory of the single-phase rotor induction motor can be used to explain the phenomena of self oscillation of synchronous machines.

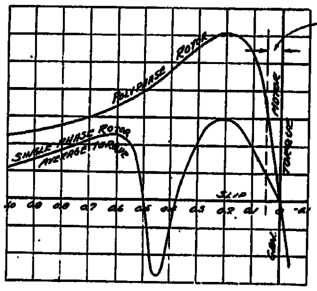

FIG. 1

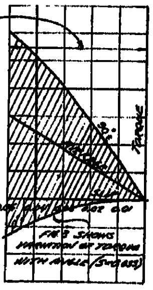

FIG. 2

FIG. 1—SLIP-TORQUE CURVES FOR AN INDUCTION MOTOR WITH A POLYPHASE ROTOR AND WITH A SINGLE-PHASE ROTOR

FIG. 2—DIAGRAM SHOWING INSTANTANEOUS RANGE OF VARIATION OF SLIP-TORQUE CURVE FOR AN INDUCTION MOTOR WITH A SINGLE-PHASE ROTOR

Fig. 1 herewith shows the speed torque curve of an induction motor with a polyphase rotor. If one rotor phase is opened the resulting curve will be that indicated also in Fig. 1 (see for instance, Arnold and La Cour "Die Inducktionsmaschinen," pages 186-196). The characteristic has the well-known dip at half speed. I wish to point out that this torque curve is, however, only an *average*. As the single-phase rotor winding rotates it alternately grabs and releases its hold on the rotating magnetic field at double slip frequency. The torque, we must then realize, is a function of *angular displacement* between the terminal potential and the axis of the single-phase winding as well as of slip. This is shown in Fig. 2 for the region from zero to 5 per cent slip. The instantaneous torque varies with the angle making an excursion over the shaded area between the boundaries and giving an average torque which is shown by the curve marked "average" on Fig. 2 and is shown over the complete range in Fig. 1. The effect of stator resistance is to cause the instantaneous torque to give generator action for small values of motor slip and angle.

Fig. 4 shows the wiring diagram for a test to show the effect of a single-phase rotor on the induction motor effect. A Westinghouse instantaneous polyphase-power oscillograph element recorded the power at the terminals of the motor with the external resistance between the infinite bus and the oscillograph. Other elements were used to record stator and rotor current, voltage of an open-circuited synchronous machine on the shaft, and a voltage from which angle could be computed.

The oscillogram, Fig. 5, shows that for part of the time the motor is acting as a generator and is actually delivering power. From this oscillogram were measured power input and angular displacement. These are plotted against each other in Fig. 3. For the no-load region about zero degrees of angular displacement the damping tendency of the induced currents is such as to cause an increase in slip rather than retard its progress. This action is the source of self-oscillation.

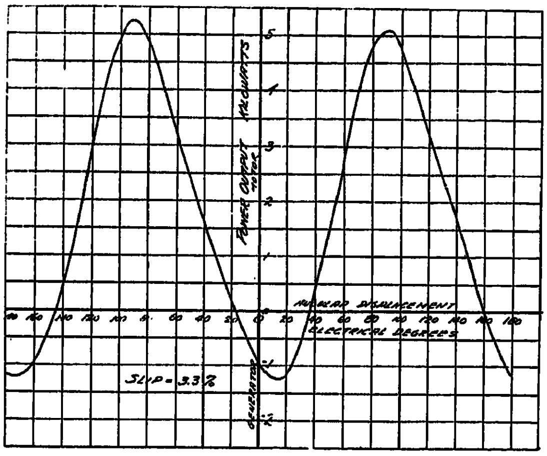

FIG. 3—EXPERIMENTAL INPUT-ANGLE CURVE FROM OSCILLOGRAM (FIG. 5) OF AN INDUCTION MOTOR WITH A SINGLE-PHASE ROTOR CIRCUIT. SLIP = 3.3%

The frequency of oscillation of a synchronous machine depends upon the inertias of the rotating machines and upon the synchronous torque-angle relations. This frequency depends upon what portion of the torque-angle curve is being used and upon the amplitude of the oscillation. The angle-time curves are not sinusoidal if the variation is large.

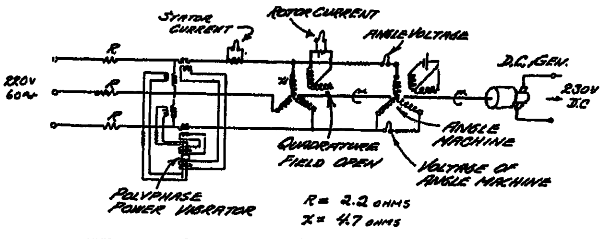

FIG. 4—WIRING DIAGRAM FOR OSCILLOGRAM (FIG. 5) TO SHOW INSTANTANEOUS VARIATION OF INPUT TO A POLYPHASE INDUCTION MOTOR WITH A SINGLE-PHASE ROTOR

The damping has very little effect on the frequency of oscillation. The damping, however, determines how fast transient oscillations are to decrease to zero or increase. Mr. Wagner shows very clearly how the energy per cycle from damping action is negative for the case where oscillations increase and positive for cases where the oscillations damp out. For sustained oscillations the energy per cycle of oscillation must be zero. That is if the damping is negative part of the time it must oscillate over enough degrees to include just enough positive damping to keep the total energy per cycle equal to zero.

If the machine oscillates about no-load it is important to notice that the maximum slip occurs at small angular displacements and so the negative damping effect, if any, will be accentuated. As the rotor swings into angular displacements which give positive damping the slip is much smaller and so the swing will need to be greater.

If the excitation is obtained from a constant voltage d-c. source, increasing the excitation has two effects: (a) It increases the frequency of oscillation because it stiffens the synchronizing torque thus influencing the manner in which the slip varies; (b) It causes the slip-torque characteristic to change because the external field resistance is decreased. A change in inertia also affects the range of self-oscillation because it causes a different variation of the slip by changing the period of oscillation.

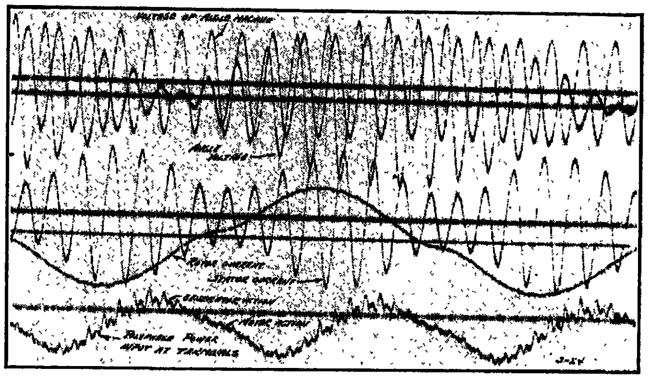

FIG. 5—OSCILLOGRAM SHOWING VARIATION OF INDUCTION MOTOR (WITH SINGLE-PHASE ROTOR) INPUT WITH ANGULAR DISPLACEMENT. CONNECTIONS FIG. 4. VALUES FROM OSCILLOGRAM FIG. 3

As Mr. Wagner points out the practical elimination of the self-oscillation phenomena depends upon the use of quadrature or amortisseur windings which tend to make the rotor appear as a balanced polyphase winding to the stator.

R. C. Bergvall: Mr. Wagner's analysis of hunting phenomena of synchronous machines has been interesting to me on account of accidentally having had experience which checks his analysis. In 1923, Mr. Evans and I wrote a paper covering tests on a miniature transmission system on which hunting was actually produced by increasing the field current with relatively high resistance in the lines. These tests were made at no load and the severity of the hunting was found to increase with an increase in field current. The same machine was tested both with and without damper windings and it required more field current to produce the same degree of hunting with damp windings. A couple of years ago while experimenting with high-speed excitation applied to a 150-kv-a. synchronous condenser which was connected to a large bus through about 15 per cent reactance, severe enough hunting to cause the condenser to pull out of step with the system was produced by the application of five times normal field current. This should not indicate that there is much possibility in actual practise of producing hunting with high-speed excitation even though five times normal voltage is applied to the slip rings during emergencies, because this high voltage is only used to obtain a rapid build-up of the field current and is dropped down when the field current reaches the proper value.

C. F. Wagner: I am very greatly interested in Mr. Edgerton's point of view regarding this problem and look foward to the time when I can analyze his method in more detail.

The 1923 tests to which Mr. Bergvall referred illustrated quite clearly the effect of excitation in armature resistance. The machine under test had no dampers and had considerable resistance in the armature circuit. We found during the test that with low excitation the machine was stable and as the excitation was increased the machine became unstable. It was unfortunate at the time that we did not continue the tests to bring out the complete cycle as we might have done. We could have increased the load on the machine and again made the machine stable after which, by decreasing the excitation, the machine could again be made unstable. Decreasing the load would have made the machine stable again. This procedure would have illustrated quite plainly the effect of both load and excitation.

The Synchronous-Repulsion Motor

A Special Development for the Photophone

BY H. C. SPECHT*

Member, A. I. E. E.

Synopsis.—This paper describes a motor which is developed specially for the operation of photophone machinery. The characteristics of this motor are analyzed and demonstrated by oscillograms. Since the well-known theory of synchronous and repulsion motor can easily be applied for this double motor, no equations are given. For illustration of the characteristics various oscillograms and test curves are given.

* * * * *

INTRODUCTION

THE exact speed of an ordinary motion picture projector is relatively unimportant provided it is kept within certain limits. Not only can the projector be operated at different speeds, but also it is possible, and usually desirable, to be able to vary the speed at the will of the operator. These simple requirements are quite easily met by an ordinary adjustable speed motor, such as a series motor or repulsion motor with some form of speed control.

Such a motor, however, would be quite unsuitable for driving a talking motion picture projector. In the first place, the sound record, whether it be on the film itself or on a disk driven synchronously with the projector, must be run at one definite speed in order that the musical sounds be reproduced at their correct pitch. Even more important than the fact that the record be run at the proper speed is the fact that once the speed is set, it must not be changed suddenly, for while only a few may have an ear for "absolute pitch," practically every one experiences a disagreeable sensation if the pitch suddenly changes. However, these requirements are easily met by a synchronous motor for the frequency of a modern power system is substantially constant.

However, a synchronous motor is not entirely satisfactory, for sometimes a theater will have mixed into its program, the ordinary silent pictures. In order to keep performances on schedule, these silent pictures must often be run at a different speed,—usually higher,—and the adjustable speed characteristics desirable with the ordinary projector are highly desirable in this case, too. Thus, the application requirement for this service is a driving set which can be operated either as a synchronous motor when desired, or as an adjustable speed motor which speeds up to, say, one and one-half times synchronous speed. These requirements could be met by a synchronous motor and an adjustable speed repulsion motor on the same shaft, but limited space requirements in a projection booth and simplicity in operation make it highly desirable to have this dual service provided by a single machine. Such a machine, which may well be styled a synchronous-repulsion motor, has been developed for this service and will be described in this paper.

Suppose two separate motors, a synchronous motor and a repulsion motor mechanically coupled in tandem to drive an external load, and further suppose both machines to be connected to a proper source of power, an external resistance or other speed-controlling means being provided for the repulsion motor. The repulsion motor can be used to bring the load and synchronous motor up to synchronous speed where the set will pull into step and operate in synchronism, provided of course that the torque of the repulsion motor at synchronous speed is less than the break-down torque of the synchronous machine as a generator plus the load torque, and more than the difference between the load torque and the breakdown torque of the synchronous machine as a motor. It is hardly necessary to mention that if the repulsion motor torque is less than the load torque, the synchronous machine will act as a synchronous motor, drawing power from the line and developing mechanical torque to assist the repulsion motor; if the repulsion motor develops torque in excess of that required by the load, the synchronous machine acts as a generator, absorbing the excess torque of the repulsion motor and feeding power back into the line.

However, a synchronous machine and a repulsion motor can be combined into a single machine using the same magnetic circuit and the same windings. Such a motor is shown diagrammatically in Fig. 1.

When the switch *g* is thrown to the upper position, the machine will operate as an ordinary repulsion motor, speed control being effected by means of the rheostat *h*, though of course this could be done by control of the line voltage by means of a transformer, resistance or reactance in the line circuit, etc. When the switch *g* is thrown to its lower position, the motor, for reasons to be explained later, will come up to speed, pull into step and operate as a synchronous motor. A flywheel is provided to prevent hunting during synchronous operation. Thus, the control of this motor is so simple that an inexperienced person can readily operate it.

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*Consulting Engineer, Westinghouse Elec. & Mfg. Co., Springfield, Mass.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

DETERMINATION OF THE SIZE OF MOTOR REQUIRED

As already mentioned, the projector must be run at speeds above synchronism. The power requirements at the highest speed fix the size of the motor for it can be designed as an ordinary repulsion motor, ample in size to carry this load at this speed as if there were no other considerations. The design of the motor being fixed, nothing remains to be determined except the direct current excitation necessary for satisfactory synchronous operation which can either be calculated or found experimentally.

CHARACTERISTICS OF THE SYNCHRONOUS REPULSION MOTOR

The characteristics of this motor are somewhat different from those of any conventional single machine, but closely resemble the characteristics of two machines in tandem as previously described.

It is generally known that when the slip-rings of such a motor are short-circuited, the armature will run at synchronous speed under certain conditions. As the action of this motor is easier to understand if the latter be excited by direct current, let us consider this case. Since the direct current is introduced into the armature winding at two points which are fixed with respect to the armature, the direct current will set up magnetomotive forces tending to set up a flux which is fixed in space with respect to the armature; that is, the direct current sets up magnetic poles similar to the poles set up in a non-salient pole turbo alternator. These poles lock in with the rotating field set up by the repulsion motor and thus the armature rotates at synchronous speed.

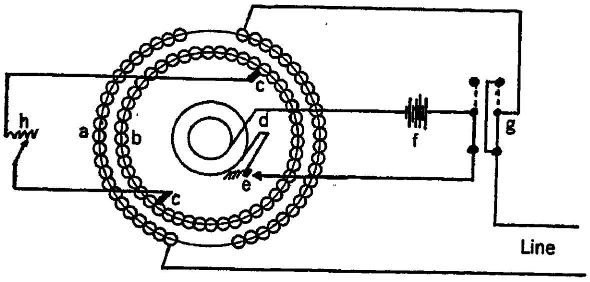

FIG. 1—COMBINED SYNCHRONOUS AND REPULSION MOTOR

a. Stator winding
b. Repulsion armature and d-c. exciting winding
c. Brushes on the commutator
d. Slip-rings and brushes on them
e. Centrifugal or automatic switch of some sort
f. Storage battery or other source of direct current
g. Double-pole, double-throw switch
h. Rheostat

Although the rotating member is excited from a direct current source, the exciting current and flux set up by the rotating member will pulsate strongly, mainly due to the high reaction of the a-c. field caused by the small air-gap, the low permeance of the laminated iron, and the distribution of the armature winding. The frequency of this pulsation is twice the line frequency, just as it is in an ordinary synchronous motor. A further pulsation is caused by a periodic short-circuiting of the slip-rings through the repulsion brushes. However, this effect is not very great since the contact resistance of the repulsion brushes is fairly high.

If the slip-rings be merely short-circuited, somewhat similar effects will be obtained, for the motor will run in synchronism with the line frequency under certain conditions and there will be pulsations of twice the line frequency in the slip-ring circuit due to having a winding rotating in the a-c. stator field. In this case, however, the effect is somewhat similar to that of a synchronous brake, as if the synchronous machine in the two-motor set used for illustration could not develop sufficient mechanical power and therefore could act only as a generator. The pull-out torque is approximately equal to the repulsion torque at synchronous speed. At light loads, the repulsion torque will overpower the synchronous braking torque, so to speak, and the machine will pull out of step and run above synchronous speed; if the slip-rings are merely short-circuited, the range of torques at which the motor will operate synchronously is rather limited and moreover the motor is unstable, tending to hunt, both of which facts make it unsuitable for photophone service. With d-c. excitation, the motor synchronizes more readily, is more stable over a larger range of loads and does not tend to hunt. Moreover with d-c. excitation, the pull-out torque will be approximately equal to the breakdown torque as a synchronous motor plus the repulsion torque at synchronous speed.

The d-c. excitation has another important effect, that of reducing the line current. The reasons for this are two-fold: First, the magnetizing component of the stator current is decreased, since the direct current sets up all or a portion of the revolving field which would normally be set up by the stator windings; that is, the repulsion motor, in effect, becomes armature-excited. Another effect, which becomes more important at light loads, is the reduction of the repulsion armature current. Perhaps an easy way to grasp the fundamental reason for this is to consider that at light loads, where the torque required by the load is less than the repulsion torque at synchronous speed, either the working flux or armature current must be less than it would be for repulsion operation at synchronous speed; the flux is essentially constant and therefore the repulsion armature current, and to some extent the stator current on account of reduced losses, is smaller than it would be if the motor were operating as a straight repulsion motor at synchronous speed.

The performance of this unusual motor can be figured from the theory of the synchronous motor and the theory of the repulsion motor. However, the many irregularities will make it difficult to obtain accurate results.

TEST DATA

Various oscillograms were taken on one of the photophone motors built. Fig. 2 shows the direct-current, line voltage, and stator current when the motor was run as a straight synchronous motor with the repulsion brushes lifted from the commutator. The oscillogram shows in a very pronounced manner the pulsation of the direct current and the distortion in the stator current. The applied a-c. voltage, of course, was not distorted.

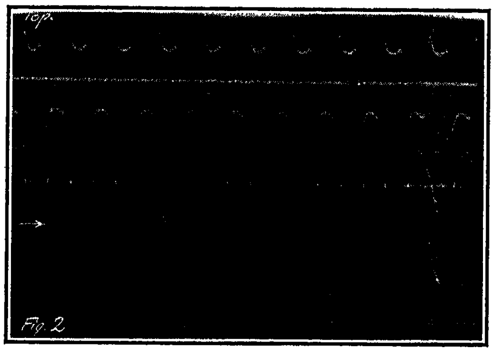

FIG. 2—AS SYNCHRONOUS MOTOR

12 amperes, d-c. excitation, repulsion brushes lifted, load = 12 oz-ft., 1.55 stator amperes. Top curve, stator a-c. amperes; middle, the line voltage; bottom = exciting current

In Fig. 3 are shown oscillograms of the direct voltage, direct current, and the alternating current through the short-circuited repulsion brushes when the motor was run as a synchronous-repulsion motor. Due to the short-circuited repulsion brushes, the pulsation of the direct current is slightly greater than shown in Fig. 2. The rotor current, or current through the brushes, shows some distortion.

In Fig. 4 are shown oscillograms of the line voltage, and of the stator and rotor currents, with the same load conditions as in Fig. 3. It will be noted that the stator current is distorted much more than the alternating rotor current, and that this distortion is quite different from that in Fig. 2 where the machine is run as a synchronous motor. This difference, of course, is due to repulsion motor action. In addition to the distortion caused by the pulsation in the exciting current, the curves show smaller ripples which are probably caused by the teeth and commutator bars. In Fig. 5 are shown oscillograms taken with the motor operated as a plain repulsion motor delivering the same output torque at a higher speed.

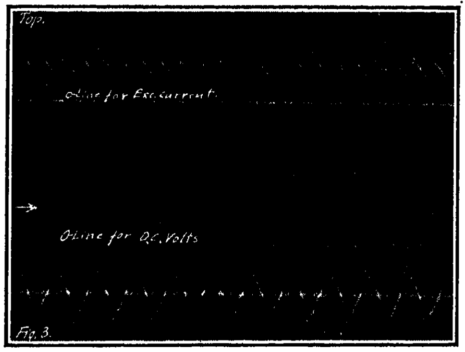

FIG. 3—AS REPULSION SYNCHRONOUS MOTOR

12 oz-ft. load, 220 volts, 1.6 amperes a-c., 1800 rev. per min. Top-d-c. volts across brushes of sling-rings; middle, d-c. amperes; bottom, rotor a c. amperes.

Figs. 6 to 11 show a few test curves of the same motor which was a ¼-hp., 60-cycle, 220-volt, 4-pole, single-phase machine. In Fig. 6 is shown the no-load saturation curve, the motor being externally driven at 1800 rev. per min. In Fig. 7 are plotted the input amperes and watts at different exciting currents when the machine was operated at no-load as a straight synchronous motor. At 15 amperes excitation, the alternating current has assumed its lowest value, and this checks with Fig. 6 where at 15 amperes the generated voltage is 220; *i. e.*, equal to the line voltage.

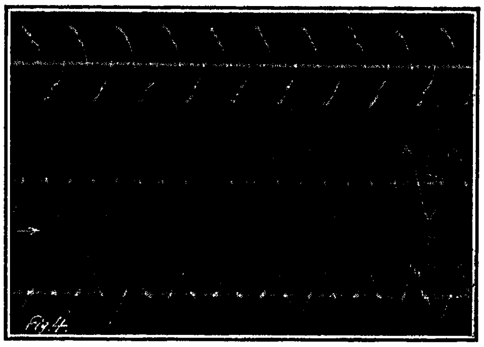

FIG. 4—AS REPULSION SYNCHRONOUS MOTOR

12 oz-ft. load, 1800 rev. per min. Top curve, stator a-c. amperes; middle 220-volt line volts; bottom, rotor a-c. amperes

Fig. 8 shows the results of a brake test as a synchronous motor with 12 amperes d-c. excitation, the repulsion brushes being lifted from the commutator. Fig. 9 shows the result of a brake test as a straight repulsion motor and Fig. 10 the result of a brake test as a synchronous-repulsion motor with 12 amperes excitation. Finally, Fig. 11 shows the results of a brake test as a synchronous-repulsion motor without d-c. excitation, the leads of the collector rings being short-circuited.

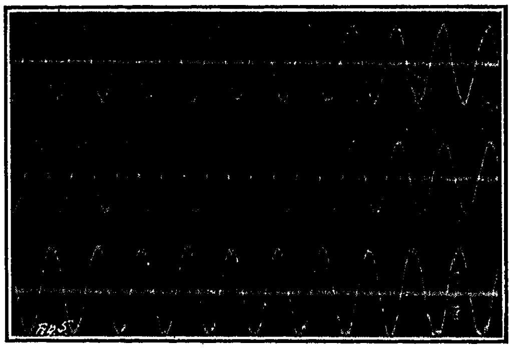

FIG. 5—AS REPULSION MOTOR

12 oz-ft. load, 220 volts, 2230 rev. per min., 2.74 stator amperes and 112.2 rotor amperes. Top, stator amperes; middle, line volts; bottom, rotor amperes

In these curves, the efficiency and watt input do not include the watts supplied from the d-c. source for excitation. The measured pull-out torque as a synchronous motor was found to be 20 oz.-ft. and the repulsion motor torque at 1800 rev. per min. was 26 oz. ft. These two torques added together give 46 oz-ft., and it was noted from test that the pull-out torque as a synchronous-repulsion motor was 50 oz-ft. This small difference may have been due partly to errors in test, for it is difficult to obtain accurate pull-out torque readings on a motor with a brake-arm.

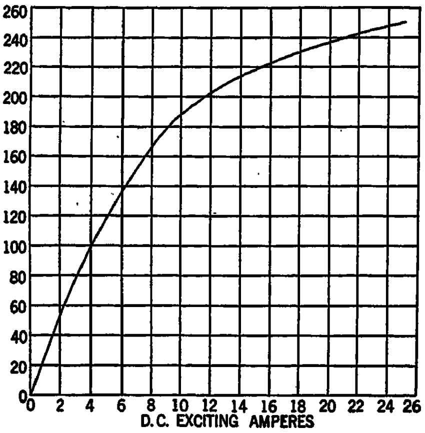

FIG. 6—SATURATION CURVE AT 1800 REV. PER MIN.

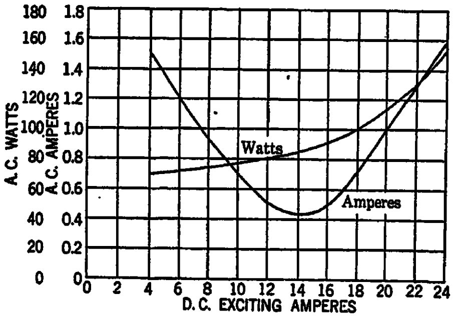

FIG. F—SATURATION AS A SYNCHRONOUS MOTOR WITH BRUSHES UP

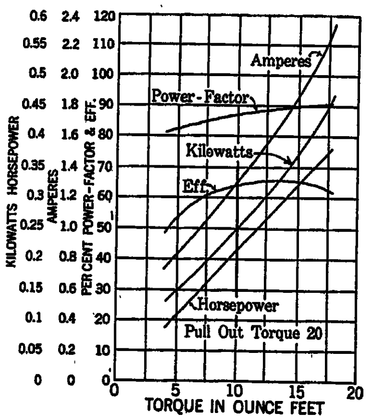

FIG. 8—BRAKE TEST AS SYNCHRONOUS MOTOR AT 1800 REV. PER MIN., 12 AMPERES DIRECT CURRENT

Further, it will be noted that under a heavy load, the performance as a synchronous-repulsion motor is the best; under a light load the efficiency as a synchronous motor is better because there are no brush friction losses nor losses in coils short-circuited by brushes as the latter are raised from the commutator. A brake test as a synchronous motor was also taken with the repulsion brushes down on the commutator and in this test, the watt input at lighter loads were as high as on the synchronous-repulsion motor test.

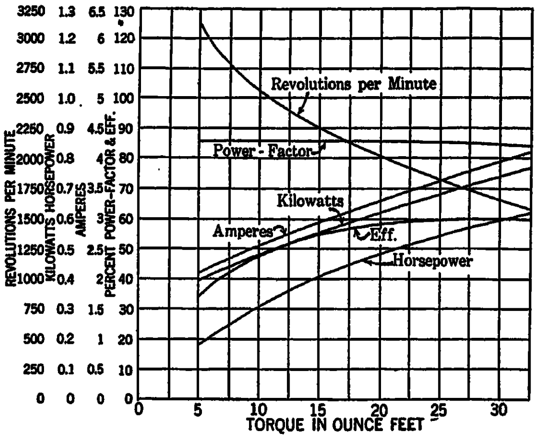

FIG. 9—BRAKE TEST AS REPULSION MOROR

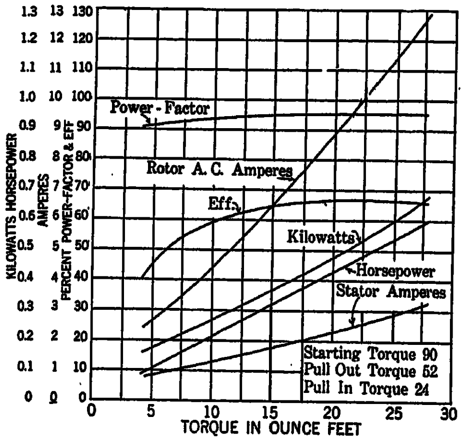

FIG. 10—BRAKE TEST AS SYNCHRONOUS REPULSION MOTOR AT 1800-REV. PER MIN., 12 AMPERES DIRECT CUENTRR

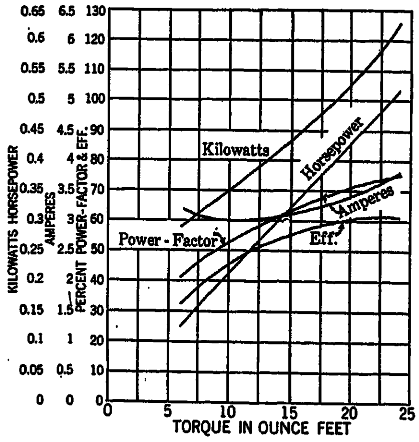

FIG. 11—BRAKE TEST AT 1800-REV. PER MIN., AS SYNCHRONOUS-REPULSION MOTOR WITHOUT D-C. EXCITATION

The pull-in torque as a synchronous-repulsion motor was observed to be 24 oz-ft. With d-c. excitation, the speed was very steady when in synchronous operation from no-load up to about pull-out, whereas, when the d-c. leads were short-circuited, the motor fell out of step at either 6 oz-ft. or 25 oz-ft.; *i. e.*, if the load torque were less than 6 oz-ft., the motor would pull out and run above synchronous speed, and at loads in excess of 25 oz-ft., the motor would pull out and run below synchronous speed. This demonstration shows quite clearly that without d-c. excitation, the machine cannot carry such a high overload torque as a synchronous motor as it can with d-c. excitation. Moreover, without d-c. excitation, hunting was quite noticeable. These conditions could have been improved somewhat by inserting some resistance between the repulsion brushes. Furthermore, it was observed that the commutator bars to which the slip-rings were connected, had a tendency to burn.

CONCLUSION

This photophone motor is very simple in operation, synchronizes readily at all loads, and can carry considerable overload. Furthermore, it is quite efficient and has a particularly good power factor. It is therefore quite satisfactory for photophone drives.

The author wishes to acknowledge gratefully the valuable assistance rendered him by Messrs. H. E. Newhouse and C. G. Veinott in performing the tests described and in preparing this paper.

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Discussion

S. Newman, Jr.: The Electric Specialty Company of Stamford, Connecticut, has recently developed a motor which combines the features of the repulsion type and the salient pole synchronous type of motor. This motor does not have any variable speed features and does not require any d-c. excitation. It is used for talking motion picture work or any application requiring synchronous operation where a small load is always connected to the motor. The small load is necessary because between no load and about 15 per cent of full load the motor runs above synchronous speed. From 15 per cent to 140 per cent of full load synchronous speed is maintained. The load at which slipping occurs, both over and under synchronism, can be controlled by shifting the position of the brushes.

The motor has very desirable characteristics as far as power factor and efficiency are concerned. The ¼ hp. and ⅓ hp. motors of the repulsion synchronous type have an apparent efficiency, *i. e.*, product of power factor and efficiency, of about 43 per cent as compared with about 20 per cent for split-phase salient pole synchronous motors of the same ratings.

The starting current on the repulsion synchronous motor of ¼ hp. is less than 15 amperes as compared with about 40 amperes for the split-phase type. This feature is especially important since the N. E. L. A. requirements allow 20 amperes starting current for motors of this size.

R. E. Hellmund: Mr. Specht has described in his paper a very interesting motor combination which is well suited for the particular application and possibly for others where similar requirements have to be met. The only drawback of this arrangement is that common to all synchronous motor arrangements, namely, the necessity for a source of d-c. power for the field excitation. For the size of motor involved, a battery as shown in Fig. 1 would serve the purpose, but it necessitates either the periodic replacement of dry batteries or means for charging a storage battery. It would therefore be much more desirable to use a copper-oxide rectifier, if the d-c. power requirements could be kept low enough to permit of the economical use of such a rectifier. In so far as the torque supplied by the motor during its synchronous repulsion operation is much in excess of the requirements, it would seem that operation at reduced voltage would give sufficient torque and at the same time permit a considerable reduction in the d-c. power requirements.

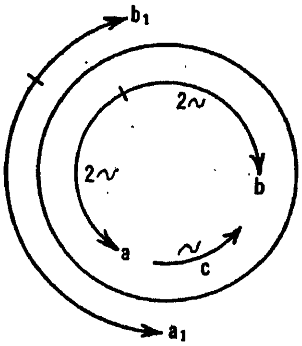

FIG. 1

The paper shows a number of oscillograms for the motor in its various operating conditions. Fig. 2, applying to the straight synchronous operation, indicates in particular a very marked third harmonic in the primary current curve. The reason for this is that the d-c. excited rotor represents at the same time a single-phase a-c. circuit. The secondary carries the double-frequency current common to motors with single-phase primaries. Since the secondary is also single-phase, the magnetomotive forces set up by the double-frequency currents can be represented in the usual way by two polyphase magnetomotive forces rotating in opposite directions relative to the rotor at double-frequency speed, as indicated by the arrows (*a*) and (*b*) in Fig. 1 herewith. Since the rotor has a mechanical synchronous speed corresponding to the line frequency, as indicated by the arrow (*c*), the speed of the rotating field (a) with regard to the stator is increased to three times synchronous speed, as indicated by the arrow (a_1), and the speed of the rotating field (b) with regard to the stator is reduced to the line frequency, as indicated by the arrow (b_1). The field (a_1) is responsible for the marked third harmonic indicated in Fig. 2 of the paper. All of this is, of course, qualitatively the same as in any ordinary single-phase synchronous machine without damper or squirrel-cage windings in the rotor. It may, however, be more marked in this case than in the usual salient-pole machines on account of the distributed windings in the rotor and the small air-gap, both of which tend to reduce the leakage coefficient and therefore the tendency of the leakage reactance to smooth out the higher harmonics.

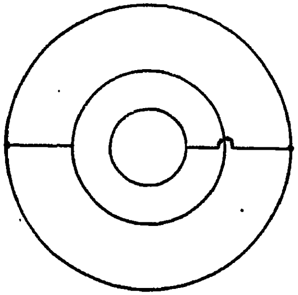

FIG. 2

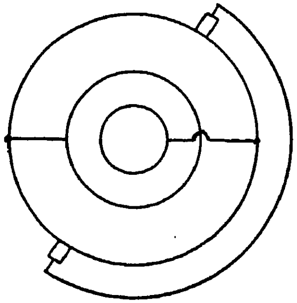

FIG. 3

As soon as a change is made from the pure synchronous operation to the synchronous repulsion operation, the short-circuited commutator brushes establish a second circuit through the rotor as shown in Fig. 3 herewith, and therefore a polyphase condition for all rotor positions except those for which the slip-ring taps happen to coincide with the commutator brush position. As a consequence, the backwardly rotating double-frequency magnetomotive force in the rotor is materially reduced, and so the third harmonic in the primary current of Fig. 4 is much less noticeable.

H. C. Specht: The synchronous repulsion motor mentioned by Mr. Newman is an ordinary repulsion motor with the exception that the periphery of the rotor possesses cutouts in the same way as is the general practise on synchronous induction motors. These cutouts are in number equal to the number of poles and the remaining projections make the salient poles. Such synchronous repulsion motors have occasionally been used for special applications for which a high starting and pull-in torque is required. The motor stays in synchronism very well at certain load and considerable overload. However, at light load, *i. e.*, at higher than synchronous speed it has a tendency to run out of step. For the application of photophone drive where the motor has to run occasionally at higher speed with greater load, the ordinary synchronous replusion motor is not adaptable.

Mr. Hellmund has mentioned the use of a rectifier for the d-c. supply, and this is certainly very desirable and there is no objection to the use of it. He also called attention to the fact that the motor I have described has considerable torque in excess when operated as synchronous repulsion motor. As a matter of fact this has been eliminated on the later motors, by a smaller d-c. excitation and by inserting a choke coil in the line supply circuit in order to reduce the applied a-c. voltage when operating as synchronous repulsion motor. By proper arrangement, the simple switching with the double throw switch is not affected. This scheme has naturally improved the performance of the motor.

Synchronous Motor Effects in Induction Machines[1]

BY E. E. DREESE[2]

Synopsis.—*Many induction motors do not have the smooth speed torque curves which are to be expected from elementary theory. Many motors at no load show a tendency to run at some speed way below that normally expected. This is sometimes called "sub-synchronous" speed; or the motor is said to be "crawling." In the present paper one cause of this phenomenon is explained. It is found that certain combinations of phases, rotor slots, and poles cause the motor to run as a synchronous motor at some intermediate speed between zero and normal no-load induction motor speed. This effect has been christened the "synchronous motor effect" in induction machines and is shown to be due to the locking of harmonic fields, generated by the stator winding with similar harmonic fields, generated by the rotor windings. The method of avoiding this trouble by the proper selection of the number of rotor slots is given.*

FROM time to time those responsible for the design of induction motors are disappointed when a new design proves to be noisy or develops what is called a sub-synchronous speed. The present paper will present a new cause for this phenomenon which will explain some cases of sub-synchronous speeds hitherto not understood.

The speed-torque characteristic of the classical induction motor is a smooth curve usually shown as in Fig. 1.

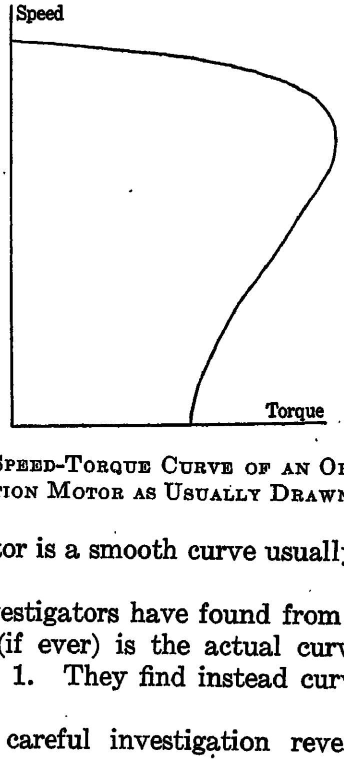

Fig. 1—The Speed-Torque Curve of an Ordinary Induction Motor as Usually Drawn

Various investigators have found from time to time that seldom (if ever) is the actual curve smooth as shown in Fig. 1. They find instead curves more like Fig. 2.

Still more careful investigation reveals that the curves are actually very "rough" near zero speed and Fig. 3 more nearly represents the true performance diagram.

In all the above curves the torque is shown as a single-valued function of the speed. This paper will show that in some cases the torque *is not* such a single-valued function and that none of the curves, such as the above, represents the true state of affairs.

The most plausible explanations advanced for these rough curves ascribe them to the presence of harmonics in the flux wave of the machine. As an example of this, consider a machine which has a prominent seventh harmonic in its flux wave. This harmonic, if due to the stator winding, will be revolving in the same direction as the fundamental and at one-seventh of the fundamental speed in a three-phase motor. It is then illuminating to consider that the motor is really a composite of two motors which tend to run at full speed and at one-seventh speed respectively. This might be shown as in Fig. 4.

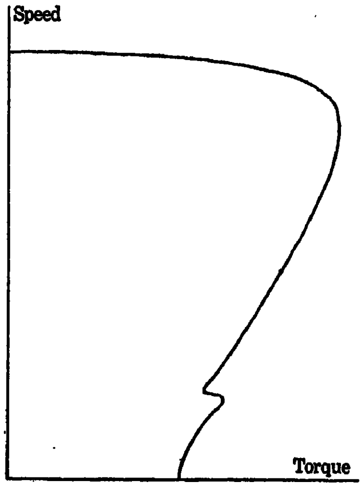

Fig. 2—The Speed-Torque Curve showing Subsynchronous "Hook" due to Induction Motor Effect of a Harmonic

This is a plausible explanation of some of the sub-synchronous speeds observed and without doubt the presence of harmonics will produce effects as shown in this figure. It will also serve to explain the roughness of the curves in the lower speeds because of the presence of higher harmonics. This effect will henceforth be called the "induction motor effect" of the harmonics.

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[1] This paper was submitted to the University of Michigan in partial fulfilment of the requirements for the E. E. Degree.

[2] Chairman, Electrical Engg. Dept., The Ohio State University, Columbus, Ohio.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Massachusetts, May 7-10, 1930.

From this point the development will be chronological as the calculations and experiments were made.

The work started on a squirrel-cage induction motor wound for the three-phase four-pole operation. The rotor had 28 bars. The motor showed a tendency to develop zero torque at one-seventh speed. The conclusion was therefore jumped at that this sub-synchronous speed was due to the "induction motor effect" of the seventh harmonic of the stator. Working with this in mind the reasonableness of this assumption was checked by making a mathematical check on the magnitude of the seventh harmonic in this particular motor by the use of Fourier's methods. Since these methods are well known they will not be gone into here and results only will be stated.

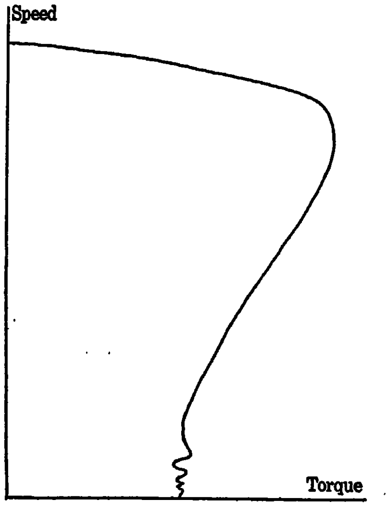

Fig. 3—Speed-Torque Curve showing Roughness Usually Found in Lower Speeds due to Higher Harmonics

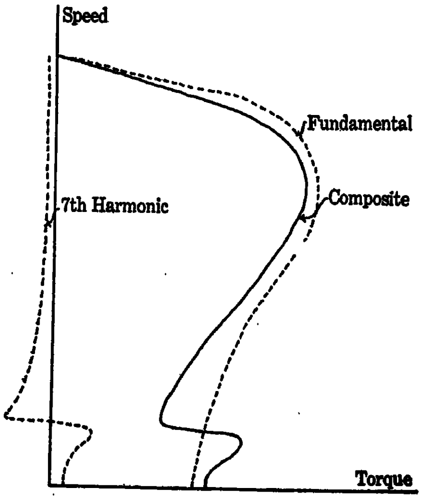

Fig. 4—Showing the Induction Motor Effect of a Forward Rotating 7th Harmonic

The stator in the present case had 24 slots. For three-phase and four poles this makes two slots per phase per pole. Now for a full pitch winding of one slot per pole per phase the relative magnitudes of the fundamental and seventh harmonic magnetomotive-forces in the stator are:

fundamental	7th harmonic
1	$\frac{1}{7} = 0.143$

The relative magnitudes for two slots per phase per pole are:

fundamental	7th harmonic
1	0.038

In addition the coils had been short spanned one slot and the two sides of a coil were 150 electrical degrees apart so that the relative magnitudes as they actually existed in the stator m. m. f. were

fundamental	7th harmonic
1	0.010

As the result of this calculation it did not seem reasonable to "assume" that a seventh harmonic of 1/100 the magnitude of the fundamental and revolving at 1/7 the speed of the fundamental could produce enough torque by the induction motor effect to overcome the torque of the fundamental at 1/7 speed.

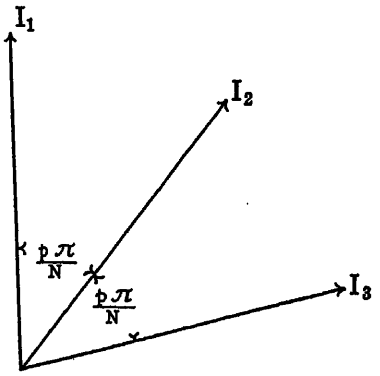

Fig. 5—N Current Vectors in Rotor Rods

Attention was then turned to the mathematical analysis of the rotor harmonics and after some false starts the magnitude, order, and speed of the harmonics originating in the rotor were worked out. The analysis follows.

Rotor Harmonics

Consider a squirrel-cage rotor of N bars being energized by a sinusoidal rotating field of p poles. The currents in the rods will then be $\frac{p\pi}{N}$ radians apart and the vector diagram of currents would be as shown in Fig. 5, in which I_1, I_2, and I_3 designate the currents in rods numbered 1, 2, and 3 respectively. For the purpose of analysis let the system of rods be replaced by N single-turn coils spanning 1 rotor slot pitch. Let the currents in the coils be i_1, i_2, and i_3 respectively (see Fig. 6). For the vector relations see Fig. 7.

It can then be seen that the actual rotor of N bars carrying currents of I magnitude $\frac{p\,\pi}{N}$ radians apart may be replaced by a rotor of N single-turn coils carrying currents of magnitude i and $\frac{p\,\pi}{N}$ radians apart provided,

$$I = 2\,i\sin\frac{p\,\pi}{2\,N}$$

It is now necessary to determine the m. m. f. waves set up by this system of N coils separated by $\frac{2\,\pi}{N}$ radians in space and carrying currents $\frac{p\,\pi}{N}$ radians apart in time phase.

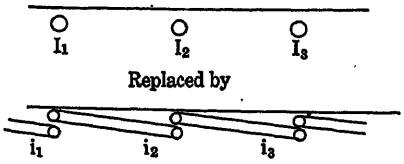

FIG. 6—SHOWING ROTOR RODS REPLACED BY COILS

First the m. m. f. set up by one such coil will be found.

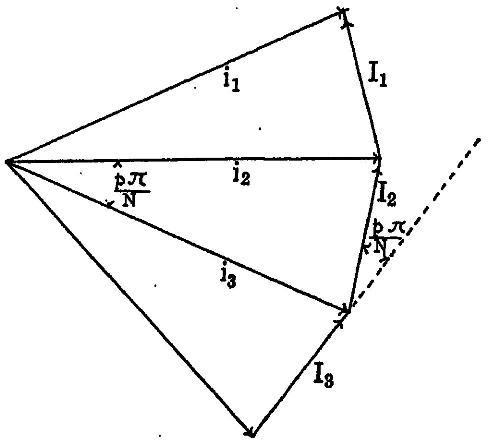

FIG. 7—SHOWING ROD CURRENTS REPLACED BY COIL CURRENTS

$I_1 = L_1 - L_2$ $\qquad I_2 = L_2 - I_3$ etc.

The form of the m. m. f. wave set up by one such coil due to an instantaneous current i is shown in Fig. 8.

The m. m. f. due to i will be divided between the two portions of the magnetic circuit in the ratio

$$\frac{\frac{2\,\pi}{N}}{2\,\pi - \frac{2\,\pi}{N}} = \frac{1}{N-1}$$

the high portion will be $\frac{N-1}{N}\,i$ and the low portion $-\frac{1}{N}\,i$. The space wave-shape shown in Fig. 8 can be expressed as a trigonometric series,

$$H = \frac{1}{2}\,b_0 + b_1\cos\theta + b_2\cos 2\,\theta + \ldots b_m\cos m\,\theta + \ldots$$

in which b_0 is obviously zero since the positive areas in Fig. 8 equal the negative areas. m is any positive integer. From the method of Fourier

$$b_m = \frac{2}{\pi}\int_0^{\frac{\pi}{N}}\left(\frac{N-1}{N}\,i\cos m\,\theta\right)d\,\theta$$

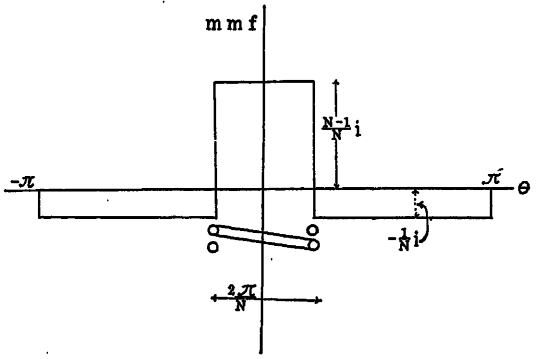

FIG. 8—M. M. F. DISTRIBUTION DUE TO CURRENTS IN ONE ROTOR COIL

$$+\frac{2}{\pi}\int_{\frac{\pi}{N}}^{\pi}\left(-\frac{1}{N}\,i\cos m\,\theta\right)d\,\theta$$

whence

$$b_m = \frac{2\,i}{m\,\pi}\sin\frac{m\,\pi}{N}$$

If the m. m. f. of coil number 1 be referred to as H_1 in which the current i is the instantaneous value of i_1

$i = \sqrt{2}\,i_1\cos\omega\,t$ (here i_1 is the magnitude only)

$H_1 = \Sigma\,b_m\cos m\,\theta$

$$= \sum\frac{2\sqrt{2}\,i_1}{m\,\pi}\sin\frac{m\,\pi}{N}\,.\,\cos m\,\theta\,.\,\cos\omega\,t$$

$$= \sum\frac{2\sqrt{2}\,i_1}{m\,\pi}\sin\frac{m\,\pi}{N}\left[\frac{1}{2}\cos(m\,\theta - \omega\,t) + \frac{1}{2}\cos(m\,\theta + \omega\,t)\right]$$

Thus each coil produces backward and forward moving waves since expressions of the type

$$\cos(m\,\theta - \omega\,t)$$

represent traveling waves of 2 m poles.

Waves of the type $\cos(m\,\theta - \omega\,t)$ will be called forward rotating and of the type $\cos(m\,\theta + \omega\,t)$ will be called backward rotating. In like manner the m. m. f. wave of the second and succeeding coils will be, $\left(\text{remembering that coils are separated by } \frac{2\,\pi}{N}\right.$ and carry currents $\left.\frac{p\,\pi}{N} \text{ apart}\right)$:

$$H_2 = \frac{2\sqrt{2}\,i_2}{m\,\pi} \sin\frac{m\,\pi}{N}\left[\frac{1}{2}\cos\left(m\,\theta - \omega\,t + \frac{2\,\pi}{N}m - \frac{p\,\pi}{N}\right) + \frac{1}{2}\cos\left(m\,\theta + \omega\,t + \frac{2\,\pi}{N}m + \frac{p\,\pi}{N}\right)\right]$$

and

$$H_3 = \frac{2\sqrt{2}\,i_3}{m\,\pi} \sin\frac{m\,\pi}{N}\left[\frac{1}{2}\cos\left(m\,\theta - \omega\,t + \frac{4\,\pi}{N}m - \frac{2\,p\,\pi}{N}\right) + \frac{1}{2}\cos\left(m\,\theta + \omega\,t + \frac{4\,\pi}{N}m + \frac{2\,p\,\pi}{N}\right)\right]$$

The resultant field is obtained by superposing m. m. fs. H_1 to H_N inclusive. Thus the rotating field of 2 m poles consists of N superimposed fields of equal magnitude since

$$i_1 = i_2 = i_3 = \ldots = i_n$$

For forward rotating fields these N superimposed fields of equal magnitude are arrayed with a phase difference between adjacent fields of

$$\frac{2\,\pi\,m}{N} - \frac{p\,\pi}{N} \text{ radians (see Fig. 9)}$$

The total angle for the N vector fields is

$$N\left(\frac{2\,\pi\,m}{N} - \frac{p\,\pi}{N}\right) = 2\,\pi\,m - p\,\pi$$

Since m is always an integer and p (the number of inducing poles) is always an even number, the pencil of vectors is always spread through an integral number of complete revolutions and the sum is therefore zero—with one exception.

That exception is the case when

$$\frac{2\,\pi\,m}{N} - \frac{p\,\pi}{N} = 2\,K\,\pi$$

where K is any positive or negative integer including zero. Hence forward rotating m. m. f. of $2\,m$ poles are produced by the rotor if

$$2\,m - p = 2\,K\,N$$

and

$$2\,m = 2\,K\,N + p \tag{A}$$

In like manner the backward rotating m. m. fs. form pencils, whose adjacent fields are separated by

$$\frac{2\,\pi\,m}{N} + \frac{p\,\pi}{N} \text{ Radians}$$

and in like manner, after the superposition, all those backward rotating fields vanish except those for which

$$2\,m = 2\,K\,N - p \tag{B}$$

The magnitude of the rotor harmonic of the 2 m poles is

$$H_m = N\left(\frac{2\sqrt{2}\,i}{m\,\pi}\sin\frac{m\,\pi}{N}\left[\frac{1}{2}\cos(m\,\theta - \omega\,t) + \frac{1}{2}\cos(m\,\theta + \omega\,t)\right]\right) \tag{C}$$

since only those fields appear in the result which are in phase in the N components.

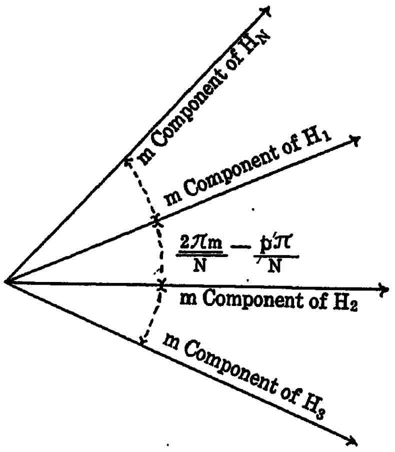

FIG. 9—SHOWING THE N COMPONENTS OF THE m FIELD

SYNCHRONOUS LOCKING

Returning now to the consideration of the 28-slot rotor, a table was made up showing the number of harmonic poles in both stator and rotor m. m. fs. In the above formulas, **A** and **B**, various values were substituted for p. These were the known fundamental and harmonic pole numbers of the stator m. m. f. In the present three-phase four-pole case, p may be 4, 20, 28, 44, 52, etc., for the fundamental fifth, seventh, eleventh, and thirteenth harmonics respectively. Then upon substituting various values for K, the various rotor harmonic pole numbers were found. These are shown in Table I.

TABLE I
28 SLOT ROTOR HARMONICS
Stator Poles P

4		20		28		44		52		68	
For.	Back.	For.	Back.	For.	Back.	For.	Back.	For.	Back.	For.	Back.
4 60 116	52 108 164	20 76 132	36 92 148	28 84 140	28 84 140	44 100 156	12 68 114	52 108 164	4 60 116	68 124 180 12	44 100 156

Soon after the above table was made up it was noticed that the four-pole stator field acting upon the 28-slot rotor produces a 52-pole backward rotating rotor harmonic (the 13th). There is also present in the stator a forward rotating 13th harmonic of the 52 poles. The question naturally arose as to how do these independently produced 52-pole fields react upon each other.

It was only a step to regard the 52-pole rotor field as the revolving field of a synchronous machine getting its excitation from the stator fundamental flux instead of from a d-c. source.

The next natural step was to find at what rotor speed these 52-pole fields had zero relative speed, that is, at what rotor speed are the two 52-pole fields locked together. Equation (C) above shows that the $2\,m$-pole field revolves at an angular velocity $\frac{\omega}{m}$ *relative to the rotor.* Where

$$\omega = 2\,\pi \text{ (rotor frequency)}$$

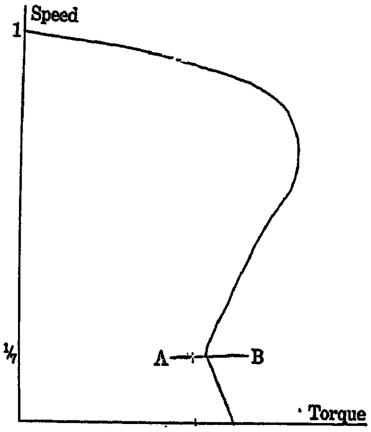

Fig. 10—Showing the Constant Speed-Torque Curve due to Synchronous Motor Effect

Now $\frac{\omega}{m}$ will be positive or negative according as the $2\,m$ field is rotating forward or backward *relative to the rotor.*

Now to find the rotor speed at which the 52-pole fields will lock, the procedure is as follows:

Let S = synchronous speed of stator 4-pole fundamental flux relative to the stator

n = speed of rotor relative to the stator

Then $\frac{S}{13}$ = speed of 52-pole stator harmonic relative to the stator

And $S - n$ = speed of fundamental stator flux relative to the rotor

Also $S - n$ = speed of fundamental rotor flux relative to the rotor

$-\frac{S-n}{13}$ = speed of 52-pole rotor harmonic relative to the rotor

$-\frac{(S-n)}{13} + n$ = speed of 52-pole rotor harmonic relative to the stator

If the speeds of the two 52-pole fields are now set equal to each other

$$\frac{S}{13} = -\frac{(S-n)}{13} + n$$

$$\frac{n}{S} = \frac{1}{7}$$

The two thirteenth harmonics lock at $\frac{1}{7}$ speed which is the observed sub-synchronous speed with the 28-slot 4-pole rotor. If this effect accounted for the observed performance of this motor is would differ from previously published explanations of sub-synchronism in being a *"synchronous motor effect"* superimposed upon the induction motor of normal operation. This synchronous motor effect suffered a setback at this point because experimental speed-torque curves available (such as those of Stiehl) showed the induction motor effect as in Fig. 4.

It was therefore decided to take the speed-torque curve of the motor carefully on a dynamometer. The speed was read with a tachometer accurate within probably 5 rev. per min. The speed torque was as shown in Fig. 10.

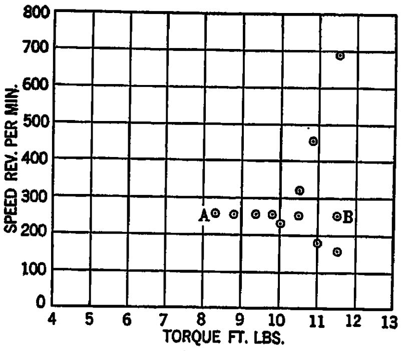

Fig. 11—Showing Actual Test Points at Constant Speed due to Synchronous Motor Effect

Within the limits of accuracy of the tachometer *there was* a synchronous motor effect evidenced here. Some representative points in the neighborhood of $\frac{1}{7}$ speed (257 rev. per min.) are shown in Fig. 11. *It appeared to be possible to vary the torque by 32 per cent at 257 rev. per min. without changing the speed.*

To make certain, the motor shaft was fitted with a stroboscopic disk with seven marks spaced at equal distances about the circumference. This was viewed through another disk with a slit in it revolving so as to give 1800 exposures per minute. See Fig. 12.

With the stroboscope the flat portion of the speed-torque curve was observed to be *absolutely flat.* The motor was running as a synchronous motor superimposed on an induction motor. The only effect on the stationary image of the seven-point disk as the load was varied was that a slight shift in the coupling angle was observed, as in a pure synchronous motor. As the torque approached points A or B in Fig. 10, the rotor would show hunting and would vibrate erratically and then drop out of step. The proof of the existence of synchronous motor effect was definitely established thereby.

At this point some definite experimental proof of the locking of the 13th harmonics was sought and a check-up on the mathematical work showed that not only were the 13 harmonics locking but a *whole infinite series of harmonics was locking at* $\frac{1}{7}$ *speed.*

This is shown as follows. In formulas **A** and **B** where

$$2m = 2KN + p \qquad \textbf{(A)}$$

and

$$2m = 2KN - p \qquad \textbf{(B)}$$

negative values of m have been inadmissible. A little study will reveal that the expression

$$2m = 2KN + p$$

will cover both Equations (**A**) and (**B**) if it be further stipulated that negative values of m denote negative rotation. It is necessary, however, that this new convention should satisfy Equation (**C**). We see that Equation (**C**) is satisfied since $\frac{1}{m} \sin \frac{m\pi}{N}$ is not changed by a change in the sign of m. The only change is in the direction of rotation as shown in $\frac{1}{2} \cos(m\theta - \omega t)$.

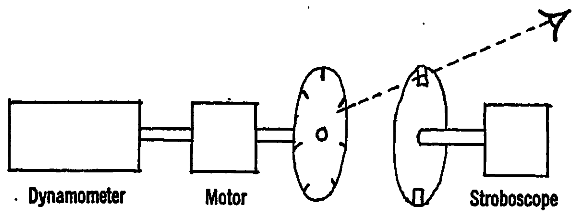

FIG. 12—STROBOSCOPE SET-UP FOR CHECKING THE CONSTANCY OF SPEED IN THE SYNCHRONOUS MOTOR EFFECT

PROOF OF EXISTENCE OF INFINITE NUMBER OF LOCKING HARMONICS

Let C be the stator harmonic which is acting on the rotor.

If C is negative it denotes negative rotation.

Now $$2m = 2KN + cp$$

If C_1 is any other stator harmonic with which $2m$ locks then

$$2m = \pm C_1 p$$

$$\pm C_1 p = 2KN + cp$$

$$\pm C_1 - C = \frac{2KN}{p} \qquad \textbf{(D)}$$

If any pair of harmonics of the stator can satisfy Equation (**D**) there will be stator and rotor harmonics of the same number of poles. Another condition is necessary for the synchronous motor effect; the relative speeds of the stator and rotor equi-polar fields must be zero.

$$\frac{S}{C} = \text{speed of } C \text{ relative to stator}$$

$$\frac{S}{C} - n = \text{speed of } C \text{ relative to rotor}$$

Now $$2m = 2KN + Cp$$

Speed of $2m$ relative to rotor is

$$\left(\frac{S}{C} - n\right)\frac{Cp}{2m} = \left(\frac{S}{C} - n\right)\frac{Cp}{2KN + Cp}$$

The speed of C_1 relative to the rotor $= \frac{S}{C_1} - n.$

Equating these speeds

$$\frac{S}{C_1} - n = \left(\frac{S}{C} - n\right)\frac{Cp}{2KN + cCp} \qquad \textbf{(12)}$$

$$\frac{S}{C_1} - n = \left(\frac{S}{C} - n\right)\frac{Cp}{\pm C_1 p}$$

$$\frac{S}{C_1} - n = \frac{S - nC}{\pm C_1}$$

Using (+) signs $$-n = -n\frac{C}{C_1}$$

Whence $C = C_1$. This gives the ordinary induction motor effect.

Using (−) signs $$2\frac{S}{C_1} = n\left(\frac{C}{C_1} + 1\right)$$

$$\frac{n}{S} = \frac{2}{C + C_1} \qquad \textbf{(E)}$$

This gives the synchronous motor effect and we use only (−) sign of **D**.

Therefore $$C + C_1 = -\frac{2KN}{p} \qquad \textbf{(F)}$$

If the stator harmonic numbers satisfy (**F**) there will be a synchronous motor effect at $\frac{2}{C + C_1}$ speed. There will be as many synchronous motor effects at

$\frac{2}{C + C_1}$ speed as there are pairs of values for C and C_1 which will satisfy (**F**). The number is infinite. As an example, take the 28 slot case considered here. The pair of values already considered were

$$C = 1 \text{ and } C_1 = 13$$

Other pairs are

C	13	19	25	31	37	etc.
C_1	1	−5	−11	−17	−23	

We thus see that there is an infinite series of locking pole numbers. The members of this series are

4 poles
20 poles
44 poles
56 poles
68 poles,
etc.

and they all lock synchronously at $\frac{1}{7}$ *speed.*

The discovery of the existence of an infinite series of synchronously locking harmonics caused a change in the point of view. Instead of regarding the operation at the sub-synchronous speed as that of a synchronous motor superimposed on the normal induction motor it is now necessary to consider the effect as due to *an infinite series* of synchronous motors superimposed upon the normal induction motor, all of these motors having the same synchronous speed. Some conception of what is going on may be obtained by imagining an induction motor and an infinite number of synchronous motors connected to the same shaft.

All experiments conducted up to this point had been with the use of a skewed rotor. Various rotors with different skews were then tried and it was found that the length of the synchronous portion of the speed-torque curves was very sensitive to skew, (see $A - B$, Fig. 10). With a reduction in skew the length $A - B$ increased until the point A crossed to the negative torque side of the speed axis. Under this condition the motor, when started under its own power, would not come up to full speed but would run at $\frac{1}{7}$ speed when started.

It was therefore decided that a rotor built in such a way that the skew could be *varied* should definitely establish whether or not the synchronous motor effect was due to one pair of harmonics or to a plurality of pairs. This can be shown by a little consideration of the nature of skewing.

EFFECT OF SKEW

If a rotor were skewed through two complete pole pitches there would be no voltage generated in the rod and as a consequence the rotor would act as though it had no winding for that particular pole pitch.

In an analogous manner it is possible to "skew out" any particular harmonic by the single expedient of skewing the rotor rod through two (or any multiple of two) poles of that harmonic. The word "analogous" is used advisedly because it is at this point where another difference between induction motor effect and synchronous motor effect is shown.

Suppose, for example, that the rotor were skewed to skew out the 13th harmonic. The skew would be through two poles of the 52-pole field. This amounts to 2/52 of the periphery of the rotor. Now with the rotor so skewed the rotor is inactive to the 13th harmonic of the stator and all rotor harmonics of which the 13th of the stator is the parent, (C) will not appear. But the rotor still has a 13th harmonic of which the stator fundamental is the parent, (see Table I). So there still exists a 13th stator harmonic rotating forward and the 13th rotor harmonic rotating backward in reference to the rotor. Now, however, the 13th harmonic of the rotor is skewed through a double pole pitch. It is just the same as if a salient pole synchronous machine had the salient poles skewed through a double pole pitch. It is obvious that under this condition no torque can be exerted between the two fields; they cannot lock.

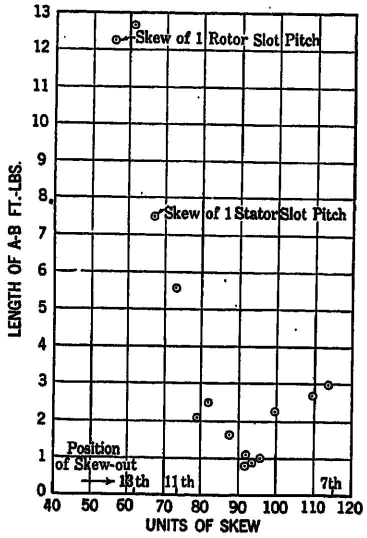

FIG. 13—SHOWING THE EFFECT OF SKEW ON THE TORQUE LENGTH OF THE CONSTANT-SPEED LINE

(Note. Accuracy of ordinates not greater than 0.5 ft. lb. Accuracy of units of skew within 2 units)

So with the rotor with variable skew it was only necessary to plot the length of the synchronous portion $A - B$ of the speed-torque against skew. If the synchronous motor effect were due to only one pair of harmonics the length of $A - B$ must go to zero when that particular harmonic was skewed out. The results of this experiment are shown in Fig. 13.

The result in Fig. 13 shows that no single harmonic is causing the synchronous torque. The minimum length for $A - B$ occurs between the points where the 11th and 7th harmonics are skewed out. There is indeed *qualitative* proof here that there are several synchronous motor effects overlaid. This is shown as follows.

Consider the theoretical effect of skew on each harmonic. As the skew is varied the length of *that portion* of $A - B$ due to a particular harmonic should vary as an attenuated sine wave. If such waves are superimposed as in Fig. 14, it is seen that the composite curve may look very much like the actual curve in Fig. 13. The 7th harmonic is neglected because of its low magnitude.

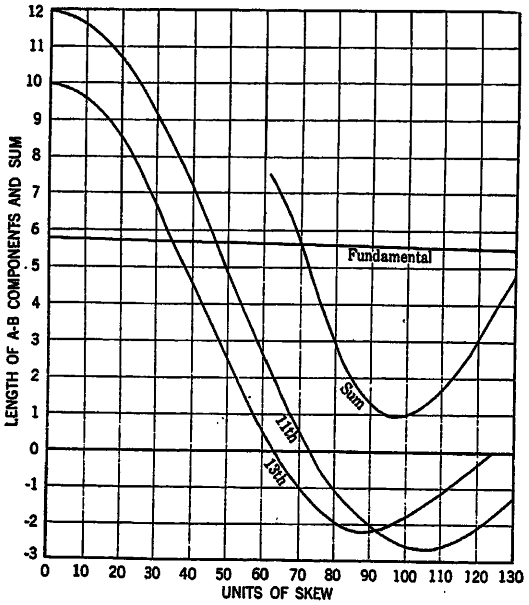

FIG. 14—POSSIBLE EXPLANATION OF FIG. 13 INVOLVING SUPERIMPOSED SYNCHRONOUS MOTOR EFFECTS

Equations for ordinates are of forms:

$A - B = \frac{\beta}{\alpha} \sin \frac{\alpha}{2}$ where β = a constant and α = angle of skew

DESIGNING TO AVOID SYNCHRONOUS MOTOR EFFECTS

It remains to predict the synchronous motor effect for other motors. It may be predicted from Equation (F).

$$C + C_1 = -\frac{2KN}{p}$$

Now in a three-phase motor

$$C = 6h + 1$$

where h is any integer and negative values of C denote negative rotation as before. Also

$$C_1 = 6h_1 + 1$$

and $$C + C_1 = 6(h + h_1) + 2 = -\frac{2KN}{p}$$

Replacing $h + h_1$ by L, any integer, and dividing by 2

$$3L + 1 = -\frac{KN}{p}$$

and $$\frac{N}{p} = -\frac{3L + 1}{K} \qquad \text{(G)}$$

The above formula gives the number of rotor bars per pole to be avoided to eliminate the synchronous motor effect. In this formula L and K are any integers. A few values are given in Tables II and III.

VALUES OF $\frac{N}{P}$ WHICH GIVE SYNCHRONOUS MOTOR EFFECTS

TABLE II

L

K	1	2	3	4	5
−1	4	7	10	13	16
−2	2	$3^{1}/_{2}$	5	$6^{1}/_{2}$	8
−3	$1^{1}/_{3}$	$2^{1}/_{3}$	$3^{1}/_{3}$	$4^{1}/_{3}$	$5^{1}/_{3}$
−4	1	$1^{3}/_{4}$	$2^{1}/_{2}$	$3^{1}/_{4}$	4
−5	$^{4}/_{5}$	$1^{2}/_{5}$	2	$2^{3}/_{5}$	$3^{1}/_{5}$

$\frac{N}{P}$

TABLE III

L

K	−1	−2	−3	−4	−5
1	2	5	8	11	14
2	1	$2^{1}/_{2}$	4	$5^{1}/_{2}$	7
3	$^{2}/_{3}$	$1^{2}/_{3}$	$2^{2}/_{3}$	$3^{2}/_{3}$	$4^{2}/_{3}$
4	$^{1}/_{2}$	$1^{1}/_{4}$	2	$2^{3}/_{4}$	$3^{1}/_{2}$
5	$^{2}/_{5}$	1	$1^{3}/_{5}$	$2^{1}/_{5}$	$2^{4}/_{5}$

$\frac{N}{P}$

When a sub-synchronous speed occurs it takes place at a speed

$$\frac{n}{S} = -\frac{p}{KN}$$

Thus in Tables II and III, all those values of $\frac{N}{P}$ in Table II give synchronous motor effects at positive speeds and those values in Table III give synchronous motor effects at negative speeds.

Corresponding tables of $\frac{N}{P}$ for two-phase motors are obtained from a formula similar to G. For two-phase the $3L$ in that formula is changed to $2L$

$$\frac{N}{P} = -\frac{2L + 1}{K}$$

The tables of $\frac{N}{P}$ are Tables IV and V:

TABLE IV

L

K	1	2	3	4	5
−1	3	5	7	9	11
−2	$1^{1}/_{2}$	$2^{1}/_{2}$	$3^{1}/_{2}$	$4^{1}/_{2}$	$5^{1}/_{2}$
−3	1	$1^{2}/_{3}$	$2^{1}/_{3}$	3	$3^{2}/_{3}$
−4	$^{3}/_{4}$	$1^{1}/_{4}$	$1^{3}/_{4}$	$2^{1}/_{4}$	$2^{3}/_{4}$
−5	$^{3}/_{5}$	1	$1^{2}/_{5}$	$1^{4}/_{5}$	$2^{1}/_{5}$

$\frac{N}{P}$

TABLE V

L

K	−1	−2	−3	−4	−5
1	1	3	5	7	9
2	$^{1}/_{2}$	$1^{1}/_{2}$	$2^{1}/_{2}$	$3^{1}/_{2}$	$4^{1}/_{2}$
3	$^{1}/_{3}$	1	$1^{2}/_{3}$	$2^{1}/_{3}$	3
4	$^{1}/_{4}$	$^{3}/_{4}$	$1^{1}/_{4}$	$1^{3}/_{4}$	$2^{1}/_{4}$
5	$^{1}/_{5}$	$^{3}/_{5}$	1	$1^{2}/_{5}$	$1^{4}/_{5}$

$\frac{N}{P}$

It is seen that in the case of the two-phase motor most synchronous motor effects occur at the same positive and negative speeds.

ACKNOWLEDGMENT

The author wishes to acknowledge the great help given by Mr. Gabriel Kron in checking the mathematics on the existence of infinite superimposed synchronous motor effects.

Since the preparation of this paper Mr. Kron has done considerable work on the causes of "hooks" and discontinuities in induction motor speed-torque characteristics. This material is in preparation for publication and should appear at an early date.

Discussion

W. V. Lyons: We are indebted to Professor Dreese for calling our attention to this interesting and little known point in the theory of induction motors. I agree entirely with his explanation of the results he observed experimentally. When we apply harmonic analysis to an induction motor we find that there is a multiplicity of flux distributions flitting to and fro in the air-gap due both to the stator and to the rotor currents. When the conditions are such that any two of these flux distributions of the same number of poles, one due to the stator and the other due to the rotor currents, are traveling at the same speed and in the same direction, these distributions will develop either forward or backward torque depending upon their relative position in the air-gap but *not* upon their direction of motion. This torque may be classed either as induction torque or as synchronous torque depending upon how the rotor component of flux is produced. Remember that only stator and rotor fluxes having the same number of poles can react to give a steady torque. If the rotor flux is produced by a rotor current that is induced by a stator flux of the same number of poles the torque is classed as induction. If, however, the rotor flux is produced by a rotor current that is induced by a stator flux of a different number of poles the torque is classed as synchronous. In the former case the two component fields always travel at the same speed regardless of the speed of the rotor. This is the characteristic of the torque developed by an induction motor. In the latter case, however, the two component fields travel at the same speed and thus develop a steady torque only at some particular rotor speed. This is the characteristic of the torque developed by a synchronous machine. For example if a seventh harmonic in the stator flux induces a current in the rotor which produces a corresponding seventh harmonic rotor flux, these two component fluxes will react to give an induction torque as shown in Fig. 4. On the other hand if, for example, the fundamental stator flux induces a current in the rotor which produces a thirteenth harmonic in the rotor flux this will react with the thirteenth harmonic in the stator flux distribution at one particular speed, *viz.*, one-seventh of synchronism, and produce synchronous torque.

It seems to me that the impression has been created that the seventh harmonic induction torque is much smaller than it really is. The value of 0.01 for the seventh harmonic does not show the true condition. Let us assume that this seventh harmonic is 1 per cent of the fundamental when there is no reaction due to the rotor, *viz.*, at no load. Now at one-seventh of synchronous speed the reaction of the rotor currents is to reduce the fundamental flux to about one-half of its no-load value but to reduce the seventh harmonic not at all. The stator current at this time is about 15 times its no load value so that the seventh harmonic flux is about 15 per cent of the *no* load fundamental flux or about 30 per cent of the fundamental flux at one-seventh of synchronous speed. It should also be noted that the seventh harmonic speed-torque curve is not like that of a motor having a constant applied primary potential. In the neighborhood of one-seventh of synchronous speed this torque is like that developed by an induction motor in which the stator current is maintained constant.

P. H. Trickey and C. R. Boothby: This paper has been of special interest to us as we have been studying the effect of the field form on the speed-torque curve of induction motors for some time. The induction motor effect has taken most of the first part of the study as the number of rotor slots does not have so much effect and the rotors which were available could be used for the tests on all the harmonics.

The special case of the 24 slot stator and the 28 slot rotor happened to occur in some experimental work and the synchronous effect was present exactly as Mr. Dreese has shown. In the unfortunate motor which we built the inner end of the cusp was negative and the outer end was 125 per cent of maximum torque. After determining that the effect was actually synchronous our first conclusions were that the 28 teeth of the rotor locked in with the 28 poles of the seventh harmonic in the same manner that an unexcited synchronous motor locks in. As Mr. Dreese has shown, the magnitude of the seventh harmonic in the three phase, two slots per pole, full pitch winding, such as we had, is only 3.82 per cent. It seemed impossible that this small value of flux could cause such a great value of torque. We then wound the primary with 5/6 chording which has only a 1.03 per cent seventh harmonic. Still the same synchronous action occurred. The tests were not accurate enough to determine whether or not the magnitude was less, at least, there was not any great difference.

The motor which we used was skewed one rotor slot. We then built a rotor which was skewed about one and three-quarters rotor slots. This reduced the length of the cusp only about 25 per cent.

We believe that Mr. Dreese has made a very valuable contribution in discovering and so clearly explaining this synchronous motor effect.

C. J. Koch: In the paper a phenomenon is described which to my knowledge has never appeared in Institute publications. It is therefore not out of place to make a few comments at this time in regard to a convenient terminology for this phenomenon. When an induction motor has a dip in its speed torque curve at some particular speed it may be said to have a tendency to crawl at that speed. If the dip reduces the torque to a sufficiently low value the motor will fail to attain full speed and therefore actually does crawl at a subsynchronous speed. If the tendency to crawl is due to the induction motor action of the rotor bars acted upon by either the stator tooth harmonics or phase belt harmonics the motor may be said to have a tendency to crawl asynchronously. If however the crawling tendency is due to the locking together of harmonic fields, the motor has a tendency to crawl synchronously. We therefore distinguish between "asynchronous crawling" and "synchronous crawling." These terms are offered simply because they have been used for some time by my associates and have proved adequate in speaking of these phenomena.

The question has been asked in the past whether some commercial use cannot be made of this phenomenon. Thus by exaggerating the synchronous crawling tendency motors could be made to run synchronously at low speeds with comparatively few poles and few slots. Very often the trouble is to get the motor synchronized. Slightly above or slightly below the crawling speed the motor torque is pulsating but the average torque is still positive due to the fundamental induction motor torque which is trying to accelerate the motor to full speed. Whether or not it synchronizes and crawls is partly a matter of chance and depends on the actual angular position of the locking fields at the instant the motor passes through the crawling speed. The inertia to be accelerated has of course an important influence. Very small single-phase motors have been made on this principle, however. Thus a shaded pole single-phase motor having two salient poles and six bars in the rotor is made to actually run synchronously at 1200 rev. per min. rather than asynchronously at 3600 rev. per min. as might be expected.

I should like to ask one question. Some years ago Mr. Alger studied this phenomenon from the point of view of permeance waves and concluded that whenever the difference between the numbers of stator slots and rotor slots was a multiple of the number of poles harmonic fields would be produced which would lock at some particular speed of the rotor. This speed would be equal to $\frac{\text{Poles}}{\text{Rotor Slots}}$ times synchronous speed and would be at either a forward or a backward speed depending on whether the motor was three phase or two phase and had more or less slots in the rotor than in the stator. These results agree with the present paper with the following exception. If the difference in slots equaled the number of poles times the number of phases the revolving harmonic fields locked when the rotor was at standstill. The commonly known tendency of some motors to tend to lock at standstill appeared to be one and the same phenomenon with synchronous crawling occurring only at a different speed. I should like to ask Mr. Dreese if in his studies he has been led to believe that the tendency to lock at standstill is the same sort of phenomenon as the synchronous crawling tendency.

L. V. Bewley: I should like to ask if it is not true that the English engineer, Chapman, many years ago wrote a very complete series of papers on the induction motor?

R. E. Hellmund: The fact which Mr. Dreese has established rather definitely, that we may meet with some synchronous motor effects in induction motors in addition to the various induction motor effects, makes an already difficult problem even more complex. Like most induction motor designers, I have previously met with the troublesome problem of cusps in speed-torque curves and as a result have given the subject a good deal of study. I have found that whenever the cusps are likely to be caused by induction motors effects, a great many possibilities exist for avoiding them. This is because an induction motor effect is impossible without a secondary current flowing in one of the members regardless of whether the superimposed fields are caused by the stator or by the rotor. This fact offers possibilities of various kinds, such as proper chording, various connections in the stator, proper pitch of rotor bars, etc., for making the flowing of any secondary currents, and therefore induction motor torques, impossible. With the synchronous locking effects described in this paper, some of these methods of attack are, however, impossible because synchronous locking is quite possible without the flowing of any secondary currents.

It is therefore evident that with the great many possible causes of both induction and synchronous torques, the number of different synchronous speeds, the necessity for designing motors for quiet operation, and the economical urge for covering various motor designs with as few slot combinations as possible, the designer's task is not a particularly easy one. However, there is a possibility of making a virtue out of a difficulty by purposely designing for subsynchronous speeds in certain small motor applications. This offers a means for designing slow-speed motors for such applications as synchronous clocks with a small number of poles and therefore a corresponding reduction in costs.

E. E. Dreese: I am glad to say that it is a great source of satisfaction to have the paper received with so much interest and comment.

The first part of Professor Lyons' remarks, as I gathered it, simply says in different language, the things that I tried to say in the paper although perhaps he says them better. The latter part of it has to do with the magnitude of the seventh harmonic that occurs in this motor. I show that the magnitude of the seventh harmonic of stator m. m. f. is about one per cent of the fundamental. He challenges that by saying that the magnitude of the torque coming from that seventh harmonic is more than one per cent and I agree but, if you will read the paper carefully, I think you will see that I simply used that one per cent to base my judgment on the fact that the seventh harmonic was not at fault. One per cent of magnetomotive force is correct. Twenty per cent of flux at that particular speed is also correct. The question of synchronous crawling at zero speed was brought up by Mr. Koch. He asks the question whether my study of this has revealed anything on synchronous crawling or locking at zero speed. In answer to that I can say that synchronous crawling at zero speed is not due to the synchronous motor effect described in this paper.

A considerable amount of water has flowed under the bridge since this paper was written a year ago. There are effects additional to those described in this paper such as the effect of revolving harmonic permeances. This effect acts much like the synchronous motor effect in its influence on the speed-torque curve and it might be called "reaction motor effect" due to the actual slotted structure of the iron, in which the revolving permeance features are brought out. There is every indication that it can explain the crawling at zero speed.

There is one important difference, I believe, between the synchronous motor effect and the reaction motor effect. The synchronous motor effect *cannot* be skewed out. I believe the reaction motor effect due revolving permeances can be skewed out.

I shouldn't close the discussion on this point without acknowledging the work of Mr. Gabriel Kron on this subject. He has gone considerably into this question and I think he has a paper now pending before the Institute which covers other causes for these cusps and hooks in induction motor curves and covers it more generally and more comprehensively than does the present paper.

The question has been asked as to whether or not some work done by Mr. Chapman in England a number of years ago did not predict this same thing. The work done by him had to do with the analysis of the harmonic waves due to the stator which is old to all of us. He went further, however, and analyzed the harmonics originating in the rotor. I discovered the paper after I had finished my analysis of rotor harmonics and I was glad to see that my results agreed with his. Mr. Chapman's work, however, did not at any time predict the thing we have described in the paper today. I have searched in vain to see any hint of it. Mr. Chapman does say there is such a thing as crawling due to harmonics but I do not believe the actual mechanism has been explained.

In this connection, you may be familiar with some experimental work done by Mr. Stiehl a few years ago, showing some very erratic speed-torque curves for induction motors. One of Stiehl's motors was a 24-slot stator, 28-slot rotor combination such as used in this paper. His curves did not show the synchronous motor effect but an induction motor effect instead, and it worried me for some time until I went into detail and found out he had got his curves on an inertia transient dynamometer and if the motor should go through the subsynchronous speed *continuously* his device would not have shown the synchronous locking. So, if any of you ever have recourse to that article of Stiehl's in which he describes these phenomena, you should bear in mind, that he was using a transient dynamometer which does not show the synchronous motor effect because of its limitations.

Transformer Ratio and Differential Leakage of Distributed Windings

BY R. E. HELLMUND* and C. G. VEINOTT†
Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper develops in as simple a manner as possible the necessity for the concept of "differential leakage" where transformer action takes place between two unequally distributed windings and explains the character of differential leakage. The value to be used for the transformer ratio in the case of such unequally distributed windings is also given. All considerations are based on the conventional stepped-wave field forms, and the mathematics used involve only the simplest linear equations. While the paper is mainly intended to be expository in nature, a general indication is given at the end as to how differential leakage and transformer ratios can be calculated for both single-phase and polyphase arrangements.

INTRODUCTION

In many a-c. motors in which transformer actions take place between two windings, certain problems arise in determining the proper transformer ratio for the two windings due to the fact that, for various reasons, the two windings are not distributed or arranged alike over the circumference. Furthermore, it is found expedient in such cases to introduce the concept of "differential leakage" in order to make the motors and their transformer actions subject to the ordinary analysis and calculating methods used for transformers.

Although Adams[3] called attention to the existence of differential leakage under the name of "belt leakage" as early as 1904, the nature of this phenomenon seems to be very little understood even today. This may be partly attributed to the fact that very little has been published on this subject in the technical press of this country. In Europe the subject matter has received a good deal of attention, as can be seen from the accompanying bibliography. However, many of the papers given there are difficult to follow, requiring considerable mathematical skill involving harmonic analysis and manipulation of Fourier's Series. Other methods as given by one of the authors of this paper show graphical solutions of the problem and are well adapted for deriving practical results for work with polyphase motors, but they are less suitable for demonstrating and explaining the fundamental nature of the phenomena involved.

Considerable confusion has also been caused by the fact that quite a variety of names has been applied to the same phenomenon. Adams calls it "belt leakage;" Hellmund and Krondl, "differential leakage;" Rogowski and Simons, "doubly-interlinked leakage;" Sumec, "unequally-interlinked leakage;" and Baffrey, "voltage of the higher harmonics." Further complication has arisen because, with some methods, it is more convenient to derive the combination of differential leakage and what is known as "zigzag" or "tooth-top leakage," and as a result differential leakage has sometimes been referred to as part of the zigzag or the tooth-top leakage, and the like. In view of this situation, this paper is offered principally as a contribution to clarify certain points regarding the transformer ratios for distributed windings, to set forth and explain the concept of differential leakage, to demonstrate the need for this concept in certain cases, and to show how it is comparable to other forms of leakage. The considerations used are based on the conventional stepped-wave, representing field forms with which every electrical student is familiar, and although it has not been found possible to avoid mathematics entirely, that used is of the simplest kind, involving nothing but a few simple equations with one or two unknown quantities. No attempt has been made to deduce complete new calculating methods or to add another long set of formulas to an already long set of existing ones. Incidentally, the outline given shows how the transformer ratio and differential leakage of a single-phase motor can be computed, but the paper is intended mainly to be expository in nature.

CONCENTRATED AND EQUALLY DISTRIBUTED WINDINGS

Leading up to the case of unequally distributed windings, brief reference may first be made to concentrated and equally distributed windings. The simplest example of concentrated windings magnetically coupled is that found in transformers. In this case there is always some flux which links only one winding in addition to the flux common to both windings. The former flux is called "leakage flux" and it is very easy to obtain a correct physical concept of it. Moreover, in such a case, the transformer ratio is simply the ratio of turns. The various relations are generally well known. By using the leakage coefficient, as described and defined in Appendix I, and the ordinary conception of transformer ratio, it is possible to calculate the voltages or currents in either winding regardless of which of the windings is used as the primary, and it is also easy to determine the various relations between currents and

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*Chief Electrical Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

†Electrical Engineer, Westinghouse Elec. & Mfg. Co., East Springfield, Mass.

3. For references see Bibliography.

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voltages when the secondary is open or when it is short-circuited.

The relations are equally simple if the two windings are distributed either evenly or unevenly over the circumference of a motor, as long as the distribution and general arrangement is the same in both windings, as shown, for example, in Fig. 1. Here again, the transformer ratio is simply the ratio of the number of turns. It will be seen that in this figure there is always a wound secondary slot opposite a wound primary slot, and the ratio of turns is the same in all pairs of opposite slots, that is, 2 to 1 in this particular case. It is also evident that the field set up will have the form shown in Fig. 2 regardless of whether it is built up by the primary or by the secondary winding. Furthermore, let us neglect slot and coil-end leakages and the following relations will apply:

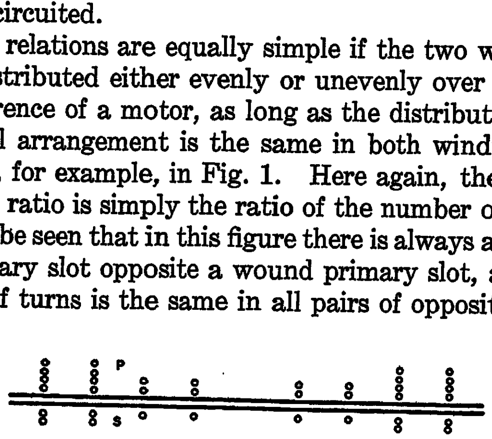

Fig. 1—Equally Distributed Windings

Open-circuit Condition. (a) Ratio of currents producing the same total flux. Since the winding S has half as many turns as the winding P, and since the field form set up by both windings is the same, it is evident that it requires twice as much current in winding S as it does in winding P to set up the same total flux. For any value of such flux, we have

$$\frac{I_1}{I_2} = \frac{1}{2} \qquad (1)$$

where I_1 = current in winding P

I_2 = current in winding S

(b) Ratio of open-circuit voltages—winding P excited. The voltage induced in the two windings is evidently proportional to the number of turns. Therefore, with winding P excited, we have

$$\frac{E_1}{E_2'} = 2 \qquad (2)$$

where E_1 = voltage impressed on winding P

E_2' = voltage induced in winding S

(c) Ratio of open-circuit voltages,—winding S excited. It is equally evident that, with winding S excited, we have

$$\frac{E_1'}{E_2} = 2 \qquad (3)$$

where E_2 = voltage impressed on winding S

E_1' = voltage induced in winding P

It is thus seen that, neglecting slot and coil-end leakages, the voltage ratio of the two windings, if either of the windings is excited, is the same as the ratio of the turns, and the current ratio is the reciprocal of this ratio.

Short-circuit Condition. Furthermore, if we assume the secondary winding as short-circuited and having zero resistance and zero leakage reactance, the voltage induced in it must be zero. The currents in the two windings flow in opposite directions, and so the resultant field as a whole, as well as at all parts of the gap surface, is zero. The voltage induced in the primary winding, if the latter is without resistance or leakage reactance, is zero as well. (If the well-known slot leakages and coil-end leakages are not neglected, the usual relations, as set forth more in detail in Appendix I, obtain.)

The above-mentioned obvious conditions existing in equally distributed windings have been recited principally for the sake of comparison with conditions to be found later for unequally distributed windings, and also to show that, with concentrated or equally distributed windings, the ordinary ratio of turns is sufficient for all ordinary calculations, and that the concept of differential leakage in such cases is not needed since the phenomena coming under this name do not exist.

Unequally Distributed Windings

For the purpose of studying how these conditions change in the case of two unequally distributed windings, a very simple arrangement of this kind, as shown in Fig. 3, will be considered. To bring out more clearly the points to be made, it is assumed that the primary and secondary have the same number of slots and that such slots are opposite each other. This eliminates the possibility of any so-called zigzag or tooth-top leakage and thus avoids any complication which would be introduced by its existence. The ratio of turns is again assumed to be 2 to 1, but the distribution of the turns is different in the two windings, as shown in the figure.

Open-circuit Conditions. With these assumptions for the unequally distributed windings, some of the same relations which were previously determined for the equally distributed windings will now be studied.

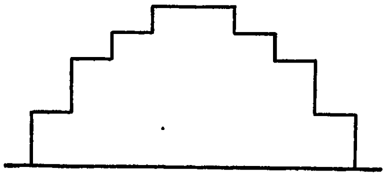

Fig. 2—Field Form of Either Winding of Fig. 1

(a) Ratio of currents producing the same total flux. In order to determine the ratio of the respective currents which, when flowing in the two windings will produce the same total flux across the air-gap, a current (I), flowing in winding P only, may first be assumed. It will set up a field as represented by the area ($a\,b\,c\,d\,e\,f\,g\,h$) in Fig. 4, and the total flux will be proportional to this area. The magnetization factor $K\,m$, defined as the ratio of the actual flux produced by the distributed winding to that which would be produced by the same total turns concentrated in the outermost slots, is 0.75 for this particular case (see Appendix II).

If we assume the same current flowing in winding S, a field proportional to the area $(m\,n\,o\,p\,q\,r\,s\,t)$ in Fig. 5 will be set up. The magnetization factor for this case is found to be 0.625 (see Appendix II).

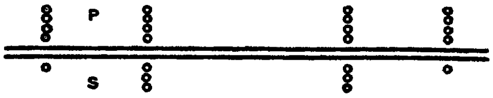

FIG. 3—UNEQUALLY DISTRIBUTED WINDINGS

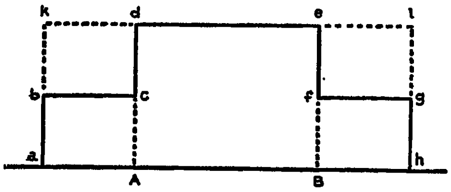

FIG. 4—FIELD FORM OF WINDING P, FIG. 3

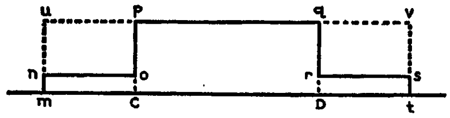

FIG. 5—FIELD FORM OF WINDING S, FIG. 3

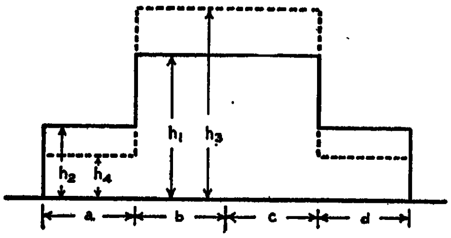

FIG. 6

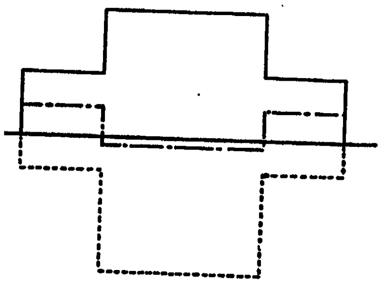

FIG. 7—FIELD FORMS, P EXCITED, S SHORT-CIRCUITED

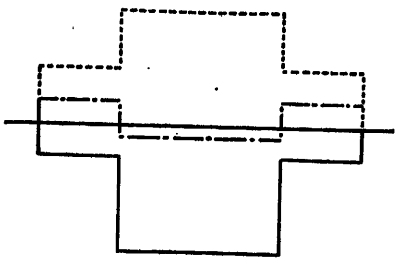

FIG. 8—FIELD FORMS, S EXCITED, P SHORT-CIRCUITED

From the above definition of the magnetization factor, it follows that the total flux produced by a distributed winding is equal to the flux which would be produced if all the turns in the distributed winding were concentrated in the outermost slot, multiplied by the magnetization factor. The fluxes which would be produced if the windings were concentrated are proportional to the areas $(a\,k\,l\,h)$ and $(m\,u\,v\,t)$ respectively. If the fluxes set up by the two windings are to be equal, the following obtains:

$$k_{m1} \times \text{area } (a\,k\,l\,h) = k_{m2} \times \text{area } (m\,u\,v\,t) \tag{4}$$

It is further evident that the area $(a\,k\,l\,h)$ is proportional to the total ampere-turns in winding P, and the area $(m\,u\,v\,t)$ to the total ampere-turns in winding S. Therefore, from Equation (4),

$$k_{m1} \times I_1 \times T_1 = k_{m2} \times I_2 \times T_2$$

or

$$\frac{I_1}{I_2} = \frac{k_{m2}\,T_2}{k_{m1}\,T_1} \tag{5}$$

if T_1 and T_2 are the numbers of turns in winding P and S respectively.

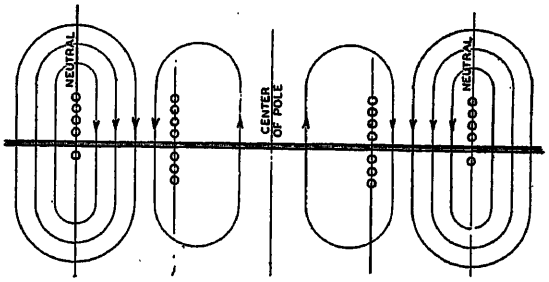

FIG. 7A—LINES OF FORCE FOR CONDITIONS OF FIG. 7

In our particular case, we find

$$\frac{I_1}{I_2} = \frac{0.625 \times 4}{0.750 \times 8} = 0.417 = \frac{1}{2.4} \tag{6}$$

In other words, the ratio of the currents producing the same total air-gap flux is 1 : 2.4, while the ratio of the actual turns is 2 : 1.

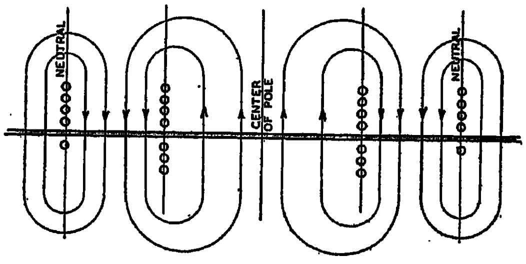

FIG. 8A—LINES OF FORCE FOR CONDITIONS OF FIG. 8

(b) Ratio of open-circuit voltages,—winding P excited. The voltage induced in either winding is proportional to the total number of turns multiplied by the voltage factor k_p, a factor which takes into account both the distribution of the air-gap flux and the distribution of the turns in the winding. The ratio of voltages is, therefore,

$$\frac{E_1}{E_2'} = \frac{T_1 \times k_{p1}}{T_2 \times k_{p2}'} \tag{7}$$

where k_{p1} = voltage factor for winding P
k_{p2}' = voltage factor for winding S
with the field set up by winding P.

As shown in Equations (65) and (66) of Appendix II, k_{p1} and k_{p2}' are equal to 0.8333 and 0.75 respectively for this particular arrangement of coils, and so the ratio of voltages becomes

$$\frac{E_1}{E_2'} = \frac{8 \times 0.8333}{4 \times 0.75} = 2.222 \qquad (8)$$

(c) Ratio of open-circuit voltages,—winding S excited. In this case the voltage ratio is

$$\frac{E_1'}{E_2} = \frac{T_1 \times k_{p1}'}{T_2 \times k_{p2}} \qquad (9)$$

where k_{p2} = voltage factor for winding S
k_{p1}' = voltage factor for winding P
with the field set up by winding S.

Using the voltage factors found in Equations (67) and (68) of Appendix II, this ratio becomes

$$\frac{E_1'}{E_2} = \frac{8 \times 0.90}{4 \times 0.85} = 2.11765 \qquad (10)$$

Thus far, the following has been established:

1. Ratio of total turns = 2.000
2. Ratio of currents producing the same total field = 0.417 = $\frac{1}{2.4}$
3. Ratio of voltages if the field is built up by currents in winding P only = 2.222
4. Ratio of voltages is the field is built up by currents in winding S only = 2.11765

It is obvious from this comparison that the calculations just made give no definite indication regarding the proper value to be used for the transformer ratio with these unequally distributed windings.

Short-circuit Conditions. A study of the short-circuit conditions will indicate how these apparent discrepancies can be eliminated.

First, let it be assumed that the secondary winding S is short-circuited and that, as before, its resistance and leakage reactance caused by slot and coil-end leakage can be neglected. Under these assumptions, the net voltage induced in this winding must be zero. The currents in the two windings will, therefore, assume such values as to satisfy this condition. Fig. 6 shows again the field of Fig. 4 as produced by a current in the primary winding in full lines, and also the field of Fig. 5 as produced by a current in the secondary windings in dotted lines, increased so as to have about the same value as the field shown in full lines. The currents I_1 and I_2 under the assumed short-circuit conditions, and therefore the height of the two field forms, are at present unknown, but we do know that

$$\left.\begin{aligned} h_1 &\sim{}^{*} 8\, I_1 \\ h_2 &\sim 4\, I_1 \\ h_3 &\sim 4\, I_2 \\ h_4 &\sim 1\, I_2 \end{aligned}\right\} \qquad (11)$$

The resultant flux densities in the teeth a and d are proportional to

$$D_a = D_d \sim (h_2 + h_4) \sim (4\, I_1 + I_2) \qquad (12)$$

and those in teeth b and c to

$$D_b = D_c \sim (h_1 + h_3) \sim (8\, I_1 + 4\, I_2) \qquad (13)$$

With uniform teeth, the fluxes in the teeth are proportional to the densities found above. The voltage induced by these fluxes in the secondary winding is proportional to

$$1\,(D_a + D_b + D_c + D_d) + 3\,(D_b + D_c)$$
$$= D_a + D_d + 4\,(D_b + D_c)$$

or $$E_{2s} \sim 2\,(4\, I_1 + I_2) + 8\,(8\, I_1 + 4\, I_2) \qquad (14)$$

but the value of secondary voltage must be zero, so therefore from (14)

$$72\, I_1 + 34\, I_2 = 0 \qquad (15)$$

Calling K the current ratio, that is, $\frac{I_2}{I_1}$, it follows from (15) that

$$K = -\frac{72}{34} = -2.11765 \qquad (16)$$

The voltage induced by the same fluxes in the primary winding is proportional to

$$4\,(D_a + D_b + D_c + D_d) + 4\,(D_b + D_c)$$
$$= 4\,(D_a + D_d) + 8\,(D_b + D_c)$$

or $$E_{1s} \sim 8(4\, I_1 + I_2) + 16(8\, I_1 + 4\, I_2) = 160\, I_1 + 72\, I_2 \qquad (17)$$

Since $I_2 = -2.11765\, I_1$

$$E_{1s} \sim 160\, I_1 - 2.11765 \times 72\, I_1$$
$$\therefore E_{1s} \sim 7.53\, I_1 \qquad (18)$$

The field forms can now be shown to scale as has been done in Fig. 7, because we now know that

$$\left.\begin{aligned} h_1 &\sim 8\, I_1 \\ h_2 &\sim 4\, I_1 \\ h_3 &\sim -\ 2.11765 \times 4\, I_1 \\ h_4 &\sim -\ 2.11765 \times I_1 \end{aligned}\right\} \qquad (19)$$

and therefore

$$\left.\begin{aligned} D_a &= D_d \sim 1.88\, I_1 \\ D_b &= D_c \sim -\ 0.47\, I_1 \end{aligned}\right\} \qquad (20)$$

We see that, while the resultant voltage induced in the secondary winding is zero, there is nevertheless a resultant flux mutual to both windings, as shown by the dot-and-dash lines in Fig. 7; the actual corresponding lines of force are indicated in Fig. 7A. This flux has both positive and negative portions in the air-gap of the same pole face, but the positive portions predominate and so the total resultant flux is positive. The reason why this flux induces no resultant voltage in the secondary winding is simply that the smaller negative portions

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[*] $\sim$ Proportionality sign.

interlink with three turns, while the total resultant flux, which is larger and positive, interlinks with but one turn; the negative voltage induced in the three inner turns is exactly equal and opposite to the positive voltage induced in the outer turn. With a primary winding (P) distributed in a different manner, the negative voltage induced in the four inner turns only partly neutralizes the positive voltage of the four outer turns, and so there is a definite positive voltage induced in the primary winding just as there would be if there were a leakage flux interlinking the primary winding. In other words, the resultant flux existing under the assumed short-circuit conditions, although interlinking with both windings, has the same effect as a leakage flux in so far as it induces a resultant voltage in only one of the windings; it may therefore be treated the same as a true leakage flux for all practical purposes. Since it is caused by the difference in distribution or arrangement of the windings, it may be called "differential leakage flux." As will be seen, the study of this flux and its characteristics has been materially facilitated by the investigation of the short-circuit condition. The reason for this is that, with this condition and the other conditions chosen, there is no flux other than the differential leakage flux in existence, and as a consequence, it is possible to obtain a true picture of the phenomenon, without having it obscured by the presence of any other fluxes.

Differential Leakage Coefficient. We have found that there is a voltage induced in the primary winding when the secondary is short-circuited. Let us find the voltage induced in the primary when the same current is passed through it with the secondary open. The ratio of the former voltage to the latter, we will call the "differential leakage coefficient," for in a transformer this ratio is numerically equal to the total leakage coefficient, as is shown in Equation (62), Appendix I.

With open secondary and a current I_1, I_2 is zero. So, from Equation (17),

$$E_1 \sim 160\, I_1 \tag{21}$$

Dividing Equation (18) by Equation (21),

$$\frac{E_{1s}}{E_1} = \frac{7.53\, I_1}{160\, I_1} = 0.047 = \sigma \tag{22}$$

where σ is the differential leakage coefficient.

Let us assume for the purpose of further investigation that the winding P is short circuited, while the winding S is connected to the supply circuit, and let us calculate the differential leakage coefficient for this condition. In this case, E_{1s} must be equal to zero, so from Equation (17),

$$160\, I_1 + 72\, I_2 = 0$$

Letting $K = \frac{I_2}{I_1}$ as before,

$$K = -\frac{160}{72} = -2.222 \tag{23}$$

The voltage induced in winding S follows from Equations (14) and (23)

$$E_{2s} \sim \frac{72}{-2.222}\, I_2 + 34\, I_2 = 1.596\, I_2 \tag{24}$$

The fields for this last case are shown in Figs. 8 and 8A, and it will be seen that they differ from the fields in Figs. 7 and 7A.

The voltage induced in the winding S, with the same current I_2 flowing through it and winding P open-circuited, ($I_1 = 0$), follows from Equation (14) or (15) to be

$$E_2 \sim 34\, I_2 \tag{25}$$

Dividing (24) by (25), we obtain the differential leakage coefficient for this condition

$$\sigma = \frac{E_{2s}}{E_2} = \frac{1.596\, I_2}{34\, I_2} = 0.047 \tag{26}$$

which is the same as previously found, although the fields are different.

Transformer Ratio. Using this value of differential leakage coefficient just found, let us calculate the transformer ratio, using the formulas derived in Appendix I for a transformer with leakage. (In making use of the formulas in this appendix, σ_1 or σ_2 will be replaced by $\frac{\sigma}{2}$.) With S shorted and P open, we obtain from Equation (52)

$$I_2' = -I_1 \times R \times \left(1 - \frac{\sigma}{2}\right)$$

or

$$R = -\frac{I_2'}{I_1}\, \frac{1}{1 - \frac{\sigma}{2}} \tag{27}$$

which becomes for our case

$$R = 2.11765 \times \frac{1}{1 - \frac{0.047}{2}} = 2.17 \tag{28}$$

If the winding P is short-circuited and a voltage impressed upon S, we should have, according to Equation (53) in Appendix I,

$$I_1' = \frac{-I_2}{R}\left(1 - \frac{\sigma}{2}\right)$$

or

$$R = \frac{-I_2}{I_1'}\left(1 - \frac{\sigma}{2}\right) \tag{29}$$

which, using our values, becomes

$$R = 2.222\left(1 - \frac{0.047}{2}\right) = 2.17, \text{ as before.}$$

We have now found a transformer ratio which, when used in connection with the differential leakage coefficient, applies for the calculation of primary and secondary current ratios in either direction, as should be the case.

Let us see if the same transformer ratio holds for the voltage ratio in either direction, as it should. With the secondary open and the primary connected to the line, we should have, according to Equation (41), Appendix I,

$$R = \frac{E_1}{E_2'}\left(1 - \frac{\sigma}{2}\right) \tag{30}$$

We previously found $\frac{E_1}{E_2'} = 2.222$ in Equation (8), so, therefore, with winding P excited and S open,

$$R = 2.222\left(1 - \frac{0.047}{2}\right) = 2.17, \text{ as before.}$$

Finally, if we excite S and leave P open, we should have, according to Equation (42),

$$R = \frac{E_1'}{E_2}\left(\frac{1}{1 - \frac{\sigma}{2}}\right) \tag{31}$$

but, from Equation (10), $\frac{E_1'}{E_2} = 2.11765$

$$\text{therefore } R = \frac{2.11765}{1 - \frac{0.047}{2}} = 2.17, \text{ as before.}$$

Conclusions and General Discussion

Thus we see that by introducing the concept of differential leakage and by determining a differential leakage coefficient as shown, it is possible to derive a transformer ratio for distributed windings which applies to open-circuit and short-circuit conditions in either direction. The method previously shown for determining the differential leakage coefficient by finding the ratio of short circuit and open circuit voltage for the same primary current, Equation (22), may be used for single-phase motors in general; also, the transformer ratio may be determined as indicated, Equations (27) and (29).

However, after the concept of differential leakage coefficient has been established and the nature of the differential leakage demonstrated, it is possible, and frequently easier, to compute the differential leakage coefficient from the no-load voltages. In Appendix II it is shown how the no-load voltages, with either winding excited, can be determined. After this has been done, the differential leakage coefficient can be found from Equation (43), Appendix I, and the transformer ratio follows from Equation (44). In our example this gives the following results:

$$\frac{\sigma}{2} = 1 - \sqrt{\frac{E_1' \times E_2'}{E_2\,E_1}} = 1 - \sqrt{\frac{2.11765}{2.222}} = 0.0235$$

or $\sigma = 0.047$

$$R = \sqrt{\frac{E_1' \times E_1}{E_2 \times E_2'}} = \sqrt{2.11765 \times 2.222} = 2.17$$

which agree with those previously obtained.

In using Equation (43) it is necessary to evaluate the quantity under the radical with great accuracy since this quantity is nearly 1.0 and is subtracted from 1.0, the difference being small. A simpler formula based on the well-known approximation that $\sqrt{1 - A} = 1 - \frac{A}{2}$, if A is small, can be derived from Equation (43), giving

$$\sigma = 1 - \frac{E_1' \times E_2'}{E_2\,E_1} \tag{43a}$$

For polyphase motors the same general principles apply and the same general methods of calculation may be used. However, there is one condition existing in polyphase motors which does not occur in the ordinary single-phase case. In polyphase motors, differential leakage may occur even though the slots and turns may be equally distributed in the two members. This is due to the fact that in a wound-secondary polyphase motor, for instance, the rotor can assume positions where its phase belts do not coincide with those of the stator, as indicated in Fig. 9 for a three-phase motor. With the phase belts thus displaced, the conditions in the two members are not equal; as a result the fields set up by the two windings are different for any given axis, which in turn is the cause of differential leakage. It was this type of leakage caused by the difference in the phase belts that Adams[3] first pointed out under the name of "belt leakage."

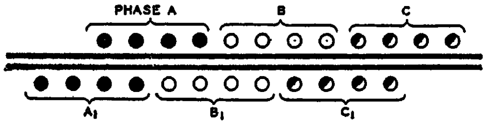

Fig. 9—Displacement of Phase Belts in a Three-Phase Machine

The conditions in connection with polyphase motors are further complicated by the fact that the field form of each of the members changes while the field rotates, which, with the methods indicated above for single-phase motors, makes it necessary to study the field forms for the various time and space relations, as has been done by a number of authors. As an alternative, the no-load conditions can be more readily studied graphically by the vector analysis given by one of the authors[5] of this paper in a previous article.

Up to this point, and particularly in the example given above for single-phase motors, the slots for the two windings have been assumed to coincide, which was done in order to eliminate the existence of zigzag or tooth-top leakage. With this assumption, all of the fluxes across the gap interlink with both windings, a fact which apparently has led to the name "doubly-interlinked leakage" used by some authors. If the slots are assumed no longer to coincide, there will be certain fluxes flowing across the gap from one member to the other which interlink with only one winding. These fluxes are usually referred to as "zigzag" or "tooth-top leakage fluxes." However, the methods indicated in this paper for determining the leakage coefficient will give the sum of the zigzag and the differential or doubly-interlinked leakage, and in many cases the separation of the two types of leakages is difficult. This is the reason why some authors have given the two leakages combined and have referred to the combination under any of the various designations given above. In the previously-mentioned article by one of the authors of this paper is derived the combination of the two for polyphase motors having an equal number of slots in both members, and Fig. 10 reproduced here is from such previous work. In this case it is readily seen that the area under the curve *B* represents the differential or doubly-interlinked leakage varying with the relative positions of the phase belts, while the areas between curves *A* and *B* represent zigzag leakage varying with the relative positions of the slots. For cases with an unequal number of slots, the separation cannot be accomplished in so simple a manner, and whenever for some reason or other the separation of the two seems desirable, it will be found expedient to determine the combination by the methods outlined in this paper, to calculate the zigzag leakage by any of the existing methods for its determination, and then subtract it from the sum of the two in order to obtain the differential or doubly-interlinked leakage.

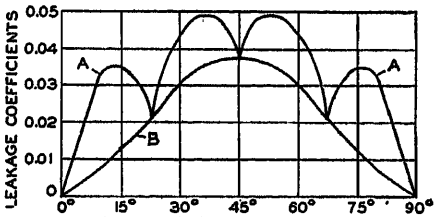

FIG. 10

The practical importance of the differential leakage varies considerably with the case under consideration. We have pointed out that it is present with any arrangement where the two windings are unequally arranged either on account of the distribution of the turns or the position of the phase belts, or on account of both. It may further be pointed out that differential leakage may even exist in a single winding with primary and secondary taps. A practical example of this kind is the armature of a frequency-changer of the commutator type where certain taps are connected to slip-rings and the other end of the winding to the commutator. For any position of the armature where the commutator brushes do not happen to be in the same location as the taps to the slip-rings, there will be differential leakage.

Generally speaking, the more the field forms of the two windings, or of the same winding with different taps, differ from each other, the larger the differential leakage will be. An appreciable difference will be found in single-phase commutator motors, for instance, where the armature has a single set of brushes with a full-pitch winding resulting in a triangular field form, while the stator is wound to give it a trapezoidal field form or is arranged so as to approach a sinusoidal field form rather closely. Another case where differential leakage can be very appreciable in single-phase motors is where there is transformer action between an armature coil short-circuited by a brush and a distributed stator winding. The short-circuited armature coil gives a rectangular field form which differs considerably from the field forms set up by distributed stator windings.

In three-phase induction motors with the most favorable chording, differential leakage can be made almost negligible, while with full-pitch three-phase windings, it is often of practical importance. In two-phase motors it is generally larger than in three-phase motors, and with full-pitch two-phase windings, it may be of great practical importance. Another instance in which differential leakage can be quite large is that of two-phase motors operated with a three-phase secondary. In most cases, the relative importance of differential leakage is, of course, governed by its magnitude relative to that of the other types of leakage fluxes. Thus, in small, slow-speed induction motors where the other types of leakage are very appreciable, the differential leakage will very often be of negligible quantity. On the other hand, in large high-speed motors where the other leakages are relatively small, the differential leakage may be, and frequently is, of practical importance.

The general statements just preceding regarding the relative importance of differential leakage, while not covering all possible cases, may be of assistance in so far as they may indicate where the detailed calculation of the leakage coefficient is desirable. While the methods indicated in this paper for determining the leakage coefficient are very simple in principle, they become quite cumbersome and time-consuming where the number of slots per pole is larger than in our example, as is usually the case in practise. If, wherever such procedure is possible, the two windings involved are designed to have field forms which are very similar to each other, the detailed calculation of the differential leakage may often be eliminated merely by making a small allowance for the differential leakage to be added to the other leakages. On the other hand, where for some reason or other the field forms materially differ from each other, an exact predetermination of the performance of the apparatus will necessitate the determination of the differential leakage by one of the methods indicated, unless the desired or similar data are available from previous investigations.*

Appendix I

General Conventions About Leakage

Although various fundamental relations between leakage fluxes and transformer ratio have been derived in literature, a number of different factors has been called the "leakage coefficient." This has led to some confusion, and, moreover, since most of these relations were left in the form most suited to the particular method of analysis employed by the one deriving them, it may be well to define the leakage coefficient as used in this paper. Incidentally, a number of other relations used in the paper or convenient for other purposes is developed.

A two-winding transformer, as shown in Figs. 11 to 14, is assumed for deriving these relations. Although all the leakage fluxes and mutual fluxes may in practise not link all the turns, the actual fluxes are replaced by the equivalent fluxes which would induce the same voltage if they linked the full number of turns. The reluctances, of course, are fictitious, but serve well for the purpose for which they are intended.

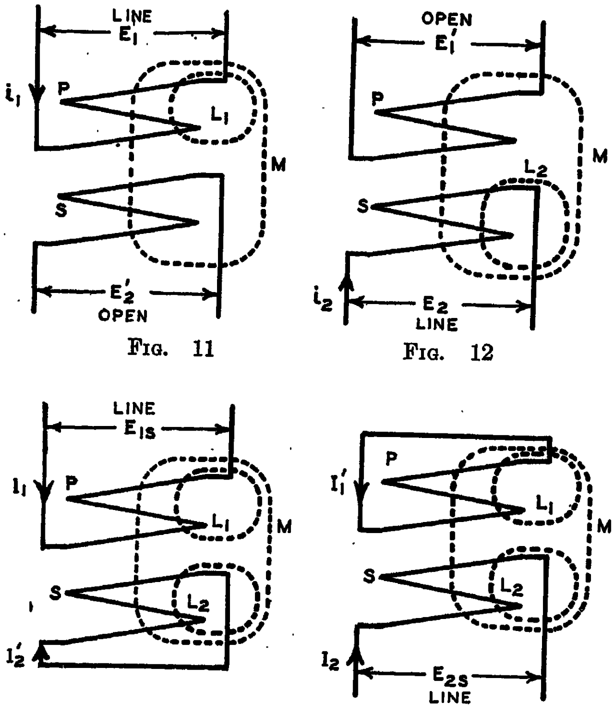

Fig. 11 Fig. 12

Fig. 13 Fig. 14

Figs. 11, 12, 13, 14—Fluxes in a Two-Winding Transformer under Various Conditions

Open-Circuit Conditions

If the coil P in Fig. 11 carries a current i_1, while the coil S is open-circuited, this current will set up a mutual flux M interlinking with both coils and a leakage flux L_1 interlinking with coil P only. Both of these fluxes induce in coil P voltages proportional to their magnitude, and the primary leakage coefficient σ_1 is defined by

$$\sigma_1 = \frac{E_{1l}}{E_{1m} + E_{1l}} = \frac{E_{1l}}{E_1} \tag{32}$$

where E_{1l} = leakage reactance voltage in coil P
E_{1m} = voltage induced in P by mutual flux
E_1 = total reactance voltage in coil P.

The leakage coefficient may also be given by

$$\sigma_1 = \frac{L_1}{M + L_1} = \frac{\mathcal{R}_m}{\mathcal{R}_m + \mathcal{R}_1} \tag{33}$$

where
$\mathcal{R}_m$ = reluctance of path of mutual flux
$\mathcal{R}_1$ = reluctance of path of primary leakage flux.

Similarly, if we assume coil P open and a current passed through S as shown in Fig. 12, the secondary leakage coefficient σ_2 is defined as

$$\sigma_2 = \frac{E_{2l}}{E_{2m} + E_{2l}} = \frac{E_{2l}}{E_2} \tag{34}$$

where E_{2l} = voltage induced in coil S by the leakage flux
E_{2m} = voltage induced in S by the mutual flux
E_2 = total reactance voltage in coil S

The voltage induced in the open-circuited winding can now be expressed in terms of the voltage induced in the winding connected to the line. In Fig. 11, the mutual flux induces in the open-circuited winding S a voltage

$$E_2' = \frac{E_{1m}}{R} \tag{35}$$

where
E_2' = voltage induced in coil S when coil P is excited
R = ratio of turns in coil P to turns in coil S.

The total voltage E_1 induced in coil P is $E_{1m} + E_{1l}$, and since, from Equation (32),

$$E_{1l} = \sigma_1 E_1 \tag{36}$$

we have

$$E_{1m} = E_1 (1 - \sigma_1) \tag{37}$$

Substituting this value of E_{1m} in Equation (35) gives

$$E_2' = E_1 \frac{(1 - \sigma_1)}{R} \tag{38}$$

In a similar manner it can be shown that the voltage induced in coil P when coil S is excited, the former being open, is

$$E_1' = E_2 R (1 - \sigma_2) \tag{39}$$

where
E_1' = voltage induced in coil P when S is excited

If the voltages in Equations (38) and (39) are known from tests, and the turns ratio R is given, the primary and secondary leakage coefficients can be calculated directly from these equations. If the ratio R is unknown, as it often is in motors with distributed windings, Equations (38) and (39) contain three variables, so neither the two leakage coefficients nor the transformer ratio can be found. Usually, however, σ_1 and σ_2 are nearly equal, and so we have

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*See Bibliography.

$$\sigma_1 = \sigma_2 = \frac{\sigma}{2} \tag{40}$$

where σ = total leakage coefficient = $\sigma_1 + \sigma_2$

Making this substitution in Equations (38) and (39), we get

from (38) $$R = \frac{E_1}{E_2'}\left(1 - \frac{\sigma}{2}\right) \tag{41}$$

from (39) $$R = \frac{E_1'}{E_2}\left(\frac{1}{1 - \frac{\sigma}{2}}\right) \tag{42}$$

Now we have two equations with two unknowns, both of which can be found by ordinary algebraic processes to be, for the leakage coefficient

$$\frac{\sigma}{2} = 1 - \sqrt{\frac{E_1' \times E_2'}{E_2 \times E_1}} \tag{43}$$

and for the transformer ratio

$$R = \sqrt{\frac{E_1' \times E_1}{E_2 \times E_2'}} \tag{44}$$

It is interesting to note that, if the primary and secondary leakage coefficients are equal, the transformer ratio is the geometric mean of the two voltage ratios obtained by first exciting one winding, then the other.

SHORT-CIRCUIT CONDITIONS

Many useful relations dependent upon the leakage coefficients can be derived from a consideration of short-circuit phenomena. In Fig. 13, coil S is short-circuited, and it is assumed that a current I_1 is passed through the primary. (It is assumed that the resistance of both windings can be neglected.) In this case, the mutual flux M will induce a voltage in S, causing a current I_2' to flow. The mutual flux M will be proportional to the net ampere-turns acting on the magnetic circuit divided by the reluctance of the latter,

or $$M \sim \frac{I_1 + \frac{I_2'}{R}}{\mathcal{R}_m} \tag{45}$$

Leakage fluxes are set up in each winding as follows:

$$L_1 \sim \frac{I_1}{\mathcal{R}_1} \tag{46}$$

$$L_2 \sim \frac{I_2}{\mathcal{R}_2 R} \tag{47}$$

where $\mathcal{R}_2$ = reluctance of path of L_2

(Dividing by R in the last equation puts L_2 on the same basis as M).

Since the secondary was assumed to have no resistance, the total voltage induced in it must be zero. This can be true only if

$$L_2 = -M \tag{48}$$

Substituting the values of L_2 and M from Equations (45) and (47) in Equation (48) and solving for I_2', gives

$$I_2' = -I_1 R \frac{\mathcal{R}_2}{\mathcal{R}_m + \mathcal{R}_2} \tag{49}$$

It would, however, be more convenient to have I_2' in terms of the leakage coefficient rather than in terms of the reluctances. By analogy to Equation (33)

$$\sigma_2 = \frac{\mathcal{R}_m}{\mathcal{R}_m + \mathcal{R}_2} \tag{50}$$

and $$1 - \sigma_2 = \frac{\mathcal{R}_2}{\mathcal{R}_m + \mathcal{R}_2} \tag{51}$$

Therefore, substituting Equation (51) in Equation (49) gives

$$I_2' = -I_1 R (1 - \sigma_2) \tag{52}$$

By a similar analysis based on short circuiting coil P and sending a current through coil S as shown in Fig. 14, it can be shown that

$$I_1' = -\frac{I_2}{R}(1 - \sigma_1)^* \tag{53}$$

A comparison of short-circuit conditions with those obtaining on open circuit leads to interesting relations. The voltage induced in the winding P in Fig. 13 where the secondary is shorted, is given by

$$E_{1s} \sim M + L_1 \tag{54}$$

in which M and L_1 can be taken from Equations (45) and (46), giving

$$E_{1s} \sim \frac{I_1 + \frac{I_2'}{R}}{\mathcal{R}_m} + \frac{I_1}{\mathcal{R}_1} \tag{55}$$

Introducing the value of I_2' from Equation (52) and simplifying, gives

$$E_{1s} \sim I_1 \frac{\mathcal{R}_1 \sigma_2 + \mathcal{R}_m}{\mathcal{R}_m \times \mathcal{R}_1} \tag{56}$$

This still leaves E_{1s} expressed in terms of the

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*Two more relations, interesting but not essential to this paper, can be derived by again assuming that primary and secondary leakage coefficients are equal and solving for the unknowns in Equations (52) and (53), obtaining

$$\frac{\sigma}{2} = 1 - \sqrt{\frac{I_1' \times I_2'}{I_2 \times I_1}} \tag{a}$$

$$R = \sqrt{\frac{I_2 I_2'}{I_1' I_1}} \tag{b}$$

Note the similarity to Equations (43) and (44) reluctances, which is undesirable. If we solve for $\mathcal{R}_1$ in Equation (33), we obtain

$$\mathcal{R}_1 = \frac{\mathcal{R}_m}{\sigma_1}(1 - \sigma_1) \tag{57}$$

Introducing this value of $\mathcal{R}_1$ into Equation (56) gives us

$$E_{1s} \sim \frac{I_1}{\mathcal{R}_m} \; \frac{\sigma_1 + \sigma_2 - \sigma_1\,\sigma_2}{1 - \sigma_1} \tag{58}$$

When S is open-circuited as in Fig. 9, the voltage induced in P is

$$E_1 \sim M + L_1 \sim \frac{i_1}{\mathcal{R}_m} + \frac{i_1}{\mathcal{R}_1} \tag{59}$$

and, by again utilizing Equation (57) and simplifying,

$$E_1 \sim \frac{i_1}{\mathcal{R}_m}\left(\frac{1}{1 - \sigma_1}\right) \tag{60}$$

Now, if it be assumed that the same current is passed through P when S is shorted as when open, *i. e.*, that $i_1 = I_1$, division of Equation (58) by Equation (60) gives

$$\frac{E_{1s}}{E_1} = \sigma_1 + \sigma_2 - \sigma_1\,\sigma_2 \tag{61}$$

Usually the product $\sigma_1\,\sigma_2$ is small, and therefore if $\sigma_1 + \sigma_2 = \sigma$ as previously assumed, we may write

$$\sigma = \frac{E_{1s}}{E_1} \tag{62}$$

This, of course, is a well-known and familiar relation used in many theories of transformers and motors with transformer actions.

Appendix II

MAGNETIZATION AND VOLTAGE FACTORS OF A DISTRIBUTED WINDING

Magnetization Factor. The magnetization factor of a distributed winding is the ratio of the actual flux set up to the flux which would be set up if all the turns in the winding were concentrated in a coil having a spread of a full pole-pitch.

For the windings shown in Fig. 3,

$$k_{m1} = \frac{\text{area } (a\,b\,c\,d\,e\,f\,g\,h)}{\text{area } (a\,k\,l\,h)} = 0.75 \tag{63}$$

$$k_{m2} = \frac{\text{area } (m\,n\,o\,p\,q\,r\,s\,t)}{\text{area } (m\,u\,v\,t)} = 0.625 \tag{64}$$

Voltage Factor. The voltage factor of a distributed winding is the ratio of the voltage actually induced in the winding to that which would be induced if all the flux linked all the turns.

It is evident from this definition that the voltage factor is dependent not only upon the distribution of the winding itself, but also upon the distribution of the air-gap flux. Thus, there will be two voltage factors for the primary winding, one for the condition where the air-gap flux is set up by the winding P itself, and the other for the condition where the air-gap flux is set up by the winding S. The former will be denoted by k_{p1} and the latter by k_{p1}', while the corresponding factors are denoted by k_{p2} and k_{p2}'.

When the air-gap flux is set up by winding P in Fig. 3,

$$k_{p1} = \frac{4 \times \text{area } (a\,b\,c\,d\,e\,f\,g\,h) + 4 \times \text{area } (A\,d\,e\,B)}{8 \times \text{area } (a\,b\,c\,d\,e\,f\,g\,h)}$$

$$(\text{see Fig. 4}) = \frac{(4 \times 120) + (4 \times 80)}{8 \times 120} = 0.8333 \tag{65}$$

$$k_{p2}' = \frac{1 \times \text{area } (a\,b\,c\,d\,e\,f\,g\,h) + 3 \times \text{area } (A\,d\,e\,B)}{4 \times \text{area } (a\,b\,c\,d\,e\,f\,g\,h)}$$

$$(\text{see Fig. 4}) = \frac{(1 \times 120) + (3 \times 80)}{4 \times 120} = 0.75 \tag{66}$$

When the air-gap flux is set up by winding S in Fig. 3

$$k_{p1}' = \frac{4 \times \text{area } (m\,n\,o\,p\,q\,r\,s\,t) + 4 \times \text{area } (C\,p\,q\,D)}{8 \times \text{area } (m\,n\,o\,p\,q\,r\,s\,t)}$$

$$(\text{see Fig. 5}) = \frac{(4 \times 50) + (4 \times 40)}{8 \times 50} = 0.90 \tag{67}$$

$$k_{p2} = \frac{1 \times \text{area } (m\,n\,o\,p\,q\,r\,s\,t) + 3 \times \text{area } (C\,p\,q\,d)}{4 \times \text{area } (m\,n\,o\,p\,q\,r\,s\,t)}$$

$$(\text{see Fig. 5}) = \frac{(1 \times 50) + (3 \times 40)}{4 \times 50} = 0.85 \tag{68}$$

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Discussion

H. C. Specht: The authors of this paper have illustrated the transformer action of distributed winding in a way which gives a very clear physical concept of this phenomenon.

There is one item that may be not so easily understood—that dealing with the short circuit condition of two equally distributed windings, as in Fig. 1, where it is assumed that the resistance and leakage reactance are zero. The voltage and flux and, consequently, the current must be zero. Nevertheless the authors figure a current in the primary and secondary in order to determine the current transformer ratio. There is nothing wrong with the method although in reality such currents do not exist and cannot be obtained by actual test. The conditions are of an imaginary nature and represent only means to arrive at the results given in the paper. The conditions obtaining in Fig. 3 are, however, not so fictitious. This imaginary condition is certainly of great help in obtaining the conception of differential leakage.

The authors have chosen in their analysis the same number of slots and coils in both members, primary and secondary, and directly opposite to each other in space. This is done, of course, for simplicity. However, it might have been well to give a simple illustration of a few cases in which the number of slots is not the same and in which one member rotates as is the case in induction motors. This would make the paper more complete and more useful for practical design work.

R. E. Hellmund: The purpose of the present paper was, as indicated, to attack the same problem, which is somewhat involved, from a different point of view and to present as simple an analysis as possible. We have purposely refrained from including in the paper a great deal of additional available material and have excluded everything except what appeared to be essential to a simple explanation of the phenomena and a few general remarks relating to the practical importance of the differential leakage and transformer ratio. The previous remarks may also serve as a reply to a part of Mr. Specht's discussion.

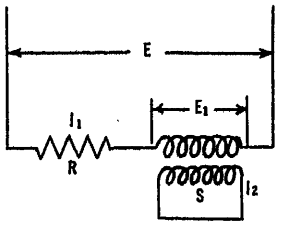


Fig. 1

In further answer to the latter, reference may be made to Fig. 10 of our paper. It is reproduced from one of my previous articles, and it will be seen that the leakage coefficient is given for all rotor positions of a three-phase motor. The average of the curves given represents the leakage for the rotating motor condition. This figure was derived for an equal number of slots in the stator and rotor. The method by which it was derived can also be utilized for an unequal number of slots, but in such cases the zigzag and differential leakage cannot be as readily separated as in Fig. 10. It will be necessary to figure the zigzag leakage by one of the existing methods and subtract it from the total in order to obtain the differential leakage.

Mr. Specht has further drawn attention to the fact that, on account of certain hypothetical assumptions made in our paper, it is concluded that in the case of Fig. 1 there is a current flowing although the voltage impressed is zero. This difficulty can be readily overcome by assuming the circuits as indicated in Fig. 1 herewith, including a resistance R in series with the primary motor winding P. The secondary winding S is short-circuited. We can now impress a total voltage E and regulate R until any desired current (I_1) is flowing. It will then be found that in the case of Fig. 1 in the paper, where there is no differential leakage, the voltage E_1 across the primary winding is zero, while in the case of Fig. 3 where there is differential leakage, there will be a definite value E_1 across the primary winding. In other words, in the circuit of Fig. 1 herewith it is no longer necessary to assume that a current is flowing with a voltage of zero impressed on the circuits.

When I was originally working on differential leakage, my principal interest was, of course, the performance calculation of motors and the influence which differential leakage had upon such values as power factor, pull-out torque, starting torque, etc. During more recent years my interest in the phenomenon was revived because it entered rather prominantly into certain difficult commutating problems. As is well known, the commutation of certain a-c. motors is not perfect because there may be a definite voltage induced in the short-circuited armature coil which is not compensated for. In such cases there is, of course, a short-circuit current flowing in the coil which has to be interrupted when the circuit is opened.

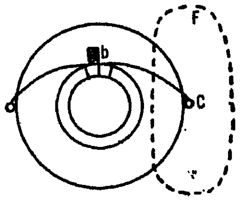


Fig. 2

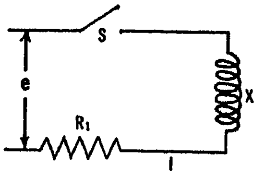


Fig. 3

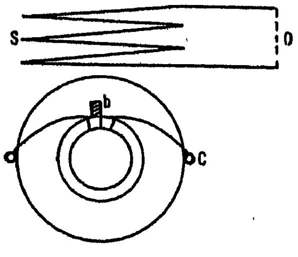


Fig. 4

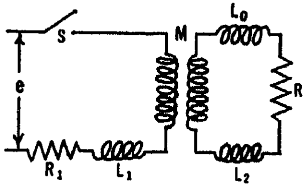


Fig. 5

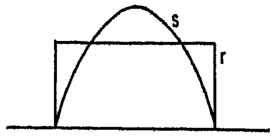


Fig. 6

Such a case is illustrated in Fig. 2 herewith. The circuit of the short-circuited coil of Fig. 2 can be represented as shown in

Fig. 3 herewith, with a definite voltage (e) induced in the circuit which contains an inductance X and a resistance R_1. The opening of the circuit at the brush is represented by a switch S. If we thus look at the commutation problem as a switch problem, we know that the sparking at the switch is not governed merely by the induced voltage (e) and the current I flowing in the circuit, but to a very great extent by the magnetic energy stored in the inductance X. If we assume now a case as shown in Fig. 2, without a stator winding or any other damping circuit, the inductance of the short-circuited coil will be very large because it is formed by the entire main flux path F. In other words, with a given voltage e of possibly three or four volts, very bad commutating conditions may be encountered with such a large amount of stored magnetic energy.

Assuming now the more common case with a stator winding S as shown in Fig. 4, of this discussion the circuit of which is closed, either through the armature, as is often the case, or through an external circuit, we have an equivalent switching circuit as shown in Fig. 5 herewith. It shows the mutual inductance M between the short-circuited armature coil and the secondary stator circuit. Both primary and secondary circuits have resistances, R_1 and R_2, and leakage reactances L_1 and L_2, and the secondary circuits may have in addition an external reactance L_0, the effect of which is the same as that of the leakage reactance L_2. If with such a circuit the leakage reactances are small, the sparking conditions will be materially improved over those in Figs. 2 and 3 because a large part of the stored magnetic energy can be dissipated in the resistance R_2 of the secondary circuit. However, in the same measure as the leakage reactances increase, the sparking conditions grow worse, and it is for this reason that the leakages between the short-circuited armature coil and the stator winding, and among them the differential leakage, become of practical importance. If the stator coil S in Fig. 4 herewith is a single coil located in stator slots opposite the armature slots containing the short-circuited coil, the leakage is likely to be small and the commutating conditions fairly good. On the other hand, if the stator winding is distributed, as is often the case, to approach a sinusoidal field form (as shown by s in Fig. 6), rather large differential leakage will be encountered because the short-circuited coil has a rectangular field form r. With such widely different field forms, I have found that the differential leakage effect can be as high as nine to ten per cent of the reactance of the main flux path, which in turn can result in very bad commutating conditions.

Induction Motor Performance Calculations

BY P. L. ALGER*
Fellow, A. I. E. E.

Synopsis.—By means of solutions of the induction motor exact equivalent circuit in the form of infinite power series, approximate formulas for the important motor characteristics are derived. These formulas enable the effects of magnetizing current, core loss, and primary resistance on maximum torque, starting current, and so forth, to be readily taken into account with engineering accuracy. A new form of power factor chart is presented, which enables the power factor–load curve of any motor to be conveniently determined from no load test data. Exact values of power factor can be obtained from the chart by applying differential corrections, formulas for which are given.

The methods outlined were developed to secure an optimum combination of speed, accuracy, and consistency in design calculations, and have been in regular use for the past seven years.

I. AVAILABLE METHODS OF CALCULATION

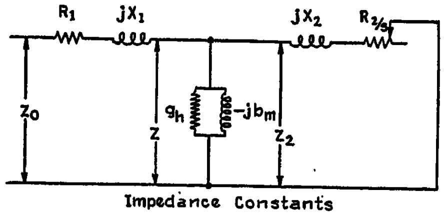
THE art of calculating the load characteristics of an induction motor may be said to have begun with the invention of the circle diagram by Behrend and Heyland. Later, the theory of the transformer was developed, and thence Steinmetz derived the equivalent circuit of the induction motor. These two methods have since been used with various modifications, but no entirely new methods have been devised.

In the author's experience, the equivalent circuit method of calculation has proved best suited for all purposes, except that of ready visualization. The circle diagram is useful, especially to the novice, for visualizing the motor's performance over its entire range of operation; but the equivalent circuit enables the characteristics of widely different motors to be determined with the greatest speed and accuracy. The calculations in the solution of the equivalent circuit must be carried out in logical sequence, and preferably with a power factor slide rule,[2] to obtain both speed and accuracy, but other means than straight-forward calculations can be employed with advantage.

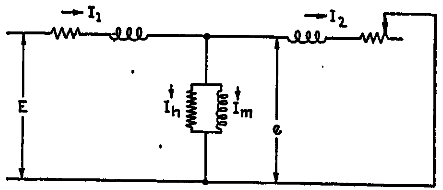
In design work, many questions require only approximate, while others require accurate answers. A single method which will yield approximate results quickly, and will also give accurate results with a greater expenditure of time is, therefore, very desirable. Such a method can be developed for most design problems by deriving fundamental equations that take into account only the major phenomena, and also deriving correction formulas which take into account deviations caused by secondary phenomena. The induction motor is particularly suited to this method of design, since many of its characteristics differ only slightly from those of an ideal motor without core loss, primary resistance, or magnetizing current.

It is the purpose of this paper, therefore, to show how the general solution of the equivalent circuit of the polyphase induction motor can be expressed in infinite power series, of which the first order terms are identical with the usual text book formulas, and the higher order terms represent differential corrections to those ideal formulas. Approximate correction factors to apply to the simple ideal formulas are thence derived, which give results to engineering accuracy, and which are very convenient for many design purposes.

The method proposed is an alternative to that recently presented by Mr. W. J. Branson,[1] algebraic formulas supplemented by a single chart being used instead of a series of approximately 100 charts. Both methods assume the equivalent circuit (Fig. 1) to truly represent the motor's performance. The only difference in the basis of the two methods is that Mr. Branson assumed the core loss branch of the equivalent circuit to be connected directly across the line terminals, instead of inside the primary impedance, as in Fig. 1. If all the core loss occurred in the primary core back of the teeth, the first assumption would be nearly exact, while if it all occurred in the air-gap surfaces the latter would be satisfactory. Neither connection is exactly correct, and both require separate allowances for stray load losses, so that these differences are unimportant.



Impedance Constants



Currents and Voltages

FIG. 1—EQUIVALENT CIRCUIT OF POLYPHASE INDUCTION MOTOR

II. BASIS OF THE NEW METHOD

The "exact equivalent circuit" of a polyphase induction motor is shown in Fig. 1. In using this circuit, several assumptions are involved, which will be explained in the next section, but these do not lead to any marked departures from experimental results.

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*General Electric Company, Schenectady, New York.

[2] For references see Bibliography.

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There are six independent constants of the circuit, which can be reduced to two principal and two subsidiary constants, by assuming the leakage reactance to be equally divided between primary and secondary ($X_1 = X_2$), and by recognizing that the secondary resistance, R_2, and the slip only occur together as the ratio $\frac{R_2}{s}$, so that $\frac{R_2}{s}$ can be used as the independent variable. The principal constants are then the magnetizing and leakage reactances, X_m and X, and the subsidiary constants are the primary resistance, R_1, and the core loss conductance, G_h. There is, in fact, one other constant, the friction and windage, which must be specified to determine the motor efficiency, but this can easily be taken care of by subtracting it from the output indicated by the circuit, without affecting in any way the circuit calculations.

The problem now is to derive simple means of determining any particular characteristic of the motor by use of the equivalent circuit, for any set of values of the four motor constants. To solve this problem, the four constants will be expressed as voltage or current ratios, and the circuit will be solved for the general case by developing it in the form of an infinite power series. Since the two auxiliary constants have much smaller values than the other two, the series need only be carried to the first or second order terms in these constants, though it must be carried out to include the fourth or even sixth order terms of the principal constants. While the algebraic expressions resulting from this process are rather formidable, a little familiarity with them leads to means of using them in convenient forms, and, if only approximate results are desired, they become very simple.

In order to obtain series converging as rapidly as possible, it is important to arrange the constants in the order of their normal magnitudes. The magnitudes depend, of course, upon the operating range of the motor under consideration; the magnetizing current being a very small part of the total under starting conditions, for example, although it is a very large proportion of the whole at light loads. For this reason, it is convenient to carry out the solution separately for the three different ranges of operation which are of interest, corresponding to normal operation under load, the region of maximum torque, and the region of starting. In the following sections of the paper, the process indicated is carried through for these three conditions in turn.

III. ASSUMPTIONS AND NOMENCLATURE

In conformity with the usual practise, the following assumptions will be made:

(1) $X_1 = X_2$, or primary reactance equals secondary reactance. This assumption is made for the reason that the departures from it met with in practise can cause only small errors; that it is difficult to determine X_1 and X_2 separately by test; and that to keep X_1 and X_2 separate in either design or approximate equivalent circuit calculations involves a great deal of extra labor.

(2) $I_m = I_1$ at no load, or the running light power factor is very low. This assumption is made for the reasons that it is closely true for motors above 5 hp. rating; that the departure from fact is greatest in high power factor motors, where the value of I_m is of least importance; and that the approximation greatly simplifies the work.

(3) I_m is proportional to e, or the magnetizing current is proportional to e. This is true except for the effect of saturation, which cannot be analytically expressed.

(4) The core loss is proportional to e^2. This has been shown by many tests to be nearly true at magnetic densities usual in induction motors. At high voltages, the exponent is usually greater than 2, while at low voltages it may be a little less than 2.

(5) All impressed current and voltage harmonics are neglected, or sinusoidal wave forms are assumed throughout.

(6) Stray load losses are neglected, although these can readily be taken into account by increasing the effective primary and secondary resistances by appropriate amounts.

A list of the symbols employed follows:

NOMENCLATURE

$a = \frac{I_m}{I}$ = ratio of no load magnetizing current to current at assumed load

$b = \frac{I\,X}{E}$ = ratio of total reactance drop to impressed voltage

$c = \frac{I\,R_1}{E}$ = ratio of primary resistance drop to impressed voltage

$d = \frac{I\,R_2}{E}$ = ratio of secondary resistance drop, based on primary current, to impressed voltage

e = induced voltage per phase due to air-gap flux

E = impressed voltage per phase

$F = \frac{\text{friction loss}}{\text{output}}$ = ratio of friction and windage losses to output

f = frequency in cycles per second

g_h = core loss conductance in mhos per phase

G = conductance in mhos per phase

$h = \frac{\text{core loss}}{\text{volt amperes input}}$ = ratio of core loss watts per phase to volt amperes input per phase

I_m = no load current, or magnetizing current, in amperes per phase

I = current in amperes per phase
j = symbol for $\sqrt{-1}$
$k = \dfrac{R_2}{s\,Z_0} = \dfrac{d}{s}$ = ratio of per cent secondary resistance drop to per cent slip
R = resistance in ohms per phase
s = ratio of slip to synchronous speed
X = reactance in ohms per phase
Y = admittance in mhos per phase
Z = impedance in ohms per phase

In general, subscripts 1, 2, m, and 0 refer to the primary, secondary, magnetizing, and total or line constants. When no subscript is used, the combined characteristics of the magnetizing and secondary circuits in parallel are indicated. The arrangement of the equivalent circuit is illustrated by Fig. 1.

The quantities b, c, and d, which vary directly as the line current, and a and h, which vary inversely, are the essential variables that determine a motor's performance, both as affected by load changes and by variations in the design constants.

IV. SOLUTION IN THE NEIGHBORHOOD OF FULL LOAD

At full load as a motor, s is quite small, and R_2/s is the largest part of the total motor impedance, Z_0. Hence, we may most conveniently calculate all the impedances of the circuit as products of R_2/s by a power series. Having done this, we may find an expression for $k = \dfrac{R_2}{s\,Z_0}$ by successive approximations, and, by substituting its value in the other expressions, obtain the ratios of all the impedances to R_2/s. These ratios and quantities derived from them will give the power factor, torque, and other quantities it is desired to know.

As a prelude to the calculations, it is desirable to note the following equalities:

$$X_1 = X_2 = \frac{b\,R_2}{2\,k\,s} \tag{1}$$

$$e_m = E\left(1 - \frac{a\,b}{2}\right) \tag{2}$$

$$b_m = \frac{s}{R_2}\left(\frac{2\,a\,k}{2 - a\,b}\right) = \frac{s}{R_2}\,a\,k\left(1 + \frac{a\,b}{2} + \frac{a^2\,b^2}{4}\right) \tag{3}$$

$$g_h = \frac{k\,h_s}{R_2}\left(1 + a\,b + \frac{3\,a^2\,b^2}{4}\right) \tag{4}$$

The first step is to express the secondary circuit admittance, Y_2, as an infinite series, derived by taking the reciprocal of Z_2.

$$Z_2 = \frac{R_2}{s}\left(1 + \frac{j\,b}{2\,k}\right) \tag{5}$$

$$Y_2 = \frac{s}{R_2}\left[\left(1 - \frac{b^2}{4\,k^2} + \frac{b^4}{16\,k^4} - \frac{b^6}{64\,k^6}\right) - j\left(\frac{b}{2\,k} - \frac{b^3}{8\,k^3} + \frac{b^5}{32\,k^5}\right)\right] \tag{6}$$

Adding the magnetizing admittance, Y_m, to Y_2, and taking the reciprocal, we have their combined impedance, Z:

$$\begin{aligned}
Z = \frac{R_2}{s}\Bigg[\Bigg[&1 - \left(a^2\,k^2 + a\,b + k\,h\right) + \Big(a^4\,k^4 + a^3\,k^2\,b + 3\,a^2\,k^3\,h \\
&+ \frac{a^2\,b^2}{4} + 2\,a\,b\,k\,h + k^2\,h^2 + \frac{b^2\,h}{4\,k}\Big) - \Big(a^6\,k^6 + a^5\,k^4\,b \\
&+ 5\,a^4\,k^5\,h + \frac{a^4\,b^2\,k^2}{4} + \frac{a^3\,b^3}{2} + 2\,a^3\,k^3\,b\,h + \frac{3\,a^2\,b^2\,k\,h}{4} \\
&+ 6\,a^2\,k^4\,h^2 + 4\,a\,b\,k^2\,h^2 + k^3\,h^3 + \frac{3\,b^2\,h^2}{4}\Big)\Bigg] + j\Bigg[\left(a\,k + \frac{b}{2k}\right) \\
&- \left(a^3\,k^3 + a^2\,b\,k + \frac{a\,b^2}{4\,k} + 2\,a\,k^2\,h + b\,h\right) + \Big(a^5\,k^5 \\
&+ a^4\,k^3\,b + \frac{a^3\,b^2\,k}{4} + 4\,a^3\,k^4\,h + 3\,a^2\,k^2\,b\,h + \frac{a\,b^2\,h}{2} \\
&+ 3\,a\,k^3\,h^2 - \frac{b^3\,h}{4\,k^2} + \frac{3\,b\,k\,h^2}{2}\Big)\Bigg]\Bigg]
\end{aligned} \tag{7}$$

The addition of the primary impedance, $R_2/s\left(\dfrac{c}{k} + j\,\dfrac{b}{2\,k}\right)$, to Z, gives the total motor impedance, Z_0:

$$\begin{aligned}
Z_0 = \frac{R_2}{s}\Bigg[\Bigg[&1 - \left(a^2\,k^2 + a\,b + k\,h - \frac{c}{k}\right) + (a^4\,k^4 + a^3\,k^2\,b) \\
&+ \left(3\,a^2\,k^3\,h + \frac{a^2\,b^2}{4} + 2\,a\,b\,k\,h + k^2\,h^2 + \frac{b^2\,h}{4\,k}\right) \\
&- \Big(a^0\,k^6 + a^5\,k^4\,b + 5\,a^4\,k^5\,h + \frac{a^4\,b^2\,k^2}{4} + \frac{a^3\,b^3}{2} \\
&+ 2\,a^3\,k^3\,b\,h + \frac{3\,a^2\,b^2\,k\,h}{4} + 6\,a^2\,k^4\,h^2 + 4\,a\,b\,k^2\,h^2 \\
&+ k^3\,h^3 + \frac{3\,b^2\,h^2}{4}\Big)\Bigg] + j\Bigg[\left(a\,k + \frac{b}{k}\right) \\
&- \left(a^3\,k^3 + a^2\,b\,k + \frac{a\,b^2}{4\,k} + 2\,a\,k^2\,h + b\,h\right) \\
&+ \Big(a^5\,k^5 + a^4\,k^3\,b + \frac{a^3\,b^2\,k}{4} + 4\,a^3\,k^4\,h + 3\,a^2\,k^2\,b\,h
\end{aligned}$$

$$+\frac{a\,b^2\,h}{2}+3\,a\,k^3\,h^2-\frac{b^2\,h}{4\;k^2}+\frac{3\,b\,k\,h^2}{2}\Big)\Big]\Bigg] \qquad (8)$$

The absolute value of Z_0, found by extracting the square root of the sum of the squares of the resistance and reactance components, is:

$$|Z_1|=\left(\frac{R_2}{s}\right)\Big[1+\Big(-\frac{a^2\,k^2}{2}+\frac{b^2}{2\,k^2}+\frac{c}{k}-k\,h\Big)+\Big(\frac{3\,a^4\,k^4}{8}$$

$$+\frac{3\,a^2\,k^3\,h}{2}-\frac{a^2\,k\,c}{2}-\frac{a^2\,b^2}{4}-\frac{a\,b\,c}{k}-\frac{a\,b^3}{4\,k^2}$$

$$+k^2h^2-\frac{b^2\,h}{4\,k}-\frac{b^2\,c}{2\,k^3}-\frac{b^4}{8\,k^4}\Big)+\Big(-\frac{5}{16}a^6k^6-\frac{17}{8}a^4k^5h$$

$$+\frac{3}{16}a^4k^2b^2+\frac{3}{8}a^4k^3c-\frac{3\,a^3\,b^3}{8}+2a^3k^3bh+\frac{a^3\,k\,b\,c}{2}$$

$$+\frac{5\,a^2\,b^2\,k\,h}{8}-\frac{9\,a^2\,k^4\,h^2}{4}+a^2k^2hc+\frac{a^2\,b^2\,c}{2\,k}+\frac{3\,a^2\,b^4}{32\,k^2}$$

$$+\frac{a^2\,c^2}{2}-\frac{a\,b\,k^2\,h^2}{2}+a\,b\,h\,c+\frac{3\,a\,b^3\,c}{4\,k^3}+\frac{a\,b^5}{8\,k^4}$$

$$+\frac{a\,b\,c^2}{k^2}-k^3\,h^3+\frac{b^2\,h^2}{4}-\frac{b^4\,h}{4\,k^3}+\frac{3\,b^4\,c}{8\,k^5}$$

$$+\frac{b^6}{16\,k^6}+\frac{b^2\,c^2}{2\,k^4}\Big)\Big] \qquad (9)$$

The complete solution of the equivalent circuit has now been formally obtained, and it remains to reduce the expressions to useful forms. The first objective we will set is the calculation of the slip, s, for a given primary current, from known values of the circuit constants. Taking the reciprocal of (9), and multiplying through by R_2/s, we find:

$$k=\frac{R_2}{s\,Z_0}=1+\Big(\frac{a^2\,k^2}{2}-\frac{b^2}{2\,k^2}-\frac{c}{k}+k\,h\Big)$$

$$+\Big(-\frac{a^4\,k^4}{8}-\frac{a^2\,k^3\,h}{2}-\frac{a^2\,k\,c}{2}-\frac{a^2\,b^2}{4}+\frac{a\,b\,c}{k}$$

$$+\frac{a\,b^3}{4\,k^2}-\frac{3\,b^2\,h}{4\,k}+\frac{3\,b^2\,c}{2\,k^3}+\frac{3\,b^4}{8\,k^4}$$

$$+\frac{c^2}{k^2}-2\,c\,h\Big)+\Big(\frac{a^6\,k^6}{16}+\frac{5\,a^4\,k^5\,h}{8}+\frac{a^4\,k^3\,c}{8}$$

$$+\frac{a^4\,k^2\,b^2}{16}+\frac{5\,a^3\,b^3}{8}-2\,a^3\,k^3\,b\,h+\frac{a^3\,k\,b\,c}{2}$$

$$+\frac{a^2\,b^2\,k\,h}{8}-\frac{a^2\,k^4\,h^2}{4}+\frac{a^2\,b^2\,c}{2\,k}+\frac{a^2\,b^4}{32\,k^2}$$

$$+\frac{a\,b\,k^2\,h^2}{2}-\frac{9\,a\,b^3\,c}{4\,k^3}-\frac{3\,a\,b^5}{8\,k^4}-\frac{3\,a\,b\,c^2}{k^2}-\frac{b^2\,h^2}{4}$$

$$+\frac{b^4\,h}{k^3}-\frac{15\,b^4\,c}{8\,k^5}-\frac{5\,b^6}{16\,k^6}-\frac{3\,b^2\,c^2}{k^4}-c\,k\,h^2$$

$$+\frac{a\,b^3\,h}{2\,k}+\frac{7\,b^2\,c\,h}{2\,k^2}+a\,b\,c\,h-\frac{c^3}{k^3}+\frac{3\,c^2\,h}{k}\Big) \qquad (10)$$

As the value of k in the neighborhood of full load is not far from unity, we may readily solve (10) by successive approximations, obtaining:

$$k=1+\Big(\frac{a^2}{2}-\frac{b^2}{2}+h-c\Big)+\Big(\frac{3\;a^4}{8}-\frac{a^2\;b^2}{4}$$

$$+\frac{a\,b^3}{4}-\frac{b^4}{8}+a\,c\,(b-a)+h\Big(a^2-\frac{b^2}{4}\Big)$$

$$+h(h-2c)\Big)+\Big(\frac{5\,a^6}{16}+\frac{5\,a^3\,b^3}{8}-\frac{3\,a^4\,b^2}{16}-\frac{7\;a^2\;b^4}{32}$$

$$+\frac{a\,b^5}{8}-\frac{b^6}{2}-c\Big(a^4+\frac{a^2\,b^2}{4}-a^3\,b+a\,b\,c-a\,b\,h$$

$$+\frac{5\,b^2\,c}{2}\Big)+h\Big(\frac{5\,a^4}{4}-2\,a^3\,b-\frac{a^2\,b^2}{4}$$

$$+\frac{3\,a^2\,h}{4}-3\,a^2\,c+\frac{a\,b\,h}{2}-\frac{b^2\,h}{4}+\frac{a\,b^3}{4}$$

$$+\frac{9\,b^4}{8}-2\,c\,h+h^2\Big)-c^3\Big) \qquad (11)$$

For practical purposes, the second and higher order terms in (11) may be replaced by the single term $2\,a^2\,b^2$, which is closely equal to them for normal motors, so that we finally obtain:

$$k\simeq 1+\frac{a^2-b^2}{2}+h-c+2\,a^2\,b^2 \qquad (12)$$

Equation (12) has been tested in comparison with the results of complete circuit calculations over a wide range of motor constants, and it has been found to be accurate within 1 per cent for usual values of k (1 to 1.2), and to 5 per cent for abnormal values of k (up to 1.4).

This equation is extremely useful in making performance calculations from test results. For there is no suitable means of directly measuring R_2, and a knowledge of this is essential for the start of the equivalent circuit calculations for a definite value of slip. Measurement of R_2 by standstill impedance tests involves large errors due to iron losses and eddy currents, and, on slip ring motors, its measurement with direct current applied across the rings involves transformer ratio calculations which may lead to errors.

The normal procedure in testing is, therefore, to operate the motor under load, and take simultaneous readings of line current and slip, establishing a slip-current curve. This test does not require any torque or wattmeter measurements, and so is conveniently and accurately performed. The usual primary resistance, running light, and blocked rotor tests determine the values of c, a, h, and b for any assumed value of primary current, I_1. Substituting these values in (12) gives the value of k, which in conjunction with the test $s - I_1$ curve gives R_2 from the identity $R_2 = k\, s\, Z_0$. A check may be obtained by repeating the calculations for several values of current, since the same value of R_2 should be found in all cases. Thus, a single pair of test values of s and I_1 is sufficient, instead of the whole curve. Without using Equation (12), the same answer can be found by guessing the value of k, thence finding R_2, carrying through an equivalent circuit calculation for the known slip, and comparing the value of I_1 thus found with the measured value. By repeating the process with successively closer values of k until agreement is reached, the correct answer can finally be obtained, and this was the method formerly used.

Other useful expressions are those for the secondary current, secondary induced voltage, and power factor. The secondary current is readily determined from the identity $I_2 = I_1 \,|\, Y_2 Z \,|$. $|\, Y_2 \,|$ is the reciprocal of the absolute value of (5) and $|\, Z \,|$ is the absolute value of (7). Evaluating these quantities with the aid of (11), and multiplying them together gives:

$$\frac{I_2}{I_1} = 1 - \left(\frac{a^2}{2} + \frac{a\,b}{2} + h\right) + \left(- \frac{a^4}{8} + \frac{a^3\,b}{4} + \frac{a^2\,b^2}{2} + a^2\,c + \frac{a\,b\,h}{2} + \frac{b^2\,h}{2} + c\,h\right) \qquad (13)$$

which is approximately equal to:

$$\frac{I_2}{I_1} \simeq .98 - \frac{a^2}{2} - \frac{a\,b}{2} \qquad (14)$$

This equation is convenient for determining the actual secondary copper loss, and hence the torque and rotor heating, for any slip when the usual no load test data are available.

Similarly, the induced secondary voltage, referred to primary, is given by evaluating the identity $e = E\, Y_0\, Z$ with the aid of Equations (9), (7), and (11), whence:

$$\frac{e}{E} = 1 - \left(\frac{a\,b}{2} + \frac{3\,b^2}{8} + c\right) + \left(\frac{3\,a^2\,b^2}{8} + \frac{a^2\,c}{2} + \frac{7\,a\,b^3}{16} + \frac{b^2\,h}{2} + \frac{5\,b^2\,c}{8} - \frac{9\,b^4}{128} + 2\,c\,h\right) \qquad (15)$$

which is approximately equal to:

$$\frac{e}{E} \simeq 1 - \left(\frac{a\,b}{2} + \frac{3\,b^2}{8} + c\right) + a^2\,b^2 \qquad (16)$$

Equation (16) is convenient for determining the actual secondary flux densities and the air-gap flux under load conditions. At no load, the per unit primary reactance drop is $\frac{a\,b}{2}$, so that, assuming the core loss to vary as e^2, the ratio of actual core loss under load to that at no load is:

$$\frac{e^2}{E^2\left(1 - \frac{a\,b}{2}\right)^2} = (1 + a\,b)\left(\frac{e^2}{E^2}\right)$$

$$= 1 - \frac{3\,b^2}{4} - 2\,c, \text{ approx.} \qquad (17)$$

This equation does not make any allowance for stray load losses, which are due to the leakage fluxes produced by the load currents, but it does give the same value for core loss as found by the formal solution of the equivalent circuit of Fig. 1.

All the equations given can be applied to induction generators by simply changing the signs of the h and c terms in the equations, wherever they appear.

V. DETERMINATION OF THE POWER FACTOR

The power factor may be calculated from the identity, power factor $= R_0 \,|\, Y_0 \,|$, R_0 being given by the resistance component of (8), and $|\, Y_0 \,|$ by the reciprocal of (9). Carrying out the operations indicated, and substituting (11) in the result to eliminate k, we find:

$$\begin{aligned}\text{Power factor} = 1 &- \frac{(a+b)^2}{2} + \Big(\Big(- \frac{a^4}{8} + \frac{a^3\,b}{2} + \frac{3\,a^2\,b^2}{2} + \frac{3\,a\,b^3}{4} - \frac{b^4}{8}\Big) + b\,h\,(a+b) \\ &+ 2\,a\,c\,(a+b)\Big) + \Big(\Big(- \frac{a^6}{16} + \frac{a^5\,b}{8} - \frac{9\,a^4\,b^2}{16} - \frac{15\,a^3\,b^3}{8} - \frac{39\,a^2\,b^4}{32} + \frac{a\,b^5}{4} - \frac{b^6}{2}\Big) - c\Big(4\,a^3\,b \\ &+ \frac{9\,a^2\,b^2}{2} + a\,b\,c + 3\,a^2\,c + \frac{5\,b^2\,c}{2} + c^2\Big) - c\,h\,(3\,a\,b + b^2 + c - h) + h\Big(\frac{a^4}{4} - 2\,a^2\,b^2 - 2\,a\,b^3 \\ &+ \frac{9\,b^4}{8}\Big) - h^2\Big(\frac{3\,a^2}{4} + \frac{a\,b}{2} + b^2\Big)\Big)\end{aligned} \qquad (18)$$

The complete form of Equation (18) is much too long for practical use, and unfortunately no way has been found for reducing it to an approximate expression of accuracy better than 2 or 3 per cent. It can be expressed in a variety of approximate forms, however, which are useful in rough calculations, such as:

$$\text{Power factor} \simeq 1 - \frac{(a+b)^2}{2} + 3\,a^2\,b^2 \qquad (19)$$

This equation indicates clearly the symmetrical way in which a and b determine the power factor, but it is not accurate enough for performance calculations.

It is important to have a ready method of finding the power factor for any load; when no load test data only are available, so that we must resort to a new expedient. By assuming definite values of the variables c and h, we can reduce the variables determining the power factor to three: a, b, and R_2/s. This enables a power factor chart to be plotted, the power factor being given as a function of R_2/s for various values of a and b. Such a chart, shown in Fig. 2, can readily be drawn with sufficient accuracy to enable the power factor to be read off to about 0.2 per cent for any conditions. The values read directly will, of course, be in error due to the differences between the actual and assumed values of c and h, but these errors can be corrected for by means of the proper terms of Equation (18). By constructing two or three such charts for particular values of c and h with definite increments between, the true power factor can be determined with sufficient accuracy for most purposes by a single chart reading, and can be found to within about 2 per cent either by interpolating between two chart readings or by adding a small correction to the reading from a single chart.

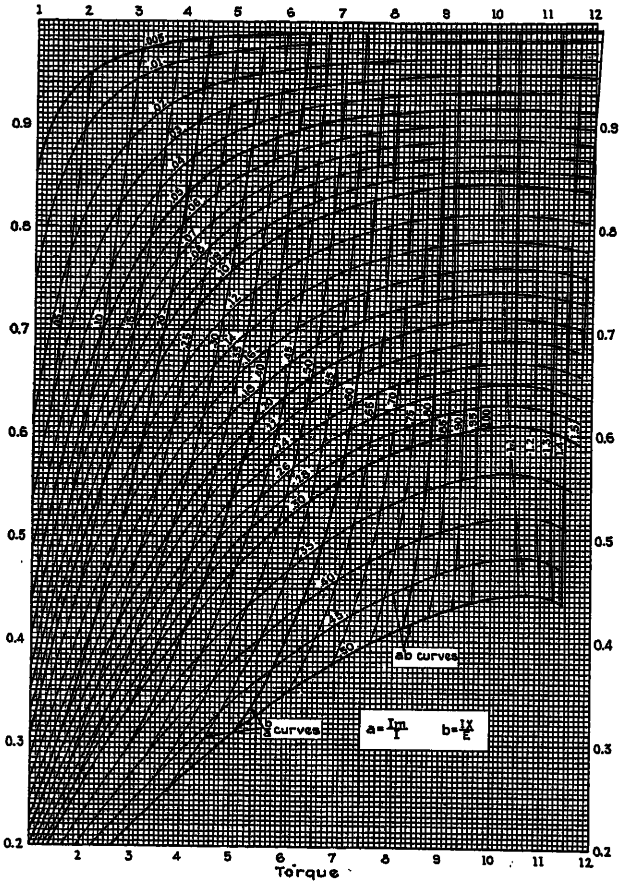

FIG. 2—POWER-FACTOR CHART FOR MEDIUM SIZED MOTORS

The power factor chart shown in Fig. 2 has proved extremely useful in design work, as it enables rapid and consistent predictions of performance to be made with good accuracy. For establishing results from guarantee tests, however, the complete equivalent circuit calculations (Fig. 1) are generally preferable, as they are more basic, and the extra time required is justified in such cases.

Somewhat similar power factor charts have been disclosed by others,[3] but no adequate methods for locating power factor load curves and for correcting the chart results to exact values have heretofore been given. The chart here described was first devised by Mr. H. Maxwell about 1910, and was successively modified by Mr. C. Macmillan and others of the author's associates before it was put into its present exact form in 1923.

The detailed procedure in making the chart is substantially as follows, the numerical values given being those used in making Fig. 2:

1. Arbitrary numerical values for the impressed voltage, E, and the leakage reactance, X, of the circuit in Fig. 1 are assumed, that will be convenient in calculation.

2. A series of values of the product $a\,b$, covering the range from 0 to 0.50, is assumed.

3. The primary resistance for any value of $a\,b$ is determined by arbitrarily assuming a definite value for c, as 0.04, at a load current corresponding to a definite ratio of a to b, as 2. These values fix R_1 for any value of $a\,b$, the numbers cited giving $R_1 = 0.04\,X\sqrt{\frac{2}{a\,b}}$.

4. The magnetizing reactance for any value of $a\,b$ is

$$X_m = \frac{X}{2}\left(\frac{2}{a\,b} - 1\right).$$

5. An arbitrary value, as 0.02, is assumed for core loss current ratio, h, at a load current corresponding to a definite ratio of a to b, as 2. These values fix g_h for any value of $a\,b$, the numbers cited giving

$$\frac{0.01\sqrt{2\,a\,b}}{X\,(1 - a\,b)}$$

6. For each value of $a\,b$, a circuit is set up with the constants given above, as shown in Fig. 3, and calculations of the circuit impedance are made over a range of values of R_2/s from approximately X to ∞.

7. A curve of circuit power factor *versus* secondary input is plotted for each value of $a\,b$, the curves for different values of $a\,b$ being put on a common scale by taking the secondary power input at the load where $b = a$, or where $Z_0 = \frac{X}{\sqrt{a\,b}}$, as 10 for each curve.

8. On each curve, points corresponding to definite ratios of b to a, as 0.1, 0.2, 0.3, ... are marked, and points for each value of b/a are connected by other curves.

9. If desired, a correction for normal friction and windage loss may be made by an appropriate shift of the horizontal scale.

The use of the chart may best be illustrated by an example. Suppose no load tests on a motor have shown that its running light current is 10 amperes, and that its leakage reactance, determined from standstill tests, divided into normal voltage, gives a current of 200 amperes; and it is desired to find its power factor at a load current of 30 amperes, and its maximum power factor. The value of $a\,b$ is evidently 10 divided by 200, or 0.05, and the value of b/a at 30 amperes input is 0.15 divided by 0.333, or 0.45. The intersection of the two corresponding curves on Fig. 2 gives a power factor of 0.902, and following the $a\,b = 0.05$ curve to its summit gives the maximum power factor as 0.917. The ratio of abscissas at the two points, 6.63 to 10.03, indicates that the torque at maximum power factor is 1.53 times the torque at 30 amperes, and this ratio divided by the power factor ratio indicates that the current at maximum power factor is approximately 45.3 amperes. At half the load torque corresponding to 30 amperes input, the abscissa is half of 6.63, and the power factor is, therefore, 0.792.

If it is desired to obtain exact power factor values, it is necessary to correct the chart values for the errors due to discrepancies between the actual and the chart values of c and h, as indicated above. By referring to Equation (18), and taking out all the terms that involve c, we find that the coefficient of c, or the per cent change in power factor due to 1 per cent change in c, is:

$$\left(\frac{\Delta \text{P. F.}}{\Delta c}\right) = 2\,a\,(a+b) - \left(4\,a^3\,b + \frac{9\;a^2\;b^2}{2} + 2\,a\,b\,c \right.$$
$$\left. + 6\,a^2\,c + 5\,b^2\,c + 3\,c^2 + 3\,a\,b\,h + b^2\,h + 2\,c\,h - h^2\right) \quad \textbf{(20)}$$

This may be approximately represented by:

$$\left(\frac{\Delta \text{P. F.}}{\Delta c}\right) = 2\,a\,(a+b) - 8\,a^3\,b \qquad \textbf{(20)}$$

and this equation gives the correction accurately enough for all practical purposes.

Similarly, the coefficient of h in Equation (18) is:

$$\left(\frac{\Delta \text{P. F.}}{\Delta h}\right) = b\,(a+b) + \left(\frac{a^4}{4} - 2\,a^2\,b^2 - 2\,a\,b^3 \right.$$
$$\left. + \frac{9\,b^4}{8} - \frac{3\,a^2\,h}{2} - a\,b\,h - 2\,b^2\,h - 3\,a\,b\,c - b^2\,c - c^2 - 2\,c\,h\right) \quad \textbf{(21)}$$

which reduces to the following approximate form:

$$\left(\frac{\Delta \text{P. F.}}{\Delta h}\right) = b\,(a+b) - a^2\,b^2 \qquad \textbf{(21)}$$

Thus, if the motor used in the example given above had a per unit primary resistance drop at 30 amperes of 0.06, and a no load core loss current of 0.9 amperes per phase, the values of c and of h at this load, 0.06 and 0.03, respectively, would be in excess of the chart values, and so the true power factor would be higher than that indicated. The chart value of c is 0.04 when $a = 2\,b$, or $0.04 \sqrt[8]{\frac{2\,b}{a}}$ at any other load, hence is 0.038 at 30 amperes input, where $b/a = 0.45$. Hence, by second (**20**), the discrepancy in c makes the true power factor greater than the chart value by:

$$(0.060 - 0.038)\;(2\;(0.333)\;(0.333 + 0.15)$$
$$- 8\;(0.333)^3\;(0.15)\;) = 0.0061$$

Similarly, the chart value of h at 30 amperes input is $0.02 \sqrt{\frac{a}{2\,b}}$, or 0.021, and the increase in power factor due to the discrepancy between the motor's and the chart constants is, by second (**21**):

$$(0.030 - 0.021)\;(0.15\;(333 + 15) - (0.05)^2) = 0.0006.$$

The true motor power factor at 30 amperes input is, therefore, 0.0067 greater than the chart indicates, or is 0.909.

Equation (18) can also be used to find the location of maximum power factor. By differentiating (18) with respect to current, and recognizing that the numerical per cent variations of a, b, c, and h for 1 per cent change in current are all the same, but that a and h decrease while b and c increase with increase in current, the per cent change in power factor due to 1 per cent change in primary current is found to be:

$$\frac{\Delta\,(\text{P. F.})}{\dfrac{\Delta\,I_1}{I_1}} = (a^2 - b^2) + \frac{1}{2}\,(a^4 - 2\,a^3\,b + 3\,a\,b^3 - b^4)$$
$$- 2\,a\,c\,(a - b) - b\,h\,(a - b) \qquad \textbf{(22)}$$

Equating this to 0, the point of maximum power factor is found to occur when $b/a = 1 + \frac{a\,b}{4}$ very nearly. The corresponding locus is indicated by the dashed line on Fig. 2.

Other relations obtainable from (18) show that the effect on power factor of an increase in frequency at constant line current and voltage is practically the same as the effect of an equal per cent increase in line current at constant voltage and frequency. Also, an increase in voltage with fixed frequency and current has exactly the same effect on power factor as an equal per cent decrease in current with voltage and frequency constant, neglecting saturation.

All the results given apply equally well to induction generators and motors, if the signs of the c and h terms are changed wherever they appear. However, the power factor chart of Fig. 2 is drawn for positive values of c and h, so that the corrections to it for the negative values corresponding to generator operation will be relatively large. The power factor so obtained for an induction generator is, of course, leading with respect to the delivered power.

VI. EFFICIENCY DETERMINATION

The readiest exact method of calculating efficiency from test data is by summation of losses. First, the core loss plus friction and windage are determined from no load tests, and then the motor power factor is found with the aid of the chart, for an assumed primary current. The primary copper loss is calculated from this known current, and the measured primary resistance. The secondary resistance is determined from the measured values of slip and current, by (**12**), and the secondary current is found by (**14**), whence the secondary copper loss is known. The sum of these, plus the stray load loss allowance, if any, give the total losses. The input being known from the assumed current and the calculated power factor, the efficiency is at once known. Repetition for different currents establishes a curve of efficiency against output, from which values at the desired loads may be taken.

Instead of using the power factor chart, and calculating the efficiency for an assumed primary current, it may be preferable to use the equivalent circuit directly, and calculate the efficiency for a given slip. When R_2 is found from the test-slip-current curve and Equation (**12**), and the rest of the circuit constants are found from no load tests, the equivalent circuit can be solved by ordinary complex number calculations for successive assumed values of slip, and the efficiency and power factor will both then be found at once. In this method stray load losses are usually allowed for by an appropriate increase in the primary resistance.

In making all these calculations by whatever method, it must be remembered that the reactive component of the standstill impedance of the motor does not represent the leakage reactance to be used in the equivalent circuit, but must be corrected for three distinct reasons:

(a) When saturation of the leakage paths occurs, the reactance decreases with increasing current, so that the test value of standstill reactance measured at the current corresponding to the particular load under consideration must be used in each case.

(b) When the secondary current distribution varies with the frequency, as in multiple squirrel cage and deep bar windings, the standstill reactance must be increased in proportion to the increase in the inductance corresponding to the reduction in frequency.[4]

(c) In all cases, the shunting of the secondary leakage by the magnetizing reactances must be allowed for, so that the true total leakage reactance for use in the equivalent circuit is approximately $\left(1 + \frac{a\,b}{4}\right)$ times its apparent value found by standstill test (see Equation (**44**)).

VII. SOLUTION IN THE NEIGHBORHOOD OF MAXIMUM TORQUE

At maximum torque, R_2/s is nearly equal to X, and the line current is usually about 4 times full load current. Hence, the normal values of the circuit variables are quite different in this region from their values near full load. b is now large (about 0.7), a is quite small, c is of the same order as a, and h is so small as to be almost negligible. By repeating the previous equivalent circuit calculations, but with the terms rearranged to allow for their new orders of magnitude, we can obtain useful formulas for the motor characteristics in this region. As less accuracy is needed here, the calculations need not be carried out to as many terms as before. In developing the final equations for characteristics in this region, it is important to use only expressions whose values are functions of the motor design constants and not variables that depend on the load. The products $a\,b$, $b\,h$, c/b, $a\,c$, and so forth, fulfil these requirements, and, therefore, the results are all expressed in terms of these quantities. Thus, although the equations involve a, b, c, and h, which vary with load, these variables only appear in relations which make them invariant, and, therefore, the equations hold no matter what value of current is assumed in calculating the particular values of a, b, and so forth.

Equations (**1**) to (**5**) inclusive are valid here, as at full load, but Equation (**6**) is no longer satisfactory, as the large value of b makes it converge too slowly. In place of (**6**), we can write:

$$Y_2 = \frac{s}{R_2}\left[\frac{(4\,k^2 - j\,2\,b\,k)}{4\,k^2 + b^2}\right] \qquad (23)$$

Adding Y_m to (**23**), and taking the reciprocal, we find the apparent secondary impedance to be:

$$Z = \frac{R_2}{s}\Bigg[\bigg[1 - a\,b + \left(\frac{a^2\,b^2}{4} + \frac{b^2\,h}{4\,k} - k\,h - a^2\,k^2\right) + \left(a^3\,k^2\,b + \frac{a^3\,b^3}{4} + 2\,a\,b\,k\,h\right)\bigg] + j\bigg[\frac{b}{2\,k} + \left(a\,k - \frac{a\,b^2}{4\,k}\right) - (b\,h + a^2\,b\,k) - \bigg(a^3\,k^3 + 2\,a\,k^2\,h - \frac{a\,b^2\,h}{2} - \frac{a^3\,b^4}{16\,k}\bigg)\bigg]\Bigg] \qquad (24)$$

Adding the primary impedance to Z gives the total motor impedance:

$$Z_0 = (R + R_1) + j(X + X_1) = \frac{R_2}{s}\Bigg[\bigg[1 + \left(\frac{c}{k} - a\,b\right) + \left(\frac{a^2\,b^2}{4} + \frac{b^2\,h}{4\,k} - k\,h - a^2\,k^2\right) + \bigg(a^3\,k^2\,b + \frac{a^3\,b^3}{4} + 2\,a\,b\,k\,h\bigg] + j\bigg[\frac{b}{k} + a\,k - \frac{a\,b^2}{4\,k}\bigg) - (b\,h + a^2\,b\,k) - \bigg(a^3\,k^3 + 2\,a\,k^2\,h - \frac{a\,b^2\,h}{2} - \frac{a^3\,b^4}{16\,k}\bigg)\bigg]\Bigg] \qquad (25)$$

The square of the absolute value of Z_0 is found by adding the squares of the real and j components of (25):

$$
\begin{aligned}
|Z_0|^2 = \frac{R_2}{s^2}^2 \Bigg[\left(1 + \frac{b^2}{k^2}\right) + \left(\frac{2\,c}{k} - \frac{a\,b^3}{2\,k^2}\right) \\
+\left(\frac{c^2}{k^2} - \frac{2\,a\,b\,c}{k} - a^2\,b^2 - 2\,k\,h - a^2\,k^2 - \frac{3\,b^2\,h}{2\,k} + \frac{a^2\,b^4}{16\,k^2}\right) \\
+\Bigg(\frac{a\,b^3\,h}{k} + \frac{a^3\,b^3}{2} + \frac{a^3\,b^5}{8\,k^2} - 2\,a^2\,k\,c \\
+\frac{a^2\,b^2\,c}{2\,k} + \frac{b^2\,c\,h}{2\,k^2} - 2\,c\,h\Bigg)\Bigg] \qquad (26)
\end{aligned}
$$

From (26) and the indentity $R_2 = s\,k\,|Z_0|$, we find:

$$
\begin{aligned}
(k+c)^2 = 1 - b^2 + \frac{a\,b^3}{2} + \Bigg(2\,a\,b\,c\,k + a^2\,b^2\,k^2 \\
+ 2\,k^3\,h + a^2\,k^4 + \frac{3\,b^2\,k\,h}{2} - \frac{a^2\,b^4}{16}\Bigg) - \Bigg(a\,b^3\,k\,h \\
+\frac{a^3\,b^3\,k^2}{2} + \frac{a^3\,b^5}{8} - 2\,a^2\,k^3\,c + \frac{a^2\,b^2\,c\,k}{2} + \frac{b^2\,c\,h}{2} - 2\,k^2\,c\,h\Bigg) \qquad (27)
\end{aligned}
$$

which, when solved by successive approximations, reduces to:

$$
\begin{aligned}
k = \sqrt{1-b^2} - c + \left(\frac{a\,b^3}{4} + \frac{a^2}{2} + h + a\,b\,c - \frac{a^2\,b^2}{4}\right) \\
+\left(\frac{a\,b^5}{8} + \frac{3\,a\,b^7}{32} - \frac{3\,a^2\,b^4}{32} - \frac{5\,a^2\,b^6}{64} - a^2\,c - \frac{b^2\,h}{4}\right) \\
+\left(\frac{a^3\,b^3}{8} - 2\,c\,h - a\,b\,c^2 - \frac{a^2\,b^2\,c}{4} + \frac{5\,a\,b^9}{64}\right) \qquad (28)
\end{aligned}
$$

or, very approximately, to:

$$
k \simeq \sqrt{1-b^2} - c + \left(\frac{a\,b^3}{4} + \frac{a^2}{2} + h\right) \qquad (29)
$$

which corresponds to Equation (12). The torque developed is equal to:

$$
\begin{aligned}
T = e^2\,G_2 = \frac{e^2\,R_2}{s\,Z_2^2} = \frac{I_1^2\,R_2}{S}\left(\frac{Z}{Z_2}\right)^2 \\
= \frac{s\,E^2}{R_2}\left(\frac{k^2\,Z^2}{Z_2^2}\right) = \frac{E^2}{X}\,\frac{(b\,k\,Z^2)}{Z_2^2} \qquad (30)
\end{aligned}
$$

which reduces to:

$$
\begin{aligned}
T = \frac{b\,E^2}{X}\Bigg(\sqrt{1-b^2} - (a\,b+c) + \left(\frac{3\,a\,b^3}{4} + \frac{a\,b^5}{4} + 2\,a\,b\,c\right) \\
+\left(\frac{3\,a^2\,b^2}{2} - \frac{a^2}{2} - h + a\,b\,h + \frac{7\,b^2\,h}{4} - \frac{27\,a^2\,b^4}{32}\right) \\
+\Bigg(2\,c\,h + 2\,a^2\,c + \frac{5\,a\,b^7}{32} + \frac{a^3\,b}{2} - \frac{19\,a^2\,b^6}{64} - \frac{13\,a^3\,b^3}{8} \\
-2\,b^2\,c\,h - a\,b\,c^2 - \frac{9\,a^2\,b^2\,c}{2} + \frac{a\,b^9}{8} + \frac{25\,a^3\,b^5}{32} - \frac{11\,a\,b^3\,h}{4}\Bigg)\Bigg) \qquad (31)
\end{aligned}
$$

This equation is not useful for calculating the torque at any particular point, but is extremely useful for the information it conveys in regard to the maximum torque. By differentiating (31) with respect to current (remembering that the rates of per cent change of b and c are equal to the rate of per cent change in current and the rates of per cent change of a and h are numerically the same but are negative), equating to zero, and solving by successive approximations, we find the approximate expression for b at maximum torque:

$$
\begin{aligned}
\sqrt{1-b^2} = 0.707\left(1 - \frac{a\,b}{4} - \frac{11\,a^2\,b^2}{16}\right) - \frac{5\,b\,h}{8} \\
+\frac{3\,c}{8\,b} - \frac{c^2}{16\,b^2} + \frac{a\,c}{16} \qquad (32)
\end{aligned}
$$

Substituting (32) in (31), the value of the maximum torque itself is found to be:

$$
T_{max} = \frac{E^2}{2\,(R_1 + X)}\left[1 - \frac{3\,a\,b}{4} + \frac{a^2\,b^2}{4} - \frac{c^2}{2\,b^2} - \frac{b\,h}{4} + a\,c\right] \qquad (33)
$$

This expression agrees perfectly with those found in standard textbooks, if $a\,b$ is made equal to zero. The usual formulas simply neglect the influence of the magnetizing current on the maximum torque, but (33) shows that this influence, expressed by the reduction factor

$$
1 - \frac{3\,a\,b}{4} + \frac{a^2\,b^2}{4},
$$

is by no means negligible. By reference to the power factor chart of Fig. 2, it will be seen that normal motors with full load power factors of 0.90, 0.80, and 0.70 (full load generally occurs when $b = 0.5\,a$, approximately) have $a\,b$ values of 0.053, 0.11, and 0.17, respectively, and consequently have maximum torques respectively 4, 8, and 14 per cent below the values indicated by the standard text book formulas. Low-speed motors, which have particularly low power factors, have to be especially liberally designed to secure normal overload torque margins on this account.

From (28) and (32) the value of k at maximum torque is found to be:

$$
\begin{aligned}
k = 0.707 - \frac{a\,b}{200} + \frac{a^2\,b^2}{2} + \frac{5\,b\,h}{8} - \frac{c}{3\,b} \\
+\frac{5\,c^2}{16\,b^2} + \frac{3\,a\,c}{8} \qquad (34)
\end{aligned}
$$

and from this the slip at maximum torque is:

$$s = \frac{I\,R_2}{k\,E} = \frac{b\,R_2}{k\,X} = \frac{R_2}{X}$$

$$\left(1 + \frac{a\,b}{4} - \frac{c}{16\,b} - \frac{a^2\,b^2}{4} - \frac{c^2}{8\,b^2} - \frac{a\,c}{4}\right) \quad (35)$$

or, approximately:

$$S = \frac{R_2}{X}\left(1 + \frac{a\,b}{4}\right) \quad (36)$$

The power factor at maximum torque is:

$$\text{P. F.} = \frac{k\,s\,R_0}{R_2} = k\left[1 + \left(\frac{c}{k} - a\,b\right) + \left(\frac{a^2\,b^2}{4}\right.\right.$$

$$\left.\left. + \frac{b^2\,h}{4\,k} - k\,h - a^2\,k^2\right) + \left(a^3\,k^2\,b + \frac{a^3\,b^3}{4} + 2\,a\,b\,k.h\right)\right]$$

$$= 0.707 - \frac{7\,a\,b}{10} - \frac{a^2\,b^2}{40} + \frac{b\,h}{10}$$

$$+ \frac{3\,c}{8\,b} - \frac{c^2}{16\,b^2} + \frac{7\,a\,c}{8} \quad (37)$$

approximately, or is normally a little below 70 per cent. By a similar procedure, the maximum output is found to be:

$$\text{Maximum output} = \frac{E^2}{2\,(R_1 + R_2 + X)}\left[1 - \frac{3\;a\;b}{4}\right.$$

$$\left. + \frac{a^2\,b^2}{4} - \frac{(c\,+\,d)^2}{2\,b^2} - \frac{b\,h}{4} + a\,c\right] \quad (38)$$

and it occurs at a slip equal to:

$$s = \frac{R_2}{X}\left[1 + \frac{a\,b}{4} - \frac{(c + d)}{16\,b} - \frac{a^2\,b^2}{4}\right.$$

$$\left. - \frac{(c + d)^2}{8\,b^2} - \frac{a\,c}{4} - \frac{5\,a\,d}{16}\right] \quad (39)$$

and at a power factor equal to:

$$\text{P. F.} = 0.707\left(1 - a\,b + \frac{c + d}{2\,b}\right) \quad (40)$$

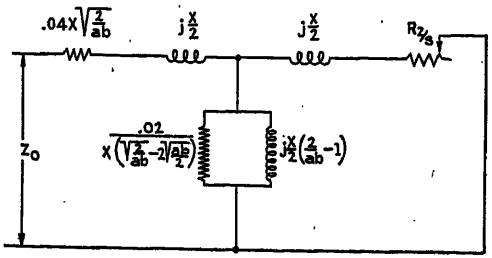

FIG. 3—EQUIVALENT CIRCUIT CONSTANTS USED IN CALCULATING POWER-FACTOR CHART

VIII. SOLUTION IN THE NEIGHBORHOOD OF STANDSTILL

Near standstill, and in reversed rotation, the secondary reactance is larger than R_2/s, so that the circuit should be developed in powers of d instead of powers of b. Here, also, the core loss can be neglected, and a is much smaller than before.

The apparent secondary impedance on this basis is:

$$Z = X_1\left[\left(\frac{2\,d}{b\,s} - \frac{2\,a\,d}{s}\right)\right.$$

$$\left. + j\left(1 - \frac{a\,b}{2} + \frac{2\,a\,d^2}{b\,s^2}\right)\right] \quad (41)$$

and the total impedance is:

$$Z_0 = X_1\left[\left(\frac{2\,d}{s\,b} + \frac{2\,c}{b} - \frac{2\,a\,d}{s}\right)\right.$$

$$\left. + j\left(2 - \frac{a\,b}{2} + \frac{2\,a\,d^2}{b\,s^2}\right)\right] \quad (42)$$

the absolute value of which is:

$$|Z_0| = X_1\left[2 - \left(\frac{a\,b}{2} - \frac{d^2}{b^2\,s^2} - \frac{2\,c\,d}{b^2\,s} - \frac{c^2}{b^2}\right)\right.$$

$$- \left(\frac{d^4}{b^4\,s^4} + \frac{4\,c\,d^3}{b^4\,s^3} + \frac{6\,c^2\,d^2}{b^4\,s^2} - \frac{a\,d^2}{b\,s^2}\right.$$

$$\left.\left. + \frac{4\,c^3\,d}{b^4\,s} - \frac{a\,c^2}{b} + \frac{c^4}{b^4}\right)\right] \quad (43)$$

or, very approximately:

$$Z_0 \simeq R_1 + R_2\,(1 - a\,b) + j\,X\left(1 - \frac{a\,b}{4}\right) \quad (44)$$

and:

$$|Z_0| \simeq X\left(1 - \frac{a\,b}{4}\right) + \frac{(R_1 + R_2)^2}{2\,X^2} \quad (45)$$

The torque is, approximately:

$$\text{Starting Torque} = \frac{2\,E^2\,R_2}{X^2 + (R_1 + R_2)^2}\left(1 - \frac{a\,b}{2}\right) \quad (46)$$

Thus, the torque at low speeds is reduced by the factor $1 - \frac{a\,b}{2}$ as a result of the presence of the magnetizing current, or by about two-thirds as much as the maximum torque is reduced. This reduction factor is very important in low-speed motors, and in the induction starting of synchronous motors.

It is important to note that Equation (44) shows that the true leakage reactance for use in the equivalent circuit, X, is greater than the apparent reactance by the factor $1 + \frac{a\,b}{4}$, approximately. Some writers use the apparent value of X directly in all their calculations; in an analogous way to that in which the actual magnetizing current, I_m, is used here in place of the magnetizing current which would flow if full voltage were impressed at the air-gap. These variations in usage are likely to cause confusion in comparing results if they are not observed.

IX. Numerical Results

It is difficult to define the accuracy with which the foregoing formulas determine the motor characteristics, since the errors depend on the relative values of all the variables. The formulas have been used with satisfaction in regular design work for the past six years, however, and the errors have been found well within allowable tolerances for most purposes. To illustrate their use, and to indicate the magnitude of error to be expected in ordinary practise, the numerical results obtained in the cases of five motors of widely different characteristics are presented in comparison with the values obtained by regular circuit calculations.

The selected motors had the following ratings, all for 60 cycles:

Motor No.	Poles	Horsepower	Rev. per min.	Voltage
1	4	500	1800	2200
2	8	10	900	440
3	10	125	720	2200
4	14	15	514	440
5	20	500	360	440

In each case, the usual no load test data and one point on a test slip-current curve are given. Thence the equivalent circuit constants are determined, and the characteristics at a point near full load are calculated, first by the approximate formulas, and then by the exact equivalent circuit. In all cases, the reactance, X, was determined from standstill tests, with the corrections described at the end of section VI. Many comparisons of actual load tests with equivalent circuit calculations, using reactances so determined and proper load loss allowances, have shown that agreement well within the accuracy of measurement is secured. Thus, the variations between the parallel columns are true indications of the errors of the new formulas.

Motor No.	1	2	3	4	5
			Measured Values		
E	2200	254	1270	254	254
I_m	16.9	5.75	9.62	16.0	267
W_h	2170	83	847	233	3900
R_1	0.349	0.85	1.16	0.380	0.0048
X	5.71	3.50	7.40	2.03	0.070
s	0.0130	0.0320	0.0343	0.0320	0.0132
I_1	65.6	13.2	29.2	24.0	620

W_h is the no load core loss in watts per phase; s and I_1 are corresponding points on the test slip-current curves.

CALCULATED CIRCUIT DATA

	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact
I_1	65.6	65.6	13.2	13.2	29.2	29.2	24.6	24.6	620	620
a	0.258		0.435		0.329		0.650		0.431	
b	0.170		0.182		0.170		0.197		0.171	
c	0.0104		0.0442		0.0267		0.0368		0.0117	
h	0.0150		0.0247		0.0228		0.0373		0.0247	
k	1.028	1.029	1.070	1.072	1.039	1.039	1.22	1.30	1.101	1.106
R_2	0.449	0.449	0.658	0.659	1.55	1.55	0.404	0.430	0.00596	0.00599

I_1 is an assumed primary current at which the characteristics are desired. k is determined by Equation (12) and R_2 by the formula $\frac{s\,k\,E}{I_1}$. The exact values of k and R_2 were found by repeated trials with the equivalent circuit.

CALCULATED VALUES AT THE ASSUMED CURRENTS

	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact
P. F.	0.924	0.916	0.842	0.843	0.896	0.891	0.698	0.700	0.846	0.837
Corr. P. F.	0.917	0.916	0.845	0.843	0.892	0.891	0.704	0.700	0.836	0.837
I_2	60.6	61.0	11.2	11.2	26.2	26.3	17.2	17.1	526	528
e	2108	2108	231	233	1188	1192	229	230	240	240

The power factor values are from Fig. 2, and the corrected values are from Equations (20) and (21). I_2 and e are from Equations (14) and (16).

Motor No.	1		2		3		4		5	
	CALCULATED MAXIMUM VALUES									
	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact
Max. P. F.	0.927	0.920	0.872	0.875	0.908	0.904	0.804	0.809	0.881	0.871
Max. torque	389	385	6.92	6.86	89.8	89.4	122	120.4	409	408

The maximum power factor is from Fig. 2, and the maximum torque, in synchronous kilowatts per phase, is from Equation (33).

CALCULATED STANDSTILL VALUES

	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact	Approx.	Exact
Starting current	386	386	68.2	68.0	164	164	121.5	120.2	3660	3710
Starting torque	63.6	63.8	2.81	2.84	39.2	39.2	52.0	54.3	74.4	73.0

The current and torque are both determined by Equation (44). The torque is given in synchronous kilowatts per phase.

X. CONCLUSIONS

A method of solving the polyphase induction motor equivalent circuit has been developed, which enables simple algebraic formulas to be used for the calculation of all the useful motor characteristics except power factor, with satisfactory accuracy. Heretofore complete circuit calculations, circle diagram constructions, or multiple chart references have been required to secure the same results, the formulas ordinarily presented in text books being too inaccurate for regular design use. Also, a comprehensive and accurate power factor chart has been shown, which is extremely useful in design work.

The "differential correction method," as it may be called, is believed to possess unique advantages in combining speed with accuracy, and in facilitating determination of the effects of independent changes in particular motor constants on any motor characteristic.

The most useful formulas developed are believed to be (12), (14), (16), (20), (33), (36), (38), (40), (44), and (46), each of which involves the product of the simple formula for an ideal motor by a correction factor based on the per unit reactance and loss constants of the motor.

A prime object of presenting this paper has been to lay a groundwork for ultimate agreement on standards for the determination of motor performance from test results. The A. I. E. E. Standards do not now give precise instructions for doing this, and it is therefore important that more complete methods should be agreed upon. While no exact procedure for this purpose is recommended here, it is believed that Equations (12), (14), (33), and (44) may be useful in this connection.

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Discussion

H. C. Specht: In the paper by Mr. Alger the assumption is made that the measured total leakage reactance can be divided equally for the primary and secondary. This has been the practise for many years. This is fairly correct for wound rotor motors with slip rings; however, for most squirrel-cage motors this is seldom accurate enough. It may be well to split up the measured leakage reactance in the ratio of the primary and secondary leakage as has been obtained by calculation.

Further Mr. Alger assumes that the core loss is proportional to the square of the induced voltage and that at high voltages the exponent is usually greater. I take for granted hereby is meant the core losses at high saturation.

Strictly sinusoidal wave forms are assumed. Although this has been a common practise for many years it does not represent the real case and often the results are badly affected by the high harmonics which more or less exist in all motors.

For the ohmic resistance, the resistance obtained by direct current method is generally used. In fact the effective a-c. resistance is somewhat higher and it seems that Mr. Alger is considering this fact but does not give a method to determine the accurate increase. By some authors on induction motors an empirical percentage of 15 to 25 per cent is assumed. It seems to me it would be worth while to do some research work on this subject.

The various equations developed by Mr. Alger with a few exceptions seem to me too long for practical use and by shortening these, simply by neglecting some members of the equations the accuracy may not be sufficient for all cases. In particular the determination of the power factor does not seem to me practical, the equation itself is much too long for practical use as stated in the paper, and even the assistance of a chart as given by Mr. Alger does not represent a quick method.

In my opinion I do not see any advantage in using an entirely algebraic method of calculating the performance. I am in favor of figuring only some values that can easily be obtained and obtaining the others by the circle diagram. In all my experience I have obtained in this way quick and sufficiently accurate results.

I do not believe that complicated calculations for the determination of motor performance from test results will lead to an agreement on standards. I agree with Mr. Alger that Equations **12, 14, 33,** and **44** may be acceptable.

The fact remains that the customer will judge motors by the results obtained on test rather than by theoretical performance calculations.

R. E. Hellmund: Mr. Alger's paper and Mr. Specht's discussion thereof merely confirm in my mind the usual experience that every engineer likes his own calculating and design methods best. While a standardization of calculating methods may have certain advantages, I am not at all convinced that such advantages would be worth while in view of the disadvantages which such standardization would bring with it. Any calculating method for everyday use must of necessity make certain approximations to be practical. Whatever such approximations may be, they are usually quite satisfactory for the average case, but there are always extreme cases where the same approximations would lead to unsatisfactory results. This in itself makes standardization dangerous, especially in the hands of the younger and inexperienced engineer. Furthermore, engineers are differently constituted mentally, and therefore methods giving one engineer the most satisfactory conception and understanding of his work will not prove nearly as useful to another engineer as some other available methods. Finally, the standardization of working, or, as you might call them, "thinking methods" would be likely to stifle the individuality and originality of the engineer.

Having thus granted that various methods should preferably be used by different engineers, I do not hesitate to state that my preference is in the same direction as that of Mr. Specht's with regard to the use of the circle diagram. The use of this method has become so common in the working up of performance from test results that it should certainly be retained as one of the generally recognized methods. I have even made it a practise to use the circle diagram almost exclusively for design purposes, because after the calculations required for any method are made the diagram can be constructed in a few minutes and is then available for deriving a complete motor performance. By using the ratio $\frac{A\ B}{A\ C}$ as the leakage coefficient and figuring the angle (E) of the circle displacement, the circle $B\ D\ C$ can be drawn. This gives for any arbitrarily assumed point D a primary current $A\ D$ and a secondary current $B\ D$. It is then possible to figure the primary copper losses $D\ E$ and draw the line $E\ F$. Its intersection with $C\ D$ locates the torque circle $B\ F\ C$. By figuring the secondary losses $F\ G$, the output circle $B\ H\ C$ can be found. The constant losses give the distance of the line $L\ K$ from $A\ C$, and we are now ready to find currents and power factor for any desired load. The efficiency also can be taken from the diagram, but may have greater accuracy if calculated in the usual way after the currents have been determined.

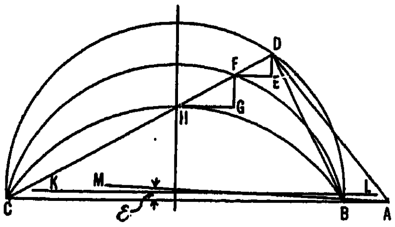

Fig. 1

I prefer the circle diagram method not only because it gives a better visual picture of the motor performance and the possibilities for changes in design, but also because it represents a method which can be easily remembered. Although I have not designed any induction motors for about fifteen years, I could readily calculate the motor performance from the diagram as shown without referring to any notes. This obviously would be impossible with a set of formulas and curves as given in Mr. Alger's paper.

C. J. Koch: Mr. Specht has raised the question as to whether the measured total leakage reactance can be divided equally for the primary and secondary. Of course, the probability is that in a very few cases that is true, primary and secondary reactances are exactly equal and in all motors of the double type it is quite far from being true. However, the error induced by moving the magnetizing current to the right or left in that figure so as to make $X\ 1$ greater than $X\ 2$ is true and as a matter of fact, except in low speed motors the result would hardly be modified. If the magnetizing current was assumed to be low, the assumption is that the core loss variation is not as much as it might appear. What we do, is take the value of core loss with the dynamometer when the motor is running light and it draws so many watts from the line. The air-gap goes down slowly and it is assumed that it is involved and that the actual L. U. T. voltage varies only perhaps ten per cent. So the assumption which we are making does not involve any very great error.

In regard to measuring the resistance by direct current I presume Mr. Specht had reference to the primary resistance. We know in large motors it may be higher than the resistance to direct current. However, the means for calculating this current are so well known to everyone that it has been more or less taken for granted in this paper. In small motors it is entirely absent and there is no need for increasing the primary resistance when it is carrying alternating current. I don't think it was Mr. Alger's intention to recommend these design methods to be used by everyone. As Mr. Hellmund says, everyone has his own preference in his every day work.

However, Mr. Alger's idea was rather this: at the present time various manufacturers give guarantees on efficiency of motors which are not based on full load tests of the motors but are based on so-called running light tests and block rotor tests and the manufacturers then deduce from these tests what the motor will actually do under load. In the organization I represent we use equivalent circles, others use the circle diagram, but the recommendations of the Institute are not very clear as to just what should be done or how it is being done. I think Mr. Alger's offering in the conclusion was the intention of improving the A. I. E. E. Standards, by giving a more definite method for going from the running light load tests to the actual power tests under full load.

P. L. Alger: I believe the equivalent circuit of Fig. 1, with the branch impedances properly determined, represents the motor performance with great accuracy, and that the straight algebraic solution of the circuit for a single load is both quicker and more accurate than a circle diagram solution. The circuit impedances are not truly constant, however, as all the reactances decrease with increasing current, and in double squirrel cage motors especially, the secondary resistance increases and the secondary reactance decreases markedly with decreasing speed. Hence, different values of the impedances must be used for different operating ranges, which is another way of saying that the imperfections of the circuit can be conveniently allowed for by varying the "constants."

The circle diagram shown by Mr. Hellmund assumes the primary copper loss to be the secondary current squared times the primary resistance, plus the no load copper loss. This gives rise to relatively large errors. In addition, the diagram is no more a true circle than the equivalent circuit is correct, when the branch impedances vary from saturation or other causes. Thus, for most accurate calculations, a different set of values for the circuit constants and a different circle diagram should be drawn for each region of motor operation.

I have found the formulas given in the paper, and especially the power factor chart, to be very convenient for regular use.

I have recently had the pleasure of meeting Mr. D. B. Hoseason, who has developed a very similar method of calculation, and who informed me that he is using it for regular design work.

Voltage Irregularities in D-C. Generators

BY J. T. FETSCH*
Associate, A. I. E. E.

***Synopsis.**—This paper enumerates the voltage irregularities which may occur in d-c. generators. It gives a description of these irregularities, the cause of their occurrence, the magnitude of their values, their effect on radio circuits, and states the problem encountered in their measurement.*

A method is described for the measurement of these voltage irregularities, by the use of an oscillograph, a blocking condenser, and a current transformer.

A number of oscillograph records, illustrative of the voltage irregularities existent in various d-c. generators, is shown, and an analysis of the wave form made for each component of slow and fast ripple. These records were taken under various electrical and mechanical conditions of the generators. Comparisons between these records are made.

Conclusions are drawn as to features desirable in d-c. generators.

* * * * *

GENERAL INTRODUCTION

THE general idea is held that the voltage from a d-c. generator consists of a continuous smooth produced electromotive force with practically no fluctuations. This opinion is decidedly a misapprehension, since in some cases the voltage irregularity may be as large as 5 to 10 per cent, although in the majority of cases the range is around 1 to 2 per cent. The value of this irregularity encountered, depends almost wholly on design, and is governed principally by the type of winding, shape and size of poles, the type and number of slots, to a degree upon the size of the machine, the air-gap, the electric and magnetic loading, the occasional lack of the interpole feature, and the method of fabrication. This last item is important, as will subsequently be brought out in this article.

For ordinary commercial power work where machines of comparatively low voltage are used, a ripple voltage of one per cent is usually not harmful, but where the d-c. generator is of the high-voltage type and is used for a special purpose such as the source of supply for plate power and bias of vacuum tubes, the following harmful effects may ensue: First, there may be modulation of the radio frequency output of the vacuum tube, causing sidebands and an accompanying broadness of the signal transmitted. This may be especially objectionable where telephone is being used. Second, generator hum in nearby receivers may be experienced by the setting up and the radiating of highly damped radio frequency oscillations in the power supply leads of the high-voltage source. Third, unduly high peak voltages on radio equipment being supplied with such a source.

The problem at hand is to measure this ripple voltage by some method, and whatever method is used, must be sufficiently sensitive and owe its successful operation to the utilization of the very small currents available for measurements. Several methods were tried, but failed on account of inadequacy or the lack of sufficient degree of sensitivity. The reason for this is because the ripple voltages present in high-voltage machines are only of the order of 10 to 20 volts, and the ripple current which flows through the circuit impedances only of the order of one or two milliamperes. Now, if a current transformer is used for stepping up this minute current to about 100 milliamperes, then the standard element of an oscillograph can be actuated to a degree sufficient to give a very good record of the voltage irregularity. By utilizing this method of transformer and oscillograph, it was possible to measure ripple voltages of the order of ¼ of one per cent and sometimes better. In addition, the oscillograph record of the wave form is very valuable, as it is the basis for the analysis of the causes of these voltage irregularities.

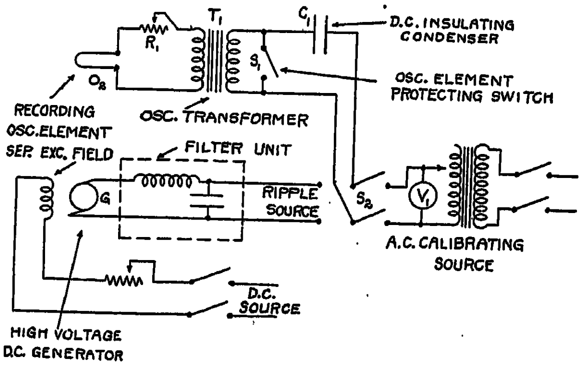

FIG. 1—DIAGRAM OF CONNECTIONS FOR MEASUREMENT OF VOLTAGE IRREGULARITIES

V_1 = 0-15 volts a-c. 25-2000 cycles
S_2 = D. P. D. T. transfer switch
C_1 = 2-3 microfarads, insulated to withstand d-c. generator voltage
S_1 = S. P. S. T. protecting switch
T_1 = oscillograph current transformer ratio approx. '95 to 1
R_1 = 2 ohm variable resistance
O_2 = standard oscillograph element 1 cm./50 ma.
G = high-voltage d-c. generator under test

2. DESCRIPTION OF METHOD FOR MEASURING VOLTAGE IRREGULARITIES

This method gives a photographic method of the voltage irregularity under investigation by the combined use of an oscillograph, a current transformer, and a condenser. Referring to Fig. 1, the oscillograph element O_2 is that used in the standard type of oscillograph, and has a sensitivity of one cm. deflection on the screen per 50 milliamperes. In

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*Asst. Radio Engr., Naval Research Laboratory, Anacostia, D. C. Submitted with the approval of the Navy Department.

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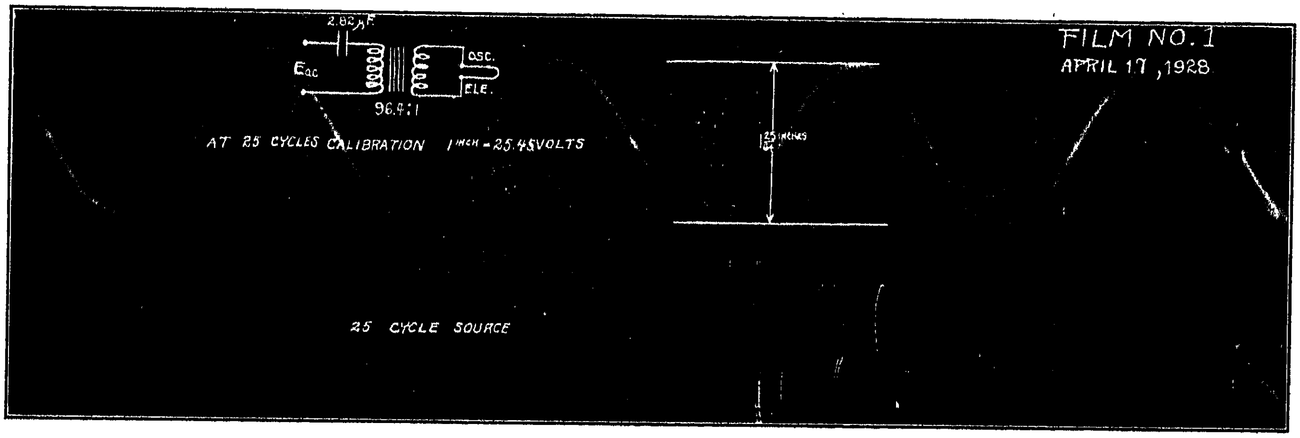
conjunction with this oscillograph element O_2 is used a current transformer T_1, so that the minute value of primary current composing the ripple may be stepped up to a useful value to operate the element O_2. A condenser C_1 of 2 to 3 microfarads is inserted in series with the primary of the transformer to prevent direct current from flowing, yet allowing the ripple current to pass through. This condenser must have negligible leakage, else sufficient direct current might pass through the transformer primary, thus causing the iron core to saturate and thereby prevent the faithful reproduction of the voltage irregularities being studied. The switch S_1 is for protecting the element O_2 and the transformer T_1, and is always kept closed except during observations and photographic recording of the ripple wave form, and while the element is being calibrated. The variable resistance R_1 is about two ohms and is used to regulate the amplitude of vibration of element O_2. The switch S_2 is used as a transfer from the unknown ripple source to the a-c. calibrating source. G is a generator, with separately excited field, whose ripple voltage is under investigation. A filter unit is shown connected into the high-voltage circuit, but may be removed from the circuit if desirable. The calibrating circuit shown comprises a sinusoidal source of known frequency with the possibility of variation, and a transformer for reducing the voltage to a desirable value. This voltage is indicated by meter V_1, and is then fed to the oscillograph circuit.



FILM NO. 1
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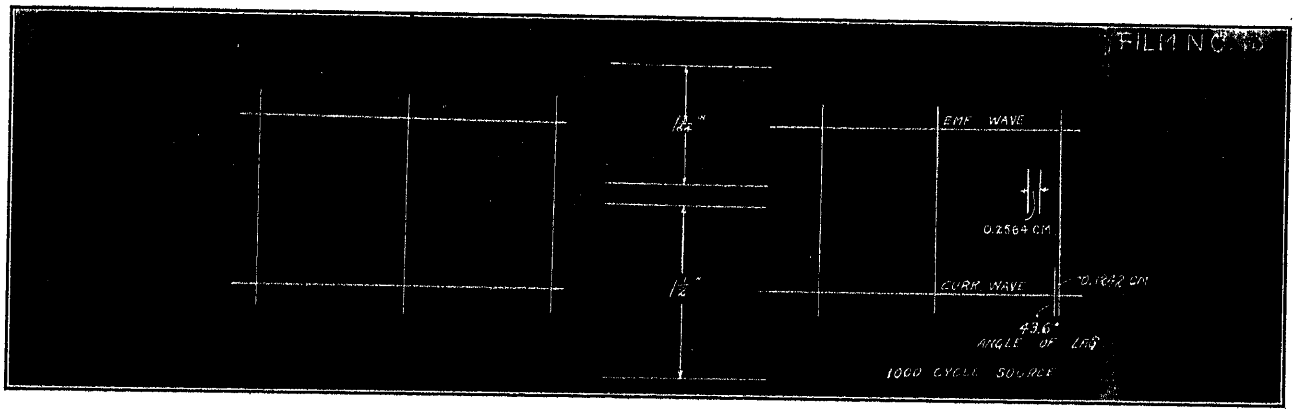


FILM N

The procedure in obtaining records is first to calibrate the element O_2, second to take an oscillographic record of the ripple, and third to recalibrate the element O_2. These calibrations are then averaged, and applied to the trace of the film taken during calibrations. This must be done for each film record to ensure accuracy, as there are physical factors which disturb the permanent calibration of the oscillograph element. For calibration, voltages of the proper amplitude and frequency corresponding to those inherent in the ripple source are used. These can be determined by observations on the screen and by simple calculations.

In manipulation of the apparatus, the frequency of the a-c. source is first adjusted to that belonging to a certain component of the ripple voltage. The voltage V_1 as read on the meter is then adjusted by varying the supply source until observations show proper amplitude of vibration as seen on the scanning screen.

The amplitude of vibration may be recorded photographically or measured on the oscillograph screen. Examples of those taken by the former method are shown in Film No. 1, where a 25-cycle source was used for calibration and of Film No. 15, where a 1000-cycle source was used. These calibrations at different frequencies are applied to their respective ripple components found in the oscillographic trace. It is necessary to apply a different calibration constant for each frequency, because the calibration varies somewhat with the frequency.

In preparation for taking an oscillographic record of a voltage irregularity, after the calibration has been obtained, Switch S_2 is placed to the left, and the apparatus placed in operation for taking a record of the generator ripple. The field of the d-c. generator G is gradually increased until normal generated voltage is indicated. After conditions are steady S_1 may be opened, and the voltage irregularity observed on the oscillograph screen. If this is judged satisfactory for photographing, a record may be made, care being taken to omit no part in the operating procedure of the oscillograph instrument, lest negative results follow. From here on the calibration procedure is once again gone through as explained previously under calibration.

In the interpretation of the film records, for obtaining the per cent ripple voltage, the question arises as to what shall be designated as the per cent ripple. This is to be designated as the voltage variation from the highest positive peak to the lowest negative peak on a trace divided by the average d-c. potential of the generator as read on a D'Arsonval instrument.

As a definition for the components of the ripple voltage, the slow speed components will be all of those due to grain direction, speed variation, and other armature reluctance variations of any kind, while high-speed ripple will be termed that caused by the slots and that due to commutation.

In finding the per cent ripple, the low-speed and the high-speed components of the ripple voltage have their individual calibrations applied, and then, either the per cent ripple can be given for each component, or added together to give the total.

3. VOLTAGE IRREGULARITIES WHICH OCCUR AND AN EXPLANATION OF ORIGIN OF EACH

An analysis of this problem by means of oscillographic records proves that the following voltage irregularities exist collectively or in part in high-voltage d-c. generators.

(a) Slot ripple.

(b) Commutator ripple.

(c) Armature reluctance variation ripple due to grain direction in steel.

(d) Armature reluctance variation ripple due to improper physical alinement.

(e) Speed variation due to improper shaft alinement of motor and generator.

(*A*) *Slot Ripple.* Slot ripple is due to resultant pulsations in magnetic flux which surge through the armature windings of the generator, thereby causing periodic voltages to appear at its terminals. These pulsations arise by virtue of the fact that the slots on the armature periphery are moving by the leading, and trailing pole tips, thus effecting changes in magnetic reluctance and field distribution in the air-gap as they do so. It can easily be seen that the periodicity of this pulsating electromotive force will be the product of the total number of slots times the rev. per sec. of the armature.

An explanation of the resultant flux pulsation and its accompanying induced electromotive force in the armature winding can be seen by studying the diagram shown in Fig. 2. Here is a developed simplex, singly re-entrant drum winding for a two-pole machine. There are eight slots on the armature of this machine on which all even conductors are located at the bottom of the slots, and all odd conductors located at the top of slots, or vice versa.

Considering the direction of rotation of the generator to be to the right (as shown in Fig. 2), the leading pole tips will be the left hand edges of the N and S poles and the trailing pole tips the right hand edges of the N and S poles.

Now, whenever an armature tooth passes by a leading pole tip into the magnetic field there will be a decrease in reluctance of the magnetic circuit at this point. Correspondingly, when a slot passes by a leading pole tip into the magnetic field, there will be an increase in the reluctance of the magnetic circuit at this point.

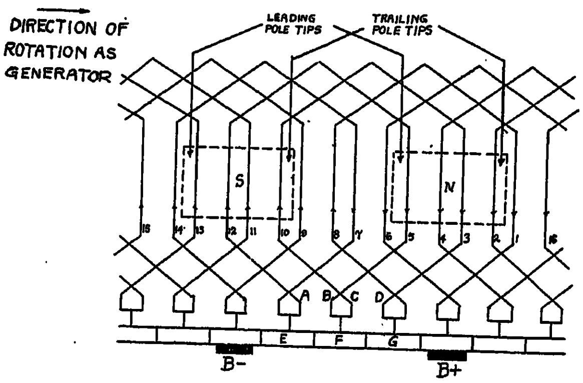


FIG. 2—DEVELOPED TWO-POLE SIMPLEX SINGLE RE-ENTRANT DRUM WINDING

Also, when the above armature teeth or slots pass out of the magnetic field from under the trailing pole tips the reluctance of the magnetic circuit is changed similarly to those cases explained above.

Referring to Fig. 2, and especially with regard to conductors A B and C D, let us imagine that a tooth is passing by the trailing tip of the S pole and at the same time imagine that a tooth is passing by the leading tips of the N pole. This means a decrease in reluctance in both cases. The effect of this change in reluctance will be to increase the magnetic flux through the armature coils A B and C D. In turn, a pulsating electromotive force will be induced in the windings and will appear across commutator bars E F and F G. As these coils are part of the total number included between the brushes B B, this pulsation will appear at the brushes of the machine.

Now, going back a bit, let us imagine that conditions were such as enumerated in the above case, except that instead of the tooth passing by the leading tip of the

N pole, a slot is passing by. Here, in one instance, the reluctance is increased and in the other instance, the reluctance is decreased. These will tend to offset each other. If they are equal in magnitude and oppositely directed, no pulsating voltage will be induced in the windings *A B*, and *C D*, hence no resultant slot ripple voltage will appear at the machine terminals.

The point to be made in the above explanation is that slot ripple can be entirely eliminated by proper design. This depends on the proper per cent of pole enclosure with respect to the armature slots. The distance from leading to trailing tips of adjacent poles should be proper so as to eliminate the magnetic pulsation enumerated above. Moreover, the pole coverage should be such as to include an integral number of slots or teeth. Of course, the fringing effect may change this stipulation somewhat, but the condition desired in attaining is that there be no resultant flux change through the generator windings for the complete obliteration of slot ripple.

A quick test can be made on generators to ascertain whether they will have slot ripple. If the conditions as set forth above are existent, there will be no resultant variable forces on the armature during rotation due to the slots. Keeping this in mind, if an armature is slowly rotated by hand, with full field applied, there will be no variation in torque. This means that here the generator will be devoid of slot ripple. On the other hand, if variations in torque occur as the armature is rotated, this means unequal changes in the magnetic forces at the leading and trailing tips, and hence ripple voltages will be inherent in the generator.

B. Commutator Ripple. Commutation ripple is that voltage irregularity arising from the reversal of current flow in the armature coils due to the brush passing from bar to bar of the commutator. When this current is changed in an armature coil, a reactance voltage is set up. This reactance voltage must be compensated for, else it will appear at the terminals of the generator.

In earlier types of d-c. machines, it was taken care of, at least partially, by brush advance, and in modern design by interpoles mainly, although even now many small machines are built with the omission of the interpole feature.

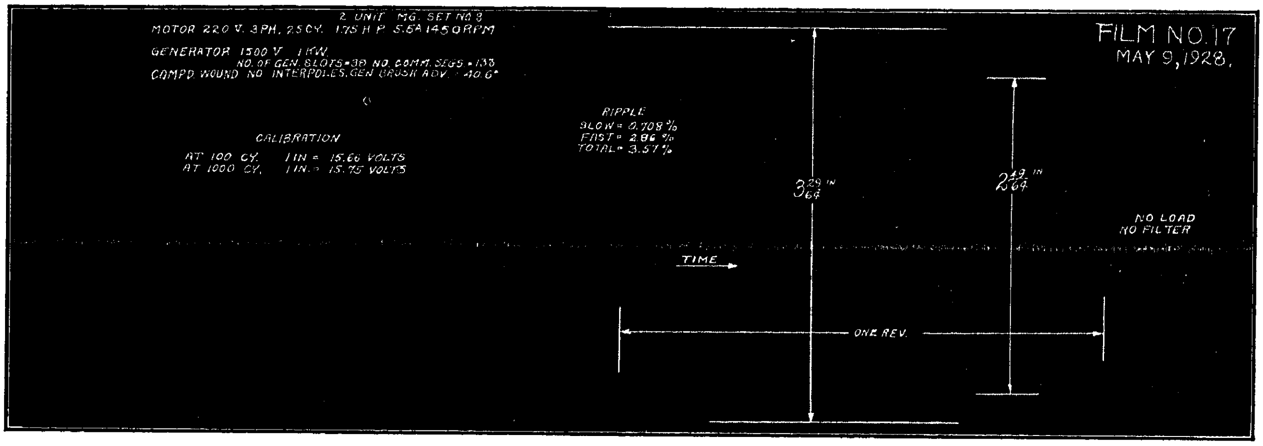
In order for a machine devoid of interpoles to commutate successfully, the coil undergoing commutation is moved into an adverse field. By so doing, a voltage is induced in the coil, large enough not only to overcome the reactance voltage set up in the coil, but of sufficient strength to make a current flow in the opposite direction in the coil equal in value to the load current. For this condition little or no sparking will occur at the brushes, and no voltage is left over to affect the terminal voltage of the machine. The disadvantage of this type of correction for commutation is that the adjustment holds only for a definite load current through the generator. Any other value of generator current produces some sparking at the brushes and some commutation voltage irregularity.

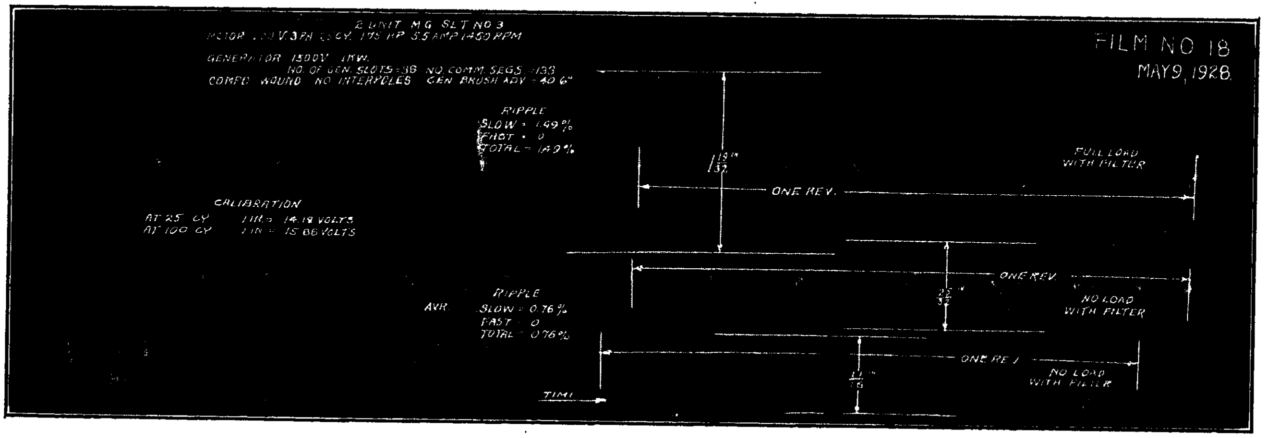
This drawback is overcome by the use of the interpole for commutation purpose. As the strength of the interpole is proportional to the load current, the induced voltage in the coil under the interpole will thus be proportional to the load current, as is also the reactance voltage in the coil. Thus, with proper adjustment, commutation irregularities practically do not exist. Experience so bears out the above statement with regard to the advantages of the interpole feature that all machines used as d-c. high-voltage sources should be equipped with interpoles.

C. Armature Reluctance Variation Due to Grain Direction. There is another voltage variation which is noted in d-c. high-voltage generators due to grain direction in the sheet steel used for the armature material. This grain direction is a property imparted to the sheet steel during manufacture and causes the steel to have somewhat better magnetic properties in the direction of sheet rolling, than in other directions. Thus, this permits pulsations in the magnetic circuit as the armature rotates, and therefore ultimately affects the terminal voltage of the generator.

In the use of sheet steel for armature laminations, certain methods of manufacture are employed in the stamping of the armature laminations and in the fabrication (construction) of the armature from these laminations. Usually, where small d-c. motors and generators are concerned, the armature generally consists of a single lamination stamped from a piece of sheet steel. Moreover, a place for a keyway is incorporated in this lamination by having the punching die so made as to cut out a rectangular place in the steel. At the factory where these laminations are punched, the sheet silicon steel is fed into a machine at one end, and the lamination stamped out of this material, the lamination being always stamped in the same relative direction with respect to the grain of the material. This direction is usually along the longest dimension of the sheet. Hence, all the laminations, when stamped, have the same relationship between the direction of the grain in the sheet steel and the slot made for the keyway.

Now, it can be seen that when these laminations are all assembled in the form of an armature, the magnetic properties of the armature are going to be somewhat better between those points which include the line coincident with the grain direction, than with any other direction. Thus, with an armature of this type (and up to this time all small armatures are built up of a stack of single disks with slots and keyway stamped therein) there will be a pulsation in the reluctance between the pole faces as the armature rotates. This pulsation will cause a variation in the flux threading the armature, the rate of pulsation being determined by the speed and number of poles of the machine. The period will be twice the rev. per sec. of the machine because it must be noted that for every revolution of the armature the grain direction is coincident with the line of the poles twice, considering a two-pole machine. Thus the periodicity of this variation will be twice the product of the rev. per sec. times half the number of poles. Good examples of this are Films Nos. 16, 20, and 24. These all show this grain direction voltage irregularity, and are examples of four-pole generators. The periodicity of this voltage irregularity proves to be as stated above, *i. e.*, the rev. per sec. times half the number of poles. While these few films show in a very pronounced fashion the effect of grain on the terminal voltage of a generator, every film included herein shows this effect to a greater or less extent.





While in general the magnetic path from pole to opposite pole through the armature is best when the grain direction in the armature laminations is coincident with a line through the centers of these poles, nevertheless a digression from this condition is sometimes existent. For machines with very small armatures it may happen that on account of the diameter of the shaft being comparable with that of the armature, the path of least reluctance of the armature and poles does not occur when the grain direction is coincident with a line through the centers of the main poles, but occurs both before and after this position is reached. In this case, the periodicity of this voltage irregularity would be four times the product of the rev. per sec. times half the number of poles, instead of twice as in the preceding explanation. A striking example of this is afforded by Films Nos. 17 and 18, and all subsequent films taken with motor generator set No. 3. Here the pulsation, due to grain direction, is twice the frequency of what would be expected with a large diameter armature. Owing to the fact that the diameter of the shaft of this generator is nearly 30 per cent the armature diameter, this accounts for the doubled periodicity of the magnetic pulsation.

In connection with the effect of grain direction, the following may be pointed out. If the generator is of the interpole type, there will be an additional point of minimum reluctance besides that stated above and this will have some relationship with the grain direction of the armature and a line through the centers of the interpoles. This reluctance variation, unlike the one above, may or may not produce a voltage irregularity, depending on certain factors such as the design of the machine. What can happen is that the wave form of the voltage irregularity may change with load, as the magnetomotive forces acting, and the flux distribution at no load are different from those acting at various loads. These magnetomotive forces can disturb the flux distribution in the air-gap and in conjunction with the grain direction, bring about a change in wave form of the ripple. The effect of these magnetomotive forces with an interpole machine is much more likely to disturb the flux configuration in the air-gap than with a machine devoid of interpoles.

A description of the effect of grain in a machine can be better elucidated, perhaps, by means of Figs. 3, 4, and 5. Fig. 3 shows a two-pole interpole machine with the grain direction indicated. When the grain direction is in line with the main poles, the reluctance of the magnetic circuit is minimum, and is somewhat greater in all other positions of the armature. Fig. 4 shows the field distribution under the main poles. As the machine rotates, and the grain direction comes under position *A*, Fig. 4, there is a sudden increase in magnetic flux. This flux surges through the generator winding and induces a voltage as shown in Fig. 5, at position *D*. When the grain direction reaches *B*, the flux increase no longer is taking place, hence there is no electromotive force induced in the generator winding. This is the point where the ripple voltage is zero, (position *E*, Fig. 5). Later on when the grain direction reaches *C*, Fig. 4, there is a diminution in the magnetic flux, and hence an electromotive force of opposite polarity is induced in the generator windings as is indicated by position *F*, Fig. 5. Throughout the remainder of the revolution an exact repetition of the above changes takes place. Examples can be seen by examination of any of the records presented herein. In particular Film No. 2 bears out this explanation.

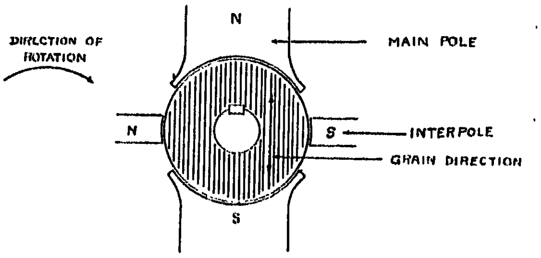


FIG. 3

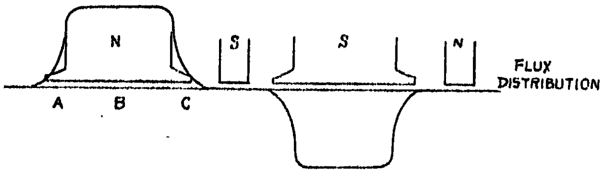


FIG. 4

D. Armature Reluctance Variation Due to Improper Physical Alinement. If an armature is not running true, so that the armature periphery and the field bore are both eccentric to the shaft, then the conditions represented may be such as to produce a reluctance variation, and so a voltage irregularity. Also, if the armature shifts to and fro longitudinally, so that the pole face does not always cover the ends of the armature, the effective flux acting through the armature windings may be reduced at certain instants, causing voltage irregularities. The periodicity of the former will be equal to the rev. per sec., whereas that of the latter may be some integral number times the rev. per sec.

E. Speed Variation Due to Improper Shaft Alinement of Motor and Generator. As the steadiness of the generated d-c. voltage depends on the constancy of the generator speed, any factor causing variation in speed will influence the terminal generated e. m. f. A few possibilities can be mentioned as first, a variation in motor speed, second, a variation in torque on the generator armature, and third, misalinement of the motor and generator shafts. It is this latter factor that sometimes may play an important part in causing a voltage irregularity, sometimes being the greatest in magnitude, compared to any other.

This may come about, if the shafts of the motor and generator are out of alinement, and a flexible coupling connects them. What takes place is that a to and fro oscillation is set up, due to the pliable material of the coupling flexing during certain parts of the shaft revolution. A little later, this flexure changes, and this pliable material between the two faces of the steel couplings takes on another configuration. The contact points between the leather and the couplings are continually changing their effective radii, and fluctuations in speed arise thereby. Immediately upon alinement of the shafts of such a motor generator set, the voltage irregularity due to this cause disappears.

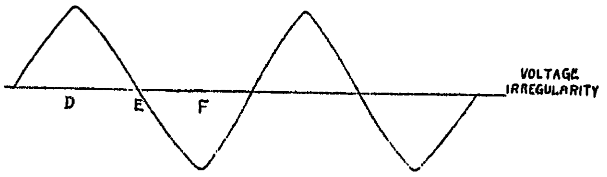


FIG. 5

5. OSCILLOGRAPHIC RECORDS OF VOLTAGE IRREGULARITIES IN HIGH-VOLTAGE D-C. GENERATORS

Herewith are presented examples of oscillograph records of voltage irregularities taken by the method described in this article. The records were taken from generator sources used as d-c. high-voltage supplies for radio equipment. These films show quite a few unexpected phenomena, and each film will be described separately as almost every record brings out and substantiates some particular fact.

Concerning the film records, complete data such as calibration values of volts per inch, ripple percentage, and other facts have all been recorded directly on the oscillograph film.

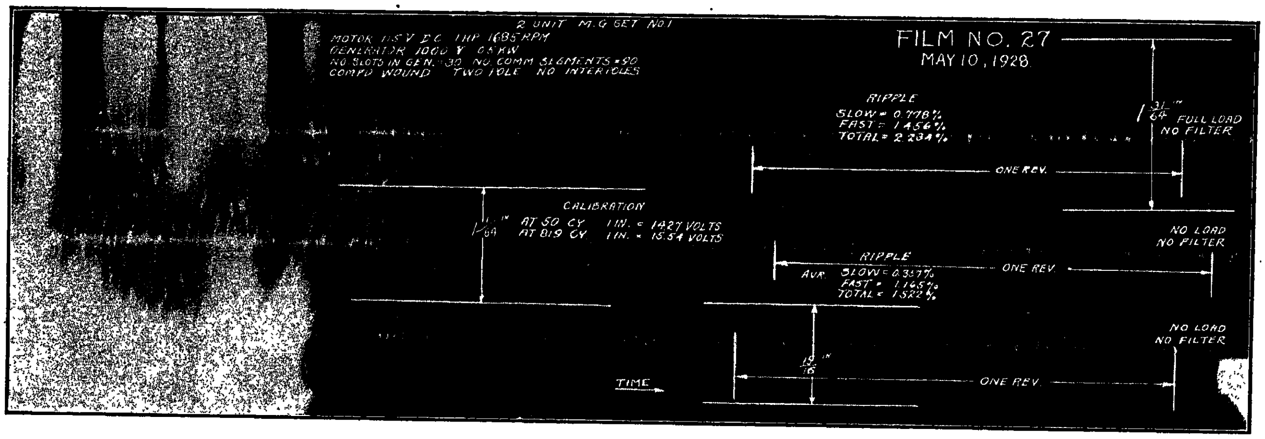
Film No. 27. The first oscillographic record to be presented is that of the generator of set No. 1, taken on Film No. 27. This two-pole generator has 30 slots and 90 commutator segments. It is devoid of the interpole feature, and does not have the proper pole coverage with respect to the slots for eliminating slot ripple.

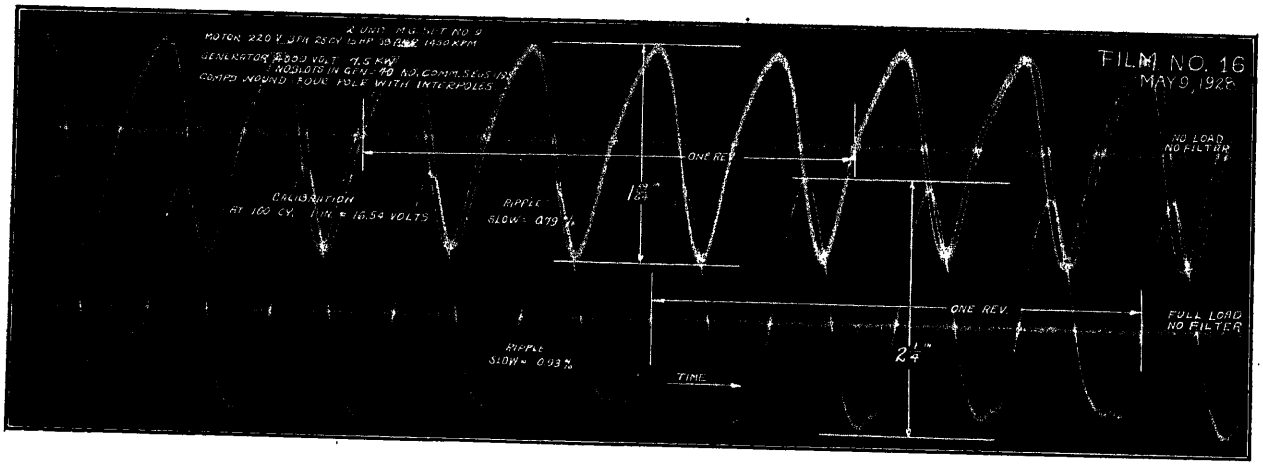
It is shunt wound, and has a single high-voltage commutator.

With a study of Film No. 27, it can be seen that slot ripple exists as is shown by the 30 pulsations occurring per revolution of the armature. This is a high-frequency ripple of 819 cycles per second, and is not extremely difficult to filter.

Some commutator ripple is also present due in part to the lack of the interpoles, and to the division of the winding. This too, like slot ripple, is of high frequency, but is not generally of very great amplitude as compared with other kinds of ripple.

The effect of grain direction in the armature can be observed and is seen to consist of four pulsations instead of two as is generally obtained. This is caused by the smallness of the generator armature, the machine being of only 0.5 kw. rating. This is a slow speed voltage irregularity and requires a very large filter to eliminate effectively.





No other irregularities are discernible in this machine, and hence indicate either the entire absence or the extreme smallness of their values.

A point to be noted is the variation in the amplitude of the ripple with load. In this particular generator, the ripple increased about 30 per cent from no load to full load.

Film No. 16. The four-pole generator used in obtaining this film record is that of motor generator set No. 9. It has 49 slots and 195 commutator segments. There are two high-voltage commutators connected in series; the machine has interpoles to take care of commutation, and has a compound field.

Slot ripple is non-existent in this machine as the design of the pole coverage with respect to the armature slots is such as to eliminate it.

Commutator ripple is also non-existent, principally on account of the interpole feature.

An armature reluctance variation e. m. f. due to grain direction, is seen to consist of four pulsations per revolution, which is in agreement with the number of poles. The amplitude of this ripple increases about 10 per cent from no load to full load, and a small amount of saturation effect can be noticed on the full load trace.

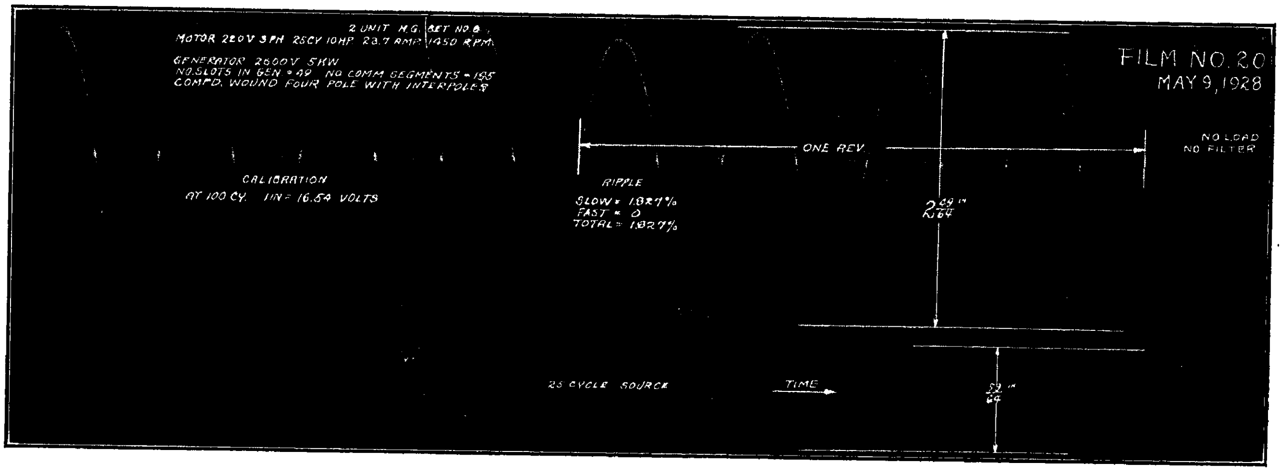
There is a slight indication that the speed of the machine varies with the revolution of the armature, as the peaks of the ripple wave form recorded on the trace are not all of the same height. This, of course, may be due to slight misalinement of the armature and poles rather than due to speed variation.

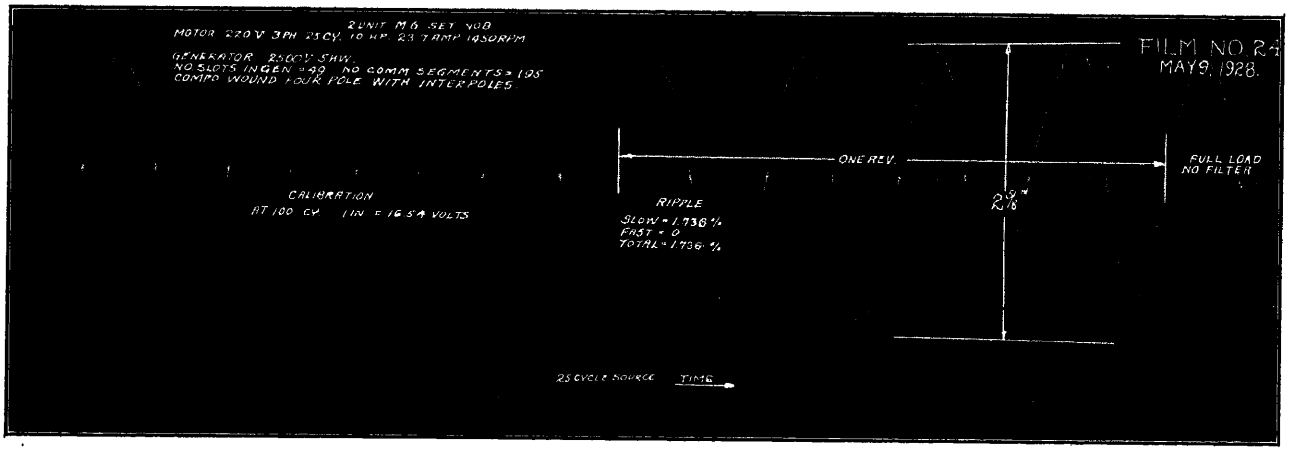
Films No. 20 and No. 24. These two records were taken from the four-pole generator of set No. 8. This machine, with the exception of a possible different shunt field winding and of more turns per armature coil, is exactly similar to the generator of motor generator set

No. 9 whose oscillographic record is shown in Film No. 16. It has 49 slots and 195 commutator segments. There are two high-voltage commutators connected in series; the machine has interpoles and has a compound field.

Both slot ripple and commutator ripple are non-existent for the same reasons given in the above explanation for Film No. 16.

Similarly, as in the previous example, armature reluctance variation due to grain direction is seen to consist of four pulsations per revolution, which is also in agreement with the number of poles of this machine. Here, unlike the results shown on Film No. 16, the amplitude of this ripple decreases slightly (about 5 per cent) with load and a small amount of saturation effect can be noticed on the full load trace, but is not so pronounced as it is on the full load trace of Film No. 16.





In the comparison of generator No. 9 with generator No. 8, whereas these machines are so nearly identical, yet the former is a 4000-volt 7.5-kw. generator, and the latter a 2500-volt 5.0-kw. generator. For continuous full load operation, the 4000-volt machine has a 50 deg. cent. temperature rise, and the 2500-volt machine has a 40 deg. cent. temperature rise.

Obviously, this points to the fact that the electric and magnetic loading of the 4000-volt generator is much greater than that of the 2500-volt generator. If the full load traces of these two machines are compared, the above fact is substantiated by the greater amount of saturation indicated on the trace of the 4000-volt machine than is indicated on that of the 2500-volt machine.

Films No. 25 and No. 26. These two records, one of which is at no load and the other at full load, were obtained from the two-pole generator of motor generator set No. 5. The machine has 22 slots and the 89 commutator segments. There are two high-voltage commutators connected in series; the machine has interpoles, and has a compound field.

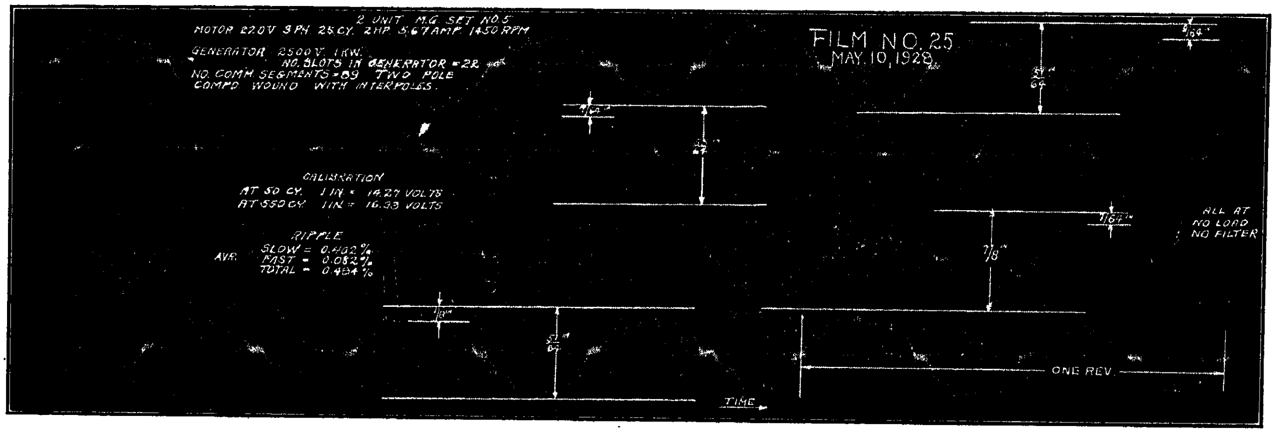
Slot ripple is present in this machine, and the number of pulsations per revolution of the armature is 22, corresponding to the number of slots. This is due to improper pole coverage with respect to the armature slots.

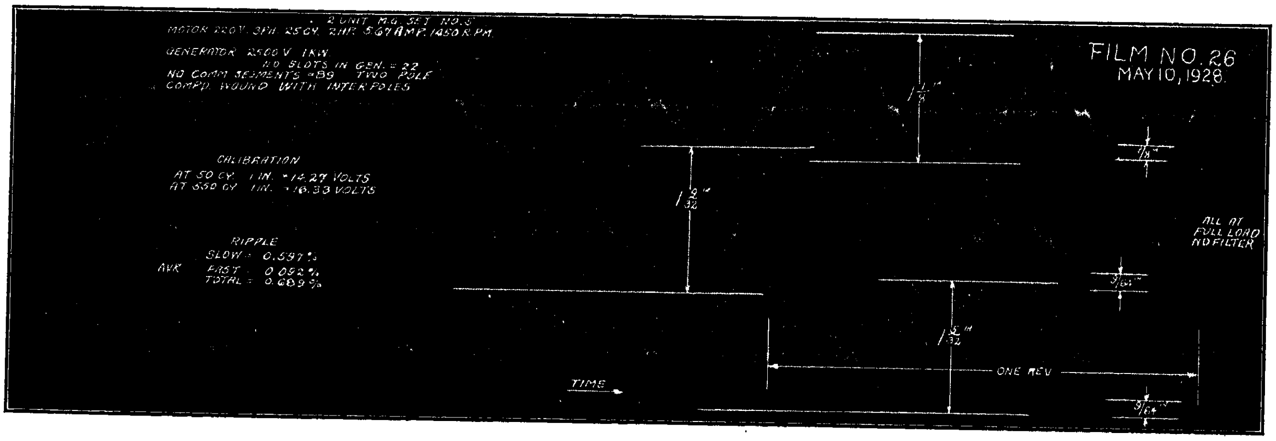
Referring back to machines No. 8 and No. 9, the idea might occur that the reason for the absence of slot ripple in these machines is on account of the presence of the two commutators in series. If the ripple voltage from each commutator were equivalent in wave form and 180 deg. out of phase, then the terminal voltage would contain no ripple, but as can be seen by consideration of the directions of the generated voltages, these voltage irregularities, if present, are in phase, and add up directly.

Thus, in a two-commutator armature, whatever voltage irregularity appears across each of the commutators, should appear collectively across both when they are connected in series with the d-c. voltages additive, and such is the case with motor generator set No. 5.

An armature reluctance variation voltage due to grain direction appears in Films No. 25 and No. 26, consisting of two pulsations per revolution, which is in agreement with the number of poles. The amplitude of this ripple increases about 30 per cent with load.

Here also is some indication that the speed of the machine varies with the revolution of the armature, as the successive peaks of the ripple wave form recorded on the trace are of different amplitude.



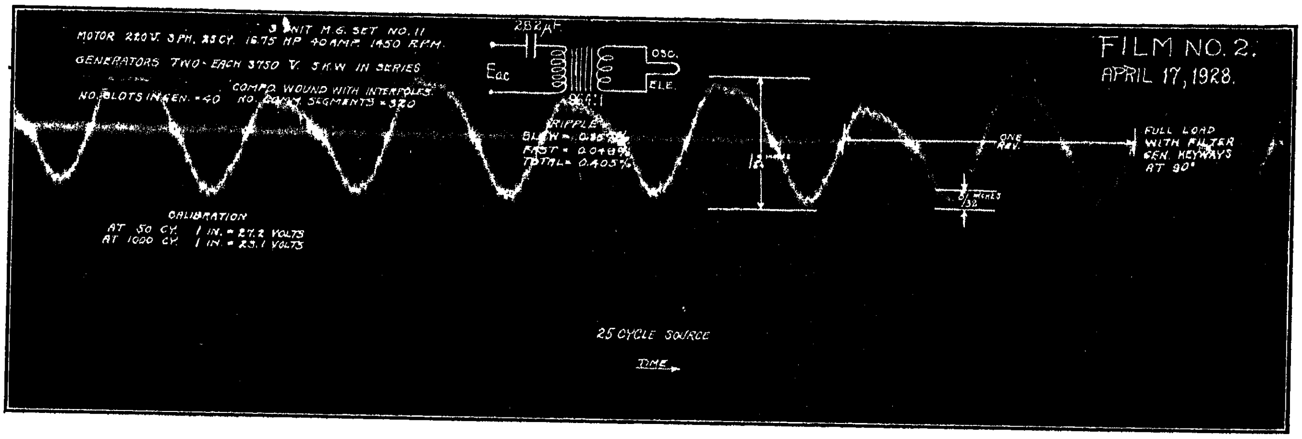


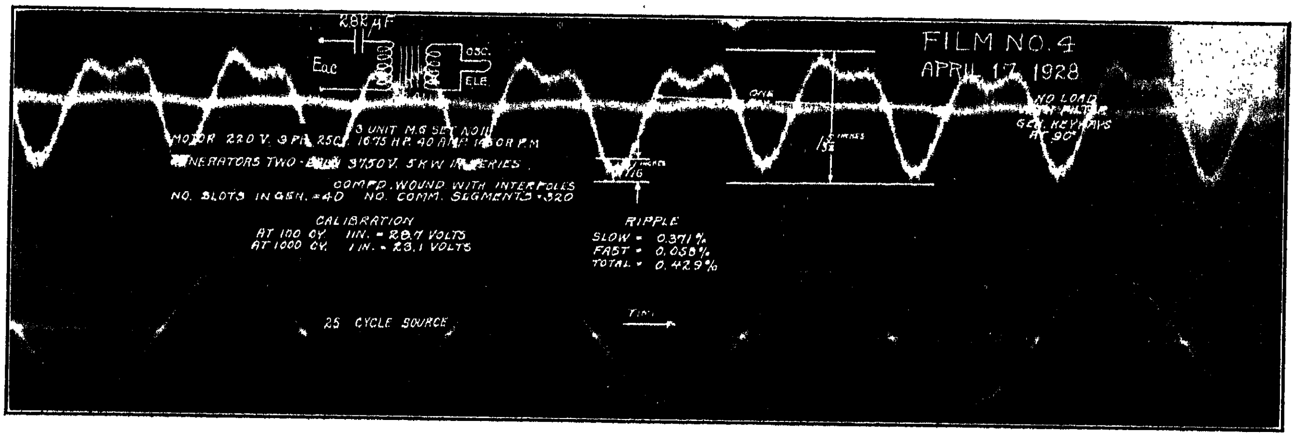
Films No. 2 and No. 4, No. 8 and No. 14, No. 11 and No. 12. These records all appertain to the same source, that is, motor generator set No. 11. This equipment consists of three units, one motor with two generators. These two-pole generators each have 40 slots and 320 commutator segments. Each generator unit has a single high-voltage commutator, interpoles, and a compound field. The 7500 volts is obtained by connecting these two 3750-volt units in series.

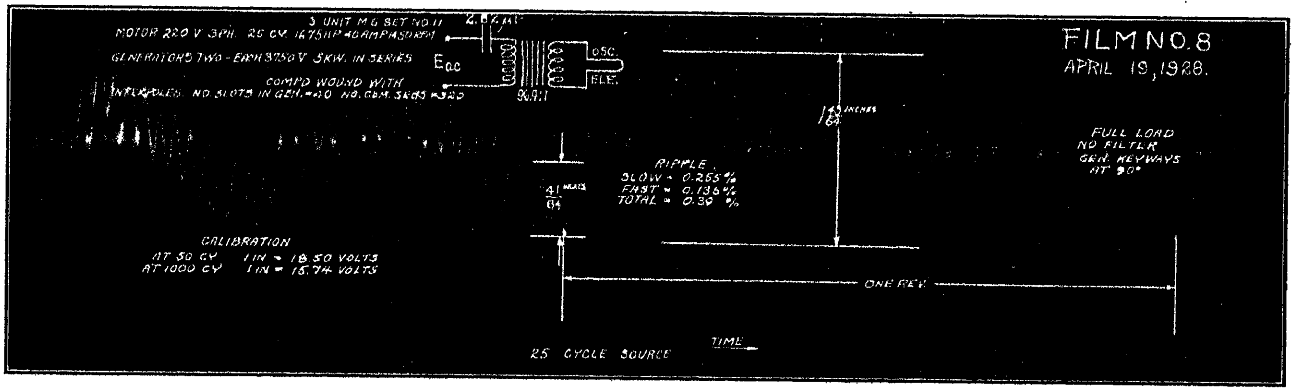
Films No. 11, No. 12, and No. 14 were taken with the armature grain direction of the two generator units at 90 deg. relative to each other, while with Films No. 2, No. 4, and No. 8, the armature grain direction of the two generator units was in like direction, relative to each other. For the former condition, the generator keyways were actually in a 180 deg. relationship, while for the latter condition the keyways were in a 90 deg. relationship.

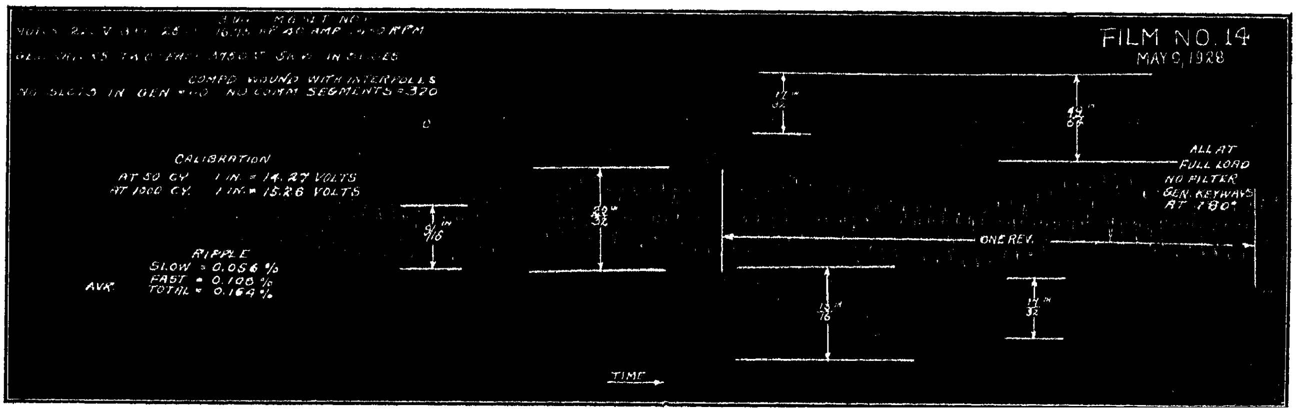
Attention is called to the fact that when the grain direction in the armatures of the two generators are at 90 deg. relative to each other, the voltage irregularities due to grain do not add up directly, but geometrically at right angles. Hence the ripple should be of a smaller magnitude for this condition than when the grain of the armatures is in like direction. This is borne out by examination of Films No. 11, No. 12, and No. 14, all showing a smaller ripple percentage as compared with corresponding Films No. 4, No. 2, and No. 8, respectively.

Analysis of these films discloses the fact that slot ripple and armature grain ripple, with a minute amount of commutation ripple, are present. Moreover, the percentage of ripple and its wave form change with load. The wave form is also different for the generator keyway relationships of 90 deg. and 180 deg. respectively. The latter fact is by virtue of the way in which the ripple voltage wave forms of the two generators add when in series and undisplaced, and when displaced from each other by 90 deg. The reason for the change in wave form with load is due to the collective influence


3 UNIT M.G. SET NO. 11
MOTOR 220 V. 3 PH. 25 CY. 16.75 HP 40 AMP. 1450 R.P.M.
GENERATORS TWO-EACH 3750 V. 5 K.W. IN SERIES
COMPD. WOUND WITH INTERPOLES.
NO. SLOTS IN GEN. = 40 NO. COMM SEGMENTS = 320
2.82 μF
Eac
OSC.
ELE.
96.6:1
RIPPLE
SLOW = 0.357%
FAST = 0.048%
TOTAL = 0.405%
FILM NO. 2.
APRIL 17, 1928.
ONE REV.
FULL LOAD
WITH FILTER
GEN. KEYWAYS
AT 90°
3/32 INCHES
CALIBRATION
AT 50 CY. 1 IN. = 27.2 VOLTS
AT 1000 CY. 1 IN. = 23.1 VOLTS
25 CYCLE SOURCE
TIME

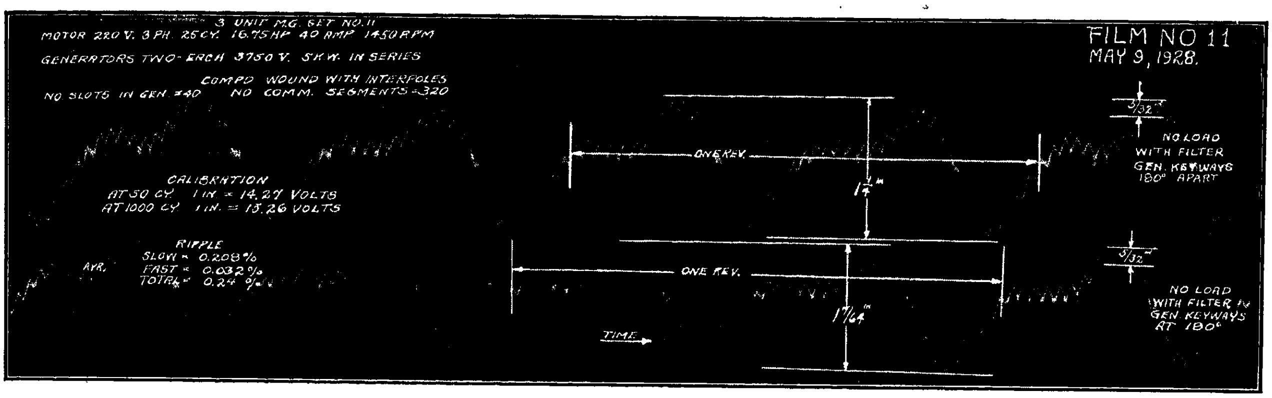

2.82 μF
Eac
OSC.
ELE.
FILM NO. 4
APRIL 17, 1928
ONE REV.
NO LOAD
NO FILTER
GEN. KEYWAYS
AT 90°
3 UNIT M.G. SET NO. 11
MOTOR 220 V. 3 PH. 25 CY. 16.75 H.P. 40 AMP. 1450 R.P.M.
GENERATORS TWO-EACH 3750 V. 5 K.W. IN SERIES.
COMPD. WOUND WITH INTERPOLES
NO. SLOTS IN GEN. = 40 NO. COMM. SEGMENTS = 320
1/16 INCHES
INCHES
CALIBRATION
AT 100 CY. 1 IN. = 28.7 VOLTS
AT 1000 CY. 1 IN. = 23.1 VOLTS
RIPPLE
SLOW = 0.371%
FAST = 0.058%
TOTAL = 0.429%
25 CYCLE SOURCE
TIME

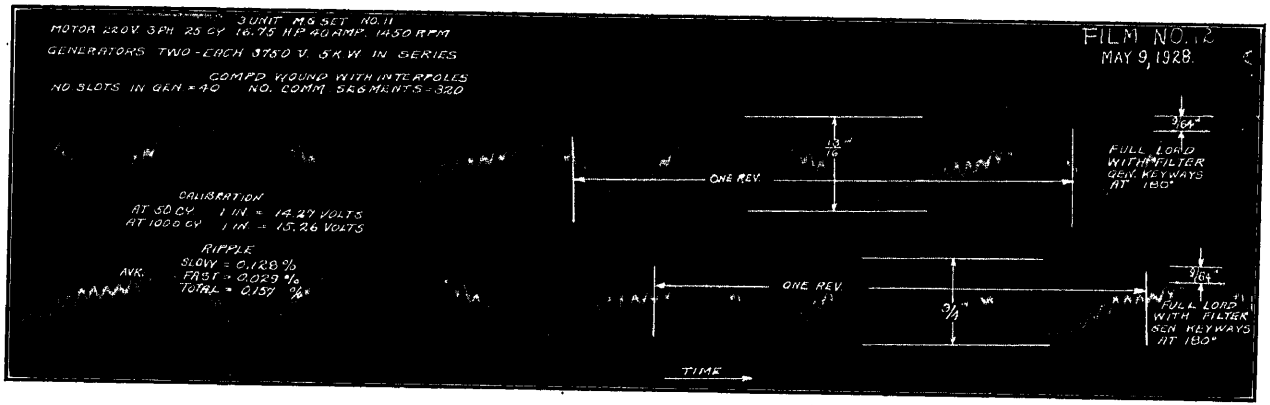

3 UNIT M.G. SET NO. 11
MOTOR 220 V 3 PH. 25 CY. 16.75 HP 40 AMP 1450 RPM
GENERATORS TWO-EACH 3750 V 5 KW. IN SERIES
COMPD WOUND WITH
INTERPOLES. NO. SLOTS IN GEN. = 40 NO. COM. SEGS = 320
2.82 μF
Eac
OSC.
ELE.
96.6:1
FILM NO. 8
APRIL 19, 1928.
1 43/64 INCHES
FULL LOAD
NO FILTER
GEN. KEYWAYS
AT 90°
41/64 INCHES
RIPPLE
SLOW = 0.255%
FAST = 0.136%
TOTAL = 0.39 %
CALIBRATION
AT 50 CY 1 IN = 18.50 VOLTS
AT 1000 CY 1 IN = 15.74 VOLTS
ONE REV.
25 CYCLE SOURCE
TIME

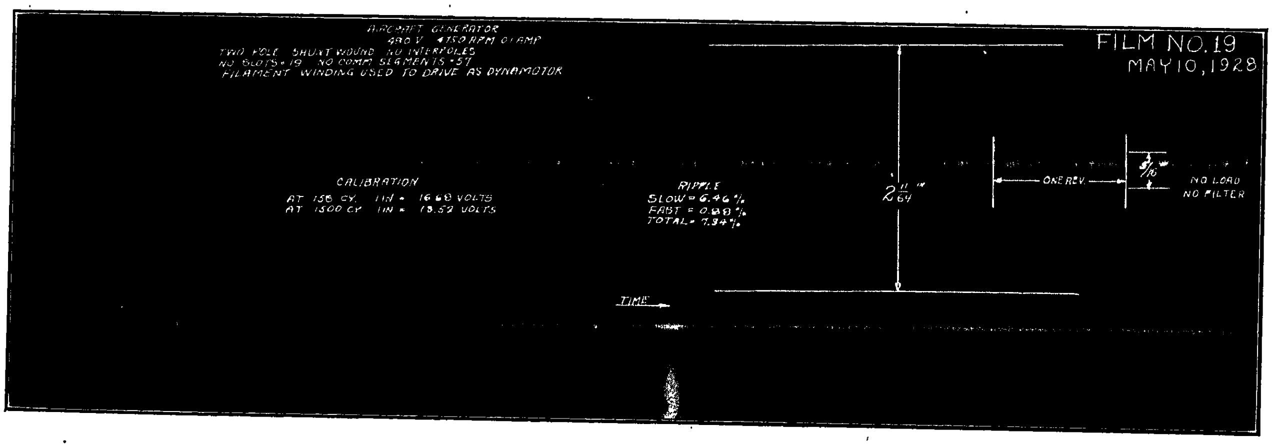

16.75 HP 40 AMP RPM
COMPD WOUND WITH INTERPOLES
NO SLOTS IN GEN NO COMM SEGMENTS = 320
FILM NO. 14
MAY 9, 1928
17/32"
49/64"
ALL AT
FULL LOAD
NO FILTER
GEN. KEYWAYS
AT 180°
CALIBRATION
AT 50 CY 1 IN. = 14.27 VOLTS
AT 1000 CY. 1 IN. = 15.26 VOLTS
9/16 IN
29/32"
ONE REV.
RIPPLE
SLOW = 0.056 %
FAST = 0.108 %
AVR. TOTAL = 0.164 %
13/16"
17/32"
TIME

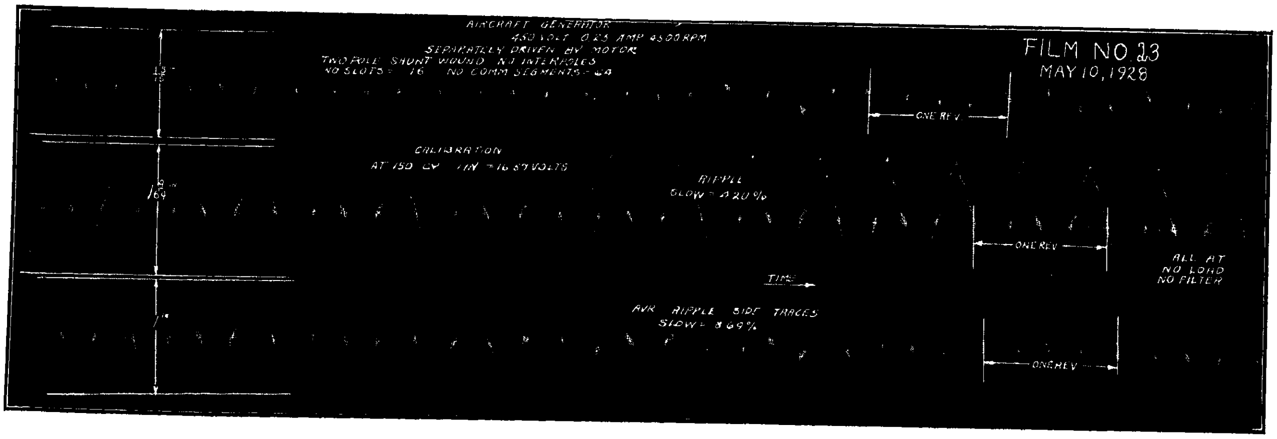
of the interpoles, armature reaction, and grain direction, and the added possibility of speed variation due to some factor such as variable motor torque or the effect of the coupling.

Films No. 19 and No. 23. Both of these records are taken from aircraft generators, each two-pole machines. On Film No. 19, the wave form divulges the presence of slot ripple, that of grain direction, and a slight amount







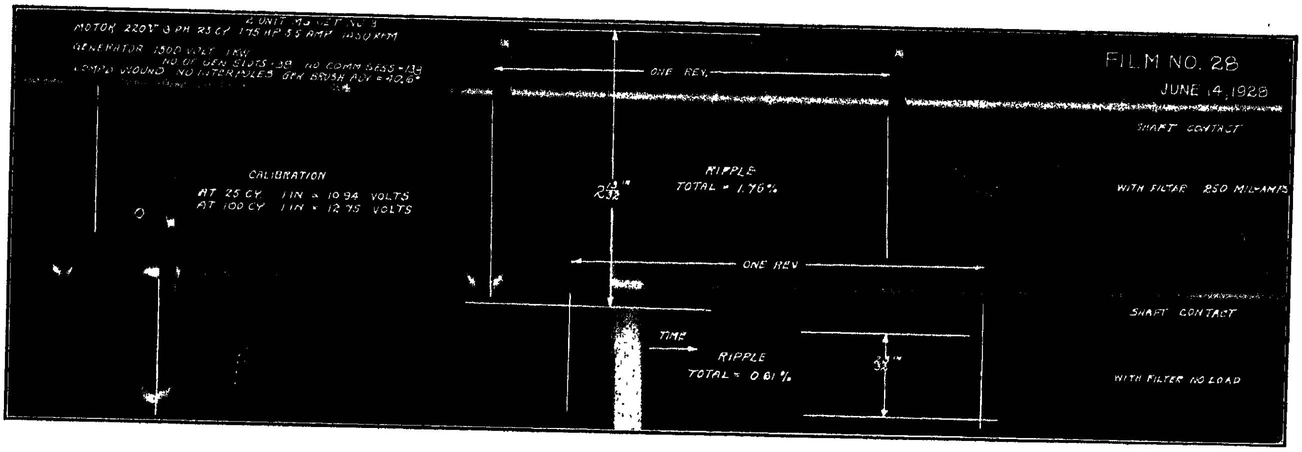


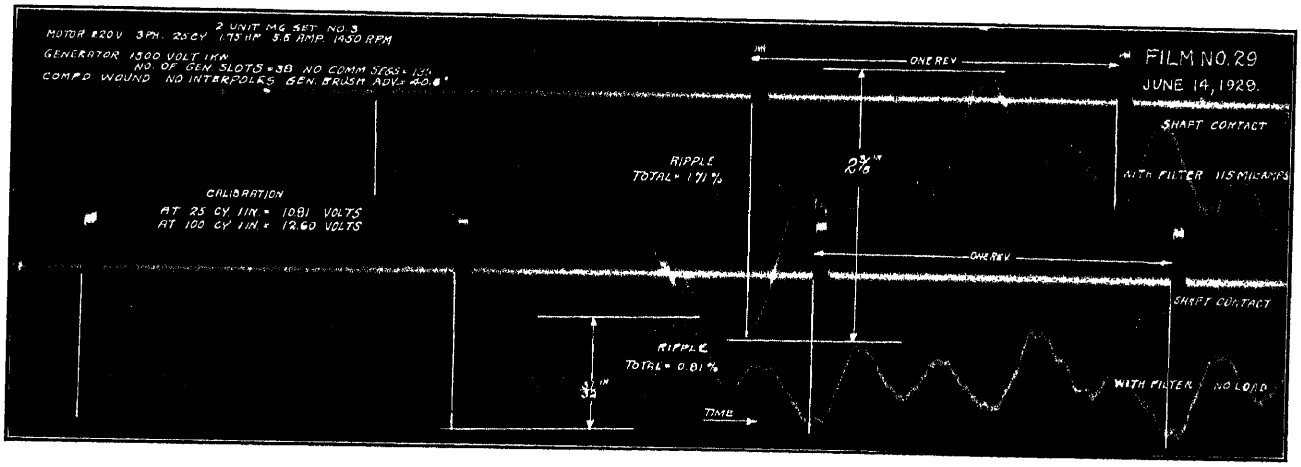
of commutator ripple. On Film No. 23 is indicated slot ripple and that due to grain direction. The largeness in percentage or ripple present in these machines is due to the design and to the economy with which the materials are used, *i. e.*, the electric and magnetic loading are extremely high.

Films No. 17 and No. 18. These two film records were taken from the two-pole generator of set No. 3. This machine has a compound field and its armature has 38 slots and 133 commutator segments. It is devoid of interpoles, and so as to ensure reasonably good commutation, the brushes are advanced about 40 deg. in the direction of rotation.

Film No. 17 was recorded without a filter in the high-voltage generator supply. Here the slot ripple preponderates by far over the grain direction irregularity. Commutation ripple can be seen here and there on the trace, superimposed on the slot pulsations, but with regard to amplitude, it is of minor importance. Film No. 18 was recorded with the filter in the high-voltage generator supply circuit and here the slot ripple has been almost entirely eliminated, but the grain direction ripple still persists. It should be noted that the no load and full load traces on Film No. 18 are somewhat dissimilar. At no load the grain direction component can be noticed as four pulsations per revolution, instead of two as would be expected on an average machine. This difference is due to the smallness in armature diameter. These same four grain pulsations also appear on the full load trace in Film No. 18, but they are superimposed on another variation that does not occur in the no load trace. The explanation of this additional irregularity will be given in connection with the next four film records.

Films No. 28, No. 29, No. 30, No. 31. These four records were taken from the same generator as were the above Films No. 17 and No. 18. The object was to ascertain how the ripple wave form varied as load was applied to the generator. Loads of 115 milliamperes, 250 milliamperes, 480 milliamperes, and 667 milliamperes (full load) were applied and the ripple wave forms recorded for these conditions as shown in Films No. 29, No. 28, No. 30, and No. 31 respectively. The cause of the great change in wave form and amplitude from no load to full load is to be attributed to the misalinement of the motor and generator shafts.



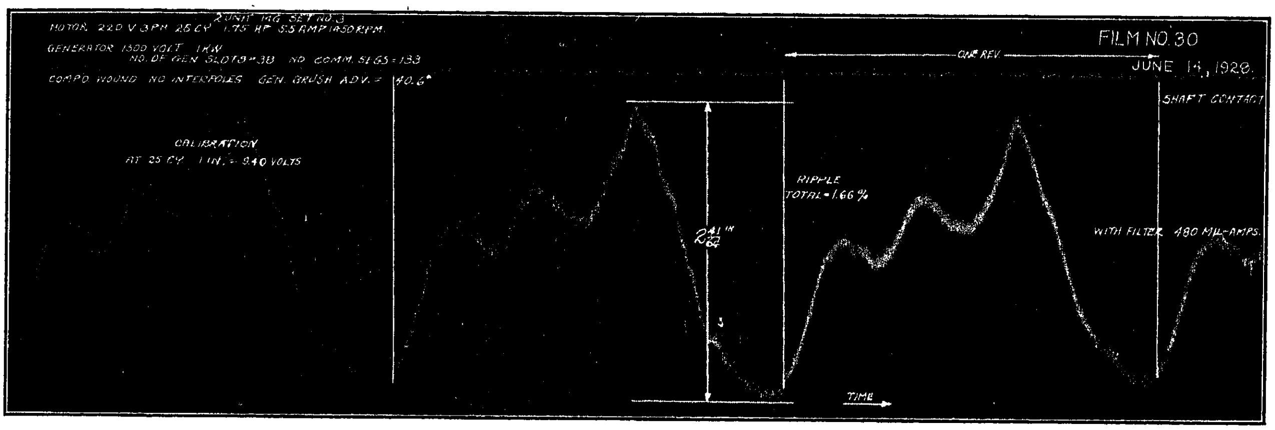


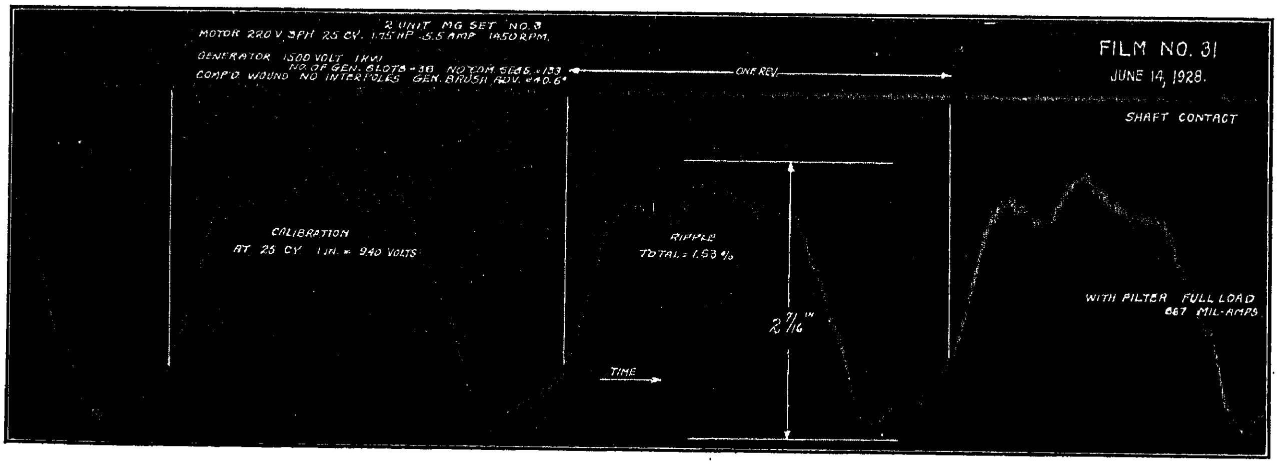
The dissimilarity between Films No. 18 and No. 31 arises from the fact that the operating condition of motor generator set No. 3 was poorer for the latter film. This one was taken about a month later so that the maladjustment of the set had increased during that interval of time.

For comparison of the above four films, a shaft contact trace was included on each film. This was done by having a contactor, attached to the shaft, close an oscillograph circuit for a very short interval once per revolution.

One additional fact that may be mentioned is that for light loads, the generator armature of motor generator set No. 3 apparently shifts longitudinally once every two revolutions, as is shown on Film No. 29.

Films No. 43, No. 44, No. 45, and No. 46. These four film records show the ripple wave form of the generator of set No. 3, taken with the coupling plates of the motor and generator out of alinement by approximately 1/16 in. Corresponding to Films No. 43, No. 44, No. 45, and No. 46 are four relationships of the motor coupling with the generator coupling, namely 0 deg., 90 deg., 180 deg., and 270 deg. respectively.



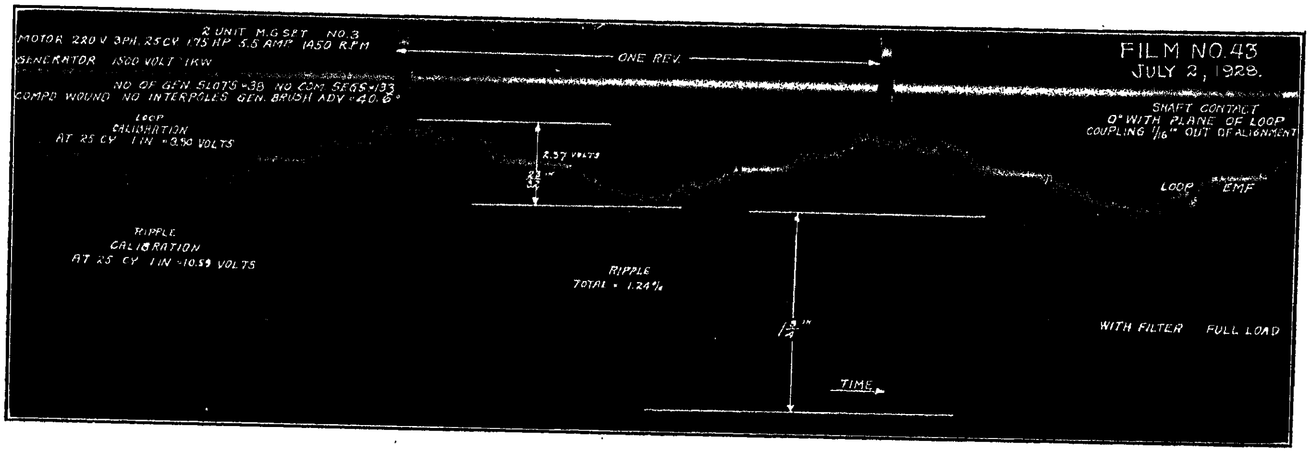


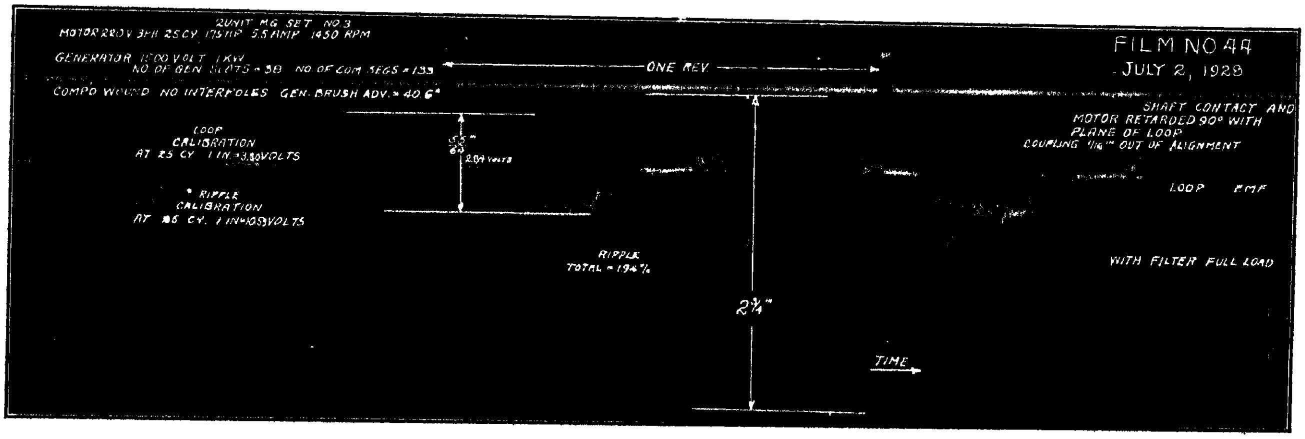
For each of these records, the ripple by inspection is seen to be of different wave form, but it must be noticed that the grain direction component appears in each film unchanged. The component which changes in wave form is caused by the pliable part of the coupling flexing, thus introducing variations in speed, and thereby voltage fluctuations.

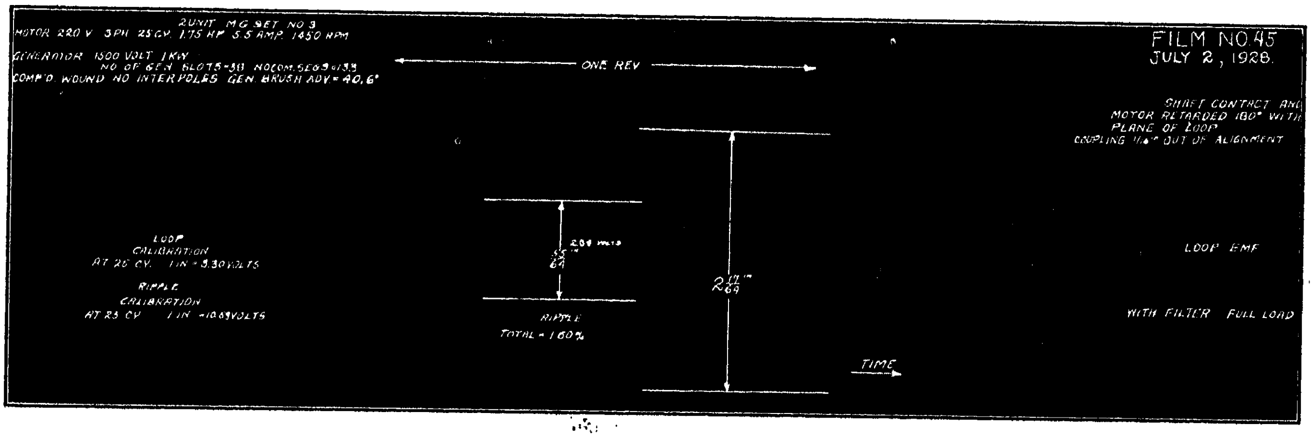
In addition, at this point of the experiment, a one turn loop was placed around the armature, the ends of this loop being connected to a pair of slip rings. By means of brushes, the potential generated in the loop was carried to an oscillograph element for recording. The recorded wave form shows directly the flux distribution in the air-gap, and indicates how very ragged and irregular this can be. With such a distribution it is surprising that the terminal voltage of this generator is as smooth as shown in Film No. 17, this record having been taken without filter.

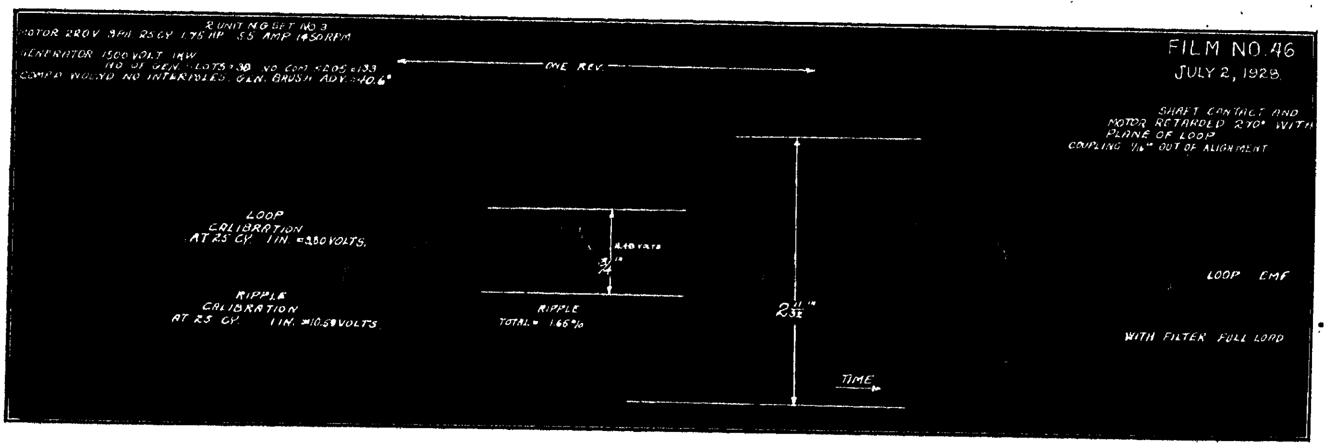
For purposes of comparison, it will be noticed that the shaft contact trace used as a reference appears to have been moved to the right in Film No. 44 by 90 deg. electrically over that of Film No. 43, and that of Film No. 45 moved 180 deg., and that of No. 46 moved 270 deg. Here the loop wave form should be used as reference, as this is kept intact and the various points on the ripple traces and the loop traces have the same relationship in each of the above four films.

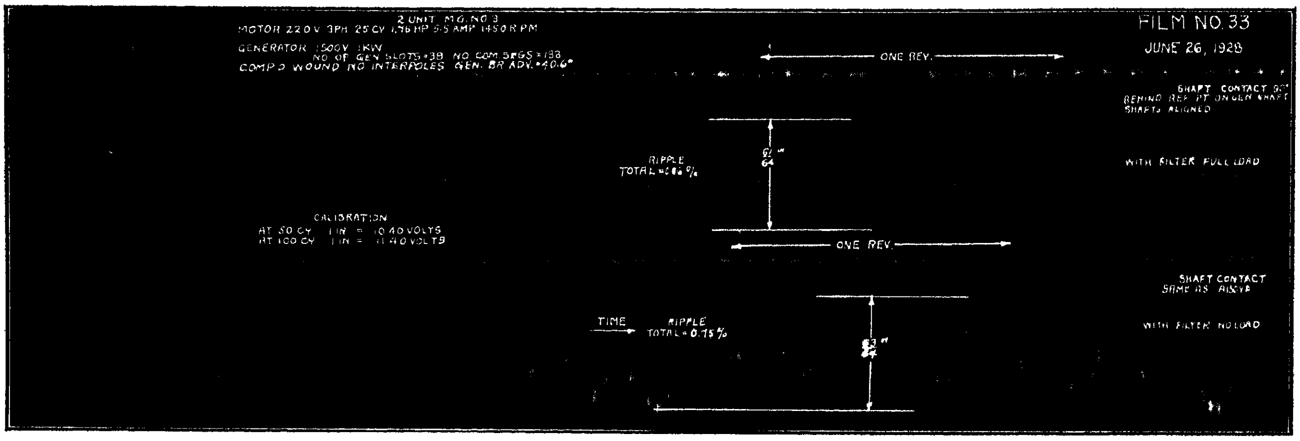
Films No. 33, No. 34, No. 35, and No. 36. These four film records were taken from the same machine as above, that of motor generator set No. 3, but with the couplings on the shafts of the motor and generator placed in good alinement, and with the above four relationships of the motor coupling with the generator coupling, *i. e.*, 0 deg., 90 deg., 180 deg., and 270 deg. The voltage irregularity which appears on these four films is mainly that of grain direction variation, the other components of ripple present in the preceding four records having been eliminated by proper alinement. Moreover, the percentage ripple has decreased about

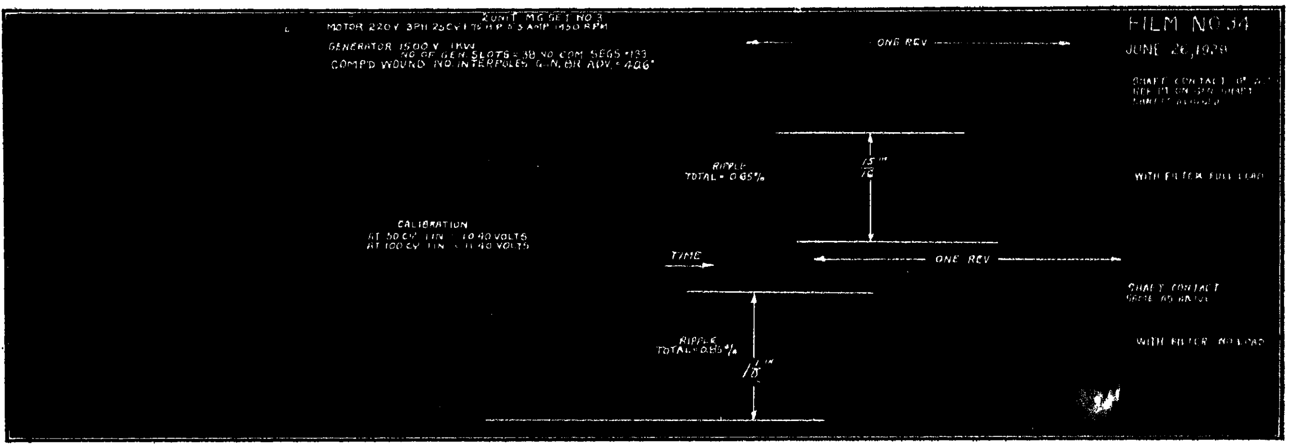

2 UNIT M.G SET NO.3
MOTOR 220 V 3PH. 25CY 1.75 HP 5.5 AMP 1450 R.P.M
GENERATOR 1500 VOLT 1KW
NO OF GEN. SLOTS = 38 NO. COM. SEGS = 133
COMPD WOUND NO INTERPOLES GEN. BRUSH ADV = 40.6°
LOOP
CALIBRATION
AT 25 CY 1 IN = 3.30 VOLTS
RIPPLE
CALIBRATION
AT 25 CY 1 IN = 10.59 VOLTS
ONE REV.
2.57 VOLTS
22/32"
RIPPLE
TOTAL = 1.24%
1 3/4"
TIME
FILM NO.43
JULY 2, 1928.
SHAFT CONTACT
0° WITH PLANE OF LOOP
COUPLING 1/16" OUT OF ALIGNMENT
LOOP EMF
WITH FILTER FULL LOAD

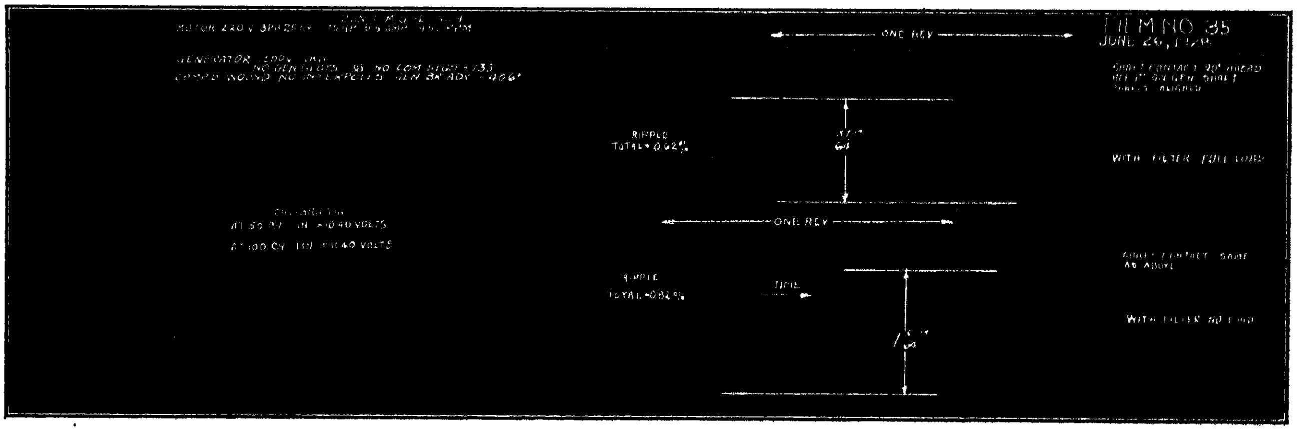

2UNIT MG SET NO.3
MOTOR 220 V 3PH 25CY 1.75HP 5.5AMP 1450 RPM
GENERATOR 1500 VOLT 1KW
NO. OF GEN SLOTS = 38 NO. OF COM SEGS = 133
COMPD WOUND NO INTERPOLES GEN. BRUSH ADV. = 40.6°
LOOP
CALIBRATION
AT 25 CY 1 IN = 3.30 VOLTS
RIPPLE
CALIBRATION
AT 25 CY. 1 IN = 10.59 VOLTS
ONE REV.
55/64"
2.84 VOLTS
RIPPLE
TOTAL = 1.94%
2 1/4"
TIME
FILM NO 44
JULY 2, 1928
SHAFT CONTACT AND
MOTOR RETARDED 90° WITH
PLANE OF LOOP
COUPLING 1/16" OUT OF ALIGNMENT
LOOP EMF
WITH FILTER FULL LOAD

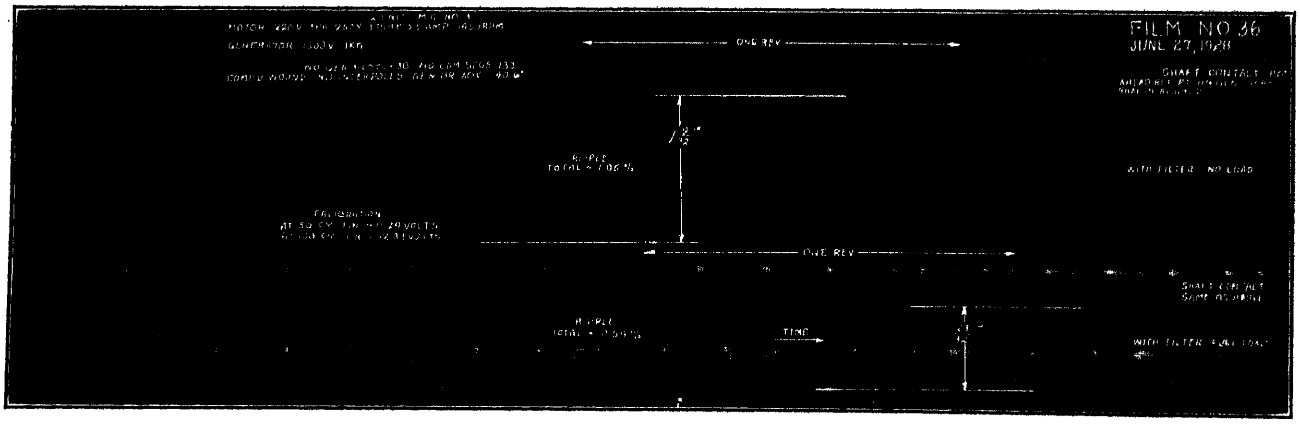

2UNIT MG SET NO 3
MOTOR 220 V 3PH 25CY. 1.75 HP 5.5 AMP. 1450 RPM
GENERATOR 1500 VOLT 1KW
NO OF GEN SLOTS = 38 NO COM. SEGS = 133
COMP'D. WOUND NO INTERPOLES GEN. BRUSH ADV = 40.6°
ONE REV
LOOP
CALIBRATION
AT 25 CY. 1 IN = 3.30 VOLTS
RIPPLE
CALIBRATION
AT 25 CY 1 IN = 10.59 VOLTS
2.84 VOLTS
55/64"
RIPPLE
TOTAL = 1.60%
2 17/64"
TIME
FILM NO.45
JULY 2, 1928.
SHAFT CONTACT AND
MOTOR RETARDED 180° WITH
PLANE OF LOOP
COUPLING 1/16" OUT OF ALIGNMENT
LOOP EMF
WITH FILTER FULL LOAD

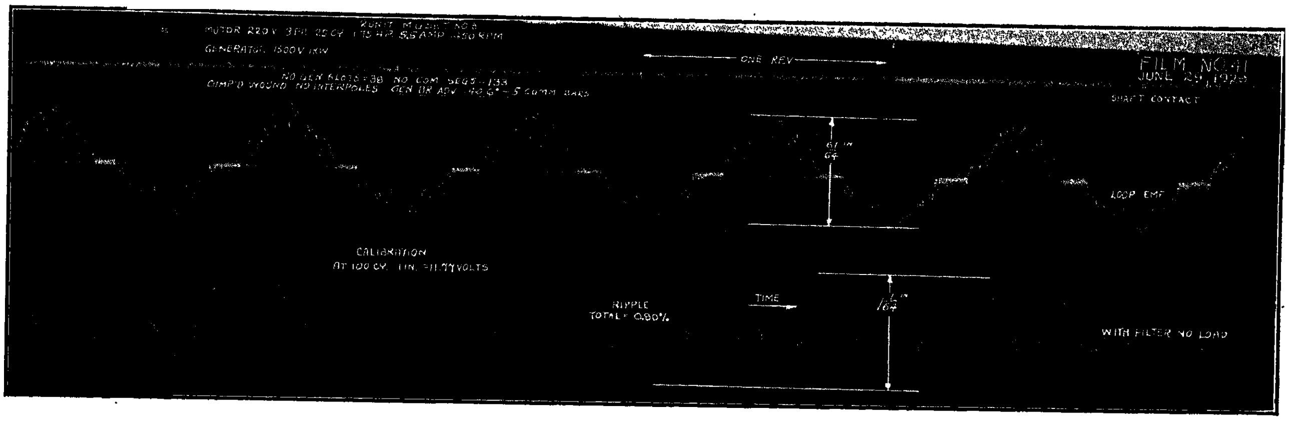

2 UNIT MG SET NO 3
MOTOR 220 V 3PH 25 CY 1.75 HP 5.5 AMP 1450 RPM
GENERATOR 1500 VOLT 1KW
NO OF GEN. SLOTS = 38 NO COM SEGS = 133
COMPD WOUND NO INTERPOLES. GEN. BRUSH ADV. = 40.6°
ONE REV.
LOOP
CALIBRATION
AT 25 CY. 1 IN. = 3.30 VOLTS.
RIPPLE
CALIBRATION
AT 25 CY. 1 IN. = 10.59 VOLTS
2.40 VOLTS
3/4"
RIPPLE
TOTAL = 1.66%
2 11/32"
TIME
FILM NO.46
JULY 2, 1928.
SHAFT CONTACT AND
MOTOR RETARDED 270° WITH
PLANE OF LOOP
COUPLING 1/16" OUT OF ALIGNMENT
LOOP EMF
WITH FILTER FULL LOAD

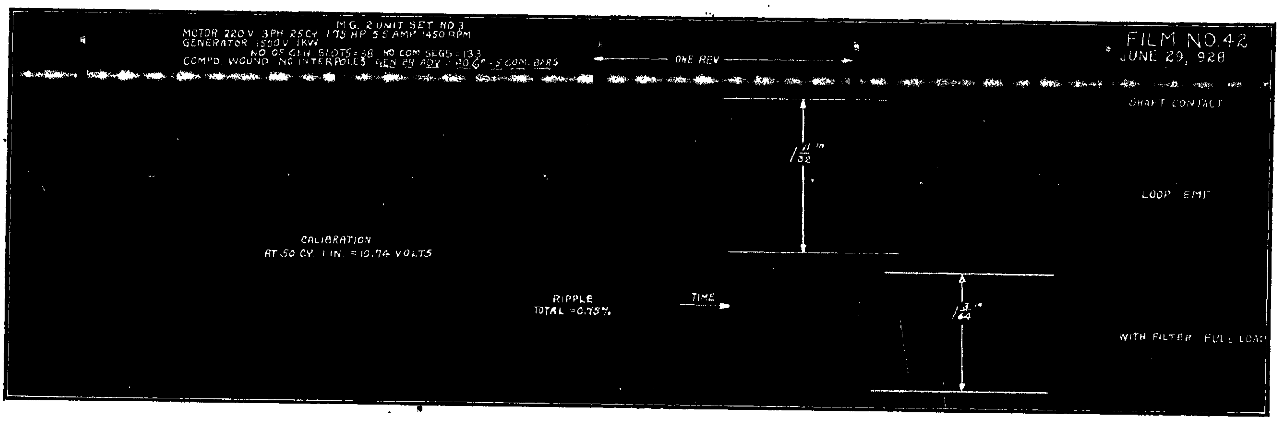

2 UNIT M.G. NO 3
MOTOR 220 V 3PH 25 CY 1.96 HP 5.5 AMP 1450 R.P.M.
GENERATOR 1500 V 1 KW
NO OF GEN SLOTS = 38 NO COM SEGS = 133
COMP'D WOUND NO INTERPOLES GEN. BR ADV. = 40.6°
FILM NO. 33
JUNE 26, 1928
ONE REV.
SHAFT CONTACT 90° BEHIND REF PT ON GEN SHAFT SHAFTS ALIGNED
RIPPLE TOTAL = 0.86 %
61/64"
WITH FILTER FULL LOAD
CALIBRATION
AT 50 CY 1 IN = 10.40 VOLTS
AT 100 CY 1 IN = 11.40 VOLTS
ONE REV.
SHAFT CONTACT SAME AS ABOVE
TIME
RIPPLE TOTAL = 0.75 %
WITH FILTER NO LOAD

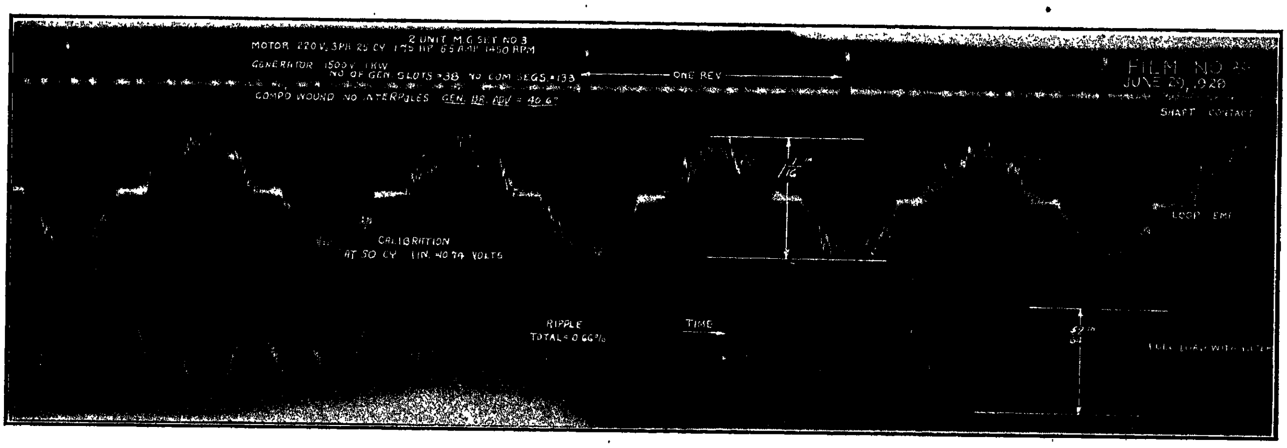

2 UNIT M.G. SET NO 3
MOTOR 220 V 3PH 25 CY 1.96 H.P. 5.5 AMP 1450 R.P.M.
GENERATOR 1500 V 1 KW
NO OF GEN. SLOTS = 38 NO COM SEGS = 133
COMP'D WOUND NO INTERPOLES GEN. BR ADV. = 40.6°
FILM NO 34
JUNE 26, 1928
ONE REV
RIPPLE TOTAL = 0.65%
15/16"
WITH FILTER FULL LOAD
CALIBRATION
AT 50 CY 1 IN = 10.40 VOLTS
AT 100 CY 1 IN = 11.40 VOLTS
TIME
ONE REV
SHAFT CONTACT SAME AS ABOVE
RIPPLE TOTAL = 0.85 %
1 1/8"
WITH FILTER NO LOAD

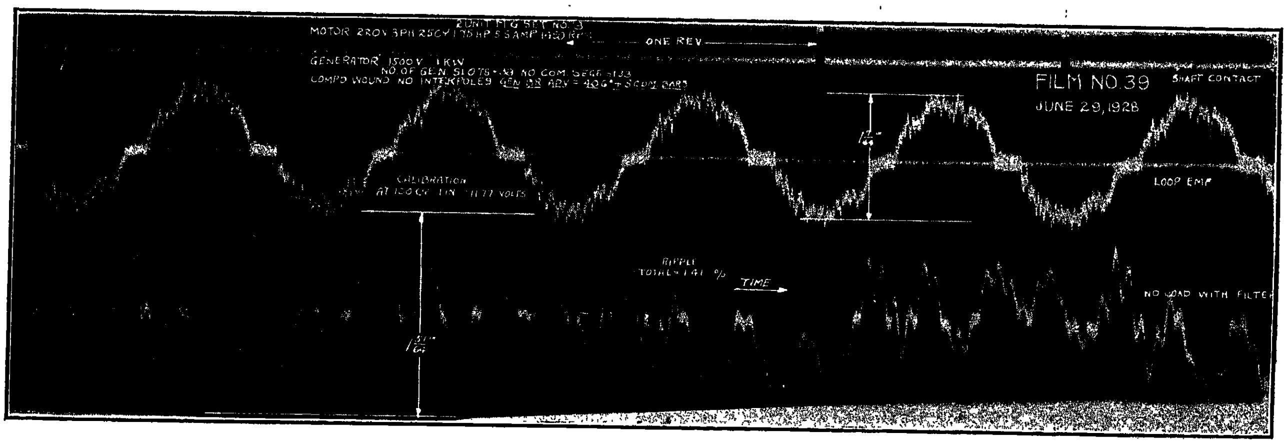

MOTOR 220 V 3PH 25 CY
GENERATOR 1500 V 1 KW
NO GEN SLOTS 38 NO COM SEGS = 133
ONE REV
FILM NO 35
JUNE 26, 1928
SHAFT CONTACT 90° AHEAD REF PT ON GEN SHAFT SHAFTS ALIGNED
RIPPLE TOTAL = 0.62%
57/64"
WITH FILTER FULL LOAD
AT 50 CY 1 IN = 10.40 VOLTS
AT 100 CY 1 IN = 11.40 VOLTS
ONE REV
SHAFT CONTACT SAME AS ABOVE
RIPPLE TOTAL = 0.82%
TIME
WITH FILTER NO LOAD

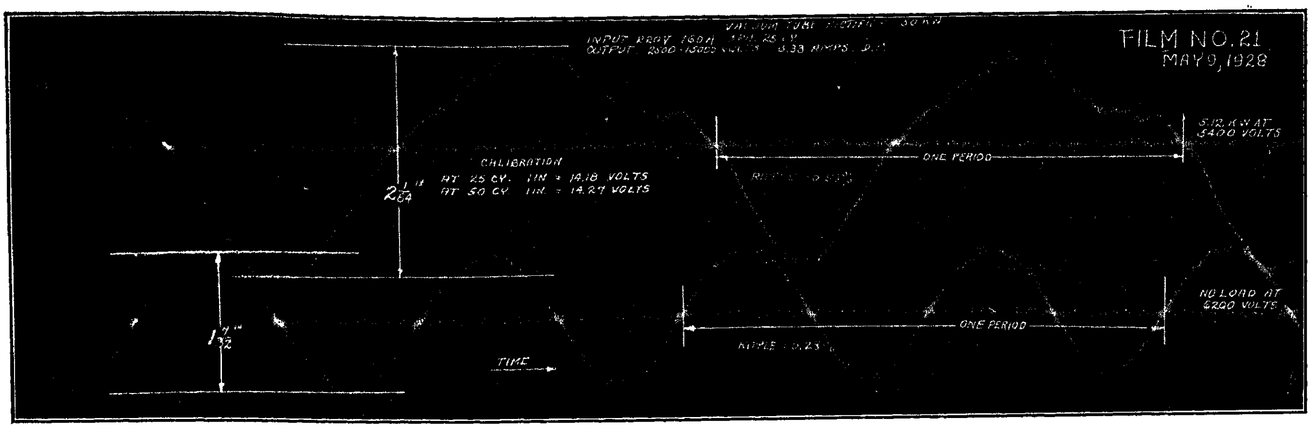

M.G. NO. 4
GENERATOR 1500 V 1 KW
NO GEN SLOTS = 38 NO COM SEGS 133
ONE REV
FILM NO 36
JUNE 27, 1928
SHAFT CONTACT 90°
RIPPLE TOTAL = 1.06 %
WITH FILTER NO LOAD
CALIBRATION
AT 50 CY 1 IN = 10.29 VOLTS
ONE REV
SHAFT CONTACT SAME AS ABOVE
RIPPLE TOTAL = 0.54 %
TIME
WITH FILTER FULL LOAD


2 UNIT M.G. SET NO 3
MOTOR 220 V 3PH 25 CY 1.75 HP 5.5 AMP 1450 RPM
GENERATOR 1500 V 1 KW
ONE REV
NO GEN SLOTS = 38 NO COM SEGS = 133
COMP'D WOUND NO INTERPOLES GEN BR ADV = 40.6° = 5 COMM BARS
FILM NO. 41
JUNE 29, 1928
SHAFT CONTACT
61/64 in
LOOP EMF
CALIBRATION
AT 100 CY. 1 IN. = 11.77 VOLTS
RIPPLE
TOTAL = 0.80%
TIME
1/64 in
WITH FILTER NO LOAD


M.G. 2 UNIT SET NO 3
MOTOR 220 V 3PH 25 CY 1.75 HP 5.5 AMP 1450 RPM
GENERATOR 1500 V 1 KW
NO OF GEN. SLOTS = 38 NO COM SEGS = 133
COMPD. WOUND NO INTERPOLES GEN BR ADV = 40.6° = 5 COM. BARS
ONE REV
FILM NO. 42
JUNE 29, 1928
SHAFT CONTACT
11/32 in
LOOP EMF
CALIBRATION
AT 50 CY. 1 IN. = 10.74 VOLTS
RIPPLE
TOTAL = 0.75%
TIME
3/64 in
WITH FILTER FULL LOAD


2 UNIT M.G SET NO 3
MOTOR 220 V. 3PH 25 CY 1.75 HP 5.5 AMP 1450 RPM
GENERATOR 1500 V 1 KW
NO OF GEN. SLOTS = 38 NO COM SEGS = 133
ONE REV
COMPD WOUND NO INTERPOLES GEN. BR. ADV = 40.6°
JUNE 29, 1928
SHAFT CONTACT
1/16 in
LOOP EMF
CALIBRATION
AT 50 CY 1 IN. 40.74 VOLTS
RIPPLE
TOTAL = 0.66%
TIME
52/64 in
FULL LOAD WITH FILTER


2 UNIT M.G SET NO 3
MOTOR 220 V 3PH 25 CY 1.75 HP 5.5 AMP 1450 RPM
ONE REV
GENERATOR 1500 V 1 KW
NO OF GEN. SLOTS = 38 NO COM SEGS = 133
COMPD WOUND NO INTERPOLES GEN BR ADV = 40.6° = 5 COM BARS
FILM NO. 39
JUNE 29, 1928
SHAFT CONTACT
LOOP EMF
CALIBRATION
AT 100 CY 1 IN. = 11.77 VOLTS
RIPPLE
TOTAL = 1.41 %
TIME
NO LOAD WITH FILTER
51/64 in



50 per cent as compared with the four previous films, This gives an idea of the importance of proper alinement of the motor generator set.

Films No. 41 and No. 42. Here the generator brushes were retarded the distance of 5 commutator bars from their normal advanced position, and two records taken, one at no load, and one at full load. The former shows a small amount of commutation ripple, which disappears when load is placed on the machine. In addition the loop indicates the voltage induced in individual armature coils, and shows the sudden small irregular changes in flux which are continually taking place.

Films No. 38 and No. 39. Film No. 38 was taken with normal brush advance, while Film No. 39 was taken with the brushes advanced a distance of 5 commutator segments beyond this position. Here a voltage irregularity appears with 19 pulsations per revolution, corresponding to half the number of slots. This gives some indication of the importance of having the brushes set for the proper condition of commutation, whether the generator is either of the non-interpole type or a machine with interpoles.

Film No. 21. This film was added by way of parenthesis, since it is a record of the wave form of a six-phase kenotron rectifier rather than that of a generator. There is one trace showing the no load ripple at 6200 volts direct current and the other trace at 5400 volts with 5.12-kw. load. Though the power supply frequency is 25 cycles per second, the frequency of the no load trace is 50 cycles per second. The light load trace shown is somewhat irregular, but is at a frequency of 25 cycles per second. With additional load the ripple becomes sinusoidal at a frequency of 25 cycles per second and its amplitude increases.

CONCLUSIONS AS TO DESIRABLE FEATURES IN D-C. GENERATORS FOR MINIMIZING VOLTAGE IRREGULARITIES

This investigation of voltage irregularities has shown that it is desirable to have the following features inherent in d-c. high-voltage generators:

1. The armature laminations should have their grain oriented in various directions normal to the shaft, so as to cause the armature reluctance to average to uniformity. In the case of circular punchings for small machines, this orientation of grain will permit cross flux from one plate to another, due to permeability difference, with the accompanying slight increase in eddy current losses, but this will not be of a serious nature. For segmental punchings as used on large generators, the grain direction should parallel the teeth, on account of the high teeth flux densities used.

2. Proper design as regards pole coverage and armature tooth and slot dimensions should be employed in the physical make-up of the generator so as to eliminate slot ripple.

3. All generators to be used as high-voltage d-c. supplies should have interpoles, as the characteristics of machines so equipped are much better than the non-interpole machines both from the standpoint of commutation and voltage irregularity.

4. Motor generator sets should have a coupling of such design and characteristics so as not to produce side shifting of the generator armature under fluctuating or continuous load. Moreover, the coupling should not affect the generator speed and cause periodic fluctuations therein.

Discussion

L. V. Bewley: On page 1070, Mr. Fetsch gives five causes of voltage irregularities:

1. Slot ripple,
2. Commutator ripple,
3. Armature reluctance variation ripple due to grain direction in steel,
4. Armature reluctance variation ripple due to improper physical alinement,
5. Speed variation due to improper shaft alinement.

To these may be added:

6. Ripple due to the oscillation about its mean position of the belt of conductors between brushes.

Mr. Fetsch's study is experimental. It is of interest to see if these ripples can be calculated, and to consider how they may be eliminated. In my paper, *Induced Voltage of Electrical Machines*, (A. I. E. E. TRANS., Vol. 49, April 1930, p. 456), I gave a general equation for the induced voltage of electrical machines having parallel coil sides and in which the flux distribution can be specified as Fourier series. It is

$$E = -2L \frac{\tau}{\pi} \frac{n N^1}{10^8} \sum_{1}^{\infty} C_{ck} C_{dk} C_{pk} C_{sk} \left[\frac{1}{k} \frac{d \beta k}{d t} \sin k \left(\frac{x_0 \pi}{\tau} + \gamma_k + \psi_k + \psi_{k^1} \right) + \left(\frac{d \gamma_k}{d t} + \frac{\pi}{\tau} \frac{d x_0}{d t} \right) \beta_k \cos k \left(\frac{x_0 \pi}{\tau} + \gamma_k + \psi_{k^1} + \psi_k \right) \right]$$

in which the C's are the harmonic reduction factors due to skew, pitch, belt distribution, and phase connection; k is the order of the space harmonic, β_k the kth harmonic of flux density, γ_k its position, and x_0 the position of the phase belt. The other factors are defined in the paper, but are not pertinent to this discussion. There are thus three ways in which a voltage can be induced in the windings:

1. Pulsation of the flux harmonics as standing waves $d\,\beta_k/d\,t$
2. Rotation of the flux harmonics as traveling waves $d\,\gamma_k/d\,t$
3. Rotation of the armature conductors at speed $d\,x_0/d\,t$

The first four causes of ripples in d-c. generators are due to $d\,\beta_k/d\,t$ and the other two causes to speed variation $d\,x_0/d\,t$. Now it happens that the frequency of the commutator ripple b and the belt oscillation ripple f are the same, but since they are due to different causes, if the harmonic reduction factors are adjusted for the cancellation of the kth harmonic voltage when caused by $d\,\beta_k/d\,t$ it does not follow that this voltage will be cancelled when due to $d\,x_0/d\,t$.

A complete analysis of the ripples in d-c. generators involves flux plots of the field form for different relative positions of the slots with respect to the poles, the specification of the instantaneous distribution as Fourier series, and finally the numerical solution of the general equation at enough successive points so that the ripples can be plotted.

J. T. Fetsch, Jr.: The additional voltage irregularity that Mr. Bewley mentioned was not discernible on the records obtained in my tests. Perhaps the ripple obtained by the oscillation of the belt of conductors between brushes about its mean position would be more pronounced in machines of low voltage, with an armature of just a few slots, coils, and commutator bars.

It is of interest to note that some of the records of ripple show only that of grain direction effect. Since this can be eliminated by various degrees of orientation of the armature lamination, it should be possible to design d-c. generators devoid of ripple e. m. fs.

Cooperative Courses—Their Development

and Operating Principles

BY KARL L. WILDES[1]

Member, A. I. E. E.

Synopsis.—*The Cooperative Idea has been the motivating spirit in the various attempts to bring together the educational and industrial elements in engineering. The outstanding milestones of progress in this undertaking are the "Sandwich System" of Scotland, the establishment of shops in schools, the rise of educational opportunities in industrial concerns, the report of Sir William White's committee of practising engineers and educators in England, and finally, the conception and inception of the cooperative courses at the University of Cincinnati and at the Massachusetts Institute of Technology.*

These courses are taken as examples of two types of cooperative course called respectively the "Cincinnati Plan" and the "M. I. T. Plan." Each of these possesses a structure and a set of operating principles determined by the purpose for which it was instituted and the conditions under which it is carried out. The operating principles of the M. I. T. Plan are discussed in detail in this paper. The graduates of both these plans are demonstrating that the results sought are being accomplished.

* * * * *

"THEORY is the commander, practise the soldiers," said the far-sighted Da Vinci. The cooperative idea, or the principle of coordinating theory with practise, has struggled for expression down through the ages. As time goes on, it is bound to express itself with increasing forcefulness, and at the present time effective applications of the principle are in use in this country, especially in the field of engineering education. With the growing interest in the relative functions of the technical colleges and the electrical industries, it would seem to be timely that a review of the development and operating principles of this type of training be presented before the Institute.

THE COOPERATIVE IDEA—DEVELOPMENT

The Antagonism between Theory and Practise. The engineering profession arose out of the crafts, and naturally enough the early engineering processes were the craftsman's rule-of-thumb methods. Such empirical information as was found practicable was handed down and augmented by later engineers, and thus was the fund of engineering knowledge built up. Practise, then, was the only means of entrance into this profession, and it became the custom for young men, usually with little scientific education, to serve several years as apprentices with their elders, after which they might be recognized as civil engineers. For many years the young man who had studied the then existing sciences was not too welcome in engineering circles, for he was bothersome; he had to be divested of his academic ideas before he could begin to learn anything practical. The cooperative idea was facing, in this situation, the apparently insurmountable task of reconciling the practising engineering profession with the technical college, an outgrowth of the schools of science in the universities.

Conciliatory Moves. The first gesture of the cooperative idea in technical education was probably seen in Germany. The notion that boys finishing at the *"gymnasium"* were too immature and lacking in educational appreciation to make the most of their studies in the *"polytechnikum,"* led to the system of placing them in mechanical workshops for at least one year before entrance into the scientific school. A certain evaluation of materials and methods and an increased desire for scientific knowledge were thought to result from this practise; in fact, the results were so convincing that the plan spread into neighboring countries and still persists, notably in the higher technical colleges of Germany and Scandinavia, as a requirement for a degree.

The "Sandwich System" came into use nearly a century ago, not as an educational move, as some have supposed, but rather from financial expediency. According to a Scottish engineer,[2] Scotchmen of former generations could not afford attendance at the university for more than six months in the year, which explains the short six months' session of the Scottish universities. As Glasgow University was one of the first to teach engineering science, the short session and the large engineering community naturally evolved the sandwich system, which gives the student six months at the university and six months in the workshop. There was no attempt here to coordinate the school work with that of the shops, or vice versa, but it has been remarked by employers that this type of education produces "men who can make a good living." Here the cooperative idea almost came to light.

The Move of the School toward Industry. Inspection and instruction trips into industrial establishments have been featured by technical schools in this country from the very first[3] (1824). In the case of Worcester Polytechnic Institute (1868), a small manufacturing plant was established on the campus. Skilled journeymen were employed and articles of commercial value were produced by students and journeymen working together. About a decade later, instructional shops were established at the Massachusetts Institute of Technology, for the purpose of teaching the fundamental shop operations, without consideration of factory production. Both these types persist in their respective institutions and one or the other has been adopted by practically every engineering college in this country. These, and other moves on the part of the schools to develop the industrial side, were prompted by the cooperative spirit in them reaching out for the practical.

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[1] Asst. Professor of Elec. Engg., Massachusetts Institute of Technology, Cambridge, Mass.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

[2] Captain J. M. Scott-Maxwell, *I. E. E. Jl.*, Vol. 57, 1919, p. 466.

[3] "History of Rensselaer Polytechnic Institute," P. C. Ricketts, John Wiley & Sons, Inc., 1895, p. 43.

The Move of Industry toward the School. But the cooperative spirit also reached out from industry toward education. This is perhaps best evidenced by the establishment of courses of classroom instruction in the great manufactories, first for trades instruction and later for engineering instruction. The classroom instruction provided by the industries has in some cases risen to a high type of theoretical instruction, as exemplified by the "Out-of-Hour Courses" of the Bell Telephone Laboratories, the "Advanced Course in Engineering" of the General Electric Company, and the "Design School" of the Westinghouse Electric and Manufacturing Company. The establishment of scientific research laboratories in the large companies was also a decided advance toward recognition of the closer relationship between pure and applied science.

Eminent Engineers and Educators Express Themselves. In 1903, a committee of very eminent British engineers[4] and educators, under the chairmanship of Sir William H. White, was appointed "to consider and report. . . as to the best methods of training for all classes of engineers; it being an instruction to this committee that the principle shall be maintained that the education of an engineer must include both practical experience and scientific training." An exhaustive inquiry, similar in nature to the recent one made by our Society for the Promotion of Engineering Education,[5] asking opinions of practising engineers and professors, was used as a basis of this committee's 13 recommendations, which include the following: a year's experience in a mechanical engineering workshop, termed the "Introductory Workshop Course," one academic year common to all types of engineering students, in which mathematics and mechanics would be stressed, two academic years allowing some specialization, and three years of practise in a particular field (Civil Engineering, Mechanical Engineering, etc.). Finally, if post graduate work became desirable, it was to follow the practical experience. It was also mentioned that for mechanical engineers it might be advisable to place all the practical training before the college course, in which case it was very important to provide evening classes, for "otherwise boys would lose seriously during four or five years' suspension of systematic study." For those who take the Introductory Workshop Course only, "the Committee thinks it is most important that all boys should at least maintain their scholastic acquirements." It was also held feasible to cut down the total time by sandwiching the practise, as at Glasgow. The greatest of the British engineering minds were striving to accomplish a plan which was a shadow of the cooperative idea.

The Idea Born. None of the foregoing arrangements merits the cognomen of "cooperative education" as it is practised in its most effective forms in America at the present time. Finally, in 1906 the "idea" succeeded in breaking the shell and appearing as a reality. During the years occupied by Sir William White's investigation, Mr. Herman Schneider had conceived the scheme of coordinating engineering theory, best taught in the schools, with engineering practise, best taught in the industries, and he was having difficulty in putting his proposal into operation. Great credit is his due, however, for his persistent and energetic labors, for finally in Cincinnati he discovered an attractive situation in which he could launch his project. Thus came into being the world's first real cooperative course, recognized in America and in Europe alike as the birth of the cooperative idea.

THE COOPERATIVE IDEA—OPERATING PRINCIPLES

The Cincinnati Course—Aim. The peculiar situation into which the cooperative idea was born had much to do with the structure and therefore with the operating principles of this first course. Cincinnati is a city of great industrial plants, like those of the present Cincinnati Grinders, the Union Gas and Electric Company, and the Cincinnati Traction Company. It is also a public-spirited city having at heart the progress of its public institutions, including its municipal university as well as its private business enterprises. It is at once evident that the local industries furnish not only exceptional opportunities for training young men and women, but for utilizing their services after they are trained. The demand upon the university is therefore chiefly for the production of engineers to occupy the responsible positions in these industries. To quote President Schneider,[6] "The aim of the course is not to make a so-called pure engineer; it is frankly intended to make an engineer for commercial production."

The Cincinnati Plan—Original Structure. Originally, the course was laid out to cover six years, alternating the student week by week between shop and college during the academic year of eight months, and placing him in the shops also during the four months of vacation. This arrangement gives a total of four years in the shops and 24 months in college, as compared with the 32 months of the regular four-year plan. President Schneider reports that more, rather than less, scholastic work was accomplished under the cooperative plan than under the regular one, due chiefly to the eagerness for study engendered by the contact with shops, and to the fact that the boys did some studying on their own initiative in the industrial periods. The story of the experimental period of this undertaking is most fascinating,[7] and in it some important changes took place, as will be seen; but with only a passing reference to this story, we will pass to the final set-up of the course as it exists today, and we will call this scheme the "Cincinnati Plan" of cooperative education. Several other prominent institutions, about 20 in number, such as the University of Akron, Georgia School of Technology, and Antioch College, have established courses after this plan.

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[4] *Inst. Civil Eng. Proc.*, Vol. CLXVI, Part IV, 1906, p. 159.

[5] Bulletin No. 13 of the Investigation of Engineering Education, S. P. E. E., Dec. 1927.

[6] "The Cooperative Course in Engineering at the University of Cincinnati," Herman Schneider, *Proc. S. P. E. E.*, Vol. XV, 1907, p. 391.

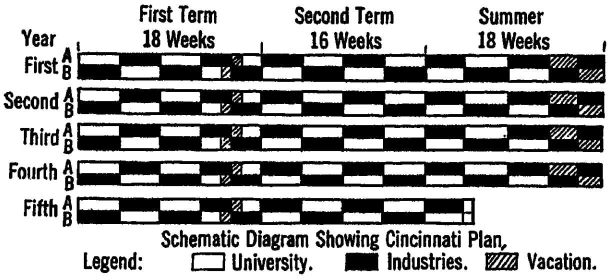

FIG. 1—SCHEMATIC DIAGRAM SHOWING CINCINNATI PLAN

The Cincinnati Plan—Present Structure. The present structure of the Cincinnati Plan is this: a summer practise period is advised previous to entrance to the course, and beginning with the fall term of the first year the student alternates in four-week periods between college and shops throughout the five-year course, with four weeks of vacation per year. The shop work is coordinated by a staff of coordinators in such a way that the jobs whose chief value lies in shop discipline and simple operations are given to the freshmen, and those requiring more judgment and training are given to the upper classmen. These coordinators also hold classes in coordination at the University and visit the men on the job, in both cases answering questions and pointing out important practical and theoretical aspects of the jobs. In some cases the employment may be all with one concern, but in cases where the work supplied by one company is not extensive enough, the student may be moved to other companies. Jobs at the plants are kept filled by the principle of alternation in pairs, one or the other of the students of a pair being held responsible for filling the job at all times while this pair is assigned to this job. No university classes are held for the students at the shops, but a comprehensive report is required of each student upon his return to the University.

The M. I. T. Plan—Inception and Purpose. A plan, involving a more intimate application of the cooperative idea, differing from the Cincinnati Plan in purpose and scope and therefore in operating principles, was developed for electrical engineering conjointly by Mr. Magnus W. Alexander, then of the Lynn Works of the General Electric Company, and Professor Dugald C. Jackson, of the Massachusetts Institute of Technology, and was put into operation in 1917. The World War interrupted this initial attempt, but it was re-established in the summer of 1919 and its administration placed in the hands of Professor William H. Timbie. It was the result of ten years' observation of the Cincinnati Plan, consideration of European practises and opinions, and a careful study of the adaptation of the cooperative idea to the needs of the company and the educational facilities of both the company and the Institute. The purpose was to train a selected group of promising and scholarly young men for the technical and executive positions of the electrical manufacturing industry. It was apparent that the academic training needed by such men would involve not only a thorough study of the fundamental sciences and the application of those principles to the design and operation of machines, but at least an additional year of post graduate study in which creative ability and acquaintance with the frontiers of scientific knowledge would be developed. It was equally apparent that a prolonged and haphazard round of shop duty was a waste of time for the type of young man under consideration. For this reason, a course of "Manufacturing Practise" was laid out, covering the essential details of the manufacturing problem from the machine shop through the various tests of apparatus, production offices, and engineering development and research laboratories. Furthermore, a good foundation in mathematics, physics, chemistry, English, and drawing, was seen to be necessary before entering this coordinated course in theory and practise.

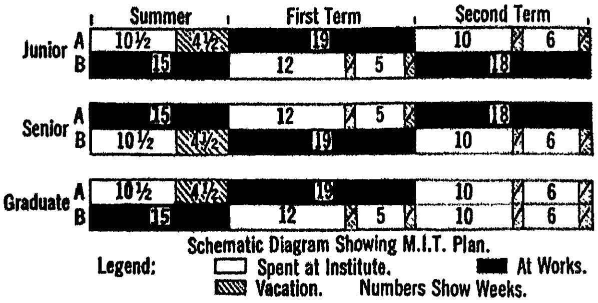
Summer
First Term
Second Term
Junior A B
Senior A B
Graduate A B
10½
4½
19
10
6
15
12
5
18
Schematic Diagram Showing M.I.T. Plan.
Legend: Spent at Institute. At Works.
Vacation. Numbers Show Weeks.

FIG. 2—SCHEMATIC DIAGRAM SHOWING M. I. T. PLAN

The M. I. T. Plan—Structure. In view of the above considerations, the Cooperative Course in Electrical Engineering, known at the Institute as Course VI-A, was constructed as follows: The successful completion of the first two years of basic scientific and humanistic studies is required for entrance to the cooperative years. At the beginning of the summer following the second year, the class jointly selected by the cooperating company with which the particular students will be related for three years, and the Electrical Engineering Department of the Institute, is divided into two groups, designated Groups A and B, Group A remaining at the Institute and Group B going to the industrial plants. The groups alternate between industry and the Institute at the end of each academic term, fitting into the regular Institute calendar of studies, activities, and vacations when at the Institute, but responsible to their company during the complete term while on industrial assignments. The project includes continued scientific and general study, and instruction by members of the Institute staff, for students at the Works. The two groups are reunited at the Institute for the final term of the graduate year, during which they complete the requirements for the Master's degree. This last term away from industry places the student in an unbiased atmosphere at the time he is considering permanent employment, and promotes class activities preceding and including the commencement season. This plan has been called the "M. I. T. Plan," and has been adopted also by the Civil Engineering Department of the Institute in cooperation with the Boston and Maine Railroad, and is pending adoption by one or two other institutions having good graduate departments. We shall now discuss some of the operating principles of this plan, with special reference to Course VI-A.

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[7] "The Cooperative System of Education," Clyde W. Park, University of Cincinnati Studies, Series II, Vol. XI, Part 1, 1925.

The M. I. T. Plan—Operating Principles. It is desirable, although not requisite to the successful operation of the M. I. T. type of cooperative course, that a non-cooperative course carrying in its curriculum the same scientific training, be maintained for the following reasons:

1. In cases of more mature or experienced men coming out of industry for training in the fundamental sciences, it is usually advisable for them to pursue these studies as rapidly as is consistent with their ability and preparation. In some instances these men take heavier schedules than would be advisable for younger men, and by continuing their studies through the summers, they may return to their employment in the minimum possible time.

2. Ten per cent of the students in electrical engineering (1928-'29) at M. I. T. have come from foreign countries and a large proportion of these intend to return to their home countries after completing their technical education. The cooperating companies cannot as a matter of policy take these students into training, except by special arrangements in case they have foreign departments.

3. Some students cannot meet the physical or certain personal requirements of the cooperating companies, but are otherwise qualified to carry on engineering study.

4. Many students transfer to the Institute in the fourth year, too late to include a substantial course in industrial practise.

5. Many students desire an undergraduate engineering education but are neither desirous nor qualified to pursue graduate work; others prefer the scientific studies for themselves, and are not attracted by the works experiences at the time.

In some institutions operating on the cooperative plan only, some of these reasons have become urgent and are causing a serious consideration of establishing a non-cooperative, or "full-time" course, in parallel with the cooperative course.

The cooperating company must meet the following specifications for the M. I. T. cooperation:

1. It must be large enough and complete enough in its organization, training facilities, and range of production to provide a comprehensive course in its particular field. While a small company may be intensely interested in the project and may give very excellent experience in a limited field, the more important considerations involved in the organization and administration of a large enterprise and association with multifarious but related production cannot be gained. This point is well illustrated by a comparison of the small and the large manufacturing companies. A small concern tends to produce artisans, men who become adepts in some narrow field such as die-making, for example. It is an incorrect principle, however, to confine the early practical education to a highly specialized phase of a business. A general course in a well-ordered machine shop would give a broader view of this class of work and a good foundation upon which die-making or any other specialty could be built subsequently. Certainly the contact with the several departments of a large manufacturing concern with its diversified problems of materials, processes of fabrication, research, and personnel, will give a young man, under competent direction, a practical equipment for a professional career far superior to that obtainable in a small factory under the best possible conditions. The modern trend toward consolidation also suggests the importance of seeing commercial problems in the large.

2. The company must be competent and willing to assume its share of the responsibility in the administration of the project. This is probably the most vulnerable spot in the whole plan, and unless a responsible officer of the company constantly sees that the practical program is carried out, the ideals of the cooperation will not be realized. This company responsibility also means more than the interest of one man; it means the interest and effort of the whole organization from the president to the foreman under whom the students work.

3. The company must have a specific need for the type of man trained. If this is not the case, it is training men only for competitors, or at best for other concerns. Plainly enough, this is not good economy and cannot be justified on any ground except possibly that of creating good-will. Moreover, if there is no interest in the product, there can be little interest in its making, which means that eventually the cooperation must terminate. If, on the other hand, a substantial number of outstanding men is added to the organization from year to

year from this course, the course will be highly respected and will continue to be well administered.

These specifications are believed to be met by the companies associated with Course VI-A, each company occupying a prominent place in a special field of electrical engineering. The General Electric Company gives an unexcelled course in Manufacturing Practise, including shop work, testing of electrical and mechanical apparatus, factory production studies, and development and research on products and manufacturing processes. The Public Utility Practise is given in any one of three companies, the Edison Electric Illuminating Company of Boston, for generation and utilization of electric power, the Boston Elevated Railway, for electric transportation, and Stone & Webster, Inc., for the planning, construction, and operation of various kinds of power projects. The Communication Practise is given by three component companies of the Bell System, the Western Electric Company, Inc., in the manufacture and installation of communication apparatus, the New York Telephone Company, in the maintenance and operation of telephone plant and the Bell Telephone Laboratories, Inc., in the development and research in communication engineering, the relation to the three communication companies being coordinated by the American Telephone and Telegraph Company.

Similarly, the educational institution must match the above specifications with the following:

1. It must maintain an adequate and competent staff of teachers, administrators, and scientific investigators, classroom and laboratory equipment, and undergraduate and graduate educational program sufficient to provide the technical and cultural groundwork for the engineering profession in general, and for the better present and prospective industrial positions in particular. The cooperative method does not reduce to any considerable extent the necessity for laboratory facilities with which fundamental principles can be demonstrated and original research carried on.

2. It must of course not only carry the entire administration of the course while the students are in academic residence, but also must enter thoroughly into the cooperative spirit in the administration of both the academic and practical content of the curriculum. This cooperative spirit brings about an interchange of ideas, beneficial to the participating organizations as well as to the students. Through the medium of student theses, investigators of both institutions lend their aid to the solution of industrial scientific problems.

3. The educational institution must be situated not too far away from the cooperating industries. An expensive practise of transferring students several times over long distances would not be economically justified, and would give them an erroneous evaluation of their own importance. This does not mean that for the long-term arrangement the distance could not be so great as a night's train journey, but it should exceed this distance only in cases of special assignments toward the end of the course. Besides the interchange of students between employment and college, there is the necessity for frequent conferences on the part of administrators, and these are facilitated by the close proximity of the institutions represented.

Before a student is accepted for this cooperative course of training, he must successfully complete the first two years of groundwork in mathematics, physics, descriptive geometry, drawing, chemistry, and English. These have been termed the a b c's of engineering and ought to be well in hand before the principles of design and operation of electrical and mechanical apparatus are undertaken. It has been found the best policy at M. I. T. to concentrate on such basic subjects in the first two years, for upon these rests the whole structure of engineering. If a man cannot master these a b c's, he should not be encouraged to pursue a course of theory and practise, the successful completion of which rests so heavily upon this foundation. These two years also serve as an orientation period during which the requirements and opportunities in the several fields of electrical engineering are brought to the students in personal conferences with members of the teaching staff, in lectures by the president and heads of departments of the Institute and by men from the industries. It is upon the achievements of this two-year period that the Electrical Engineering Department bases its recommendations to the cooperating companies.

Late in the sophomore year, the candidates recommended by the Department of Electrical Engineering of the Institute are interviewed by representatives of the company with which the cooperative training is desired. The qualifications sought by these company representatives are in general the same as those considered important for regular permanent employment. Good health, good physique, strong personality, initiative, ability to cooperate, and adaptability to the work of the particular organization concerned, are qualities of importance in this selection. The companies are not expected to accept students who are reasonably sure of eventually going with other concerns, or upon whose future services they can see no probability of substantial returns. The mutual obligations of student and company cease, however, at the end of the course.

Although no contracts are signed, a company agrees to complete the training of each student it starts, except in cases of faulty conduct, and in return it is understood that under normal conditions a student is to complete his course of training with the company. This mutual understanding is necessary to the stability of the course.

The rates of pay for students in cooperative training are substantially uniform among the companies, slight compensations being made for variations in living expenses in different localities. In order to insure the transfer of students from one job to another on the basis of the best educational value, it is held to be essential that the rate of pay be fixed according to the class in which the student is registered, rather than determined by prevailing rates for the kind of work he does. In several cases, students have been recommended by their foremen for advances in pay in recognition of exceptional ability or interest. Such advances are not allowed, as the student will soon be moved to a new job, but the incident is usually related to him by way of encouragement. A student's achievement in any cooperative assignment is recognized by the grade given in Industrial Practise, and this record is as much a requirement for graduation as a subject taken while at the Institute. Special coveted research or development assignments are often given as rewards for outstanding achievements or exceptional ability. The average total amount of money earned per student during the four cooperative periods is about eighteen hundred dollars.

During the Institute periods, the curriculum calls for a normal schedule of 48 hours per week. It is essential that the required load be normal, since overcrowding tends toward abbreviation and superficial work. Furthermore, a fair proportion of the men like to add to their normal schedule the courses of the Reserve Officer's Training Corps, or such general studies as the humanities, appreciation of music, etc., while others are interested in student activities of educational and broadening character. Several are members of the Electrical Engineering Honors Group, the object of which is to allow the men with acute intellects and sound judgment to study their required subjects most economically by permitting them to determine for themselves the amount of illustrative problem work and class attendance essential to their mastery of these subjects. The Honors men take the same quizzes and examinations as the other students, but pursue their studies with the counsel of a member of the Electrical Engineering Department, who directs their attention to more extended studies in the subjects of the curriculum or allied topics. Practising engineers and employers of engineering graduates are urging these types of broadening influences upon the engineering schools, but if the essential scientific and required cultural studies are all crowded into the Institute periods, no room is left in a five-year course for these desirable extra features. A six-year course is not proving attractive, although it has many points to recommend it, but fortunately it has been found practicable to accomplish a normal required Institute schedule by introducing studies at the plants.

During the practise periods, additional studies are required to the amount of eleven hours per week, classroom and study. Two subjects are conducted by members of the Institute staff on two afternoons or evenings each week after working hours. One of these subjects is given by the Department of Electrical Engineering and is classed as a scientific subject, and the other is given by the Department of English and History and is classed as a humanistic subject. Thus, the "Principles of Electrical Engineering" are studied almost continuously during the cooperative years, and such cultural subjects as "Modern Forms of Literature" are given while the men are in the more practical atmosphere of the plants. Cooperative arrangements often meet the criticism that they develop a materialistic attitude in the students, but examination of the results obtained under these conditions shows no such results. Besides the accomplishment of a whole term's study during the four periods at the plants, this plan performs the very important function of teaching men to use their spare time to advantage while in industry; that is, the studies are carried into the industries to produce in the young man as a worker correct habits of budgeting his outside time, and not to give him the status of a schoolboy at work. Another outstanding result of studies at the works is the alertness with which students assume the full Institute program. The two subjects in each Works term give enough mental exercise to keep the men's minds active, and yet not enough to fatigue them.

Close coordination must be maintained between the two kinds of work, academic and practical. This does not mean necessarily that every practise period must contain the practical application of the scientific principles of studies during the next preceding term; in fact, in some cases it is desirable to reverse this order, and make the contacts with certain apparatus a stimulus for the study of its principles of operation. This is not an argument, however, for a general laxity in coordination of theory with practise. It is desirable that these go hand in hand, each stimulating interest in the other. One of the important functions of the administrative officers of both the school and the company is to make sure that students take proper attitudes toward their work, and use their opportunities to observe the broader policies of the company, such as the inter-relations of departments, economic structure, and problems of personnel. In Course VI-A, members of the staff of the Department of Electrical Engineering work with administrative officers of the companies to see that the men are fitted, by age, temperament, and educational background, for the various jobs, in order that the greatest educational value may be gained.

Vacations average about five weeks a year during the cooperative years, and students are encouraged to take full advantage of these periods for rest and recreation. A vacation period never precedes an Institute term. This allows the student to take full advantage of the mental activity engendered by the two subjects pursued at the plants, as he swings into a full schedule of academic study.

Values of the Project. The ultimate value of the course to all parties concerned is best measured by its value to the students. Some of the chief benefits to the students are the following:

1. The regular four-year undergraduate course in Electrical Engineering plus an academic year of gradu-

ate study are accomplished in five years without condensation of Institute schedules. The Master of Science and Bachelor of Science degrees are awarded for this work.

2. In addition to the scholastic attainment, each student spends fifteen months, during the three years just prior to graduation, in a well-ordered course of practical training with one large company, where he learns to correlate fundamental scientific principles with their method and scope of application.

3. The opportunity to study the personnel relationships in organized industry provides a real course in industrial psychology, which should develop in the student a sound philosophy of organization and management.

4. A company loyalty is developed which is essential to the young man's progress and his value to any organization in which he is employed.

5. The whole cooperative scheme forces the student to evaluate his own personal characteristics, such as tact, persuasiveness, creativeness, cooperative ability, mechanical aptitude, and power of concentration, thus tending to improve his personality.

6. Valuable habits of evening study are attained, which persist after the young men leave college and obtain responsible positions in engineering.

7. Valuable acquaintances are formed with leading figures in a great industrial organization.

Other Examples of Alternating Study and Practise. The principle of alternating school and employment is becoming widely used. Several years ago it was introduced into the secondary schools of Fitchburg, Mass., and has since come into prominence in the Boston schools and those of other cities throughout the country. This principle is being applied in colleges of liberal and practical arts as at Cincinnati and Antioch. Some other examples approaching the cooperative idea are: Worcester Polytechnic Institute where 15 months of practise is advised between the junior and senior years, and the Pratt School of Naval Architecture at the Massachusetts Institute of Technology in which students work in the near-by navy yards during summer vacations. The School of Chemical Engineering Practise of the Massachusetts Institute of Technology represents still another variation of the cooperative feature. This school offers a graduate course, leading to the Master's degree and consisting of six months of plant study and testing at field stations of the Institute, established in the chemical industries, followed by six months advanced study at the Institute. This School of Practise is also made available to undergraduates pursuing studies leading to the Bachelor's degree only, by a special arrangement of the fourth year in which 22 weeks are spent in industry.

The outstanding features and points of difference of the Cincinnati and M. I. T. plans may be summarized as shown in the following parallel columns:

Cincinnati Plan	M. I. T. Plan
Bachelor's or professional degree in five years.	Bachelor's and Master's degrees in five years.
All students cooperative.	Cooperative students selected.
Cooperation extends entirely or nearly throughout course.	Cooperation during last three years only.
Course in practise laid out by coordinators in college.	Course in practise laid out by officials of company and professional department of the college.
Classes discontinued at works.	Classes carried on at works.
Each student may be trained with several companies.	Each student is trained with one comprehensive company.
Short alternation periods (2 to 6 weeks).	Long alternation periods (15 to 19 weeks).
Each unit of study spread over several short college periods.	Each unit of study completed in each long college period.
Practical training greatly affected by business conditions.	Practical training slightly affected by business conditions.

A clear understanding of these differences of plan will relieve many of the controversies concerning the cooperative method. For example, the question of long *versus* short alternation periods should be answered with reference to a definite situation. If freshmen and sophomores are placed in cooperative training, they should not be allowed too long at a time in the materialistic atmosphere of the shop, but on the contrary should be almost continuously under the influence of the idealism and imagination fostered at the college. Furthermore, if a youth gets a monotonous routine job for a long period, as is usually the case in the freshman year, he is likely to return to college with a sour attitude toward his studies and toward life in general. At the other end of the course the practise assignments become more interesting and are often important pieces of research which ought not to be broken up by the too early transfer from the company to college, while the work at the college should be pursued in continuous units, sometimes under the direction of specialists or visiting engineers of renown. If, then, cooperation is extended over the whole period, as the student progresses from the freshman through the graduate years, a change in length of period from four to six weeks in the early years to one of a term or more in the final years would be desirable. Clearly, from these and other arguments,[8] in the "M. I. T. Plan" where it has been found desirable to cooperate only during the junior, senior, and graduate years, the long term is educationally sound, while in the "Cincinnati Plan," where cooperation is placed in the early years, the short period becomes a logical policy.

Results. The writer has had the privilege of visiting several of the industrial plants of Cincinnati as well as the University, and believes that not only does the plan function efficiently but its graduates are filling the kind of responsible positions for which they are trained. The results of the M. I. T. Plan have been gratifying and have substantiated the ideals involved in the plan. The graduates of Course VI-A have taken prominent places in the engineering and commercial departments, and research laboratories of the large manufacturing and public utility companies. These men are all young yet, the first class having been out of college only eight years, but there is unmistakable evidence that they are proving the value of their training.

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[8] *A New Method of Training Engineers*, M. W. Alexander, A. I. E. E. TRANS., Vol. XXVII, 1908, p. 1459.

Discussion

A. Boyajian: (communicated after adjournment) I have had the opportunity of watching at very close range the progress of the M. I. T. cooperative students in a large manufacturing plant and have seen splendid results from the system. In addition to the many obvious advantages of the cooperative system, which need not be repeated here, I have observed that such a system opens the door of opportunity to the student considerably earlier and puts him in a strategic position to avail himself of the best available opportunities for advanced engineering work as they arise.

The carrying on of some studies at the works is also very desirable. One big benefit that has been mentioned is the development of the habit of study while practising, a habit that should be carried through life. There is also the added benefit that studying, or problem thinking, has a little different tone in an industrial atmosphere than in an academic atmosphere, and it is desirable that the students familiarzie themselves with both tones of analysis before their thinking gets too set. Scientific thinking in an industrial atmosphere tends towards physical analysis with ideals of simplicity and concreteness with plenty of latitude for approximations, while scientific thinking in an academic atmosphere tends towards mathematical analysis with ideals of rigor and completeness with plenty of latitude for abstractions. The students should benefit from exposure to both, whether or not they develop proficiency in both.

When systematic studies are carried on at the works, direct or indirect facilities automatically become available also for studies collateral with the practical work which the students are doing, even when the official course of study may be something different. The instructor perhaps furnishes the students the necessary guidance for such collateral study.

R. W. Chadbourn: (communicated after adjournment) Professor Wildes' discussion of the cooperative student problem particularly interests me, because we have in the laboratory of the Boston Edison Company men working under both the Cincinnati and M. I. T. plans, and have therefore had an unusual opportunity to compare the two systems. Of course, no system, however great its advantages, is ideal, and while I am heartily in favor of the cooperative scheme of education and appreciate fully the advantages of the two systems, as outlined by Professor Wildes, still I think it may not be amiss to point out one or two difficulties which have not been entirely overcome.

I will designate the school employing the Cincinnati plan as School A, that using the M. I. T. plan as School B. Men from School A work with us on alternate five week periods during the last four years of a five-year course; all their working time is spent in one department. Men from School B spend single terms of varying lengths in several departments of the company, their working periods being confined to the last three years of a five-year course. From School A we always have working with us a definite number of men who can be rotated through the department on a definite schedule. From School B, we have occasionally one or more men for an indefinite number of weeks, not exceeding three months. Our experience, covering a period of many years, has been that men from School A prove valuable helpers during their cooperative working periods, that in general they are enthusiastic over their work, and that they become favored candidates for permanent employment with us. Although their work has been restricted to one department, they have become sufficiently reconciled to the routine of business to be valuable even in other departments of the company. Their lack of variety of experience is made up to a certain extent by willingness to tackle any and all kinds of work. Unlike men without cooperative experience, who so often have to flounder around for a year or two before absorbing the atmosphere of business, they adjust themselves immediately to the job.

The men from School B are divisible roughly into two sorts. The first comprises men so keen for knowledge and so alive to all the opportunities presented to them that nothing can hold them back. Because of their ambition, their willingness to work, and their superior variety of experience and training, they become easily the most favored candidates for post-graduation employment. Undoubtedly, for this class of man, the M. I. T. system is ideal, because the system is designed to give him the broadest training possible. This very feature is, however, perhaps responsible for one of the weaknesses of the system.

For it appears that there is a second class comprising men, ambitious enough, it is true, but inclining to the belief that they are favored individuals, that their wide variety of training and experience fits them immediately for executive positions, that the routine matters of everyday work are beneath their dignity. Now, I would not decry the formation of a superiority complex so long as it does not become a "snob complex." Perhaps I may seem to be laying undue stress upon this point, but it is serious enough to have caused a distinct prejudice in my own department against men from School B. Some of these students have complained about the nature of their work, have looked down upon the more disagreeable tasks, and have conducted themselves as though they believed that the company owed them something but that they owed the company nothing in return. Some of them graduate with the idea that they should immediately qualify for executive positions. Obviously the function of cooperative education is not to qualify men to start near the top but to provide them the means for more rapid advancement than their less thoroughly trained associates.

Perhaps part of this attitude is the fault of the employing company, some departments of which may cater too much to the men and thereby tend to spoil them; part is undoubtedly due to the working schedule, which is laid out largely to benefit the student. I do not believe we should cater too much to the student. He should, during his working periods, be an employee, in every sense of the word, as well as a student; he should be made to realize his obligations to his employing company; he should be given enough routine work so that he will not develop a false sense of superiority over his fellow-employees. I believe that this difficulty can be partly overcome by a greater regularity of schedule, a redistribution of the working periods to benefit the company as well as the student,—perhaps a compromise between the Cincinnati plan, under which a student's experience is confined to one department, with frequently an excess of routine, and the present M. I. T. plan, under which a man's time is apportioned to various departments largely on the basis of benefit to himself, the result being an irregular schedule, with sometimes infrequent, sometimes overlapping, working periods in any one department. Perhaps the best average results would be obtained if the men are given the maximum variety of experience and training, and still be taught one of the most important fundamentals of business life, *viz.*, that all work has some routine, and that the widely trained man who shows a willingness and eagnerness to tackle anything which may be placed before him is the man who will eventually land on top. I believe that such a modification of the present schme might give the student a better attitude toward business, resulting in diminished antagonism on the part of the employing company to the student, and thereby both student and company would materially benefit.

D. C. Jackson, Jr.: (communicated after adjournment) The cooperative plan is still young in all engineering schools using it. The first cooperative engineering class graduated from the University of Cincinnati in 1911. Owing to the slowness of the industries and the general public to recognize certain advantages of the cooperative system of engineering education, all of the early cooperative classes at Cincinnati were small, and other institutions were cautious about adopting this method. The success of the graduates of an institution using a new method of education is a measure of the success of that method. Up to the time of the World War the number of graduates of cooperative engineering schools was too small for one to draw definite conclusions with the certainty of accuracy. Since the War the number of cooperative engineering graduates has increased to such an extent that the advocates of this method now have adequate data on which to base their claims.

There are now quite a number of engineering schools using the Cincinnati Plan, either substantially without change, or with certain modifications. Some of these modifications cause the individual cooperative course to approach in certain aspects the M. I. T. Plan.

Professor Wildes has given nine outstanding features and points of difference between the two types of plan. I agree with his comparison in the case of five points, namely, items 2, 4, 5, 6, and 9, but if the Cincinnati Plan is considered as it is employed in a number of engineering schools I am not in accord with him on items 1, 3, 7, and 8.

Item 1. There are several engineering schools using the Cincinnati Plan which grant a bachelor's degree in four years.

Item. 3. In those engineering schools having a four-year cooperative course leading to a baccalaureate degree the cooperation extends through only one or two years. These are ordinarily the sophomore and junior years.

Item 7. Many of the engineering schools conducting cooperative courses have periods longer than six weeks. At least one has a six-week period which can be combined with the succeeding period when desirable to have a twelve-week term. Others regularly use twelve-week, or even a sixteen-week period. The University of Pittsburgh uses a semester of approximately seventeen weeks as the basis of the cooperative term.

Item 8. Those cooperative engineering schools using a period of twelve weeks or more have the curricula so arranged that each unit of study is completed in a single college period.

As the two plans ordinarily are operated at present, the difference given in item 5 exists. However, this is not a fundamental difference when the engineering school is located in an industrial city, as is the case of the universities of Cincinnati, Pittsburgh, Akron, and Louisville. The question of conducting night classes during the cooperative periods, as is done under the M. I. T. Plan, has been considered by at least one of these institutions with no definite adverse decision.

In my opinion the important fundamental differences between the two plans are the following:

CINCINNATI PLAN	M. I. T. PLAN
Scholastic work is limited to undergraduate studies.	Scholastic work in the last year of the course is definitely graduate in character.
All students are cooperative at some time during the course.	Cooperative students are selected.
The course in practise is laid out by coordinators of the college, sometimes with the assistance of company officials.	The course in practise is laid out by officials of the company and the professional department of the college.
Each student may be trained with several companies.	Each student is trained with one comprehensive company.
Practical training is greatly affected by business conditions.	Practical training is only slightly affected by business conditions.

The other four points are adaptable to either plan.

There is no single best plan of cooperative engineering education. The form to be used is dependent upon many factors. Some of these are:

Whether the college is a privately endowed, a state, or a municipal institution.

Whether it is an independent polytechnic institute or the engineering school of a university.

Whether it is a large institution drawing a sizable proportion of its students from all parts of the country, a large or medium-size institution with the large majority of its students from a definite region, such as a state or group of states, or a small or distinctly local institution.

Each of these must determine first whether any sort of cooperative method should be adopted, second (if the answer to the first question is affirmative) whether all or only part of the students should have cooperative work at some time during the college course, and finally the details of the method to be used, after a thorough study of the industries and the part of the country the school serves.

K. L. Wildes: Mr. Boyajian has pointed out the difference in point of view between studies in the educational institution where emphasis is placed more on the fundamental principles underlying a problem, and in the industrial concern, where economic limitations and immediate utility are important considerations. In thesis problems of the graduate year it often happens that a part of the work is done while the student is employed with the company, and a part while in residence at the educational institution. Thus both the academic and industrial standpoints are brought to bear on a single problem, or set of problems. The instructor at the works is a most important factor in helping the student to appreciate these points of view.

Referring to Mr. Chadbourn's discussion I should like to remark that I believe excessive opportunity is often a handicap, not only to students, but to men in permanent employment. The first few years of a young man's life should be characterized by achievement. Cases arise in which the positions given to young engineering graduates are so far above their abilities that these seeming opportunities become actual handicaps, and by them the young employees are prevented from doing work of outstanding quality.

This principle also applies to the training periods of the M. I. T. system of cooperative education. Apparently Mr. Chadbourn is satisfied that the ablest and best balanced students are receiving great profit from their periods of training and experience and become worth while members of the organization after graduation. I have not much use for his second class of men and therefore I do not primarily blame either the cooperating company or the M. I. T. plan of cooperative education for their lack of correct attitude while in training, or their exalted opinion of their ability to take on executive duties immediately after they complete their course of study. These qualities are inherent in the man himself and such a man does not, in my opinion, merit any kind of cooperative training. My solution of Mr. Chadbourn's problem is to eliminate this type of student from the course. The personnel departments of all the cooperating companies are becoming increasingly efficient in the selection of students, so that it looks as though this difficulty is well on its way to solution.

Determination of Generator Speed and Retardation During Loss Measurements

BY O. E. CHARLTON* and W. D. KETCHUM*
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper describes an instrument which was developed to make graphic records of generator revolutions and time as the generator slows down under the action of its losses during retardation tests. The construction of speed-time curves from the records thus obtained is outlined.

The principle of numerical differentiation is discussed and its application to the determination of the slope of the speed-time curve is pointed out.

A complete example illustrating the application of this principle is included in the paper.

INTRODUCTION

During the past two years efficiency tests have been made upon generating units in several hydro plants on the Alabama Power Company system. In determining generator efficiency the fundamental formula

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output plus losses}} \qquad (1)$$

was used.

The losses were determined by the retardation method as described by J. Allen Johnson[1] from the equation

$$\frac{d\,E}{d\,t} = \frac{W\,R^2}{g} \,.\, 4\,\pi^2 \,.\, s \,.\left(\frac{d\,s}{d\,t}\right) \qquad (2)$$

Where

$\frac{d\,E}{d\,t}$ = power.

$W\,R^2$ = flywheel effect of rotating parts.

g = acceleration of gravity.

s = angular velocity of rotating parts.

$\frac{d\,s}{d\,t}$ = angular retardation of rotating parts.

The scope of this paper is limited to a description of the methods used in evaluating the speed (s) and the retardation $\left(\frac{d\,s}{d\,t}\right)$ for use in Equation (2).

MEASUREMENT OF GENERATOR SPEED

The precision of the results to be obtained from the retardation method depends primarily upon the accurate determination of the speed-time curve as the generator slows down and upon the most exact determination possible of the slope of this curve at various points. Experience has shown that a tachometer and a stop watch do not provide the necessary degree of precision, particularly when the generator under test has relatively low $W\,R^2$ and consequently slows down rapidly. An attempt was therefore made to develop an instrument which would record these quantities accurately.

The ideal device for this purpose would be a recording instrument, such as that suggested by V. Karapetoff,[2] which would measure retardation directly. No such instrument is on the market at the present time, however, and difficulties were encountered which prevented the construction of one for the tests.

As a substitute for this ideal instrument, a chronograph was constructed which in its original crude form, consisted essentially of three spark-gaps through which a paper chart was passed as it unrolled from a supply spool and was wound up on a motor-driven rewind spool. A contact on the shaft of the generator energized one spark coil, giving a puncture on the chart for each revolution of the generator. Another spark coil was energized through a contact made by a seconds pendulum giving a puncture every second. The third spark coil was energized through a manually operated switch and its function was to designate on the chronogram the instant at which all instruments were read.

The first form of chronograph, which is illustrated in Fig. 1, was used on several occasions and the results are believed to be of a reasonable degree of precision. Fig. 2 shows a specimen chronogram taken with this instrument and illustrates the type of record which it produced. It was felt, however, that the importance of the efficiency tests justified as high a degree of precision in the measurements as could be obtained practically, and, therefore, an instrument embodying numerous refinements was developed.

While operating on the same fundamental principle as the old chronograph, the new instrument differs widely in detail from its predecessor. It is mounted on an aluminum base approximately 16 in. by 48 in. and about 2½ in. deep. Essentially, it consists of a driving motor, an electrically operated tuning fork, and an electrically operated stylus. The record is made upon a strip of waxed paper which is unwound from a supply spool, passes under the stylus and tuning fork and continues around the two driving rollers to the rewind spool. The tuning fork is electrically driven at 100 vibrations per second, while the stylus is energized through a contact on the generator shaft. Fig. 3 is a

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*Southeastern Engg. Co., Birmingham, Ala.

1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

general view of this instrument, and Fig. 4 shows a typical record taken by it.

One refinement which has been found to contribute materially to the facility with which the records may be interpreted is the constant linear speed drive for the paper. This is accomplished by driving the paper strip directly through constant speed driving rolls, while a friction clutch automatically varies the speed of rotation of the rewind spool.

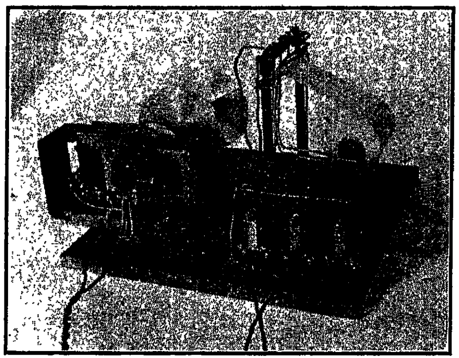

Fig. 1—Chronograph as Originally Constructed

Fig. 2—Typical Record from Chronograph Shown in Fig. 1

Between the tuning fork and the magnetic stylus there is located a spark-gap which is energized manually from a switch. The function of this gap is to designate on the chronogram the instant of taking readings on the instruments.

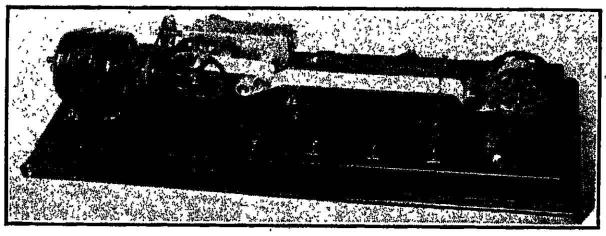

Fig. 3—Improved Chronograph

Speed-Time Curve

The data supplied by the chronogram are in the form of two graphic records, one of revolutions and one of time. In order to obtain the speed of the generator, at any point, it is, of course, necessary to count the indications of each of these records over a period of time. Since the displacement between indications of successive revolutions is far greater than that between successive cycles made by the tuning fork, it is obviously much simpler from the standpoint of interpolation, to count the cycles of the tuning fork record for a given number of revolutions. This process would have been very laborious if it had been necessary to count each cycle individually. However, by means of a transparent celluloid interpolator consisting of ten uniformly spaced radial lines, this work was facilitated materially. The data taken from the chronogram in this manner were tabulated as shown in Table I. Since it is probable that the true speed-time curve of a large machine

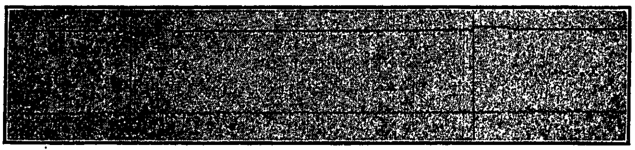

Fig. 4—Typical Record from Chronograph Shown in Fig. 3

TABLE I

I Point number	II Actual time in seconds for 3 revolutions	III Average speed in revolutions per second	IV Time elapsed since start of test (seconds)
1	2.010		
2	2.020		
3	2.025		
4	2.035		
5	2.045	1.467	10.135
6	2.055		
7	2.065		
8	2.070		
9	2.085		
10	2.090	1.435	20.500
11	2.100		
12	2.110		
13	2.120		
14	2.130		
15	2.140	1.402	31.100
16	2.150		
17	2.160		
18	2.170		
19	2.180		
20	2.190	1.370	41.950
21	2.200		
22	2.210		
23	2.225		
24	2.230		
25	2.245	1.337	53.060
26	2.255		
27	2.265		
28	2.275		
29	2.285		
30	2.300	1.304	64.440
31	2.310		
32	2.325		
33	2.330		
34	2.345		
35	2.360	1.271	76.110
36	2.370		
37	2.380		
38	2.390		
39	2.405		
40	2.420	1.240	88 075
41	2.430		
42	2.445		
43	2.455		
44	2.470		
45	2.480	1.210	100 355

as it slows down is a continuous curve, the data taken from the chronogram were plotted and a smooth curve, as shown in Fig. 5, was determined from the plotted points with the idea that this curve would be more nearly a true representation of the speed-time curve of the machine than the actual plotted points. The use of this curve also facilitated the determination of the slopes, as the method of numerical differentiation involves the use of values of speed taken at constant intervals of time.

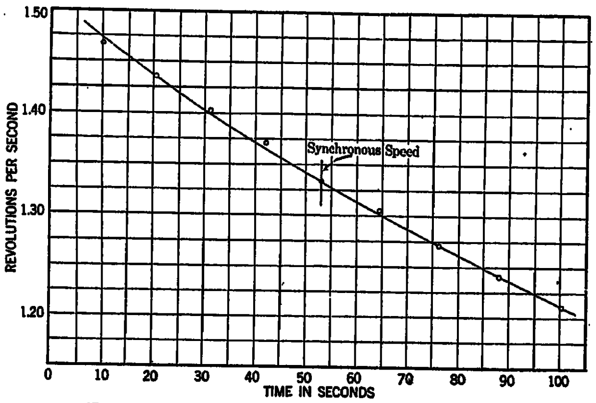

Fig. 5—Typical Speed—Time Curve

Method of Obtaining Slope of the Speed-Time Curve

The slope of the speed-time curve for any retardation run should be determined with the greatest amount of accuracy possible from the data available, because the value of the losses varies directly as this slope.

In finding the slope of a curve, the equation of which is unknown, the usual method is to construct a tangent to the curve at the point where the slope is desired. This graphical differentiation is at best inaccurate for most empirical curves and especially so when the curve approaches a straight line.

Instead of this somewhat crude and inexact method, the principles of numerical differentiation have been applied, and it is believed that a considerably higher degree of precision has been obtained in this way than would have been possible using the strictly graphical method. The principles of numerical differentiation are thoroughly described in many standard mathematical works[3, 4, 5, 6] and hence, the derivation and proof of this process lie outside the scope of this paper. However, a practical outline of the method is included in order that its application to the determination of angular retardation may be clearly understood.

In applying this method to an empirical curve of unknown equation $y = f(x)$, values of x are taken (throughout the range of the curve under consideration) with a constant increment h, as shown in Fig. 6, so that

$$\begin{aligned} x_1 - x_0 &= h; \\ x_2 - x_1 &= h \\ &\cdots \\ &\cdots \\ x_n - x_{n-1} &= h \end{aligned}$$

Corresponding values y are then taken from the curve (*i. e.*, $y_0, y_1, y_2, \ldots\ldots y_{n-1}, y_n$).

Differences between successive values of y are then computed, thus:

$$\begin{aligned} y_1 - y_0 &= a_0 \\ y_2 - y_1 &= a_1 \\ y_3 - y_2 &= a_2 \\ &\cdots \\ &\cdots \\ y_n - y_{n-1} &= a_{n-1} \end{aligned}$$

The values $a_0, a_1, a_2, \ldots\ldots a_n$ obtained in this manner are known as differences of the first order. In the same fashion, the differences of the second order are computed, thus:

$$\begin{aligned} a_1 - a_0 &= b_0 \\ a_2 - a_1 &= b_1 \\ a_3 - a_2 &= b_2 \\ &\cdots \\ &\cdots \\ a_n - a_{n-1} &= b_{n-1} \end{aligned}$$

This process is continued until the desired number of differences has been taken. The Difference Table (Table II) is then constructed, with the values thus obtained.

The question of how many differences to include in the table requires some attention. In the somewhat special case where the original curve can be exactly represented in the range under consideration, by a polynomial of nth degree, the nth order of difference will be constant and the $n + 1$st order will vanish. In such a case the slope as calculated from the Difference Table will be rigorously correct.

More frequently, however, it is found that some order of difference is approximately constant, while successive orders tend to disperse irregularly. Such behavior as this is due to discrepancies between some of the tabulated values of y and the corresponding values of a polynomial which approximately, but not exactly, coincides with the original function in the

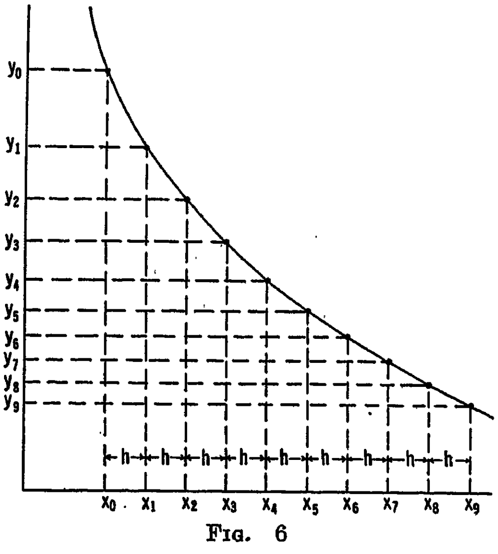

Fig. 6

range under consideration. Such discrepancies may be the result of either errors in the empirical data, or of the fact that the original function cannot be represented exactly by a polynomial in the interval being studied. The magnitude of such deviations from the polynomial function may be determined for each of the tabulated values of y by the use of a correction term (which is evaluated by trial) multiplied by the binomial coefficients.[6]

TABLE II
DIFFERENCE TABLE

x	y	Δ_1	Δ_2	Δ_3	Δ_k
x_0	y_0				
		a_0			
x_1	y_1		b_0		
		a_1		c_0	
x_2	y_2		b_1		m_0
		a_2		c_1	
x_3	y_3		b_2	.	m_1
		a_3		.	
x_4	y_4		.		.
		.		.	
.	.		.		m_{n+1-k}
		.		c_{n-2}	
x_n	y_n		b_{n-1}		
		a_n		.	
x_{n+1}	y_{n+1}				

Where:

$x_1 - x_0 = h$ $y_1 - y_0 = a_0$ $a_1 - a_0 = b_0$

$x_2 - x_1 = h$ $y_2 - y_1 = a_1$ $a_2 - a_1 = b_1$

. . . . etc.

. . . .

$x_{n+1} - x_n = h$ $y_{n+1} - y_n = a_n$ $a_n - a_{n-1} = b_{n-1}$

An inspection of Table III will make this clear.

TABLE III
SHOWING THE INFLUENCE OF AN ERROR ϵ IN ONE VALUE OF y ON THE SUCCESSIVE DIFFERENCES

x	y	Δ_1	Δ_2	Δ_3	Δ_4
x_0	y_0				
		a_0			
x_1	y_1		b_0		
		a_1		c_0	
x_2	y_2		b_1		$d_0 + \epsilon$
		a_2		$c_1 + \epsilon$	
x_3	y_3		$b_2 + \epsilon$		$d_1 - 4\epsilon$
		$a_3 + \epsilon$		$c_2 - 3\epsilon$	
x_4	$y_4 + \epsilon$		$b_3 - 2\epsilon$		$d_2 + 6\epsilon$
		$a_4 - \epsilon$		$c_3 + 3\epsilon$	
x_5	y_5		$b_4 + \epsilon$		$d_3 - 4\epsilon$
		a_5		$c_4 - \epsilon$	
x_6	y_6		b_5		$d_4 + \epsilon$
		a_6		c_5	
x_7	y_7		b_6		d_5
		a_7		c_6	
x_8	y_8		b_7		
		a_8			
x_9	y_9				

This table shows the influence of an error ϵ in any tabulated value of y on the successive differences. It will be seen that the original error is multiplied by the binomial coefficients in each order of difference.

The discrepancies between the original curve and polynomials of several different degrees may be determined as outlined above. These discrepancies will be found to be a minimum for a polynomial of say the kth degree. The Difference Table is therefore extended to include the kth order of differences, but is terminated at this point.

Having completed the Difference Table the following formula for the derivative which was obtained from Newton's Interpolation formula may be used:

$$\frac{dy}{dx} = \frac{1}{h}\left[a_0 + (2n-1)\frac{b_0}{\underline{|2}} + (3n^2 - 6n + 2)\frac{c_0}{\underline{|3}} + (4n^3 - 18n^2 + 22n - 6)\frac{d_0}{\underline{|4}} \ldots\right] \quad (3)$$

where n may be defined as $\dfrac{x - x_0}{h}$, and where a_0, b_0, c_0, d_0, etc. are obtained from the Difference Table, and h is the constant interval between successive values of x.

This formula is rigorously true for all values of n, provided some order of differences (say the kth) is constant. It is approximately true if the kth order of differences is approximately constant, particularly for fractional values of n. Since it is only the sequence of the subscripts in the Difference Table that is significant; any value of x may be designated as x_0 provided the following value is called x_1, etc. Thus it is clear that n may always be made fractional.

In the particular case where the derivative at a tabulated point is desired, n vanishes and Equation (3) reduces to

$$\frac{dy}{dx} = \frac{1}{h}\left[a_0 - \frac{b_0}{2} + \frac{c_0}{3} - \frac{d_0}{4} + \ldots\right] \quad (4)$$

Using Equations (3) and (4), the slope of the original curve may be calculated at as many points as may be desired.

In determining generator losses, the slope of the speed-time curve might be calculated for the point corresponding to synchronous speed only, since this is the only value used in the calculations. However, the slopes were calculated for a number of points along the speed-time curve, and a curve of retardation against time was plotted. From this curve the value of the slope at synchronous speed was read. A curve of retardation against time was used instead of a single calculated point in order that any deviation from its general trend might be detected, and checked for errors.

Application of Numerical Differentiation

In order to clarify the foregoing explanation and at the same time illustrate the use of "Numerical Differentiation" when applied to a practical problem, an example will be given of an actual retardation run taken on a generator during an efficiency test.

The generator revolutions and a timing wave were recorded by the chronograph shown in Fig. 3, making a record similar to the specimen shown in Fig. 4. From this record, the data were tabulated as shown in Table

TABLE IV

x Time in seconds	y Rev. per sec.	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9
10	1.4725									
		−0.0363								
20	1.4362		0.0026							
		−0.0337		−0.0004						
30	1.4025		0.0022		−0.0001					
		−0.0315		−0.0005		0.0012				
40	1.3710		0.0017		0.0011		−0.0044			
		−0.0298		0.0006		−0.0032		0.0106		
50	1.3412		0.0023		−0.0021		0.0062		−0.0188	
		−0.0275		−0.0015		0.0030		−0.0082		+0.0248
60	1.3137		0.0008		0.0009		−0.0020		0.0060	
		−0.0267		−0.0006		0.0010		−0.0022		
70	1.2870		0.0002		0.0019		−0.0042			
		−0.0265		0.0013		−0.0032				
80	1.2605		0.0015		−0.0013					
		−0.0250		0.0000						
90	1.2355		0.0015							
		−0.0235								
100	1.2120									

I. The average speed in revolutions per second was then calculated at intervals of 15 revolutions and the points plotted against time. That is, columns III and IV of Table I were plotted to give the speed-time curve of the retardation run. This curve is shown in Fig. 5.

The next step was to construct a "Difference Table" from the data of the speed-time curve. In doing this, values of time were taken every 10 seconds and the corresponding values of speed read from the curve. These values were tabulated as shown in Table IV.

An inspection of Table IV shows that there is no order of differences which is constant. The third order, however, appears to be of smaller magnitude than the succeeding differences, and this fact suggests that the table should be terminated there. In order to determine how far the tabulated values of speed deviate from a polynomial of the third degree the method previously discussed and illustrated in Table III is applied. Table V is therefore constructed, using Table IV as a base, but terminating the difference table with the third difference. The column C_1 compensates for a discrepancy in the value of y corresponding to a value of x at 60 seconds; C_2 for a discrepancy in the value of y at point when x equals 70 seconds, etc. This table, therefore, shows that the given curve differs from a cubic curve to the following extent:

Time (seconds)	Discrepancy
10	0
20	0
30	0
40	0
50	0
60	−0.0011
70	−0.0023
80	−0.0035
90	−0.0065
100	−0.0118

If desired, this process may be repeated using a Difference Table which includes Δ_4, and it will be seen that the discrepancies between the given curve and a quartic exceed those tabulated above. This shows that differences of the third order should be the highest used in the slope formula. Similarly, the Difference Table

TABLE V

x Time in seconds	y Rev. per sec	Δ_1	Δ_2	Δ_3	C_1	C_2	C_3	C_4	C_5	$C_1+C_2+C_3$ $+C_4+C_5$	$\Delta_3+C_1+C_2$ $+C_3+C_4+C_5$
10	1.4725										
		−0.0363									
20	1.4362		0.0026								
		−0.0337		−0.0004							−0.0004
30	1.4025		0.0022								
		−0.0315		−0.0005							−0.0005
40	1.3710		0.0017								
		−0.0298		0.0006	−0.0011					−0.0011	−0.0005
50	1.3412		0.0023								
		−0.0275		−0.0015	0.0033	−0.0023				0.0010	−0.0005
60	1.3137		0.0008								
		−0.0267		−0.0006	−0.0033	0.0069	−0.0035			0.0001	−0.0005
70	1.2870		0.0002								
		−0.0265		0.0013	0.0011	−0.0069	0.0105	−0.0065		−0.0018	−0.0005
80	1.2605		0.0015								
		−0.0250		0.000		0.0023	−0.0105	0.0195	−0.0118	−0.0005	−0.0005
90	1.2355		0.0015								
		−0.0235									
100	1.2120										

might have been terminated with Δ_2, in which case the discrepancies would also have been found larger than those tabulated above. This signifies that three terms of Equation (4) should be used in computing the slope.

Performing this computation for the case of $x = 30$ seconds, we have:

$$\frac{d\,y}{d\,x} = \frac{1}{10}\left[-0.0315 - \frac{0.0017}{2} + \frac{0.0006}{3}\right]$$

$$= -\ 0.00321 \text{ rev. per sec. per sec.}$$

In like manner the slopes were calculated for other points, and the results appear in Table VI.

TABLE VI

Time in sec.	Slope in rev. per sec. per sec.
10	−0.00377
20	−0.00350
30	−0.00321
40	−0.00314
50	−0.00281
60	−0.00264
70	−0.00272

These data were plotted as shown in Fig. 7. The slope at synchronous speed was read from this curve and was used to calculate the power loss by Equation (2).

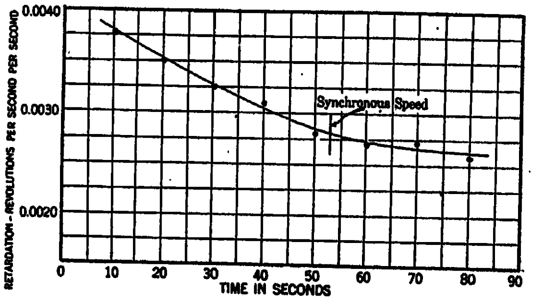

FIG. 7—TYPICAL RETARDATION CURVE

CONCLUSIONS

It is believed that the chronograph is considerably more accurate than the tachometer and stop watch. Experience has shown that the chronograph, using a tuning fork for a timing element, is far preferable to the instrument which depends upon a seconds pendulum for its timing.

In addition to being more accurate than the stop watch, the chronograph is applicable to tests upon generators whose WR^2 is relatively low and which therefore slow down rapidly, whereas the stop watch method is not well adapted to use on such tests.

Due to the inherent accuracy of the chronograph, it is not necessary to take data throughout as great a range of speed during a retardation run as in the case of the stop watch method. Experience with generator tests on the Alabama Power Company system has shown that a range of from 10 per cent above synchronous speed to approximately 10 per cent below synchronous speed is adequate. The average time required for such a retardation run is approximately 3 minutes.

An additional advantage inherent in the use of the chronograph is the fact that it produces a permanent graphical record, thus reducing the chance for personal errors to creep in.

While the method of numerical differentiation is not new, it is felt that its advantages in connection with the determination of the retardation of generators have not been generally recognized. This principle constitutes a useful tool which may be readily employed.

It is realized that the method of determining slopes as outlined above is not mathematically rigorous unless some order of difference becomes constant. The type of curves encountered when considering the speed-time characteristics of a generator under the retarding forces of its losses, in general, does not give a constant set of differences. This is not surprising when it is remembered that the windage loss, which is always present, varies as a variable power of the speed.

It is felt, however, that the results obtained from this admittedly approximate method are of a considerably higher degree of precision than would have been obtainable from a strictly graphical method, and that they are well within the range of engineering accuracy. This, of course, is all that may be expected from data obtained from measurements made in the field where laboratory methods are inapplicable.

The use of this principle has produced consistent results wherever it has been applied by the authors. Typical results of two tests, each of which were duplicated, are tabulated below:

Run No.	Retardation rev. per sec. per sec.	Per cent deviation from the average
1	−0.001763	0.45
2	−0.001747	0.45
Average	−0.001755	
1	−0.001549	0.16
2	−0.001554	0.16
Average	−0.001552	

The close agreement between the duplicate runs seems to justify considerable confidence in the accuracy of this method of recording the speed of the generator and of obtaining the slope of the speed-time curve.

Bibliography

1. *The Retardation Method of Loss Determination as Applied to the Large Niagara Falls Generators*, by J. Allen Johnson, A. I. E. E. TRANS., Vol. XLV, 1926, p. 747.

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3. "Graphical and Mechanical Computation," Part II, by Joseph Lipka.

4. "Interpolation," by J. F. Steffensen.

5. "Interpolation," by Whittaker and Robinson.

6. "Theory and Practise of Interpolation," by H. L. Rice.

J. A. Johnson: I am always glad to see a paper on improvements in the retardation method of testing generators, which I have advocated for a long time. I am particularly glad to see the development of more accurate instruments which will promote the use of this method in testing generators, where it is suitable, for it is a beautiful method and remarkably accurate and easy to apply.

I am rather disturbed, however, to see that this paper may give the impression that the application of this method calls for what appears to be a rather complicated mathematical computation in order to produce a satisfactorily accurate result. I gather that the only excuse for using this method of arithmetical differentiation is greater accuracy. Now, I should like to call attention to Table IV on page 1099. The third column of that table headed Δ_1 gives the loss in revolutions per second for each ten second interval. Dividing these figures by ten will give figures which represent the rates of retardation in revolutions per second per second which are the average rates of retardation during the corresponding ten second intervals. In the method used in evaluating the tests which I made some years ago and presented in the paper referred to by Messrs. Charlton and Ketchum, it was assumed that this average value matched up in time with the middle of the corresponding interval. Of course, this is not scientifically correct but is more nearly so the shorter the interval.

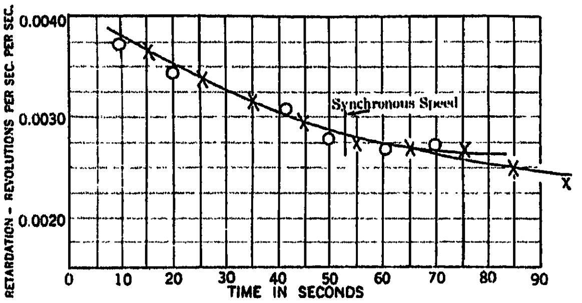

Fig. 1—Typical Retardation Curve

○ = Arithmetical Differential Method
X = Direct Method

Plotting these values (the third column of Table IV with the decimal point moved one place to the left) on Fig. 7, the first value at 15 seconds, the second at 25 seconds, and so on, it appears that these points coincide with the authors' curve more closely than their own points do above synchronous speed, and by giving two more points than the authors' method below synchronous speed, clearly indicates that the arithmetical differentiation method has given very questionable results in this part of the curve. I therefore claim that the authors have not demonstrated that their method of arithmetical differentiation gives more accurate results than the much simpler method I have used.

I bring up this point because I should be very sorry to see the retardation method handicapped by an erroneous impression that it requires laborious computations to get accurate results. On the contrary, I claim that all the accuracy necessary can be built into the chronograph recording instrument, and that this having been done, the instrument record can be interpreted in retardation and loss values in a very simple manner, involving a minimum of computation.

W. D. Ketchum: I should certainly be quite unjust if I did not acknowledge gratefully the benefits which we in our retardation tests have derived from Mr. Johnson's paper. We started out extremely inexperienced and his paper was a guide which we fell back on time and time again and it is only fair that I acknowledge it, because without it we would have had much more difficulty than we had.

As to his criticism of the method of numerical differentiation as being a needless refinement, it seems to me that this is a question which has to be answered in each specific case. It certainly appears from the curve which was flashed on the screen that in the example which we gave in our paper, a simpler method would have been almost as accurate. I say, it appears this is true because the scale at which this curve was reproduced is somewhat small and it is difficult to tell much about it. However, it seems to me that since the shape of that curve will vary somewhat with each individual test, and certainly with each individual machine, this is a matter which has to be settled in each instance.

Now, Mr. Johnson's wide experience in this type of work as compared to our relative inexperience, certainly lends weight to his criticism. We felt, however, that if we went ahead and applied the principle of numerical differentiation we would have obtained as accurate an answer as we could with the instruments and facilities at our disposal and we felt that had we stopped with the simplification which Mr. Johnson pointed out, we would have overlooked something which might or might not have changed the results. That was our justification for going into this type of method.

I frankly think that on close inspection it will be found that this method is not as intricate as my description may lead you to believe. If you will follow the example given in the paper I believe you will find you can go through the various operations in a comparatively short time.

Certainly the last thing we would want to do would be to discredit the retardation method for all the experience we have had indicates that it is a very ready and relatively simple means of determining generator efficiency and we would hate to do anything to throw it in disfavor.

O. E. Charlton: I should like to emphasize the fact that the application of numerical differentiation to the speed-time curve does not require any complicated mathematics and the increase in labor is negligible.

After the necessary information has been obtained from the chronogram to plot a speed-time curve, which is necessary before slopes can be found, by any method, the difference table can be constructed by subtraction. The retardation is then determined by substituting the proper values taken from the difference table in a simple formula. I have found from actual practise that once the data are worked up from the chronogram, the actual calculation of the slopes requires only about fifteen or twenty minutes for each point.

The error introduced in the loss measurement by assuming the speed-time curve to be a straight line between any two points, varies with the type of curve and with the size of the interval between points. We have found from a number of tests, however, that with an interval of ten seconds the error will vary from three to six or seven per cent.

It might be argued that an error of six per cent in the losses would make very little difference in the efficiency of the machine, but a six per cent error in all the losses determined by retardation if they should be accumulative, would change the resultant efficiency by one-tenth of one per cent, or more, and it is not uncommon to find a penalty clause in generator contracts for each tenth of a per cent that the efficiency drops below the guaranteed value.

Numerical differentiation of the speed-time curve gives a very accurate value for slopes and it can be applied with as much ease as the tangent method and with very little more work.

The M. I. T. Network Analyzer

Design and Application to Power System Problems

BY H. L. HAZEN,* O. R. SCHURIG,† and M. F. GARDNER*

Synopsis.—Previous network-computing devices with their uses and limitations are reviewed briefly, following which is a statement of the requisites for an effective a-c. calculating table. The M. I. T. Network Analyzer, a static miniature a-c. power system, designed and constructed jointly by the Massachusetts Institute of Technology and the General Electric Company for the computation of actual power networks, is described in detail and its operation outlined. The fields of its application are enumerated and include the study of normal operating conditions, stability, and short circuits. An example illustrating its application to the solution of a normal-operation problem is given.

* * * * *

INTRODUCTION

To determine the behavior of a power system under various conditions of operation necessitates numerous solutions for the distribution of current, voltage, real and reactive power in its network. The advantages of experimental methods over lengthy and usually impracticable mathematical calculations for the determination of these quantities have long been recognized and several types of experimental computing devices or miniature systems for network analysis have been developed and have proved their usefulness.

The following paper deals with an improved form of network-computing device designed and built jointly by the Massachusetts Institute of Technology and the General Electric Company. The Network Analyzer, as this device is called, is installed in the Electrical Engineering Research Laboratory of the Massachusetts Institute of Technology at Cambridge, Mass.

Most of the network computing devices developed have been in the form of miniature systems. These are imitations on a small scale of those features of the real system considered significant or indispensable for solution of the class of problem for which the model was developed. One of the earliest developed was the d-c. short-circuit calculating table.[1] Due to the use of direct current, it is limited in its application to those problems in which phase angles can be neglected, as for example in short-circuit studies involving only questions of current magnitudes. Its simplicity has made it almost indispensable for this purpose.

The first miniature to employ alternating current was a three-phase representation of one particular system.[2] Like the d-c. table it was a static duplication, containing no rotating elements. Although the phase angles at generating points were not adjustable, it afforded an ingenious means of investigating relay performance with unbalanced short circuits at a date when the usefulness of symmetrical-component analysis was as yet unappreciated.

The necessity for representing angle differences between synchronous machines led to the use of a-c. miniature systems with three-phase generators, motors, static loads, and lumped three-phase artificial lines. One such system[3] used synchronous machines of 3.75 kv-a. rating and normal system currents of 10 amperes at 440 volts. This scale was considered the minimum which would permit the use of the usual types of portable indicating ammeters, voltmeters, and wattmeters for measuring quantities in the circuits without the necessity of correcting for their presence.

Similar systems[4] of larger ratings, having machines ranging from 200 to 600 kv-a. and employing line voltages of 2300 volts, have been used in the study of stability. Questions of stability involve mechanical momentum and electrical damping, and the normal relationships between these cannot be retained with the customary type of rotating machine as the model scale is indefinitely reduced.

Rotating machines in general are not adapted to use in network computing devices of laboratory convenience and accuracy because of their cost, space and power requirements, and their instability when an attempt is made to employ more than three or four units.

The limitations of the foregoing schemes for network representation led to the experimental development at M. I. T. of a static type of miniature system[5] incorporating features which could be applied to the solution of normal-operation problems in extensive systems on laboratory scale. It was demonstrated that generating stations are adequately represented by static phase-shifting transformers, thereby eliminating the difficulties associated with rotating machines.

The Network Analyzer described in this paper is a comprehensive a-c. calculating table incorporating these ideas. While it has been built primarily for educational and research purposes, careful consideration has been given in its design to make it suitable for commercial engineering service.

GENERAL DESIGN CONSIDERATIONS

For a network computing device to adequately achieve its purpose it must embody such complete representation of significant electrical constants and terminal conditions that such errors in method as are inherent in the d-c. table are eliminated. Furthermore, for practical usefulness it must be sufficiently extensive and flexible to represent numerous actual systems; it

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*Massachusetts Institute of Technology, Cambridge, Mass.

†General Electric Company, Schenectady, N. Y.

1. For references see Bibliography.

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30-76

must yield results of suitable accuracy with a reasonable expenditure of time; its operation should be simple; its space and weight requirements must be reasonable; and its cost should be a minimum consistent with these requirements.

The principal features of the design follow:

1. Representation of one phase line-to-neutral.
2. Representation of the lumped impedance and admittance equivalents of lines, transformers, cables, and loads by adjustable resistors, reactors, and capacitors.
3. Representation of either the terminal or excitation voltage of a synchronous machine by the single-phase terminal voltage of a phase-shifting transformer, this voltage being adjustable in magnitude and phase.
4. Use of minimum electrical scale consistent with direct measurements by portable indicating instruments.
5. Use of such portable indicating instruments—ammeters, voltmeters, and wattmeters—that they can be plugged into or removed from the network without materially altering the voltage drop and current distribution.
6. Provision for interconnecting the units to represent complete power transmission networks.

The selection of the relative sizes and ranges of line and load units required an analysis of the constants of modern transmission systems. Moreover the determination of the voltage and current scales of the Network Analyzer called for a consideration of the impedances of series and shunt elements of measuring instruments.

The results of these analyses will be briefly presented.

Results of Analysis of Transmission System Constants. The analysis of a number of overhead as well as underground transmission systems gave the percentage distribution of line and load resistance and reactance values shown in Table I. In general, the two lower ranges of resistance and reactance are for lines, and the two higher ranges for loads.

TABLE I

DISTRIBUTION OF RESISTANCE AND REACTANCE VALUES OF TYPICAL SYSTEMS

	Per cent of lines and loads in transmission system
Resistance less than 2.5%	25
Resistance from 2.5% to 25%	38
Resistance from 25% to 1000%	27
Resistance from 1000% to 5000%	10
Reactance less than 5%*	20
Reactance from 5% to 30%	40
Reactance from 30% to 1000%	30
Reactance from 1000% to 5000%	10

*Values of per cent resistance and per cent reactance of an impedance element are computed on the basis of a reference kv-a. value, and are equal to the per cent drop in the element in terms of normal voltage when carrying a current corresponding to the reference kv-a. at normal voltage.

The electrical ranges of the line and load units were chosen in accordance with these data.

Selection of Current and Voltage Scales. Upon selection of a suitable voltage scale for the Analyzer, its current scale was chosen as small as possible consistent with the requirement that certain portable indicating instruments—voltmeters, ammeters, and wattmeters—could be inserted without materially disturbing currents and voltages in the circuit. For current measurements thermocouple type ammeters were chosen because of their relatively low series resistance in the lower current ranges when compared with other portable ammeters. Likewise thermocouple type voltmeters were chosen because of their low current requirements. Suspended-armature type wattmeters were chosen because their current and voltage elements could be made to meet these requirements. In Table II are given the voltage drops of the series elements of such instruments, and in Table III the currents taken by their shunt elements, expressed as per cents of various voltage and current bases. Corresponding data relating to other types of instruments have been published previously.[6]

TABLE II

VOLTAGE DROPS OF AMMETERS AND WATTMETER CURRENT COILS AT RATED CURRENT

	Volts drop	Percentage of base voltage			
Base volts		400	200	100	50
Thermocouple ammeter	0.12	0.03	0.06	0.12	0.24
Suspended-armature wattmeter					
1-ampere current coil	0.08	0.02	0.04	0.08	0.16
2-ampere current coil	0.04	0.01	0.02	0.04	0.08

TABLE III

CURRENT REQUIREMENTS OF VOLTMETERS AND WATTMETER POTENTIAL CIRCUITS AT RATED VOLTAGE

	Amperes	Percentage of base current			
Base amperes*		2	1	0.5	0.25
Thermocouple voltmeter	0.002	0.1	0.2	0.4	0.8
Suspended-armature wattmeter	0.003	0.15	0.30	0.6	1.2

*The base current or nominal current of the miniature system designates the general order of magnitude of the currents in the principal lines. The operating currents of these lines will of course cover a wide range, of from say one-fifth to four times the nominal value.

A high operating voltage for the Analyzer appears desirable of course to reduce the effects of series-connected instruments on line impedances at a given operating current, as will be seen from Table II. An operating voltage of 200 volts was chosen because it was not desired to subject the operators to the hazards of a higher voltage.

The voltage scale having been fixed, the nominal operating current (see footnote of Table III) was chosen at a value which would give relatively small errors due to the insertion of instruments. It was of course desirable to keep the current as low as possible so as to have a low kv-a. rating of the Analyzer equipment.

It is seen from Table II that the voltage drops in the thermocouple ammeter and in the one-ampere current element of the suspended-armature type wattmeter are 0.12 volt and 0.08 volt, or 0.06 per cent and 0.04 per cent respectively, when carrying one ampere in a 200-volt circuit. If the impedance of the line itself in which these instruments are to be placed is, say, 2.5 per cent (on a one-ampere, 200-volt basis) the increase of total impedance due to the addition of either instrument would be about 2 per cent, *i. e.*, the total impedance would be approximately 2.55 per cent instead of 2.5 per cent. For lines of smaller impedance carrying one ampere in a 200-volt system the additional impedance due to the instruments would of course become more significant and would in some cases require correction or adjustment.

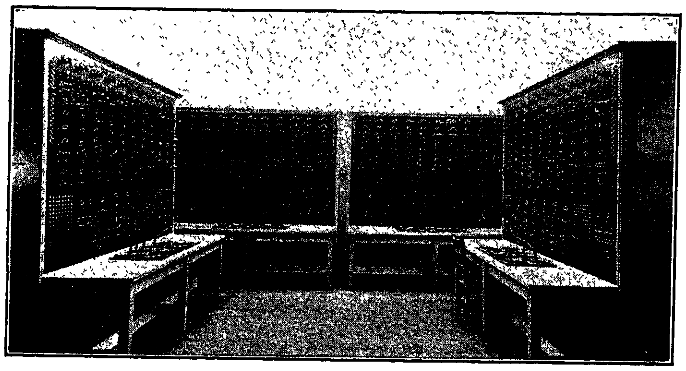

FIG. 1—THE NETWORK ANALYZER

General view of Analyzer plugged for a problem showing the layout of the four sections and the accompanying instrument tables

On the other hand, if the one-ampere wattmeter of Table II is used in a circuit carrying 0.5 ampere and having an impedance drop of 2.5 per cent at 0.5 ampere and 200 volts, the addition of the wattmeter current element would cause an increase in the circuit impedance drop of only about 1 per cent. A value of the same order of magnitude would of course apply in the case of the one-ampere thermocouple ammeter similarly used at 0.5 ampere.

These considerations point toward the selection of a nominal current of not less than 0.5 ampere for a miniature system in which significant errors due to, or frequent adjustments on account of, impedances of the series-connected instruments of the types covered by Table II are to be avoided.

DESCRIPTION OF NETWORK ANALYZER AND ITS OPERATION

Based on the analysis and general requirements set forth in the foregoing section, a device has been built which is proving very satisfactory in the solution of actual engineering problems. The equipment in the present installation includes a sufficient number of units to represent in its electrical essentials a system consisting of as many as 8 generating stations, 60 lines and cables or other connecting elements, 40 loads, 4 ratio-changing transformers for closing loops, and any desirable number of synchronous condensers. In general it may be assumed that any network whose number of elements falls within the above limits can be represented on the Analyzer with a completeness adequate for the engineering study of the problems for which it was designed.

This equipment is assembled in four steel frames each with cabinet enclosures on all sides except the front. The four sections are arranged facing inward on three sides of a square as shown in Fig. 1. Any of the assembled units can be removed from these section frames merely by taking out four screws, and as they are mechanically interchangeable a very flexible assembly results. Each unit has its own panel and when placed in the frame these panels, together with strips bearing outlets for the busses, form the front face of the section. In front of each section stands a table with a center recessed portion containing instruments. (See Fig. 2.)

Description of Component Parts. Having thus considered the more general features of the Analyzer, the various types of units and other parts will be described giving the significant design details and constants.

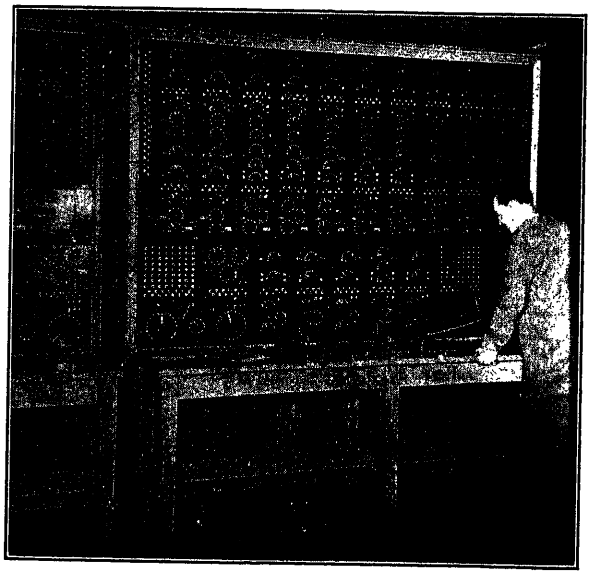

FIG. 2—A SINGLE SECTION

Line- and load-unit panels are at top and center, two phase-shifter panels are at lower left and right, and capacitor and synchronous-reactor panels are immediately above phase-shifter panels. Instruments are located in recess in the table top

a. Generators. The six principal phase-shifting transformers, used to represent an equal number of generating stations, are rated at 0-500 volts, 6 amperes with 0.5-volt steps, while two smaller additional units used in previous work rated at 0-300 volts, 1 ampere, are available if needed. Both types are very similar in design and one of the larger units will be described in detail. This phase-shifting transformer or phase shifter, as it is called, is built on a standard wound-rotor induction-motor frame of about 7 kv-a. capacity with an ordinary two-pole, three-phase, delta-connected rotor winding designed as a 230-volt primary. Its rotor can be adjusted to any desired angular position by a manual worm-gear drive operated at the unit panel, and the angle read from a convenient scale graduated in tenths of degrees. (See Fig. 3.)

The secondary two-pole single-phase winding is provided with taps for 0.5-volt steps. It is of the concentric-coil type, thus making the time phase of the secondary induced voltage essentially independent of the number of coils in the circuit, *i. e.*, of the voltage setting. The angular adjustment of the rotor provides the time-phase adjustment of the secondary voltage, hence by means of the tap switches and the angle control, the vector representing this voltage can be made to terminate at any point within a circle of 500 volts radius. This secondary voltage can be used to represent either the terminal or the excitation voltage of a synchronous machine under transient as well as steady-state conditions.

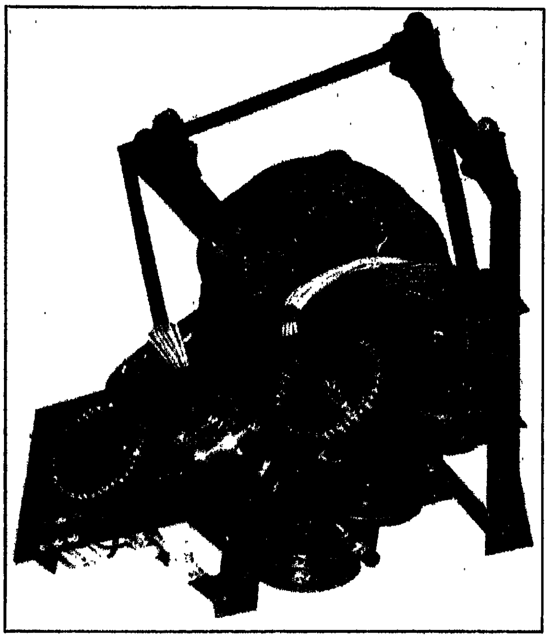

Fig. 3—Artificial Generating Station

A phase-shifter unit showing truck mounting of transformer with control panel, which bears voltage-control switches, angle-control handle and indicating dials, and terminals

By the use of low flux densities, suitable windings, and a power source of particularly good wave form, the harmonic content of the phase-shifter terminal voltage for all load conditions has been kept under one per cent. The phase shifter with its control panel is assembled similarly to a truck type switchboard section making it very accessible. Suitable jacks are provided for terminals and for the insertion of instruments to indicate its output.

b. Line and load units. These are considered together since the different sizes of resistor or reactor units differ only in winding details. Ordinarily a resistor unit is paired with a reactor unit, the two being connected in series by removable leads in the case of the line units, and in parallel in the case of the load units.

In Table IV the number, range, and principal constants of these units are given.

TABLE IV
LINE AND LOAD UNITS

	Number	Max. ohms	Ohms per step	Max. capacity
Line units	**60**			
Resistors.......	25	10	0.1	4 amp.
	35	100	1.0	2
Reactors.......	20	20	Approx. 1% of setting	4
	40	120	Approx. 1% of setting	2
Load units	**40**			
Resistors.......	30	4,000	4	1 amp. or 200 v.
	10	20,000	200	200 v.
Reactors.......	30	4,000	Approx. 1% of setting	1 amp. or 200 v.
	10	20,000	Approx. 1% of setting	200 v.

R/*X* ratios of reactors

Size	Setting	*R*/*X* in per cent at 60 cycles
20 ohms	20 ohms	3.5
	1	5
120	120	3.5
	20	4.5
4,000	All	6
20,000	All	7

Saturation. For a 20-to-1 change of ampere turns the reactance at any setting within the working range changes not more than 2 per cent.

The resistor units are assembled of resistor tubes, each consisting of high-resistivity, low-temperature co-efficient wire wound on a porcelain tube covered by vitreous enamel. The number of tubes varies from 4 in the 10-ohm unit to 28 in the 4000-ohm unit, the design depending on the amount of power which must be dissipated as heat from the most heavily loaded tube. (See Fig. 4.) All units are calibrated, and a step can be selected whose resistance is within ½ per cent of the desired value. Variations of resistance due to changes in load and ambient temperature have been kept considerably less than one per cent. Hence under all

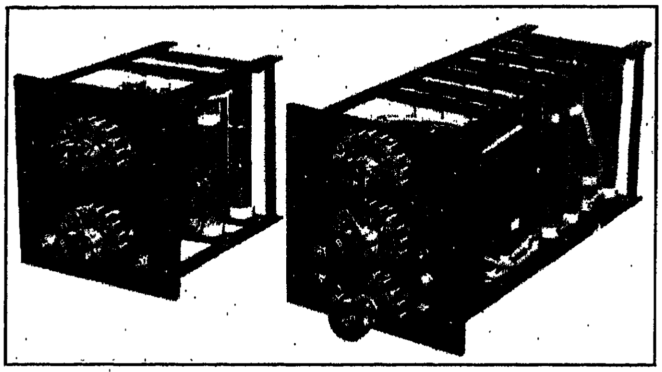

Fig. 4—Resistor Units

Line unit (left) and load unit (right) showing resistor-tube assembly and control panel bearing decade switches, taper-socket terminals, and current jack (load unit only). These units are self contained and mounted in section frame with four screws conditions of operation the actual resistance of a unit is within one per cent of the desired value.

The design of reactance units whose inductance and resistance are properly related and sufficiently independent of variation with current, and whose size and cost are at the same time reasonable, was one of the major problems in the development. All of the line- and load-reactor units are approximately identical in size and weight, varying only in the number of turns and size of wire in the coil. The core consists of two stacks of nickel-iron alloy E-punchings so assembled that the magnetic circuit resembles a wide block figure 8 split by a vertical cut forming the air-gaps. The coil is placed around the central air-gap, making the reactor look like a shell type transformer with an air-gap in each of its three legs. Each reactor weighs approximately 25 lb. and occupies a space 6½ by 7 by 8½ in. By mounting the reactors in trunions the mutual reactance between units could be adjusted to a minimum after assembly, and the space requirements considerably reduced. Three dial switches on the unit panel provide for reactance adjustment. (See Fig. 5.) Current jacks are placed in all line and load units except the line resistors, which are always in series with their corresponding reactors if used at all.

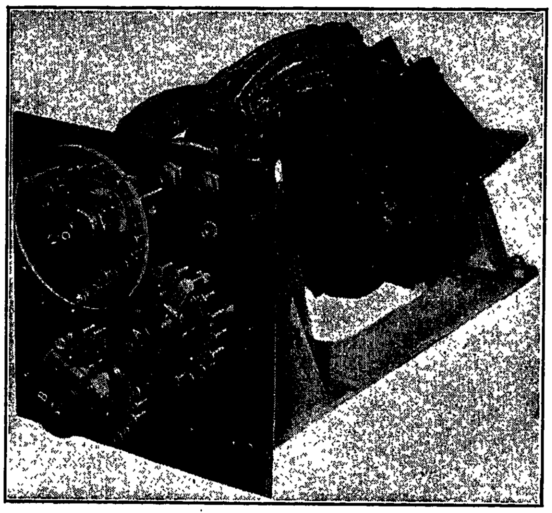

Fig. 5—Reactor Unit

Load-unit assembly showing reactor tilted in trunion mounting behind control panel bearing tap switches, terminals, and current jack. Reactor is of shell type design with central air-gaps. Glass guard protects operator from high voltage on large-step dial due to auto-transformer action.

It is essential that the ratio of resistance to reactance be small over a large range of reactance. This was accomplished with minimum space and weight by a suitable grading of wire sizes and arrangement of taps. (See Table IV.)

The calibration of reactance units, where more than one set of taps is used to adjust the reactance, is an interesting problem. Since the reactance added by the addition of a given number of turns depends on the number already in use, and the reactance varies only approximately as the square of the number of turns, it is theoretically necessary to calibrate each combination of taps of every reactor. A method was developed, however, whereby only a relatively small number of measurements was required, and a simple calibration chart for each size gives the setting for any reactance, taking into account the small effect due to current if desired. These calibrations are accurate to about ¼ per cent, which justifies the correction for current if accuracy better than one per cent is warranted by the problem under consideration.

The maximum flux density used is approximately 6000 gausses.

c. Synchronous-impedance units. Special reactor units are needed to represent the synchronous impedance of generators because of the large rating of these units, and to secure a lower R/X ratio than is needed for the line units. These units, which are about three times the size of the other reactance units, are rated at 0-500 ohms, 6 amperes or 500 volts (whichever gives the lower rating), and have practically a constant R/X ratio of 3 per cent from 50 to 500 ohms.

d. Capacitors. Considering the fact that 20 miles of present-day 132-kv-a. cable requires a charging current which is approximately equal to its allowable operating current, it will be seen that a device which is to be used in the solution of metropolitan networks must have an amount of capacitance comparable with the full-load generator ratings. There have been incorporated in the Network Analyzer 32 capacitance units of 12 μ f. each, adjustable in 0.1-μ f. steps, which at 200 volts can draw approximately 80 per cent of the combined phase-shifter ratings. This capacitance can be used not only for representing the constants of lines and cables but for the representation of synchronous condensers in all normal-operation problems.

The capacitors, which are a wax-paper type of high quality, are rated at 350 volts, 60 cycles, alternating current.

e. Auto-transformers. To represent the effect of ratio-changing transformers used to control the reactive kv-a. distribution in closed loops of transmission circuits, four auto-transformers adjustable from 0 to 10 per cent voltage buck or boost by 0.5 per cent steps are included. Their magnetizing currents, which must be small, range from 1.7 to 4 milliamperes; their impedances vary from zero at unity ratio, to 0.3, 0.7, or 1.5 ohms (depending on rating) at 10 per cent buck or boost.

f. Bus structure. At the rear of each section, 37 No. 0 B. & S. solid copper bus-bars are placed longitudinally in horizontal rows. These busses have frequent taper-socket outlets along the face of the sections between the rows of unit panels. Units are connected together by plugging their terminals to outlets of the same busses with short leads having tapered brass terminals. The bus structure and the connectors are made of such size copper that the total resistance between two units at widely separated points is at most a few milliohms, which on the scale used is entirely negligible.

g. Primary power. The primary power is obtained from a sine-wave alternator driven by a d-c. motor, the controls of both machines being available at the Analyzer. This alternator is rated 75 kw., 230 volts, three-phase, 60 cycles, and has a wave form differing from a sine curve by about 1 per cent under the worst conditions.

A three-phase auto-transformer makes voltages of 100, 30, and 10, in addition to the usual 230-volt supply, available for the phase-shifter primaries. By using one of these low-voltage taps, short-circuit studies can be made on a system set up for normal operation without changing the scale or materially altering the set-up, since the reduction of the primary voltage reduces the large network currents due to short-circuit conditions to reasonable ampere values. The power can be thrown on or off the primaries of the phase shifters by means of numerous conveniently located push-button stations.

h. Instruments. The requirement that all measurements be made by portable indicating instruments without disturbance of the network due to their presence, has been adhered to with the result that the measurement of all quantities is a simple and convenient process. There are two groups of instruments: (1) those used at the terminals of the phase shifters which are standard portable instruments, and (2) those used for making measurements in the lines and at the loads when the restrictions on current-coil impedance drops and potential-circuit currents are most severe. The instruments of the second group are portable indicating thermocouple ammeters and voltmeters and portable indicating suspended-armature wattmeters. The use of a separate set of instruments for each phase shifter greatly facilitates the adjustment of the network terminal conditions since the operators can tell at a glance the conditions at all generating stations without having to re-plug instruments and change scales. Table V is a list of the instruments with their principal constants.

The reactive power, and phase angles of currents and voltages are quantities measured readily by means of a phase-shifting transformer and a wattmeter, preferably of the sensitive suspended-armature type. With the same equipment, the components of the complex expression of a current referred to any axis are read easily.

Procedure in Solving a Network Problem. The solution of a power-network problem on the Network Analyzer involves several distinct steps: (1) the determination of the scale on which the system will be represented, (2) the connection and adjustment of units to represent this network to the scale selected, (3) the adjustment of the terminal conditions such as generator outputs, loads, and voltages to such values that they represent the desired operating condition, (4) the measurement of the desired results by means of instruments, and (5) the conversion of these results from the Analyzer base to the actual system base. These various steps will be considered in order.

TABLE V

INSTRUMENTS FOR NETWORK ANALYZER

Number, Range, and Principal Constants

Item	No. used
1. Voltmeters (used at phase-shifter terminals)	6
Ranges 300/600 volts	
Current at 200/400 volts 0.0125 ampere	
2. Ammeters (used at phase-shifter terminals)	6
Ranges 2/6 amperes	
Resistance (on both ranges) 0.30 ohm	
3. Wattmeters (used at phase-shifter terminals)	6
Potential-circuit ranges 200/400 volts	
Potential-circuit current at 200/400 volts 0.013 ampere	
Current ranges 2/6 amperes	
Resistance (on both ranges) 0.45 ohm	
4. Voltmeters, thermocouples*	2
Ranges 5, 10, 25, 50, 100, 250 volts	
Current at full scale 0.002 amperes	
5. Ammeters, thermocouples.*	4
Ranges 0.1, 0.25, 0.5, 1.0, 2.5, 5.0 amperes	
Potential drop at full scale (all ranges) 0.12 volt	
6. Wattmeters, suspended-armature type	3
Potential-circuit ranges 100/200/400 volts normal	
Normal potential-circuit current 0.003 ampere	
Current ranges† 1.0/2.0 ampere normal	
Impedance drop (normal current) 0.08/0.04 volt	
7. Wattmeters, suspended-armature type	2
Potential-circuit ranges 200/400 volts normal	
Normal potential-circuit current 0.007 ampere	
Current ranges 0.2/0.4 amperes normal	
Impedance drop (normal current) 0.1/0.05 volt	
8. Wattmeter, suspended-armature type	1
Potential-circuit ranges 200/400 volts normal	
Normal potential-circuit current 0.003 ampere	
Current ranges 0.2/0.4 ampere normal	
Impedance drop (normal current) 0.4/0.2 volt	

*The thermocouple voltmeters and ammeters will withstand a 100 per cent overload for short periods without affecting the calibration and have an accuracy of 1 per cent of full scale.

†The current coils of items 6, 7, and 8 have an ample overload current-carrying capacity. The potential circuits have a maximum rating 50 per cent above the normal voltage.

In selecting a base for setting up a system, 200 volts is chosen as nominal voltage for all but short-circuit studies. Such a base of current, and thereby of power, is selected that the actual system per cent constants, when applied to this base, result in current magnitudes which lie within the convenient working range.

After scale selection, working connection diagrams are prepared on which are noted the numbers and settings of the units selected to represent the different system parts—such as generators, lines, and loads—and from these the connections and unit settings are made. If a problem involves the study of several different connections of a system this process is repeated for each connection.

Usually for each connection, a number of different operating conditions is to be studied. These operating conditions are represented one at a time by setting the quantities at generators, loads, and other points in the system to prescribed values. With the settings made, the distribution of current, voltage, and power throughout the Analyzer circuits, is a miniature replica of that existing on the power network for the given operating condition.

After adjusting the terminal conditions to the prescribed values, the quantities desired can be read directly. The final step consists in expressing these quantities as fractions of the Analyzer base. These same fractions apply directly to the actual system.

Several methods of rapidly checking the final results are used whereby possible errors in connecting or setting the units, making readings, or converting results become apparent. These checks, and the accuracy of the Analyzer which their application shows, appear in the discussion of the example.

FIELDS OF APPLICATION OF THE NETWORK ANALYZER

The chief function of the Network Analyzer is to serve as an experimental substitute for the lengthy and generally impractical calculations of normal-frequency current, voltage, power and phase relations in electric power networks for which the resistance, inductance, and capacitance constants of lines, generators, transformers, and loads are known. Before considering its fields of application it is well to point out that it will *not* serve for the determination of circuit constants of lines, generators, transformers, or of other system apparatus, but rather requires for its use a knowledge of these electrical circuit constants. Moreover since it has lumped line units, and phase shifters to represent synchronous machines, it is not applicable to the study of high-frequency transient phenomena, corona problems, arcing grounds, lightning induction, synchronous machine characteristics, harmonics due to transformers or generators, etc.

The various applications of miniature a-c. systems have been discussed in previous publications[7] and will be only briefly reviewed here. They may be classified as follows:

1. Determination of voltage, current, and power data for normal operating conditions in a network of interconnected lines, stations, and load centers. Such data are necessary for the most economical layout of systems, and for the determination of operating procedure.
2. Solution of problems concerned with the stability and power limits of systems for loads slowly or suddenly added, during short circuits, etc.
3. Determination of currents during short circuits for relay settings, rating of current-limiting reactors, duty of oil circuit breakers, electromechanical stresses, etc.

Studies of Normal System Operation. One of the important problems requiring solution by means of a miniature system is the determination of load division among the various branches and stations of a power network. This is frequently desirable for the best design of new systems as well as for the study of additions or changes and the best operation of existing systems. While a study of this nature may require only a knowledge of the current at various points in the network, in general it requires also a knowledge of voltage, and real and reactive power.

Two problems closely related to the foregoing are (1) voltage regulation in power networks and (2) power factor correction. Power factor correction requires a study of the effect of certain classes of terminal apparatus, such as synchronous condensers, when added at different points of a system.

Studies of System Stability. The growth of power systems, the advent of wide-spread interconnection, and the consequent increase in length of transmission lines in order that the interconnected systems may be supplied from large hydroelectric developments have resulted in bringing the problem of system stability to the fore. Designers and operators are interested in system power limits for slowly-applied loads and during short circuits or other disturbances.

Problems of static stability can be studied with the Network Analyzer even beyond the stable point of the power system because synchronous machines are represented by stationary phase shifters. Although it will not serve to reproduce actually the transient swings of generating and receiving machinery, problems of this class can be studied by the point-by-point method[8] with its aid. In this method the rotor-position changes, together with field-current and prime-mover torque changes, are computed for successive arbitrary short intervals of time, the progress of the transient thus being developed by small steps. Since a change in field current and the angular position of the rotor changes the armature current and power in every machine, it is necessary to determine the values of these armature currents and powers as they exist at the beginning of each interval. These are fixed by the rotor positions and field currents found to exist at the end of the previous interval. In other words a normal steady-state electrical solution of the network with the known terminal conditions must be made for the end of each interval, and the result used to compute the changes which will occur in the following interval. The calculation of these numerous steady-state conditions is the most laborious part of the point-by-point method and is the type of calculation for which the Analyzer was developed.

As mentioned above, the Analyzer is not applicable to the determination of synchronous machine characteristics, reactance values, saturation effects, etc. These data must be separately obtained.

Short-Circuit Studies. Experimental solutions of short-circuit problems have been extensively discussed in earlier publications.[9] The principal points bearing on the application of the Network Analyzer are the following:

Since the short-circuit calculating table ordinarily serves adequately for the more common short-circuit studies of systems, the Analyzer will not supplant this simple and effective d-c. device. The errors[8] of the latter are commonly below 10 per cent and rarely in excess of 20 per cent. The errors of the Network Analyzer will, in general, be less than 3 per cent.

The solution of unbalanced short circuits on the Analyzer may be accomplished either by successive single-phase setups with the aid of symmetrical-coordinate analysis, or in an approximate manner by a three-phase setup with a three-phase power supply with or without the use of the phase shifters.

Educational Uses. Perhaps the application of the Analyzer most far reaching in its effect is its educational use not only in the solution of sample problems by classes but as a computation tool which opens fields of research to graduate students otherwise closed by excessive labor requirements. Furthermore, the presence of a device regularly used in the solution of actual engineering problems has a very real educational value. These uses make the Analyzer a valuable asset for an electrical engineering laboratory.

EXAMPLE

It is the purpose of this example to illustrate the way in which the Network Analyzer can be used in the solution of a typical problem which may arise in the growth of a power network. The example does not illustrate the limit in complexity of circuits, or range of conditions which can be studied on the Analyzer, but is given to be suggestive of the possibilities of the device. The one-line diagram of the system which is used for this example, is shown in Fig. 6. It includes three loops and a number of radial lines connecting the generating centers *A*, *D*, *F*, and *J* with the load center at *K* and vicinity, and with another system to which power is supplied at *Q*. Both *K* and *Q* represent large load and steam-generating centers to which the network furnishes the power used in excess of that generated at these points. At busses *B*, *C*, *H*, *I*, and *N* synchronous condensers serve to maintain voltage.

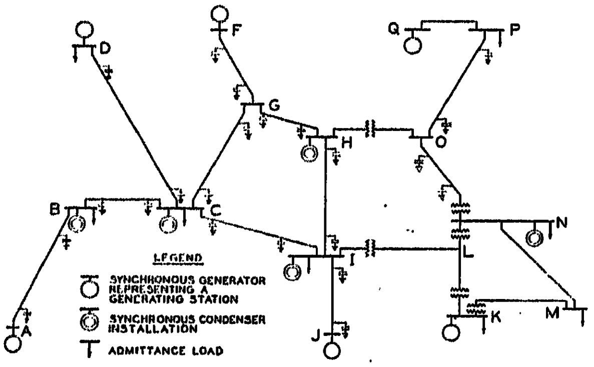
D
F
Q
P
G
H
O
B
C
N
I
L
K
M
A
J
LEGEND
SYNCHRONOUS GENERATOR REPRESENTING A GENERATING STATION
SYNCHRONOUS CONDENSER INSTALLATION
ADMITTANCE LOAD

FIG. 6—SYSTEM USED IN EXAMPLE

One-line diagram of sample network in which effect of ratio-changing transformer and phase-shifting equipment on required synchronous condenser capacity and power flow were examined. For constants see Table VI

The problem is to determine the reductions in synchronous condenser installations which can be effected by using a ratio-changing transformer in the main loop *C G H I*; also to determine what phase-shifting equipment, if any, is needed to properly distribute the power flow between the various parallel paths.

Specifically, four cases are to be examined. (1) With the network as shown in Fig. 6 and all of the stations delivering specified loads, the size of the condensers at *C*, *H*, and *I* required to maintain normal voltage is to be determined in relation to the setting of a ratio-changing transformer in line *C I*. (2) With station *D*, which is of large capacity, disconnected from the network at *C*, the size of the synchronous condensers at *C* and *I* is to be determined in relation to the transformer ratio in *C I*. This case fixes the size of these condenser installations before station *D* comes into service; thereafter they presumably will have to be considerably increased. (3) With line *C I* removed by opening the circuit breakers at both ends, voltage at the major busses and power in the principal lines are to be determined when the condenser outputs are limited to the values found most suitable in the first case. (4) If any of the lines in the major loop *C G H I* are overloaded under any of the above conditions, the beneficial effect of phase-shifting equipment, located preferably at *I*, is to be considered.

The problem having been thus briefly described, the next step is to state the constants and the operating conditions which are to be represented to obtain the desired information To accomplish this in an orderly way, the problem is resolved into the study of one or more operating conditions on each of several connections. Here a *connection* is considered as one configuration of the network, and an *operating condition* as one set of terminal conditions applied to this configuration with the resulting current, voltage, and power distribution. The data describing a connection are given first, then follow the terminal conditions which apply to each operating condition included under this connection. This is repeated for each different connection studied.

The numerical statement of the data for the various connections in the problem will be given.

CONNECTION I (corresponds to the first case mentioned above)

In Fig. 6 is shown the diagram of connections. The constants are tabulated in Table VI. The ratio-changing transformer in *C I* at *C* has 0 to 10 per cent voltage buck or boost.

Operating Condition I-1. The necessary terminal conditions to establish operating condition I-1 include the generating-station power outputs (Table VII), the power and reactive kv-a. taken by loads (Table VIII), and the voltage maintained at points where it is under control. These points must in general be busses to which are connected synchronous machines. Busses *A*, *D*, *F*, *J*, *K*, and *Q* are maintained at 100 per cent voltage by the synchronous generators, and busses *B*, *C*, *H*, *I*, and *N* at 100 per cent by synchronous condensers. The ratio-changing transformer at *C* is set at the one-to-one ratio, *i. e.*, it has no effect on the reactive kv-a. distribution.

Operating Condition I-2. The terminal conditions are identical with those for operating condition I-1, but the ratio-changing transformer is set at 5 per cent step-up of voltage from bus C to the line $C\ I$.

It may be mentioned that any other terminal conditions could be set with equal ease, but merely to make the statement of the example as simple as possible, essentially the same terminal conditions are used throughout.

TABLE VI
CONSTANTS

Constants of the equivalent $-\pi$ circuits* representing the lines, transformers, etc., connected between adjacent busses. All quantities are expressed as a per unit on a 100,000-kv-a. base.

Circuit element	Architrave impedance*	Pillar admittance* (at each end)
$A\ B$	$0.0113 + j0.132$	$+j\,103 \times 10^{-3}$
$B\ C$	$0.0082 + j0.079$	62
$D\ C$	$0.0176 + j0.208$	162
$C\ I$	$0.0098 + j0.0915$	71
$C\ G$	$0.0054 + j0.0620$	46
$F\ G$	$0.0041 + j0.0470$	35
$G\ H$	$0.0071 + j0.084$	65
$H\ I$	$0.0141 + j0.134$	103
$J\ I$	$0.0091 + j0.174$	140
$I\ L$	$0.0058 + j0.0640$	0
$L\ K$	$0.0010 + j0.0190$	0
$K\ M$	$0.038 + j0.581$	0
$M\ N$	$0.106 + j0.487$	0
$L\ N$	$0.056 + j0.513$	0
$N\ O$	$0.0195 + j0.376$	18
$H\ O$	$0.0108 + j0.160$	0
$O\ P$	$0.0109 + j0.121$	5.6
$P\ Q$	$0.024 + j0.037$	0

*For explanation of terms see L. F. Woodruff, "Electric Power Transmission and Distribution," John Wiley and Sons, Inc., 1925, pp. 139 and 155.

TABLE VII
GENERATING-STATION OUTPUTS

Power expressed as a fraction of base kv-a.

Station.....	A	D	F	J	K*†	Q*
Output.....	1.95	2.30	0.52	2.42	−2.2 −2.6	−0.62

*Bus K represents a large load center with extensive steam generating equipment, which is, however, insufficient to carry the entire load at K. The combination of generators and loads at this point is represented by an admittance load and a synchronous machine which absorbs the difference between the locally generated power and that consumed exclusive of the admittance load. The machine at Q represents a similar combination.

†The synchronous machine at K takes the difference between the power generated in the network and that consumed by losses and the other loads. It should lie within the limits given.

TABLE VIII
LOADS

Power and reactive kv-a. (lagging) taken by loads, expressed as a fraction of base kv-a.

Point	Power	Reactive kv-a.	Point	Power	Reactive kv-a.
B	0.36	0.320	K	0.82	0.82
C	0.58	0.192	M	0.45	0.238
D	0.29	0.204	N	0.38	0.345
I	0.68	0.60	P	0.32	0.206

CONNECTION II

The diagram of connections and the constants are the same as for connection I except that station D and line $D\ C$ are not connected to the system.

Operating Condition II-1. The power consumed by the synchronous machine at bus K is reduced by approximately the amount of power which was fed into the network by line $D\ C$ in connection I, otherwise the terminal conditions are unchanged. The transformer at C is set at unity ratio.

Operating Condition II-2. Same as II-1 except that the transformer at C is set to step up the voltage 5 per cent between bus C and line $C\ I$.

Operating Condition II-3. Same as II-2 except that the transformer ratio is set for 10 per cent step-up of voltage instead of 5 per cent.

CONNECTION III

Line $C\ I$ is disconnected from the network at both ends, otherwise the network is identical with connection I.

Operating Condition III-1. The terminal conditions are the same as in I-1 except that the outputs of the synchronous condensers at C and I are limited to the values fixed by the results secured in connection I. This limits the control of the voltage at busses C and I to those values which are attainable with the restricted condenser outputs. The voltages at these points are to be as near 100 per cent as possible, and the loads are to draw specified power and reactive kv-a. independent of the bus voltages.

CONNECTION IV

If the results of connection I show that line $G\ H$ carries more than 100,000 kv-a., a phase-shifting transformer is to be inserted in line $H\ I$ at I which will introduce a series voltage in this line at right angles to the line voltage, and thus change the power flow in this line.

Operating Condition IV-1. The terminal conditions are to be identical with those in I-1. The phase-shifting transformer voltage is to be adjusted to such a value that the load on line $G\ H$ is reduced to 100,000 kv-a.

This completes the numerical statement of the problem and the first step in its solution is the selection of a suitable Analyzer scale. This is usually done by inspection, noting the range of currents and impedances and then choosing a round-figure Analyzer base which will give convenient magnitudes. Since normal Analyzer voltage is 200 volts, it is necessary to consider only scales for current and impedance. An inspection of Tables VI, VII, and VIII shows that a current base of one ampere (and the corresponding 200-ohm impedance base) results in suitable impedance ranges, and magnitudes of current and power within the range of accurate measurement.

It may be noted here that with the constants of the lines given for one phase line-to-ground, power read on the Analyzer is phase power if the Analyzer voltage is considered as line-to-ground voltage, and total power if it is considered as line-to-line voltage. The latter method is slightly more convenient for the interpretation of results.

The second step in the solution is the connection and adjustment of the units to represent connection I. As the third step, the terminal conditions are adjusted to represent operating condition I-1, after which, as the fourth step, the results desired are read from the various instruments. The last step consists of converting the current, voltage, and power readings to fractions or per units of the Analyzer base, these fractions also applying directly to the system base.

These various steps have been carried out for the problem stated above, and the results obtained are tabulated in Tables IX and X, being arranged in the same order as the statement. A rather complete set of results is given for operating condition I-1, while for the others only those results are given which are of immediate use.

TABLE IX

RESULTS FOR OPERATING CONDITION *I* -1

All quantities are expressed as fractions of base quantities

a. Generating-Station Outputs (Power + *j* leading kv-a. as a fraction)

Station	Output	Station	Output
A	1.95 + *j* 0.07	*J*	2.42 − *j* 0.22
D	2.30 − *j* 0.32	*K*	− 2.22 − *j* 1.74
F	0.52 + *j* 0.09	*Q*	− 0.62 − *j* 0.47

b. Voltages at Unregulated Busses

G	*M*	*O*	*P*
1.00	0.893	0.990	1.000

c. Power, Reactive Kv-a., and Current in Branches

Power and reactive kv-a. are at point designated by the first letter, flowing in the direction from first to second letter. Leading kv-a. is positive. Architrave currents are given.

Branch	Power	Reactive kv-a.	Current	Branch	Power	Reactive kv-a.	Current
A B	1.95	+*j*0.07	1.95	*J I*	2.42	−*j*0.22	2.43
B C	1.48	+*j*0.09	1.48	*I L*	3.57	−*j*0.28	3.60
D C	1.99	−*j*0.12	1.99	*L K*	3.26	+*j*0.60	3.31
C I	1.92	+*j*0.14	1.92	*K M*	0.25	−*j*0.18	0.31
C G	0.88	+*j*0.13	0.89	*L N*	0.23	+*j*0.03	0.23
F G	0.52	+*j*0.09	0.53	*O N*	0.37	+*j*0.01	0.37
G H	1.40	+*j*0.13	1.41	*O P*	0.96	+*j*0.13	0.93
H I	0.03	+*j*0.10	0.10	*P Q*	0.63	+*j*0.45	0.78

d. Synchronous-Condenser Outputs (lagging kv-a.)

B	*C*	*H*	*I*	*N*
0.53	0.53	0.23	1.66	0.62

e. Angles of Bus Voltages

A	49.5°	*J*	42.4°
B	34.5	*K**	0.0
C	27.6	*L*	3.7
D	52.5	*M*	−9.0
F	25.8	*N*	0.2
G	24.3	*O*	4.8
H	17.5	*P*	−2.0
I	17.1	*Q*	−4.2

*Used as reference for angles.

Discussion of Results. A brief résumé of the results will be given, interpreting them in terms of the questions included in the statement of the problem. Considering connection I, it is seen that operating condition I-2 requires approximately equal synchronous condenser installations at *C* and *I*, this division of condensers being accepted as most suitable.

The synchronous condenser requirements at *C* and *I* for connection II can be plotted as a function of the transformer ratio from the tabulations and the best distribution fixed upon for this stage of the system development. Conditions in the other lines were changed inappreciably by altering the transformer ratio.

Removing line *C I* in connection III had no serious effects other than reducing the voltage at several busses. Increased synchronous condenser installations at *C* and *I* would have made possible the maintenance of normal voltage at *C*, *H*, and *I*.

As anticipated, line *G H* is overloaded in connection I (*i. e.*, carries in excess of 100,000 kv-a.), and in connection IV the magnitude of quadrature voltage necessary to remove the overload was found to be 0.17, as shown in Table X.

Since the example is merely illustrative no further analysis of the specific results will be given. Of more interest are the time requirements and accuracy. Table XI gives the approximate time spent on the various operations of solving this problem starting from

TABLE X

RESULTS FOR OPERATING CONDITIONS I-2, II-1, II-2, II-3, III-1, AND IV-1

Power and reactive kv-a. are at first point in the direction from first to second point. Leading kv-a. is positive. Architrave currents are given. All values are fractions of base quantities. The quantities in the rectangles are arranged as follows: upper left, power; upper right, reactive kv-a.; lower, current.

Operating condition	Condenser outputs					Lines				
	B	*C*	*H*	*I*	*N*	*C G*	*C I*	*G H*	*H I*	*I L*
I-2	0.53	1.06	0.23	1.09	0.64	0.89 +*j*0.12 0.90	1.90 −*j*0.40 1.94	1.41 +*j*0.12 1.42	0.041 +*j*0.08 0.09	3.56 −*j*0.26 3.58
II-1	0.49	0.23	0.08	1.03	0.64	0.325 +*j*0.066 0.33	0.53 +*j*0.09 0.54	0.85 +*j*0.095 0.85	−0.311 +*j*0.053 0.316	1.86 −*j*0.05 1.86
II-2	0.49	0.64	0.08	0.60	0.64	"	"	"	"	"
II-3	0.49	1.06	0.08	0.15	0.64	"	"	"	"	"
III-1*	0.53	1.09	0.25	1.10	0.65	2.80 −*j*0.36 2.85	0 +*j*0 0	3.29 −*j*0.54 3.43	1.52 +*j*0.19 1.65	3.15 +*j*0.28 3.34
IV-1†	0.53	1.09	0.15	1.09	0.64	0.49 +*j*0.13 0.51	2.31 −*j*0.48 2.36	1.00 +*j*0.10 1.00	−0.46 +*j*0.09 0.47	3.47 −*j*0.22 3.48

*Voltages at:	*C*	*H*	*I*	*P*	*N*	*M*
	0.99	0.931	0.950	0.990	0.968	0.871

†Phase-shifting transformer voltage is 0.17 advancing terminal voltage of line *I H* at *I* with respect to bus I.

the data as given. It should be noted that in the time stated a problem on an unfamiliar network has been solved and the results checked as outlined below to eliminate errors.

Although a direct determination of the over-all accuracy by comparison with precise computed solutions is impracticable, the possible errors introduced in various steps of the procedure can be approximately evaluated. The methods used to detect mistakes and inaccuracies and to determine the limits of normal error will be briefly considered, together with the range of these errors.

Several checks on the final results are conveniently made, among which are the summing of power and reactive kv-a. at busses, a comparison of the impedance of a branch with the quotient of the voltage drop by the current, and the closure of the line-drop vector diagrams for loops. The application of these checks to this example and to other problems show that the measured power and reactive kv-a. inflow and outflow at a bus differ by not more than two per cent and usually by less than one per cent. The impedance of network branches as measured by voltmeter and ammeter is usually within one per cent of the given value even though no correction for current is made when setting the reactors. Loop line-drop vector diagrams ordinarily close within one per cent of the line voltage.

Whereas the foregoing checks make apparent any appreciable errors in measurements or settings of units, they do not demonstrate the accuracy of the solution with respect to a perfect solution. That is, it is possible for small errors which may compensate in these checks, and errors due to small deviations of the terminal settings from stated values to appear combined in particular individual results. Thus it is difficult to determine what the maximum departure of a given experimentally measured quantity is from its true value because of the labor involved in computing a check to the necessary precision. However, the following statements can be made: Line voltages are relatively precise, and are ordinarily well within one per cent. Power and reactive kv-a., on the other hand, may differ from the true values by as much as 5 per cent when the quantities themselves are small. Currents are probably correct to 2 per cent or less. These statements apply to results as ordinarily taken. With additional precautions, which have been used in some problems, better accuracy by a factor of two or three can be obtained at the expense of considerable time, but the data seldom justify this expenditure.

TABLE XI

TIME REQUIRED FOR SOLUTION OF THE ACCOMPANYING EXAMPLE

1. Preparatory Work	
Selection of scale, conversion of constants to Analyzer scale, preparation of plugging diagrams with unit settings, preparation of data sheets	
Total (one man)	15 hr.
2. Use of Analyzer	
Plugging connections, setting units, and checking both	2
Setting first operating condition	2
Taking data first operating condition	4
Setting remaining operating conditions	5
Taking remaining results	4½
Total (2 men and Analyzer)	17½ hr.
3. Conversion of results to nominal base and checking results	
Total (one man)	10½ hr.

Summary and Conclusions

1. An improved a-c. network computing device, called the Network Analyzer, is described in this paper, its purpose being to determine by direct measurements in the laboratory the voltage, current, power, reactive power, and phase angle at any point of a network. Its scope of representation in its present form is 8 generating stations, 60 lines and cables with their capacitance, 40 loads, 4 ratio-changing transformers, and the usual number of synchronous condensers.

2. The Analyzer uses phase-shifting transformers rather than rotating machines to represent synchronous machines and generating stations. Its operation is thus greatly simplified and speeded up in comparison with previous miniature systems.

3. A voltage scale of 200 volts at 60 cycles is used, the nominal current scale being 0.5 ampere.

4. Portable indicating instruments are used throughout, including thermocouple voltmeters and ammeters and suspended-armature wattmeters, by means of which current, voltage, and power are read directly at any point of the system without materially disturbing current distribution and voltage drops in the network and without necessitating corrections or adjustments for the presence of the instruments. The accuracy of the instruments is one per cent of full scale.

5. The accuracy with which a given set of circuit constants can be reproduced is approximately one per cent. The accuracy with which the instrument readings represent the quantities which are uniquely determined by a given set of circuit constants and terminal conditions is one per cent or better in the case of voltages, approximately 2 per cent in the case of currents, and 3 per cent—with a maximum of 5 per cent for small magnitudes—in the case of power and reactive kv-a. This accuracy has been attained by providing (a) steps as low as one per cent or smaller for the adjustment of the impedance units and in the settings of the phase-shifting transformers, (b) a value of effective resistance in the reactors of the order of 4 per cent of the reactance, (c) a design of magnetic circuit in the reactors such as to give not over one per cent change of inductance due to change of current throughout the entire operating range.

6. The Network Analyzer is intended to be used:

(a) For purposes of teaching and research by students and research staff of the Massachusetts Institute of Technology, and

(b) For commercial engineering service in the solution of network problems for engineers in operating and designing work.

In its design major consideration was given to the requirements for the use by students. As a tool for engineers, it is adapted to the solution of system problems arising in normal operation, growth, and interconnection; to the point-by-point computation of stability problems including the effects of symmetrical or unsymmetrical short circuits and sudden circuit changes; to the determination of steady-state load limits and to short-circuit studies in the relatively few cases where the d-c. calculating table will not yield adequate results.

7. The ability of the Analyzer to accomplish the purpose for which it was designed has been demonstrated by the solution of actual network problems which it has been called upon to solve. Results are obtained not only with relative ease, but with a dispatch that appeals to one who has experienced the tediousness, the delay, and in some cases the hopelessness of arithmetical solutions.

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Discussion

R. C. Bergvall: I was very much interested in hearing the paper on the *M. I. T. Network Analyzer* because an a-c. calculating board was recently put into service at our East Pittsburgh works. It has been used on typical problems covering load distribution, voltage regulation, phase angle control, and stability with an over-all accuracy of from 1 to 3 per cent which checks very closely with the degree of accuracy mentioned by Mr. Hazen.

Our board differs from the M. I. T. network analyzer in that a central metering table is used and the frequency is 480 cycles. This frequency was chosen because it is the highest that could be used with dynamometer type meters and the high frequency permitted smaller reactors and condensers to be used. The board had 10 generating sources and about 240 circuits equipped with suitable units of resistance, reactance, and capacity. The voltage and phase angle control for each source is obtained from two phase shifters in series. I will not take up more time describing the board because this was covered by an article in the May 1930 issue of the *Electric Journal* by Mr. H. A. Travers and Mr. W. W. Parker.

The board was used in solving a complicated stability problem with a considerable saving in time. The generating sources, transmission and distribution network, load impedances, etc., were set up with the proper phase relations between the generating sources to obtain load and wattless current distribution corresponding to normal operation. A short circuit was then applied and the power outputs from each of the generating sources measured. The difference between the normal output and the output under short circuit conditions represents the energy available for accelerating or retarding the rotor. The angular position of each rotor after a short period of time can be calculated assuming that the output remains constant. The phase shifters are then set to the new angular position and new power output measurements obtained so that oscillations of the entire system of five or six elements can be plotted. The board gives the voltage at various points on the system and makes it possible to change the impedance of the induction motor loads to correspond with the slowing down of the motors as a function of low voltage and time. The increasing complexity of power systems will undoubtedly extend the usefulness of a-c. calculating boards in the future.

I. H. Summers: One of the fields of application of the network analyzer is that of large power system stability analysis. It is particularly valuable in transient stability analysis on a large power system where the behavior of the number of stations is to be investigated. The reasons for this may be brought out by a comparison of the methods which might be used for the analytical calculation and those which might be used on the analyzer. Analytical methods are given in *The Calculation of Alternator Swing Curves*, by F. R. Longley (see p. 1129). Equations 8 to 28 of this paper and the procedure outlined in items 1 to 8 at the end of the main body of the paper give the essence of this method.

While this method may be considerably simplified in many cases as has been brought out before in the literature, there seem to be some cases when the full analysis as given by Longley, or some equivalent, is considered necessary. An inspection of Equations 28 shows that each step of the calculation as given, requires the solution of a determinant of the order of the number of machines being included in the calculation or an equivalent amount of work, in order to find new voltages "behind quadrature reactance" in the machines after an angular swing has taken place. While this process is perfectly possible and has been to my knowledge carried out in many cases in regular work on system stability analysis, it does require the expenditure of considerable time and labor if many machines are being included in the calculation.

It would seem that this step of the procedure could be considerably shortened by the use of the network analyzer. In addition, the analyzer should shorten the work of finding the torques acting on the rotors of machines. It should also facilitate the calculation of driving point and transfer impedances in the system connections between machines.

F. O. Urban: This paper states that one of the fields of application of the network analyzer is that of system stability analysis. The analyzer affords a very attractive means for making the steady-state solutions of the network necessary at each point in a step-by-step transient study, and I have been working on this application as a Master's thesis problem at the Massachusetts Institute of Technology. Both graphical and analytical methods for a point-by-point study of machine swings have been well outlined in the literature. The method where the network analyzer is used as an adjunct is but slightly modified, although many tedious calculations are eliminated.

Briefly, the method is as follows. The network is set up on the analyzer. Certain terminal conditions of synchronous machine power and reactive kv-a. output, and system voltages are given. A steady-state solution is made and the power, excitation, and phase angle of each machine are determined. The disturbance to be studied is then reproduced on the analyzer. That is, if a generator is suddenly dropped off, the corresponding phase shifter is disconnected; if a short circuit is to be studied, the proper line-to-neutral impedance is connected at the point of fault to correctly reproduce the positive phase-sequence conditions in the network with fault on and so on for whatever disturbance is to be studied. The angle of each phase shifter is left unchanged and the magnitude of its voltage is adjusted to correspond to holding machine flux linkages constant. The electrical power of each machine is then determined by means of wattmeters connected to the phase-shifter terminals. This is the power the first instant after the disturbance, and the difference between it and the power obtaining in the steady-state solution is the differential power to accelerate or retard the rotor of the corresponding machine. Knowing the inertia of the machine rotor, the change in time-phase of the excitation voltage vector for some small time interval is calculated on the assumption that the differential power (difference between shaft and electrical power) is constant over the interval. The amount by which the field linkages change during the first interval due to decay of induced field current, exciter, and voltage regulator action is also calculated. Knowing the new linkages and phase angles of each machine at the beginning of the second interval, the corresponding phase-shifters are properly adjusted in magnitude and angle. The electrical power on each machine following these changes is determined directly by the wattmeters and the differential power effective over the second interval of time obtained as for the first interval. The process is continued for as long a time as desired. The effects of governor characteristics, if known as functions of some quantity such as speed or slip, can easily be brought into the problem at the proper point in the same manner as in a strictly analytical solution. Damping is handled similarly. Static loads, if their characteristics are known as functions of positive-sequence voltage, can also be properly handled.

In order to represent synchronous machine characteristics by means of the phase shifters the Blondel two-reaction theory as extended by Doherty and Nickle and by Park and Robertson was adopted. Saturation effects are neglected. On the basis of the theory developed by these writers, the vector diagram of a synchronous machine having no excitation in the quadrature axis and having equal transient and synchronous reactances in the quadrature axis is shown. More complicated cases can also be handled, but this one was chosen for purposes of illustration (Fig. 1).

The excitation voltage, corresponding to the field current, is e_d; x_d and x_q are direct and quadrature synchronous reactances respectively; x_d' is transient reactance in the direct axis; and e_d' is a fictitious voltage which corresponds to the total field linkages.

By inspection of the diagram it was decided that $O\,Q$ (following the notation of Park and Bancker) could most advantageously be represented by the phase-shifter. If the phase-shifter is connected to the network through a reactance x_q, $O\,Q$ will be correctly represented by the terminal voltage of the phase shifter. The magnitude of $O\,Q$ for the steady state is determined by a solution on the analyzer with the given terminal conditions. The excitation is then $O\,Q + i_d\,(x_d - x_q)$. The current i_d can be easily measured, if a spare phase shifter is available, by connecting the current coil of a wattmeter to the terminal of the phase-shifter representing $O\,Q$ and by exciting its potential coil with a unit voltage at right angles to $O\,Q$. The voltage e_d' is $O\,Q - i_d\,(x_q - x_d')$, and this, representing total field linkages, remains constant for the first instant after the disturbance. Following the adjustment of the analyzer for the steady-state conditions, the e_d' for each machine is calculated. The disturbance is then produced in the network and the $O\,Q$'s held constant in phase and adjusted in magnitude until the condition $O\,Q = e_d' + i_d\,(x_q - x_d')$ is satisfied for each machine. This can be arrived at by a series of successive approximations performed on the analyzer. The electrical power of each machine for the first instant is then determined by means of a wattmeter. This makes possible the determination of differential torque and hence the angular swing of each machine during the first time interval. The value of e_d' at the beginning of the second interval is the value at the beginning of the first interval plus the change due to field transients and exciter and regulator action, and must be determined analytically. The vector $O\,Q$, representing rotor angle, is shifted in phase in accord with the calculated swing and adjusted in magnitude so that the condition $O\,Q = (e_d'$ at beginning of second interval$) + i_d\,(x_q - x_d')$ is satisfied for each machine. The process is repeated for the successive time intervals.

At present it is not possible for me to make any comparison between the amount of time required by the analyzer in a problem of this nature and that required for a purely analytical solution where the same factors are taken into consideration. Information in this regard will, however, soon be available.

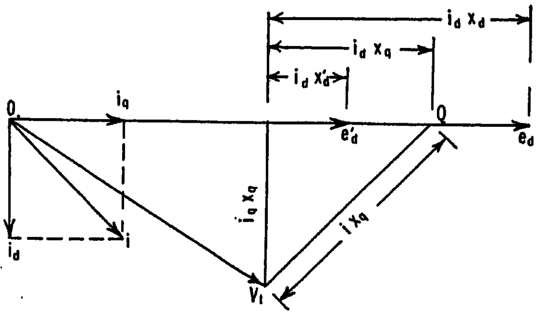

FIG. 1

C. F. Scott: The thing which has impressed me most with regard to the elaborate network analyzer has not been brought out in the general discussion although it may be inferred from the photographic illustration. That feature is the large size. A space equivalent to a good sized room is surrounded by devices that look like filing boxes of some sort. It is really a very big thing and the thing itself is much more impressive than the description.

I had opportunity a few weeks ago to see a circuit device of a comparatively very simple sort made by Mr. H. S. Baker of the Ontario Power Company at Niagara Falls. The units of the network here are not adjustable but correspond to the actual constants of the various elements of the circuit. Hence, when a new line is to be added to the system it can be added to the model and determinations made as to the characteristics of the proposed system. This device was said to be particularly useful in determining the effects which would be produced by short circuits at particular points.

O. R. Schurig: The discussions have clearly indicated the value of the network analyzer for solving in a relatively short time and with good accuracy a variety of network problems, some of which would require an enormous amount of work if solved by mathematics. Thus transmission engineers can now analyze their systems in regard to stability, load distribution, and regulation problems as they have never done before and will thereby accomplish improved service and better economy in the use of equipment.

Calculation of Protection of a Transmission Line by Ground Conductors

BY HERBERT BRISTOL DWIGHT[1]
Fellow, A. I. E. E.

Synopsis.—This paper discusses the well-known calculation of the degree of protection afforded to a transmission line by ground conductors, on the assumption of the sudden disappearance of a vertical potential gradient caused by a charged cloud. Formulas are given for the protective ratio due to one and two ground conductors. It is shown that for any number of ground conductors, it is not necessary to compute the charges on the power conductors.

* * * * *

In calculating the degree of protection afforded to a transmission line by its ground conductors, a calculation which is frequently used by electrical engineers is that based on the assumed sudden disappearance of a large potential gradient from the earth upward. Such a calculation was described by R. P. Jackson[2] in 1907, and by E. E. F. Creighton[3] in 1916.

In this paper are given some formulas for particular cases, and some methods for shortening the work of calculation.

The calculation of protective factor is based on the assumption that a vertical potential gradient is induced by a charged cloud, and that this causes charges to collect on all the conductors; then the gradient is suddenly removed when the cloud discharges. The charges on the ground conductors go to earth very quickly to a sufficient degree to bring the potential of the ground conductors to zero. The charges on the insulated conductors remain long enough to raise the potential of those conductors, and if the potential is high enough, to cause a flashover of an insulator. The protective factor is the ratio of the potential of the power conductor when protected by ground conductors, to its potential if it were not so protected.

In the calculation described by Mr. Creighton, the values of the charges on all the conductors in the system were found. It will be shown that by finding charges on the ground conductors when the power conductors are considered absent it is possible to calculate the protective factor according to the standard theory. This reduces the number of simultaneous equations to be solved and shortens the work considerably.

Let there be ground conductors $G_1\, G_2 \ldots \ldots$ and insulated power conductors $A.\ B.\ C \ldots \ldots$ all parallel to each other.

If there is a voltage gradient, each conductor takes up a charge so that when a unit charge is carried from the image of a given conductor to that conductor, the work done by all the charges is equal to the voltage gradient multiplied by twice the height of the given conductor. The power voltage is neglected. If the gradient is suddenly removed and none of the charges is changed, the voltage on each conductor is equal to the gradient times its height.

Now the ground conductors are presumed to have their charges reduced or changed very quickly, so that each ground conductor has zero potential, before any appreciable change takes place in the charges on the insulated power conductors.

The reduction in the work done by the charges in carrying a unit charge from the image of a given ground conductor to that conductor is dependent upon the reduction in charge of the ground conductors. It is not dependent upon the presence or size of the charges on the power conductors, since, according to the assumption made, those charges do not change. The reduction in the charge on each ground conductor is therefore not dependent upon the size or position of the power conductors, and may be calculated by assuming the power conductors to be absent.

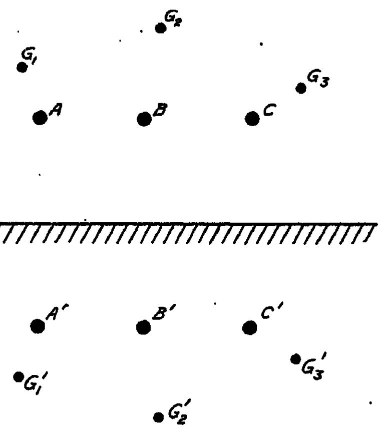

FIG. 1—GROUND CONDUCTORS AND POWER CONDUCTORS AND THEIR IMAGES

The reduction in voltage on a given power conductor such as A is equal to one-half the reduction in the work done on a unit charge carried from the image of A to A. This is dependent upon the reduction in the charges on the ground conductors, and this has been shown to be unaffected by the presence of the power conductors. Since the charge on A does not change, the reduction in voltage on A does not depend upon the shape or diameter of A, but merely on the position of A. The protective ratio of A is obtained directly from the reduction of voltage on A, described above.

It is possible, therefore, to calculate the protective

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[1] Massachusetts Institute of Technology, Cambridge, Mass., and New England Power Construction Company.

[2] R. P. Jackson, A. I. E. E. TRANS., 1907, p. 873.

[3] *Theory of Parallel Grounded Wires*, by E. E. F. Creighton, A. I. E. E. TRANS., 1916, p. 845.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

ratio for one power conductor near a group of ground conductors by assuming that the other power conductors do not exist. It is also possible to plot curves showing the protective ratio around a group of ground conductors, and these curves will be the same for power conductors of any diameter. (See Figs. 2, 3, 4, and 5.)

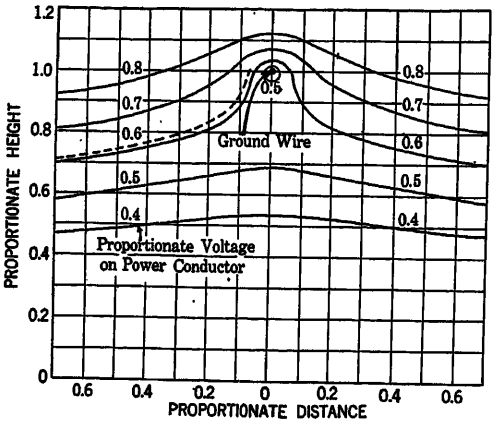

Fig. 2—Single Ground Wire

Full lines, ground wire ½ in. diameter; 80 ft. average height
Dotted line, ground wire ⅜ in. diameter; 35 ft. average height

In overhead work, when measuring the distances to the other conductors. the diameter of a conductor is negligible compared with the distance to other conductors. Similar sets of curves obtained by this same general method were given by R. P. Jackson. (Ref. 2.)

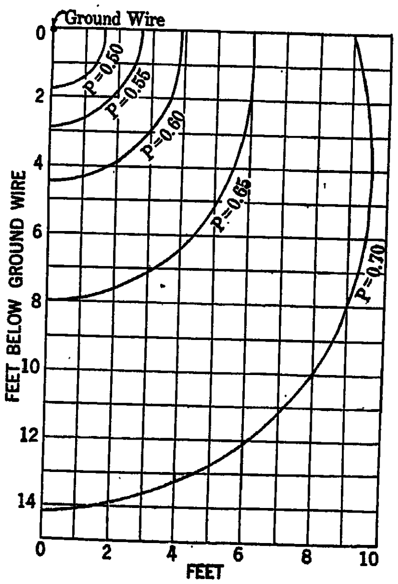

Fig. 3—Protective Factor, P, due to One 7/16-Inch Ground Wire; 65 Ft. Average Height.

Typical of a steel pole line

The above paragraphs show that in computing the protective ratio for the power conductor A, it is not necessary to find the value of the charge on A. This makes the number of simultaneous equations to be solved equal to the number of ground conductors. In most cases, the arrangement of the ground conductors is symmetrical when all the power conductors are considered absent, and some pairs of charges can be put equal to each other. The number of simultaneous equations to be solved is then equal to approximately one-half the number of ground conductors. The one main calculation will give the protective ratio for all the power conductors. Thus, the calculation becomes as follows:

Find the charges on the ground conductors due to the voltage gradient, all the power conductors being considered absent. Then carry a unit charge from the image of any of the power conductors to that conductor. This gives the reduction in voltage on that conductor due to suddenly grounding the ground conductors, assuming them to be first insulated and holding the charges left by the gradient. By subtracting this reduction from the original voltage at the power conductor A caused by the gradient, one finds the final voltage on A, and hence the protective ratio for A.

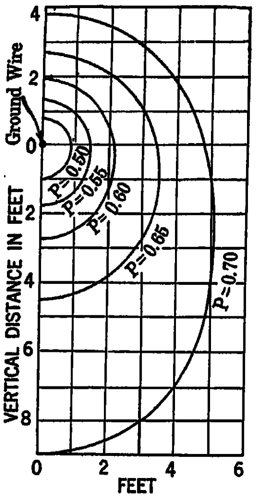

Fig. 4—Protective Factor, P, due to One ⅜-In. Ground Wire, 28 Ft. Average Height.

Typical of a wood pole line

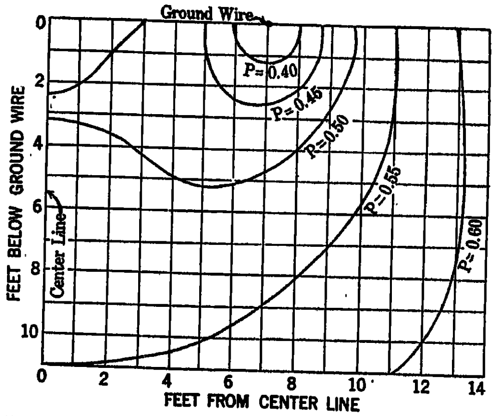

Fig. 5—Protective Factor, P, due to Two 7/16-In. Ground Wires, 14-Ft. Spacing and 40 Ft. Average Height.

Typical of H-frame transmission lines

This shorter method of calculation has been checked for some practical numerical examples by solving them by both the shorter and the longer methods, the same result being obtained by both methods.

It has been found by Mr. H. H. Spencer of the New

England Power Construction Company, that considerable time and effort in performing the calculation and checking it are saved in most cases by expressing the result of the simultaneous equations by determinants which are symmetrical about a diagonal, and then employing the Doolittle Method of evaluating the determinants.

The mirror symmetry which this method of solution of ground wire problems involves leads to a simplification in the evaluation of the charges on the ground conductors through the so'ution of the simultaneous equations.

Regardless of the arrangement of the ground conductors, the simultaneous equations for their charges can be so arranged that the determinant of the equations will be symmetrical about the principal diagonal; that is, the equations can always be so arranged that they will be of the form:

$$\begin{aligned}
a_1 Q_1 + b_1 Q_2 + c_1 Q_3 + d_1 Q_4 + \ldots + r_1 Q_n - K_1 &= 0 \\
b_1 Q_1 + b_2 Q_2 + c_2 Q_3 + d_2 Q_4 + \ldots + r_2 Q_n - K_2 &= 0 \\
c_1 Q_1 + c_2 Q_2 + c_3 Q_3 + d_3 Q_4 + \ldots + r_3 Q_n - K_3 &= 0 \\
d_1 Q_1 + d_2 Q_2 + d_3 Q_3 + d_4 Q_4 + \ldots + r_4 Q_n - K_4 &= 0 \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \\
r_1 Q_1 + r_2 Q_2 + r_3 Q_3 + R_4 Q_4 + \ldots + r_n Q_n - K_n &= 0
\end{aligned}$$

Equations of this form are much used in statistical work and a form of solution for them, well known by statistical men but not much employed by engineers, is the Doolittle Method.[4] This method consists of so tabulating and arranging the work that it becomes self-checking and at each step of the operation, columns are footed and lines extended so that the sums obtained give a numerical check on all the work done up to that point. This removes to a large measure the tediousness of solving large numbers of simultaneous equations, since the computer is relieved of the dread that an error may have been made, necessitating the repetition of a large volume of labor. In a ground wire problem involving a large number of ground conductors, 13 simultaneous equations were solved by this method in rather less than one-half the time required for solution and checking by the usual determinantal methods.

For one or two ground conductors, formulas for the protective ratio are advantageous, but for more ground conductors it is probably better to use numerical coefficients and solve the simultaneous equations.

The protective ratio with one ground conductor is

$$1 - \frac{h_G}{h_A} \frac{b}{a} \qquad (1)$$

where

$$a = \log_{10} \frac{G\,G'}{r_G}$$

$$b = \log_{10} \frac{A'\,G}{A\,G}.$$

h_G = height of ground conductor above the earth,
h_A = height of power conductor A above the earth,
r_G = radius of ground conductor,
$G\,G'$ = distance of ground conductor to its image, and so on.

Equation (**1**) corresponds to Equation (**28a**) of Reference 3, which contains an error.

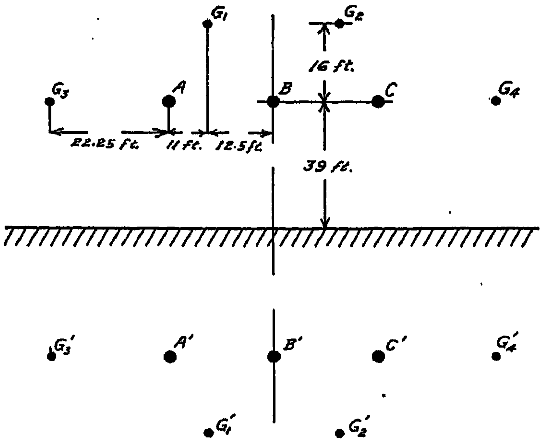

FIG. 6—TRANSMISSION LINE WITH FOUR GROUND CONDUCTORS

The protective ratio of power conductor A with two ground conductors G_1 and G_2 is

$$1 - \frac{(b\,e - c\,d)\,h_{G2} + (c\,e - b\,f)\,h_{G1}}{(e^2 - d\,f)\,h_A} \qquad (2)$$

where

$$b = \log_{10} \frac{A'\,G_1}{A\,G_1} \qquad e = \log_{10} \frac{G_1'\,G_2}{G_1\,G_2}$$

$$c = \log_{10} \frac{A'\,G_2}{A\,G_2} \qquad f = \log_{10} \frac{G_2'\,G_2}{r_{G2}}$$

$$d = \log_{10} \frac{G'_1\,G_1}{r_{G1}}$$

If $h_{G1} = h_{G2}$ and $r_{G1} = r_{G2}$

$$\text{Protective ratio} = 1 - \frac{h_G}{h_A} \frac{(b + c)}{(d + e)} \qquad (3)$$

Example—Four ground conductors (see Fig. 6).

$$r_{G1} = r_{G2} = 0.02083 \text{ ft.}$$
$$r_{G3} = r_{G4} = 0.015 \text{ ft.}$$

First assume that the power conductors are absent.

By symmetry, $Q_{G2} = Q_{G1}$ and $Q_{G4} = Q_{G3}$, these being charges per cm. of conductor.

Carry a unit charge from G_1' to G_1.

The work done against any charge such as Q_{G3} and its image is

$$4\,Q_{G3} \operatorname{logn} \frac{\text{old distance}}{\text{new distance}}$$

where "old distance" is the distance of the unit charge from the center of G_3 before the motion, and "new distance" is the distance from the center of G_3 after the motion takes place, and where logn denotes natural logarithm. Thus,

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[4] Frederick C. Mills' "Statistical Methods," Henry Holt and Company, 1924; pp. 577-581.

$$2\,g\,M \times 55 = 4\,Q_{G1}\log_{10}\frac{G_1'\,G_1}{r_{G1}} + 4\,Q_{G1}\log_{10}\frac{G_1'\,G_2}{G_1\,G_2}$$

$$+ 4\,Q_{G3}\log_{10}\frac{G_1'\,G_3}{G_1\,G_3} + 4\,Q_{G3}\log_{10}\frac{G_1'\,G_4}{G_1\,G_4}$$

where g = the gradient induced by the cloud, in statvolts per foot and $M = \log_{10} e = 0.4343$.

$$27.5\,g\,M = 4.377\,Q_{G1} + 0.694\,Q_{G3} \qquad (4)$$

Carry a unit charge from G_3' to G_3

$$2\,g\,M \times 39 = 4\,Q_{G3}\log_{10}\frac{G_3'\,G_3}{r_{G3}} + 4\,Q_{G1}\log_{10}\frac{G_3'\,G_1}{G_3\,G_1}$$

$$+ 4\,Q_{G1}\log_{10}\frac{G_3'\,G_2}{G_3\,G_2} + 4\,Q_{G3}\log_{10}\frac{G_3'\,G_4}{G_3\,G_4}$$

$$19.5\,g\,M = 0.694\,Q_{G1} + 3.835\,Q_{G3} \qquad (5)$$

Solving the simultaneous Equations (4) and (5)

$$Q_{G1} = 5.638\,g\,M$$

and

$$Q_{G3} = 4.064\,g\,M$$

To find the protective ratio of power conductor A, carry a unit charge from A' to A against the four charges on the ground conductors and their images which have just been derived, and let the voltage so found be $\overline{V}_{AA}'$.

$$\overline{V}_{AA}'\,M = 4\,Q_{G1}\log_{10}\frac{A'\,G_1}{A\,G_1} + 4\,Q_{G1}\log_{10}\frac{A'\,G_2}{A\,G_2}$$

$$+ 4\,Q_{G3}\log_{10}\frac{A'\,G_3}{A\,G_3} + 4\,Q_{G3}\log_{10}\frac{A'\,G_4}{A\,G_4}$$

$$\overline{V}_{AA}'\,M = 4 \times 9.18\,g\,M$$

$$\frac{1}{2}\,\overline{V}_{AA}' = 18.36\,g = \text{reduction in voltage on } A$$

$$\begin{array}{rl} 39\ g & = \text{voltage on } A \text{ left by gradient} \\ \hline 20.64\ g & = \text{final voltage on } A \end{array}$$

$$\text{Protective ratio of } A = \frac{20.64}{39} = 0.529$$

It is desired to make acknowledgment to Mr. L. O. Waite of the Stone and Webster Engineering Corporation for a number of contributions to this paper, among them Formulas (1) and (3) and Figs. 3, 4, and 5.

Discussion

L. V. Bewley: Ground wires are of interest to the transmission line engineer from at least seven different standpoints:

(1) Protection against induced potentials,
(2) Protection against direct strokes,
(3) Protection against incoming high-voltage surges,
(4) Effect on telephone interference,
(5) Effect on zero phase sequence reactance in connection with stability calculations,
(6) Effect on corona loss,
(7) Effect on arcing grounds.

Of these, Dr. Dwight's paper is concerned with the first, protection against induced potentials, and in common with several previous investigations along this line is based on the simplifying assumptions that:

(a) The cloud discharge is instantaneous, so that the conditions are inherently static.
(b) The ground wires are ideal, that is, perfectly grounded throughout their length.

Actually, the cloud discharge is not instantaneous and the ground wires are not ideal, but are grounded at intervals through tower resistances, so that a complete analysis of the situation involves:

(1) The law of cloud discharge,
(2) The distribution of bound charge,
(3) The presence of traveling waves, during both their formation and developed stages, on both line and ground conductors,
(4) Residual charges, up until the completion of cloud discharge,
(5) Length of the spans,
(6) Resistances of the tower footings,
(7) Successive reflections at the towers,
(8) Corona,
(9) Number of ground wires,
(10) Number of line wires.

I have recently submitted to the Meetings and Papers Committee of the A. I. E. E. the manuscript of a paper covering these points, in which it is proved mathematically that as far as ideal ground wires are concerned, the protective ratio is absolutely independent of the law of cloud discharge, distribution of bound charge, and presence of traveling waves, and depends only on the size and position of the ground wires and the position of the particular line conductor under consideration. The general system of equations involving all of these factors finally reduced, after considerable manipulation, to the static equilibrium equations of the conventional method of solution. Incidentally, as Dr. Dwight reasoned, the protective ratio of any line conductor is independent of the presence of the other line conductors,—a fact which is also true in the case of traveling waves. In this same paper there is given a general method of solution applicable to ground wires which are periodically grounded. To handle this much more difficult case it was necessary to develop the theory of traveling waves on any number of mutually coupled wires, and to devise a means for taking into account the multiplicity of reflections which occur on the spans between towers. The most important conclusions of this analysis were:

(1) The voltage crests of the traveling waves on the line wires, a span or two from the zone of bound charge distribution, are practically reduced by the same amount as is computed in the case of ideal ground wires.

(2) Within the zone of bound charge distribution, the potentials on both line and ground conductors may be of the order of four times as great as that of the traveling wave crests.

In other words, a calculation of protective ratio based on ideal ground wires is all right as far as the protection of station apparatus is concerned, but is not a correct criterion of what is happening at the center of disturbance.

The equation for the protective ratio in the case of m ideal ground wires numbered from No. l to No. m inclusive is

$$P_l\,R_l = \frac{1}{h_o}\left|\frac{\begin{matrix} p_{ll} & \ldots\ldots & p_{ml} & h_l \\ \ldots & \ldots & \ldots & \ldots \\ p_{lm} & \ldots\ldots & p_{mm} & h_m \\ p_{lo} & \ldots\ldots & p_{mo} & h_o \end{matrix}}{\begin{matrix} p_{ll} & \ldots\ldots & p_{ml} \\ \ldots & \ldots & \ldots \\ p_{lm} & \ldots\ldots & p_{mm} \end{matrix}}\right|$$

where the p's are Maxwell's potential coefficients, and the h's are the heights of the conductors. Subscripts (l) to (m) refer to the ground wires, and subscript (o) to the line wire under consideration. Note that the other line conductors do not enter. Since in most practical cases only a few ground wires will be used, it is only necessary to solve a relatively simple determinate. In the case of a great number of ground wires (Dr. Dwight mentions a case of 13 ground wires), it will undoubtedly be worth while to acquaint oneself first with the Doolittle method of evaluation.

J. E. Clem: This paper is very interesting and encouraging to me because it apparently gives a rigorous proof of certain of the principles that the writer worked out by simple algebra a few years ago and has used since that time. These are: (a) the voltage induced by a cloud field on any insulated wire is independent of the presence of all other insulated wires in the field; (b) the reduction in the induced voltage of any insulated wire occasioned by a ground wire is not affected by the presence of any other insulated wires; (c) the reduction in the induced voltage of any insulated wire by any one of a number of ground wires numerically equal to the voltage which would be induced on the insulated wire in question by a charge on the ground wire equivalent to the charge which would appear on the ground wire if the line wires were all absent. This pseudo charge on the ground wire is numerically equal to the charge on the ground wire before cloud discharge minus the charge on the ground wire after cloud discharge. These principles are useful in estimating the protective value of ground wires.

Contrary to the statement in the paper the application of (c) of the preceding paragraph does not reduce the work required for the calculation of all because, according to (b), only one line wire need be considered at a time and it is not necessary at all to calculate the charges on the ground wires or line wires. Working along these lines the writer has developed independently the following general formula for the protection afforded by any number of ground wires.

$$R_n = \frac{\overset{n+1}{D_{n+1}}}{h_{n+1}\, D_n} \qquad (1)$$

R_n = protective ratio which is defined as the ratio of the voltage induced on an insulated wire in the presence of n ground wires to the voltage which would be induced if the ground wires were not there.

h_{n+1} = height of line wire

D = determinant defined as follows

$$D_n = \begin{array}{cccccc} A_1 & B_{12} & B_{13} & \cdots & \cdots & B_{1n} \\ B_{21} & A_2 & B_{23} & & & \vert \\ B_{31} & B_{32} & A_3 & & & \vert \\ \vert & & & & & \vert \\ B_{1n} & \cdots & \cdots & \cdots & \cdots & A_n \end{array} \qquad (2)$$

$$\overset{n+1}{D_{n+1}} = \begin{array}{ccccc} A_1 & B_{12} & B_{13} \cdots & B_{1n} & h_1 \\ B_{21} & A_2 & & \vert & h_2 \\ B_{31} & & & \vert & \vert \\ \vert & & & \vert & \vert \\ B_{1n} & \cdots & \cdots & A_n & h_n \\ B_{1.n}+1 & \cdots & \cdots & \cdots & h_n+1 \end{array} \qquad (3)$$

A = Coefficient of self capacitive inductance.

$$= \frac{1}{0.03883} \operatorname{Log} \frac{2\,h}{r}$$

B = Coefficient of mutual capacitive induction (4)

$$= \frac{1}{0.03883} \operatorname{Log} \frac{R}{S} \qquad (5)$$

h_1, h_2 etc. = height of ground wire in question

r = radius of wire

R = distance from line wire to image of ground wire

S = distance from line wire to ground wire.

This is a general formula applying to any number or arrangement of ground wires. If there were one ground wire $n = 1$

$$D_n = A_1 \qquad D_2 = \begin{array}{cc} A_1 & h_1 \\ B_{21} & h_2 \end{array} \qquad (6)$$

$$R_1 = \frac{\begin{array}{cc} A_1 & h_1 \\ B_{21} & h_2 \end{array}}{h_2 \quad A_1} = 1 - \frac{h_1}{n_2}\,\frac{B_{12}}{A_1} \qquad (7)$$

If there were two ground wires $n = 2$, or if there were three ground wires $n = 3$, and then

$$R_2 = 1 - \frac{h_1}{h_3}\,\frac{D_2'}{D_2} - \frac{h_2}{h_3}\,\frac{D_2''}{D_2} \qquad (8)$$

$$R_3 = 1 - \frac{h_1}{h_4}\,\frac{D_3'}{D_3} - \frac{h_2}{h_4}\,\frac{D_3''}{D_3} - \frac{h_3}{h_4}\,\frac{D_3'''}{D_3} \qquad (9)$$

$$R_n = 1 - \frac{h_1}{h_{n+1}}\,\frac{D_n'}{D_n} - \frac{h_2}{h_{n+1}}\,\frac{D_n''}{D_n} \ldots \text{etc.} \qquad (10)$$

The determinant D_n is defined as previously noted and the "prime" determinants are obtained from D_n by writing $B_{1.n+1}$, $B_{2.n+1}$ etc. in the proper column as indicated by the prime. Thus

$$D_2' = \begin{array}{cc} B_{13} & A_{12} \\ B_{23} & A_2 \end{array} \qquad D_2'' = \begin{array}{cc} A_1 & B_{13} \\ B_{21} & B_{23} \end{array} \qquad (11)$$

From these expressions it is clear that the analytical determination of protective ratio is merely a matter of dimensions and the pseudo charge on the ground wires need not be calculated. The same results can be obtained by calculating the pseudo charge but a little algebra is needed to put the results into the same form.

The preceding formulas can be greatly simplified on the assumption that average values of the capacity coefficients can be used. The general formula will then be

$$R_n = 1 - \frac{h_g}{h_w}\,\frac{n\,B_{wg}}{A_g + (n-1)\,B_g}$$

h_g = average height of ground wires

$= \sqrt[n]{h_1\, h_2\, h_3 - - h_n}$

h_w = height of line wire

A_g = average coefficient of self capacitive induction of ground wires

B_{wg} = average coefficient of mutual capacitive induction between line wire and ground wires

B_g = average coefficient of mutual capacitive induction between ground wires

$$A_g = 25.75 \operatorname{Log} \frac{2\,h_g}{r_g}$$

$$r_g = \sqrt[n]{r_1\, r_2 - - - r_n}$$

$$B_g = 25.75 \operatorname{Log} \frac{R_g}{S_g}$$

$$R_g = \sqrt[n]{R_{12}\, R_{23} - - - R_{n1}}$$

$$S_g = \sqrt[n]{S_{12}\, S_{23} - - - S_{n1}}$$

$$B_{wg} = 25.75 \operatorname{Log} \frac{R_{wg}}{S_{wg}}$$

$$R_{wg} = \sqrt[n]{R_{w1}\, R_{w2} - - - R_{wn}}$$

$$S_{wg} = \sqrt[n]{S_{w1}\, S_{w2} - - - S_{wn}}$$

For one, two, and three ground wires the following simple expressions result:

$$R_1 = 1 - \frac{h_g}{h_w} \frac{B_{wg}}{A_g}.$$

$$R_2 = 1 - \frac{h_g}{h_w} \frac{2\,B_{wg}}{A_g + B_g}$$

$$R_3 = 1 - \frac{h_g}{h_w} \frac{3\,B_{wg}}{A_g + 2\,B_g}$$

These simplified expressions are exact for one and two ground wires when the heights and diameters are the same and give very close results for the usual configuration found in practise.

In the analytical determination of ground wire protection certain assumptions are necessary. These are as follows: (a) the surface of the earth is assumed to be the neutral plane and the capacitances considered are all in reference to this neutral plane. This is equivalent to considering the earth as a perfect conductor which obviously is not the case. However, since ratios are involved this may not introduce much error. (b) It has been assumed that the ground wire always remains at ground or zero potential. This assumption requires that the ground wire have no impedance and that the resistance of the earth connection be zero, also that the charge on the ground wire will change from its initial value to its final value in zero time. Of course, these conditions can never be realized so, if nothing else entered in, the actual protection could be expected to be lower than the calculated. Laboratory tests[1] have shown that the protection is reduced as impedance of the ground wire is increased and as the resistance of the earth connection is increased. For these factors theory and practise unite in indicating that it is desirable to use ground wires of low impedance and to have low ground resistance.

An interesting method of calculation has been included by Peek[1] in the latest revision of his book on High-Voltage Engineering.

Laboratory determination of protective ratios has been made by Peek.[1] These protective ratios are better than the analytical calculations would indicate. The writer and C. A. Woodrow about the same time independently suggested that the difference might be due to corona on the ground wires and line wires. The action of the corona would be to increase the effective diameter of the wires. Later laboratory tests indicated that the protective ratio varied with the field gradient and thereby seemed to confirm the theory that corona was a factor. Mr. Hunter has made calculations in which the effect of corona on the ground wires has been approximated on the basis of certain simplifying assumptions.[2]

During the field investigations of lightning carried on during the past few years considerable attention has been given to the effect of ground wires. It is generally conceded that the use of ground wires will materially improve service, and that the reaction of several years ago against ground wires was due to improper methods of insulation and the use of unsuitable material. It was noted previously that the laboratory tests gave better protective ratios than the analytical calculations indicated. Studies of the field data indicate that the ground wires in service have a protective ratio somewhere between the two values, tending however to be closer to the laboratory value. Protective ratios based on Peek's tests for average circuit conditions have been published[3] and in studying the effect of ground wires on the induced voltage on any line it is best to determine both values and then consider the protective ratio based on the laboratory test as the optimistic figure and the protective ratio based on the analytical calculations as the pessimistic figure.

E. M. Hunter: In the method of calculating ground wire protection described in this paper and the methods described in the references to the paper, no attempt has been made to include the effects of corona in the calculations. On most high-voltage transmission lines, corona appears at a comparatively low gradient.

The potential gradient at the surface of the conductors and ground wires under lightning conditions greatly exceeds the rupturing potential gradient of air. The air breaks down, forming corona, thus in effect enlarging the diameter of the conductors and ground wires. This in turn increases the capacitance between the conductors and ground wires, and correspondingly decreases the induced voltage on the conductors.

The potential gradient at the surface of a wire is proportional to the charge on the wire.

When a charged cloud is over a portion of a transmission line, an electrostatic field is established between the cloud and ground. Charges, of sign opposite to that of the cloud, will be on the conductors in the cloud field and on the earth. With the collapse of the electrostatic cloud field, due to the discharge of lightning, the charge which was bound by the cloud field is set free and begins to travel along the transmission line. The voltage which appears on the transmission line will depend upon the capacitance between the line and ground. It would be equal to the potential gradient of the cloud field multiplied by the height of the line above earth if the circuit were without ground wires. The presence of the ground wires increases the capacitance to earth; and with cloud gradients sufficient to cause corona, the capacitance is further increased as the effective diameter of the ground wire is increased.

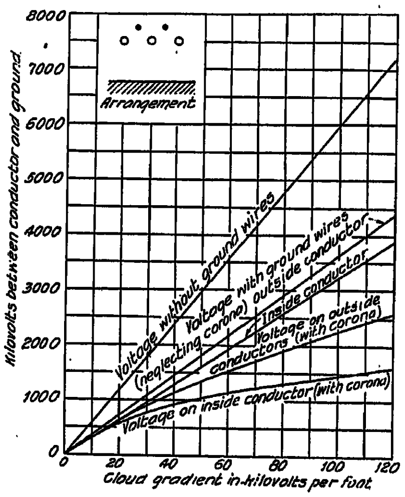

Fig. 1—Potential Between Conductor and Ground for Various Cloud Gradients

$$\text{Protective ratio} = \frac{\text{Voltage with ground wires}}{\text{Voltage without ground wires}}$$

The charge on any ground wire in a group of line and ground wires is proportional to the charges on all of the conductors in the group. Thus it is necessary to write the usual system of simultaneous equations for the charges on the conductor and ground wires, to determine the charge on the wire in question in order to determine its increased diameter. Hence the method of calculating ground wire protection described by Dr. Dwight cannot be used when the effects of corona are to be included.

An approximate method of including the effects of corona in the standard theory of ground wire protection is described in an article, "The Effects of Corona on Ground-Wire Protection of Power Transmitting Circuits" by E. M. Hunter, appearing in the February 1930 issue of the *General Electric Review.*

There exists considerable discrepancy between authorities as to the amount of protection to be expected from ground wires. There is a fairly general agreement as to the theoretical protection, as calculated from the standard formulas, but the results of tests by Mr. F. W. Peek, Jr. in the High-Voltage Engineering Laboratory in Pittsfield, Mass. show a much better protection than can be obtained from these standard formulas. However, there is a close agreement between the values of the protective ratios calculated by including the effects of corona with those obtained from tests on miniature laboratory set-ups.

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[1] "Dielectric Phenomena in High-Voltage Engineering," McGraw Hill Co.

[2] Hunter, *General Elec. Rev.*, Feb. 1930.

[3] W. W. Lewis, *Relation Between Transmission Line Insulation and Transformer Insulation*, A. I. E. E. Quarterly Trans., Oct. 1928, Vol. 47, p. 992.

The protective ratio, as calculated by the usual standard formulas, is given as constant for all cloud gradients. It is interesting to note that when the effects of corona are included in the calculations, the protective ratio is no longer a constant but varies with the cloud gradient; the higher the cloud gradient the better the protection obtained. This was also noticed in the laboratory experiments.

The induced voltage on a three-phase transmission line protected by two ground wires as a function of cloud gradient is shown in Fig. 1 of this discussion. The protective ratio as calculated by the standard formulas is a constant for any cloud gradient but when the effects of corona are included in the calculations the protective ratio varies with the cloud gradient, the higher the cloud gradient the higher the protective ratio to be expected.

H. H. Spencer: In regard to Dr. Dwight's paper I should like to say a little more about the Doolittle method of solving equations. This method has long been used by statistical engineers in getting what they call "regression coefficients in a multiple correlation equation." I have not the faintest notion what the names of those things are but a statistician associated with our organization pointed out the method used to me and it is exceedingly valuable where the protective ratio due to a large number of ground wires is desired. In one installation, a crossing over a deep gorge with a very high average distance of the power conductors above the earth, we have considered a screen of 26 ground wires surrounding 12 power conductors. This involves 26 simultaneous equations to determine the change of charge on each of the ground conductors. Actually, in this case, the symmetry of configuration enabled us to proceed with 14 simultaneous equations.

In setting up the problem for solution by the Doolittle method probably no actual labor is saved. It is probably true that substantially double the number of actual computations are made over and above the number which would be made in solving the 14 equations by the usual algebraic methods. The real advantage of the Doolittle method is that you can rule up a form on a large sheet of paper, write down the coefficients in the various equations and then pass the sheet of paper to anyone who can run a Monroe Calculating Machine and the results are more or less automatically produced. The beauty of the method lies in the fact that it is self-checking and every time a mistake is made it is automatically brought to the computer's attention, so that by the time the form is filled out, the value of each unknown has been determined and checked so that it is unnecessary to substitute back in every equation to be absolutely certain of the precision of the work. This relieves you of all the worry that you may have to do on the laborious algebra over again; and therein, to my mind, lies the real value of the Doolittle method.

R. P. Jackson: The paper presented by Professor Dwight is another one of the efforts begun by myself in 1907, in an Institute paper read in Chicago of that year to evaluate the protection against lightning obtainable by the use of overhead grounded conductors. All such calculations must, of course, be based on certain assumptions. Two of these are that the surface of the earth is of uniform zero potential and that the potential gradient upward is reasonably uniform.

The above assumptions are essential to the mathematical calculations and, furthermore, they seem reasonable and are probably near enough to the truth to afford validity to the results as far as potential values are concerned.

Another assumption has been made, however. That is that the release of the charge on the transmission as a result of the disappearance of the potential gradient is really an important source of lightning trouble on such lines and in the apparatus connected to them. Recent data collected in connection with lightning investigations have thrown grave doubt on this assumption. In fact, many engineers who have been closely associated with research on lightning disturbances have come to the conclusion that direct strokes or, at least, side strokes or filaments of the main discharge are the chief causes of trouble on high-voltage transmission lines. If this is the case such calculations become of minor importance no matter how correct mathematically.

It appears, in fact, that what we want is a "continuous lightning rod" up high enough to make it highly probable that all direct strokes will reach the rod or overhead conductor and be carried to ground thereby. My paper of 1907 and Professor Dwight's present paper would imply that several comparatively small grounded conductors spanned over a transmission line would afford a low potential field below them which should be a safe place for power conductors. If, as now appears to be the case, direct strokes, or their approximate equivalents, are the source of trouble, one grounded conductor large enough not to be easily damaged by direct-stroke, high enough to be likely to attract the stroke, and of enough conductivity to carry it away, is what we want.

All our earlier calculations in regard to lightning had largely to be based on some assumptions. This practical one, however, that direct strokes were rare and that potentials induced by the electric field of an overhead closed and then released were the commoner cause of our troubles now appears to have been incorrect. If direct strokes are the main thing to be guarded against, such interesting calculations as Professor Dwight has given us and as I indulged in over twenty years ago are beside the point. To repeat, what we need is strong, non-corrosive, sufficiently massive conductors of proper conductivity placed high enough above the line to catch the stroke and usher it politely to the ground. How high this should be affords material for further mathematical calculations and practical experiment.

H. B. Dwight: The flashovers of transmission line insulators during thunderstorms may be divided into those caused by induced voltages and those caused by strokes of lightning or branches of such strokes, reaching the transmission line. Mr. R. P. Jackson published the calculation of protective factor in 1907 on the basis of induced voltages, and it has been used by engineers for many years with little change. It is still used to a large extent.

Mr. Jackson now states in his discussion that data have been collected which show that induced voltages are of little importance and that direct strokes are the main factor. A similar statement is to be found near the end of the paper on *The Lightning Laboratory at Stillwater, N. J.*, by R. N. Conwell and C. L. Fortescue (see p. 872). A description of the data on which this conclusion is based would be of considerable interest.

The increase in protection when there is corona on the ground wires, calculated by Mr. Hunter in his article in the *General Electric Review* of February 1930, was suggested by corresponding increases measured in impulse tests on models. The calculations indicate that a ground wire with corona is very much more effective than one without corona.

The question arises as to how much the space charge within the corona envelop around a wire behaves like a charge on a very large metallic wire. When it is determined to what extent the charge represented by the corona returns to the ground wire, and how fast it returns, the items might possibly be used as an extension of the interesting calculations given by Mr. Bewley on the effect of ground conductors, using the theory of traveling waves.

A New Transmission Line Construction

Post Type Towers

PERCY H. THOMAS[1]
Fellow, A. I. E. E.

***Synopsis.**—The new construction described is intended to secure a sturdy and simple tower, not sensitive to irregularities in footing conditions; it is generally applicable where a horizontal arrangement of conductors is feasible. The tower consists of an articulated overhead transverse bridge carrying the conductors and hinged at the tops of the footings. Two or three intermediate towers are used between stabilizer towers. Transverse strength is secured by crossed internal ties and longitudinal stresses are taken up in the stabilizer towers by heavy guys and in the intermediate towers by the ground wires and conductors, which transmit these stresses to the stabilizer towers.*

The wide base structure thus secured has substantially no resultant uplift on the footings, offering a very large saving in foundation costs. This saving, together with the lesser material and erecting costs and the great ease with which it is adapted to varying ground conditions make this a very economical type of tower.

CONSIDERING the wide variety of conditions under which power transmission lines are installed, the very limited number of types of structures actually used is surprising. No one type of structure, such for example as the usual four legged, rigid, narrow base, cantilever tower, would naturally be expected to be the most satisfactory and economical design for all services and yet except for the well known so-called flexible type almost no other steel tower is used. In order to discover whether a different form of line support might not have real advantages over the usual type for some services, the author has made a careful survey of the requirements and the most available alternative forms and has worked out what may appropriately be called the "post type tower," a type contrasting sharply in a number of important features with the usual designs. It is intended primarily as a rugged, simple structure, not requiring special skill in erection and having the most favorable electrical characteristics, together with the lowest reasonable cost.

Engineers recognize the inherent weakness of the four footed rigid tower set on earth stubs, this being in effect a light, exceedingly rigid structure placed on an uncertain, non-rigid base. The difficulty of framing a wide base rigid structure with the relatively light structural sizes available, as well as side hill conditions, and sometimes also the value of the land occupied, have led to the development of the present narrow base tower, such towers having in many cases, especially on important high-voltage lines, a ratio of height to base most unfavorable for resisting the severe stresses imposed by high winds and broken conductors and resulting in greatly magnified vertical stresses on footings. The conditions imposed upon the foundations are particularly severe because the stresses on the footings alternate from compression to tension. If any inequality of bearing is thus developed, undeterminable but dangerous weakness is inevitably introduced. Unfortunately there is no way of discovering or remedying such inequality. It may not be unreasonable to surmise that were all the towers of the average existing transmission line subjected as they now stand to the maximum conditions of wind, ice, and broken conductors for which they were theoretically designed, many would fail. I think we may safely conclude that the very satisfactory results as to tower performance now secured are due to the fact that the severe conditions assumed in design very rarely occur.

Recent researches have shown that the height of a transmission line conductor above ground is one basic factor in determining the severity of the exposure to lightning, so that the vertical arrangement of conductors, helpful toward economy of right of way, is decidedly unfavorable from the point of view of service in country subject to lightning. Furthermore the difficulty in sleet country of avoiding arcing due to jumping of cables with the vertical arrangement is well known, due to widespread trouble from this source. These facts have been borne in mind in working out the new type tower.

The post type tower is fundamentally a broad base structure. Such a broad base in a transmission tower can be secured only by the use of tension members for bracing. The wide tower base and the fact that the conductors have a minimum height above ground serve practically to eliminate all direct resultant uplift on the footings.

The post type tower, which is almost equally well suited for one circuit or two circuit lines (see Figs. 1 to 4), consists of a long transverse crossarm carrying the conductors, supported near the two ends by independent upright posts, taking only axial stresses. The transverse stresses on the line are taken by internal tension cross ties located between the posts, and the longitudinal stresses by guys. The mechanical connections between the tops of the posts and the crossarm and between the bottoms of the posts and the footings are hinged, the tops for motion in a transverse plane, the bottoms for motion in a longitudinal plane. The tension members or guys taking up longitudinal stresses are connected to the crossarm in the same horizontal plane as the insulator strings and are located

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[1] Consulting Engineer, New York, N. Y.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

approximately over the posts. Therefore, while the crossarm must take up in a horizontal plane the longitudinal stresses from broken conductors transferring these stresses to the guys, there is no bending thus caused in the posts. The pull of a broken cable acting on the inclined guy, produces a downward component, but as the guys are connected near the post tops little downward strain is produced in the crossarm.

Ground wires are preferably connected to the lower chord of the crossarm in the same plane as the insulator strings and substantially at the same location as the guys. They may, however, be connected at a higher level on the crossarm, in which case the posts will have to be designed to take up the resultant bending moment developed in a vertical plane between the ground cables and the guys.

A moment's consideration will show that this structure is exceedingly favorable for supporting conductors on earth foundations, for no settling of a post footing or yielding of a guy anchor, and no error of leveling or locating of footings or anchors, within any reasonable limit, would cause any material over-stresses in the structure. At most a certain slackness of cross ties or guys may be caused, easily remedied by tightening turnbuckles.

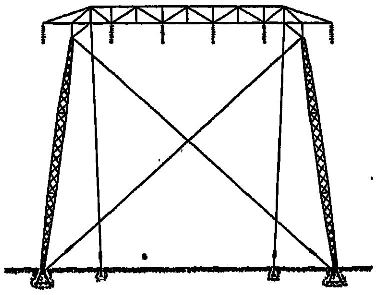

Fig. 1—Front Elevation, 132-Kv. Double-Circuit Post Type Tower Showing Guys and Cross Ties

In considering the field of utility of the post type tower the question of the right of way assumes unusual importance. It will be clear that a wider right of way is required for any horizontally arranged circuits than one which would be sufficient for a line with vertically arranged conductors. The post type is at a disadvantage, therefore, where lines must be run through woods for long distances or in thickly settled districts. On the other hand in open fields, whether used for pasture or agriculture, in which the obstruction to the land is merely that caused directly by footings and anchors, the post type tower may even have an advantage since the footings and anchors are so widely separated from one another that it is possible to work around them and very little ground area is actually obstructed. With the ordinary four footed tower, the whole area between the four footings is usually lost.

The post type tower is especially notable for its sturdiness. On account of the flexible connections at the top and bottom of the posts, movements of the foundations or stretching of guys or yielding of anchors cause no bending moments in any part of the structure. Furthermore, the stresses are all definite and calculable so that all members, including the guys, may be proportioned for their individual stresses with the assurance that no unknown and dangerous overload may be introduced by settlings of footings or irregularity of setting as with rigid towers. The single heavy crossarm further has a greater reliability than the usual single cable crossarms.

The post type tower is notable also for its simplicity and adaptability, manifest both in layout work and in the field. On moderate sidehill slopes the normal towers are merely skewed up hill somewhat and all the same material used as on level ground. On steep slopes a taller post is used on the downhill side. A limited number of unit crossarms, posts, and guys may be combined in different ways to obtain constructions suitable for all special towers, such as angle, long span, dead end, etc. This shows the flexibility of the design.

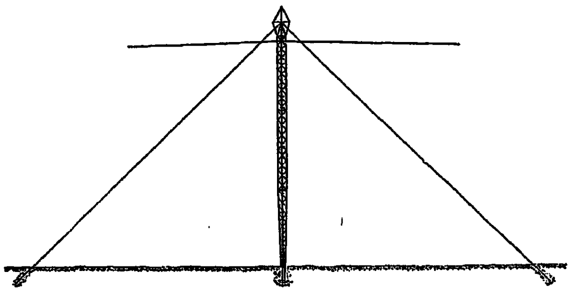

Fig. 2—Side Elevation, 132-Kv. Double-Circuit Post Type Tower

It will be noticed that the same tower, for example the tower shown in Figs. 1 to 3, can be used for two circuits 110 kv. or 132 kv. at the original installation and afterward changed to one 220-kv. circuit. In the change over, the positions of the outside conductors would be abandoned and three conductors hung between the posts instead of the four for 132 kv. As a matter of fact the total spacing distance between posts will be very closely the same for three 220-kv. conductors as for four 132-kv. conductors so that there would be very little increase in weight or complication in design in laying out the tower to serve alternatively for either voltage.

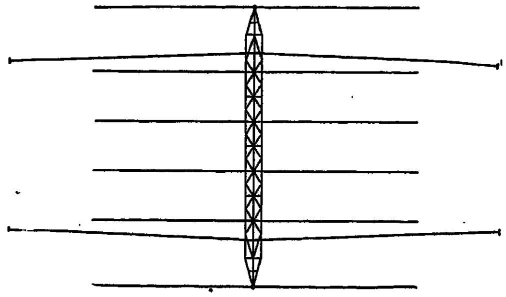

Fig. 3—Plan 132-Kv. Double Circuit Post Type Tower

Considering installation operations, since the towers are finally trued up after erection by means of turnbuckles in the cross ties and guys, there is no reason why post footings or guy anchors should be accurately located nor is there any object in carefully leveling the footings. Variations of a foot or two either in horizontal position or in the vertical setting of footings are unimportant.

Towers may be entirely assembled on the ground and ended up on the stub hinges at the bottom of the posts.

Thus where sturdiness is of value in the tower structure, where highly trained and responsible erectors are not available, or where there may be difficulty in soil conditions, the post type tower has an advantage and it was for such conditions that it was originally devised.

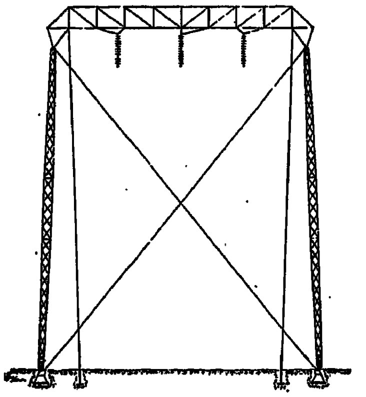

Fig. 4—Front Elevation, 220-Kv. Single-Circuit Post Type Tower

The post type tower has a further advantage in its cost saving which is very material in amount. On account of the wide base in both directions and on account of the articulated construction, there is a considerable saving in the cost of steel. Furthermore, on account of the absence of resultant uplift on the footings and the flexible connections it is not necessary for the footings to be set more than three or four feet below the surface in ordinary soil. This results in a great saving in the cost of excavation and backfill. These savings with the simpler erection and lighter weights to be transported result in a very large saving in the cost of towers erected. There cannot, of course, be any saving in the cost of conductors *per se*.

There is a modified form of the post type tower which is especially favorable from the point of view of cost and simplicity, this being especially applicable to two circuit lines. By relying on two ground wires connected respectively near the post tops and secured firmly to the crossarm to take up longitudinal stresses, it is practicable to omit the longitudinal guys on a majority of the towers, the ground wires serving the function of the guys for these towers. In this construction on account of the long spans the towers will not be held strictly rigidly in position, but being hinged at the bottom they are free to adjust themselves to the pull of the ground wires and conductor cables and no abnormal stresses are introduced. On towers where the guys are omitted a broken conductor will cause a slight displacement of the crossarm sufficient to tighten up on the ground wires in one span while slackening off in the other. No bending stresses are introduced in the posts and no mechanical instability.

Analysis shows that a series of more than three successive unguyed towers is not desirable. The latter means, however, the omission of all longitudinal guys on three-quarters of all towers on normal tangent sections.

The importance of this omission of guys, leaving the ground unobstructed except for post footings, four or five thousand feet in a stretch, is obvious. There is also a very material saving in cost. This is practically clear gain since the guyed towers serving as stabilizer towers need be no stronger than with all towers guyed.

The construction of the tower and its use on side hill slopes is shown in Figs. 5 and 6. The normal tower is used on a moderate side hill slope with the crossarm parallel to the ground. A tower on a steeper side hill is provided with an additional extension in the down hill leg as shown in Fig. 5. Where the ground slopes with the line the tower is kept perpendicular to the ground and the guys make the normal angle with the post as shown in Fig. 6. This arrangement gives the simplest and most favorable design. Small angles can be taken on the regular tower with a somewhat wider crossarm and heavier cross ties.

A tower suitable for taking a large angle in the line is shown in Fig. 7. Two pull-offs, one connected to each post, are used. Both are secured to the same anchor, but through a whiffletree arrangement by which the total pull-off stress is divided between them in the desired ratio.

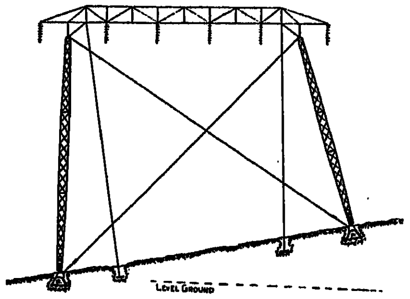

Fig. 5—Front Elevation, Two-Circuit Post Type Tower on One-in-Five Transverse Ground Slope

Proper Horizontal Spacing of Conductors

This is an important matter with post type towers since this horizontal spacing occurs three times cumulatively and largely determines the length of crossarm.

With the usual four legged two-circuit tower with a "basket" carrying the crossarms, each cable must be so far spaced from the basket that in the position of maximum swing it still has the proper clearance to tower. This condition usually determines the horizontal spacing of conductors, since it gives a spacing out in the span considerably more than necessary to prevent contact. There are other types of line, however, as for example wood pole lines and lines with six conductors horizontally spaced on one crossarm (such for example as the Davis Bridge Line of the New England Power Company) where horizontal spacing is determined by the danger of contact out in the span as with the post type tower. Practise in these cases indicates that from 6 to 12 ft. between conductors is safe on from 60 kv. to 110 kv.

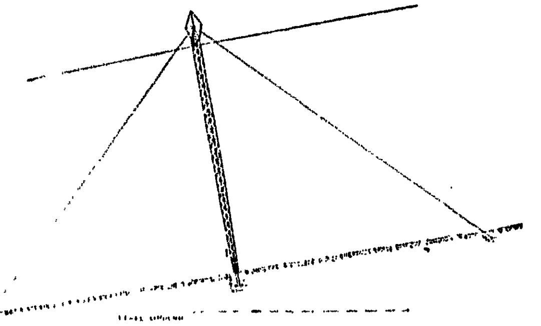

FIG. 6—SIDE ELEVATION, POST TYPE TOWER ON LONGITUDINAL SIDE-HILL SLOPE

Since a more flexible and consistent method of determing the proper spacing should, however, be available, the author suggests the formula[2] proposed by him for calculating horizontal spacing, reading as follows:

$$\text{Horizontal Spacing} = C \times \text{Sag in per cent} \times \frac{\text{Diameter of Cable in in.}}{\text{Class ``A'' Loading in lb.}} + D + \tfrac{1}{2}\,\text{Length of Insulator String in ft.}$$

Where

C = A constant chosen arbitrarily for each line to take account of the sort of metal of conductors, wind conditions, character of terrain, etc.

D = Jumping distance of line voltage—one foot for each 11 kv.

"Sag in per cent" is for class "A" loading.

Constant suggested for copper cable — 4.0

Constant suggested for A. C. S. R. — 3.5

This formula takes account of the variation in jumping distance of various line voltages, the length of insulator string, and the very important matters of cable tension and span length. A 0000 copper wire with a normal span of 800 ft. requires according to this formula about 10 ft. horizontal spacing for 110 kv. and a 336,400-cm. A. C. S. R. cable with 1000 ft. span about 11 ft. for 132 kv. These horizontal spacings fit well into the post type tower design.

STRUCTURAL DESIGN

There are many interesting design details to be considered in the post type tower, none presenting any serious difficulty. As a fundamental principle there is a horizontal plane of resistance at the bottom of the crossarm which takes up longitudinal stresses. All longitudinally acting elements, *viz.*, conductors, ground wire, guys, and insulators are attached in this plane. If this condition is not maintained there will be a resultant tendency for the crossarm to turn on a horizontal axis, which tendency can be resisted only by stiffness in the posts. The post can be of economical design only when its load is substantially axial as bending moment greatly increases the weight of members.

Fortunately this connecting of all longitudinally acting elements in this plane is generally favorable. The ground wire is then supported on the tower at a point higher than the cables by the length of the insulator string, but it is at the same time separated by a larger distance horizontally. Since the protective effect of the ground wire is increased by a location close to the conductors this relationship is favorable. A greater separation between ground wire and cable out in the span can be obtained by drawing the ground wire somewhat tighter than the conductor as is often done with other types of towers.

Attention should be called to the fact that the longitudinal guys of the post type tower are not to be compared to the usual twisted wire steep angle guys so much used in wood pole construction. They are tension members carrying definite calculated stress just as the bracing does in the usual tower. The guy anchors are designed and proportioned for their specific loads just as are the footings of the four legged tower. These guy anchors are more reliable than such footings as already stated, since the stress is always in one direction and not alternating and since any yielding of the foundation is shown by slackening of the guy and can be easily remedied by tightening the turnbuckle without disturbing the tower.

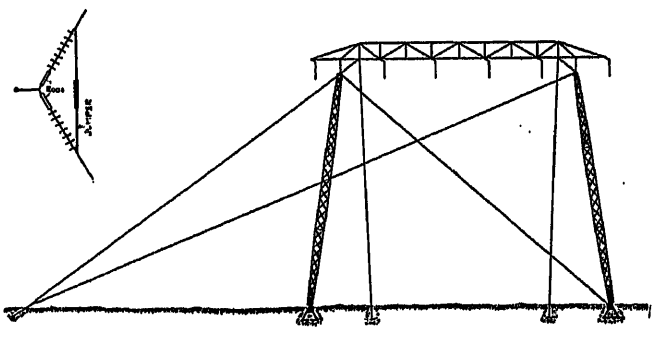

FIG. 7—FRONT ELEVATION, POST TYPE TOWER FOR HEAVY ANGLE IN LINE, SHOWING TWO PULL-OFFS AND WHIFFLETREE

The size of the tension rods used in post type towers is so large as to be substantial and not easily injured, and furthermore not greatly weakened by quite severe mistreatment. The use in the common type of tower of angles as bracing subject to compression is far less advantageous from the load carrying point of view than the tension rod.

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[2] *Formula for Minimum Horizontal Spacing of Transmission Line Conductors as Affected by Danger of Contact in the Span*, A. I. E. E. TRANS., Vol. 47, October, 1928, p. 1323.

As solid rods are not flexible special provision must be made for their convenient use. This may be secured by using the rods in fixed lengths, each length having a button upset on each end with suitable loose socket connectors for holding them together. These connectors will give an angular freedom of ten to twenty degrees each, so that a complete guy or cross tie made up of several lengths will have a very considerable flexibility. Somewhat similar methods of connection are now in use in somewhat similar service and provide a very simple and economical method of developing the full strength of the rod.

Two regular lengths of rod, 25 ft. and 12½ ft., may be used to advantage. The necessary number of such rods will be specified in the drafting room for each site and listed with the tower material. In the field the cross ties and guys will then be a few feet short. To cover this shortage there is carried by the erecting crew an assortment of lengths varying by 1 ft. steps up to 6¼ ft., which with the long screw turnbuckles will complete the ties and guys without any cutting or fabrication in the field.

Although there is no resultant uplift of material consequence on the post footings, the windward footing must take the full transverse horizontal shear on the tower, to this extent giving an advantage to the four legged tower where the shear is divided between two or more footings. If the cross ties be carried down to the earth bearing parts of the footings, the shear occurs at that low level. If connected at higher points near the ground surface there is in addition an overturning moment on the footing that must be resisted. It is important to notice, however, that a yielding of the stub to shear causes no stress in the tower unless the stub is actually pulled out of position, on account of the articulation of the tower. This is of great importance in considering the ruggedness of this design.

ERECTION

The process of erecting is simple. A tower is assembled complete with insulators on blocks on the ground with the posts hinged to the footings. A divided bridle is connected to the tops of the two posts and carried over a single shear stick in the center of the line. The tower may then be raised by a windlass or truck, the guys being used as back stays. If the construction in which the guys are omitted from a certain portion of the towers is used, temporary guys may be provided until the ground cables are strung. When the structure is erected the cross ties and guys are tightened so that the structure is made taut and rigid; in fact, it is the most rigid type of tower available.

TOWERS WITH GUYS OMITTED

The behavior of a line in which the guys are omitted from a certain portion of the towers on normal tangent line, as for example three towers out of four, is interesting. The most critical case is the breaking of a cable near the end of the crossarm, for example the outside conductor. In this case a stress equal to the final stress in the cable is imposed horizontally on the end of the crossarm. The horizontal broken cable stress pulls the end of the crossarm to one side until the other cables develop opposing stresses equal in amount. The principal opposing stress comes from the adjacent ground wire which is secured directly to the tower. The limiting stresses occur on long spans with ice and wind loading. As the crossarm moves to the side at the point of attachment of the ground cable, the stress in this cable increases in one adjacent span and decreases in the other, and the difference represents the pull opposing that of the broken cable. When the point of equilibrium has been reached it will be found that the end of the crossarm where the broken cable is located has moved some two or three ft. only. The point of attachment of the broken cable is out at the end of the crossarm while the nearest ground cable is attached several feet inside. As a result there will be a slight stress in the far ground cable in the opposite direction from that in the near ground cable. The crossarm at the far end then has a slight movement in the reverse direction.

In the case of a broken ground wire, presumably a less likely contingency, the same action occurs except that the displacement of the crossarm is somewhat greater on account of the necessity of taking up the swing of the insulator string before the development of the necessary opposing pull in the several conductors. In this case the outside conductor cable takes the greatest stress as the crossarm will as before revolve in a horizontal plane about a point near the far ground wire.

Even in the extreme position of the crossarm with a broken cable, the strength of the tower to carry stresses, transverse or vertical, is unimpaired and no weakness is introduced.

TOWER HEIGHTS AVAILABLE

It will be evident, no doubt, that a certain advantage results in spotting towers in a transmission line, where there is a large number of tower heights available differing by small steps. If, for example, a span is so located that the low point lies just over a depression in the ground, the neighboring high spots on the two sides cannot be used as tower sites to advantage unless there are available towers of the right heights.

It is difficult to determine the importance of this advantage, which apparently amounts, however, to several per cent in the total number of towers required, when as many as eight or ten heights are available over a range of 100 per cent in height. Such a large number of tower heights is very easily obtained with post type towers by the use of varying length parallel sided center sections in the posts as shown in Fig. 5.

Modifications of the various details here described may be made to advantage to adopt the post type tower to various line conditions. The general scheme is obviously very flexible in application. There are numerous other details arising in the design and use of post type towers that are of interest or practical importance but space will not permit their discussion here.

The only serious objections urged against the post type are the greater width of line structure, which is controlling in only a few cases, and the use of guys, which while exceedingly effective and economical, make a widespread base. The utility of the post type tower can be summed up very briefly as follows:

1. Sturdy and simple design.
2. No special skill or accurate leveling required in setting footings or tower erection.
3. Best arrangement of conductors electrically and as a safeguard against uneven ice loads.
4. Not sensitive to foundation movements.
5. A very material saving in weight and cost of steel and in cost of footings, anchors, and erection.

It may be added that obviously many modifications of the post type structures here shown may be made to meet different conditions or uses.

Discussion

F. E. Andrews: There are several points on Mr. Thomas' paper which I wanted to make some inquiries about. It is interesting to consider that we have a suggestion for construction like this which is radically different from the conventional designs that usually go along with tower construction; it is a healthy condition to have some suggestions like this because they are conducive to thinking in new lines which are usually very profitable.

Mr. Thomas spoke of the use of the ground wires for stabilizing intermediate towers in cases where the longitudinal guys on the intermediate towers were omitted. I should like to ask if any additional allowance is made for strength of the ground wires when they are so used, over and above that usually made for the normal wind and ice loading and whether or not any different factor of safety is used for the ground conductors like this. I should also like to ask whether or not any of these hinged structures have been actually installed and if so, whether there has been any experience with them through different weather conditions in varying temperatures, etc.

In thinking of the problem of expansion and contraction it has occurred to me that there might be some unsatisfactory results due to the fact that the guy rods would slack off in the summer time, and tighten up in the winter time. It would seem very impractical to adjust the turnbuckles seasonally and I am wondering if it was contemplated that this would be done. On page 1124, Mr. Thomas speaks of the analysis showing a series of more than three successive unguyed towers not being desirable. I should like to know the basis for determining the number of three rather than some other figure. Mr. Thomas can probably answer that question.

W. H. Colburn: Consideration of the capital cost of delivering power at the point of use shows that cost to be somewhat staggering. Figures which I have roughly in mind for a certain short line are in the neighborhood of $45 per mile per kv-a. delivered. The modern trend is toward the maximum simplification possible consistent with the maintaining of service. Hence, any construction which will lessen this cost of line construction without sacrifice of line safety should be welcome. Mr. Thomas' proposed construction seems to offer such a possibility and deserves serious study in contrast with more conventional types of construction.

R. D. Coombs: (communicated after adjournment) The "post type tower" is a guyed-mast-H frame and while it should be economical in cost, the question arises as to whether it is quite fair to compare it to a wide base tower design if the designer of the latter is held to the usual specifications. This same general idea is being used in some recent railroad electrification structures, around Philadelphia. It would be interesting to know just how far the designers of either kind of structure have permitted themselves to vary from the detail requirements of the usual wide base tower specifications.

The writer is not at all concerned over the fact that a wide base tower foundation may have alternate compression and tension, and doubts that such a tower ever failed from this cause. Undoubtedly the fine record of wide base towers is as the author says because they have never received their maximum theoretical loading. On the other hand it is also a fact that many such towers have not fallen even after wildly excessive loads or outrageous incidents. The writer refers to such cases as the three following:

(a) The conductors were doubled in size, perhaps 5 ft. pulled out of the sag, and the stubs were very badly set.

(b) A tangent tower was used at a large angle corner.

(c) The foundation under one corner descended 2 or 3 ft. from a cause unrelated to the tower.

Of course in each case the tower was partly injured, but the point is it did not fall down.

The proposed construction must of course depend for stability on the various guys. The longitudinal or ground wire guys must have a very much better clamp than any known type other than a heavy strain conductor clamp. The guys to ground depend on the guy anchorage and this seems a good occasion to inquire what relationship, if any, exists between the assumptions used to determine the strength of such anchorages and the assumptions demanded of a designer for wide base tower anchorages. The writer has claimed for many years that the foundation assumptions were all wrong or else the guy anchor figures were wrong.

What values will be permitted for the $1/r$ of the posts as a whole, the length being from crossarm to ground?

While the total weight of steel per structure may be less than for a wide base tower, its cost classification may be higher on account of the latticed masts, hinges, turnbuckles, and clevises.

The writer is inclined to think that the author has given the flat spacing a little the best of it compared to most of the horizontal separations for long span construction. In other words would a horizontal separation of 10 or 12 ft. be considered acceptable by the designers of the existing long span 110-kv. lines?

P. H. Thomas: Evidently Mr. Andrews has been thinking about this new type of tower. I am sorry we haven't more time for there are many interesting problems which have been pretty carefully studied but not touched upon. In regard to the first question as to how much we have to sacrifice in the strength of ground wires, when we omit individual longitudinal guys between anchored towers. That is more or less a matter of design within the control of the engineer. I have made about three very careful detailed designs of these towers for actual profiles and on the basis on which those have been worked out, I have assumed that the structure would have to stand one broken conductor on a six wire circuit with the maximum sleet and maximum wind, or maximum ground wire tension.

I made allowance of ten per cent in the stress on the conductor and the ground wire needs more strength which is secured by using extra high strength strand. I should like to point out that the limiting stress was taken at the condition of maximum sleet and wind. I think if a census was taken it would be found that broken conductors occur nine-tenths of the time under other conditions and less than one-tenth of the time under these maximum loading conditions, so that in nearly every actual case additional reserve strength will be available. The set-up is this, the bridge which is held only by cables in the intermediate bridge is a very flexible structure. It adjusts its position without stress to the mean position of the various cables. It should be noted that if the bridge is allowed to pass excessively in one direction from neutral, the footing being fixed in position, there is an effect from the overhanging weight which causes a horizontal thrust on the footing.

The horizontal deflection of the crossarms when a break in a cable occurs produces a higher tension in cables on one side of the arm and a lower one on the other and it is the difference in these tensions which balances the stress due to the broken cable.

Answering the last question of Mr. Andrews as to why I have suggested that three consecutive unguyed towers is as many as is likely to be suitable, I will point out that the tightening up of cables which comes from the physical movement of the crossarms due to a broken conductor is divided between the spans between the break and the next stabilizer towers, that is between the moving crossarm and the first fixed point, and the more spans there are the less tightening is there in any one of them. Again the more the crossarms overhang the neutral position the greater is the tendency to accumulate additional cable stress due to its weight. I presume two unguyed towers in succession are more conservative than three but three is the maximum limit.

The question of the temperature is a very interesting one. After giving it a good deal of thought I believe it is not a matter which will cause practical difficulty. My suggestion is that when a stabilizer tower is set up the guys should be tightened until there is a very considerable stress in the guy. This does not weaken the power of the tower to resist the pull of a broken conductor unless this stress is more than one-half the elastic limit of the guy. In this case one guy is pulling against the other very tightly. When a broken conductor occurs it produces stress on the guy on the other side and the effect is to tighten up the far guy, that loosens the near guy and if the stress of the broken cable reaches the elastic limit of the guy on the far side the stress is substantially slacked off on the near guy and the full strength of the strained member in holding the broken conductor. As to a convenient method of using solid rods for guys, my suggestion is to use a 25-foot unit rod with a butt-on forged on each end and some kind of open-sided socket in which the ends of two rods can be slipped, thus making a coupling between them. That type of construction is now in use on some of the Pennsylvania Railroad electrification and it seems a very feasible thing. In setting up a tower there may be specified for use an appropriate number of these 25-foot lengths and 12½-foot lengths for each guy, these guys being finally completed in the ground with a lot of assorted short lengths carried around by the erecting crew, the turnbuckle giving the final adjustment.

With such a construction and a U-bolt connection at the tower there is considerable flexibility and it will be found that the total elongation due to the flexibility of the connection and the stretch of the rod is considerably more than any effect due to the contraction by cold. Presumably there will be no change of length in the ground. The only effect of temperature would be the differential effect of the expansion of the steel in the post and of the steel in the guys. If the guys are longer there will of course be proportionately more expansion. While it is thus true that there will be a variation in the lengths of members due to temperature, I believe it will turn out they will not be taken into account practically.

In reply to Mr. Coombs, the paper is not intended to offer any criticism of the usual narrow base tower, merely to show that a material advantage can be obtained by the use of the new construction where the conditions do not require the other construction.

It should be noted that the duty on the cable clamps, either conductor or ground wire clamps, as the result of a broken cable is not very severe, since the stress on the clamp is only the difference in stress between the tight side and the loose side and further since the stress due to a broken ground wire is divided between two conductor clamps.

An examination of design detail shows that no special material is required for hinges or clevices other than the usual plates and bolts. All the latticing of the posts can be done with two or at most three lengths of latticing, throwing these members into quantity production. A real cost advantage comes from the use of round rods for these have a lower base price than angles.

The matter of horizontal spacing has been fully discussed in the previous paper referred to and the spacings recommended are believed to be entirely conservative for these services.

The Calculation of Alternator Swing Curves

The Step-by-Step Method

BY F. R. LONGLEY*
Associate, A. I. E. E.

Synopsis.—The paper is intended to give a thorough explanation of step-by-step calculations of the synchronous rotor oscillations and other transients, which occur in electrical power systems when faults or other sudden changes take place. It is divided essentially into two parts. In Part I a descriptive illustration of the phenomena is given in order that the reader may visualize the conditions. In Part II general formulas are developed for calculations where any number of salient or non-salient pole machines are operating at various points on an impedance network. In Appendix I a special and simplified example is set up and analyzed. In Appendices II and III the numerical calculations of the swing curve for the special example in Appendix I are given, and a brief discussion is offered on the accuracy of this step-by-step method of calculation.

* * * * *

INTRODUCTION

THIS paper is based on the method suggested in the paper *System Stability as a Design Problem*, by R. H. Park and E. H. Bancker.[2] Its purpose is to develop further the methods of making step-by-step calculations of the angular swings and other transient phenomena of synchronous machines in a power system following disturbances. Additional material from the paper *Progress in the Study of System Stability*, by I. H. Summers and J. B. McClure,[8] has been incorporated and the resulting methods have been worked out in great detail. The object of doing this is to make the theory and the practical procedure clear even to a novice, so that any engineer who finds it desirable will have at hand rules for making the calculations.

The synchronous machine formulas are based on the vector diagrams developed by Messrs. Doherty and Nickle in their series of papers[5] on synchronous machines and also as given by R. H. Park in his paper[9] on *Two-Reaction Theory*.

The need for this paper is indicated by the following review of the subject:

The two stability papers mentioned above are the only ones which, to the author's knowledge, give details of the procedure of making step-by-step stability calculations. The first mentioned paper covers a great deal of ground and suggests much of the material here presented, including the general procedure, but the information is given without much detail so that it is practically impossible for an engineer who does not have some additional information to use it. The second paper considers a special case which is often encountered and simplifies the work considerably. It gives instructions in precise detail for making calculations under the assumptions involved, and makes it possible for any interested engineer to make calculations, based on the assumption of constant rotor linkages, and equality of reactances in the direct and quadrature axes.

There are some cases, however, which do not fall into the class amenable to calculation by this method. In practise these are primarily the cases which involve variation of field excitation. It is to furnish detailed instructions for making these additional calculations that this paper is presented.

A careful descriptive illustration of the phenomena involved will be given first. This is followed by the development of general swing curve formulas for any number of salient pole machines having varying excitations and connected to an impedance network (resistances included). A special case is then taken and it is shown how the general formulas adapt themselves to it. Finally the numerical calculations of the swing curves for the special case are given. It will be seen later that this example represents a condition commonly encountered when problems of transient instability are found to exist in electrical power systems.

Part I

A DESCRIPTIVE ILLUSTRATION OF THE PHENOMENA INVOLVED

A physical understanding of the manner in which the electrical quantities and rotor position of a synchronous alternator vary during and after a transient disturbance should add to the interest of the subject and aid readers to follow the mathematical analysis. These illustrations are descriptive only and form no part of the development of the swing curve formulas.

Two or more pieces of synchronous apparatus are stable as long as they remain in synchronism. When power is exchanged between a synchronous generator and motor an electrical angle δ, exists between their rotors. Any transient disturbance will cause the rotors of rotating synchronous apparatus to oscillate or swing with respect to each other. Whether or not they remain in synchronism will depend upon the severity of the swings. The swings can be calculated by the step-by-step process, and are usually plotted to show the angle δ between rotors as a function of time.

This angle δ will depend among other things upon the excitation system used, the impedances of the system,

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*Western Massachusetts Companies, Springfield, Mass. Formerly of the General Electric Co., Schenectady, N. Y.

2. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

30-79

the character of the disturbance, the initial load, and the governor characteristics.

All quantities in this paper will be expressed in per units[6] unless otherwise stated. Per units are decimal percentages. They are percentages without the multiplier of 100 and are used in place of percentages because they make it unnecessary to watch for, and correct for, coefficients of 100 when calculations are being made.

Let us consider two large salient pole alternators synchronized through a reactive tie over which a steady power flows from one to the other, and let this tie be suddenly weakened by a single-phase fault.

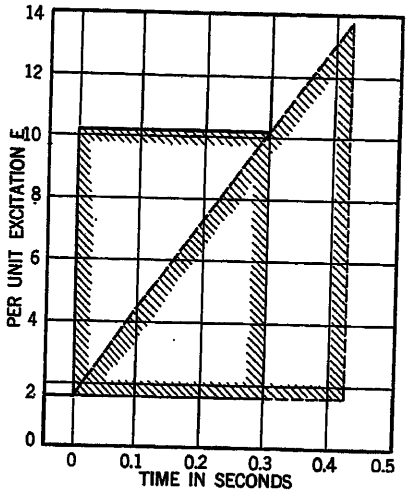

Fig. 1—Two Types of Super Excitation

Case I in solid lines. Case II in broken lines

The effect of excitation will be demonstrated on a magnified scale by applying the type often referred to as "super-excitation."[10]

In order to illustrate the effect of the *rate* of excitation rise, two types of "super-excitation" are used, and the effect of each of these two types is shown separately. Upon application of the fault the excitation in one case rises instantly from 156 volts to a ceiling (maximum value) of 1000 volts, or in per units from 1.59 to 10.2; while in the other case it rises from 156 volts to 1350 volts at a rate of 2810 volts per sec., or in per units from 1.59 to 13.77.

The rise in volt-seconds* is the same in each case and is that shown by the shaded areas in Fig. 1. The curves resulting from the instantaneous rise in excitation are in solid lines, and will be referred to as Case I, while those resulting from the excitation rise at the rate of 2810 volts per second are in dotted lines, and will be referred to as Case II.

As explained in Appendix I of Bibliography 1, a given rise in volt seconds will produce very closely a proportional rise in the field flux of large alternators during the first half second, no matter how applied.

The more quickly the volt-seconds are applied, the greater the stability. The stability, during a fault, is not a function of the rate of excitation rise alone, nor of the excitation ceiling, alone. It is a function of both these conditions, expressed together as volt-seconds.

If, at the instant the fault is applied, the alternator field flux linkages Ψ_d could rise instantly to the proper value, there would be no swing of the rotors whatsoever. This required rise in flux, for the example used here is shown by the dashed curve in Fig. 2.

An instantaneous rise in alternator flux is impossible but the nearer it is approached the greater will be the stability. Fig. 2 shows that the solid curve approaches this condition more nearly than the dotted line. Both finally arrive at the same flux when their volt-seconds become the same, at 0.3 sec. and 0.425 sec., respectively, as shown by Fig. 1, but the dotted line gives the rotor more time to swing before it is stiffened by the rise in flux.

The electrical torque T_e is proportional to the field flux, and is a function of the rotor angle δ. If this flux remains constant, the relation between T_e and δ is shown in Fig. 3, by Curve A when there is no fault, and by Curve B during the fault. Both Curves A and B will change when the field flux linkages Ψ_d change.

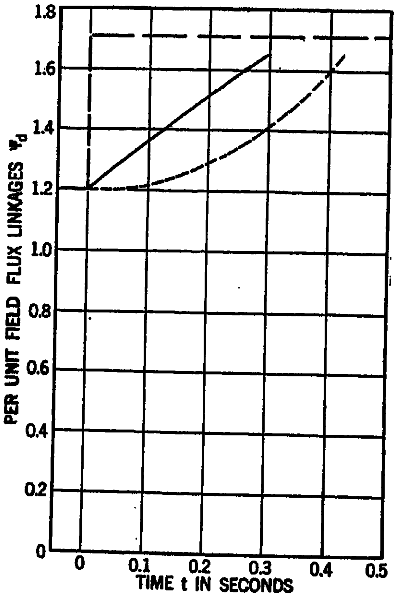

Fig. 2—Field Flux Linkages Produced by Excitation in Fig. 1

Case I in solid lines. Case II in broken lines. Curve in long dashes shows field flux linkages required for perfect stability

Before the fault occurs the electrical torque is T_1, Fig. 3, and neglecting resistance and machine losses, it is also the torque applied to the generator shaft. The prime mover torque may often be considered constant during the transient, as shown by the horizontal line T_1 in Fig. 3. This is especially true of waterwheel units with their large sluggish governors.

Before the fault occurs, the torque-angle characteristic is Curve A, the torque is 2.38, and the generator is operating at point a, a rotor angle of 35.7 deg. The moment the fault is applied, the torque angle characteristic drops to curve B and the electrical torque drops to point b. The difference between input and output torque $(T_1 - T_e) = T_a$ is $a\,b$. The excess torque, $a\,b$, is applied to the generator rotor and moves it toward a new angle of 56 deg. at point c where T_e becomes again equal to T_1.

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*Appendix I of Bibliography 1.

A magnified view of the curves within the dotted rectangle, Fig. 3, is shown in Fig. 4.

If there is no change in field flux, the rotor will move along Curve B. In swinging from b to c, the rotor receives an acceleration which forces it to swing past c to the position d, an angle of 80 deg. This limit, d, of the rotor swing is determined by the areas $a\,b\,c$ and $c\,d\,e$, which must be equal.[2] An area in Figs. 3 or 4, is the product of T_e and δ, and is therefore energy. The area $a\,b\,c$ is the energy input to the rotor, produced by the excess of input over output torque. If the machine is to remain in step, this same energy must be given up by the rotor, and this is done when the rotor swings from c to d, giving to the system the energy represented by the area $c\,d\,e$.

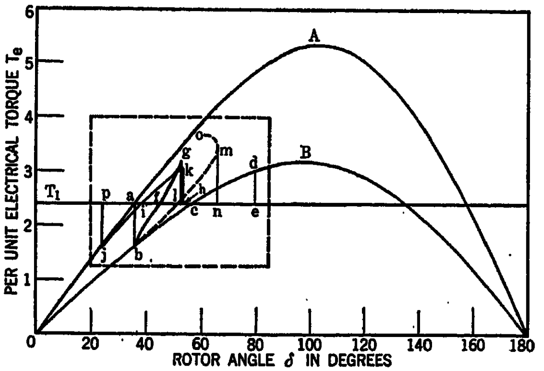

FIG. 3—ELECTRICAL TORQUE BETWEEN GENERATOR AND MOTOR, SHOWN AS A FUNCTION OF THE ELECTRICAL ANGLE BETWEEN THE ROTOR AXES

Neglecting losses and possible hunting of the governor, the rotor will continue to oscillate between positions b and d until some new transient, such as a removal of the fault, takes place.

If the excitation for case I is applied when the fault occurs, the section $b\,d$ of Curve B changes to $b\,g$. The energy input to the rotor is $a\,b\,f$, while the equal energy given up by the rotor is $f\,g\,h$.

The maximum swing of the rotor is at g, an angle of 53 deg. There appears to be no need to increase the field current after the maximum angle has been reached. It is assumed that the regulating equipment planned for this type of excitation would hold this maximum field current constant until the fault has been removed or other conditions warrant its reduction, except that the average field current held after the point g has been reached would be about 5 per cent less than that which would actually exist at point g.

Under these conditions, the rotor will swing back along the smooth Curve $g\,i\,j$ where area $i\,j\,p$ is equal to area $i\,g\,h$. In the next forward swing, since the field current is slightly lower than that which existed at g, the rotor will move along the curve $j\,i\,k$, where area $i\,k\,h$ is equal to area $i\,j\,p$. The rotor will then continue to oscillate between positions j and k in the same manner as previously described under constant flux conditions for the oscillation between b and d.

For the excitation described in Case II, the rotor will swing along curve $b\,l\,m$ where area $l\,m\,n$ is equal to area $a\,b\,l$. The maximum angle in this case is seen to be 66 deg., but since the excitation is permitted to continue rising as shown by the dotted curve, Fig. 1, the field current and, therefore, the field flux is permitted to continue rising after the maximum angle has been reached, and the motor starts its return swing along $m\,o$ instead of along a curve similar to $g\,i\,j$.

Figs. 3 and 4 give the angular swings as functions of the torque but not as functions of time. It is interesting to see how these quantities vary with time.

In Fig. 5, the difference between input and output torques is plotted as a function of time. Its initial value is $a\,b = 0.773$, the same as $a\,b$ in Figs. 3 and 4. An area in Fig. 5, is the product of torque and time and is, therefore, an impulse. But impulse is equal to momentum and:

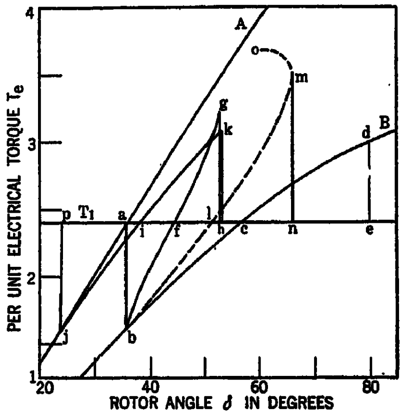

FIG. 4—ELECTRICAL TORQUE BETWEEN GENERATOR AND MOTOR, SHOWN AS A FUNCTION OF THE ELECTRICAL ANGLE BETWEEN THE ROTOR AXES

$$M w = \int T_a\, d t \tag{1}$$

where

t = time.
w = departure from synchronous speed.
M = inertia.
T_a = accelerating torque = $a\,b$.

From (1)

$$w = \frac{1}{M} \int T_a\, d t \tag{2}$$

The integral term $\int T_a\, d t$ is the area shown shaded in Fig. 5. When the positive and negative areas become equal, the total area is zero and w is zero. At the moment when w is zero, the rotor has come to rest and is ready for its return swing. The rotor comes to rest on its outward swing at its maximum angular position, which from Fig. 5, occurs at 0.295 sec. and 0.3625 sec. for Cases I and II respectively.

Integrating the shaded areas in Fig. 5, gives the departure from synchronous speed as shown by (2).

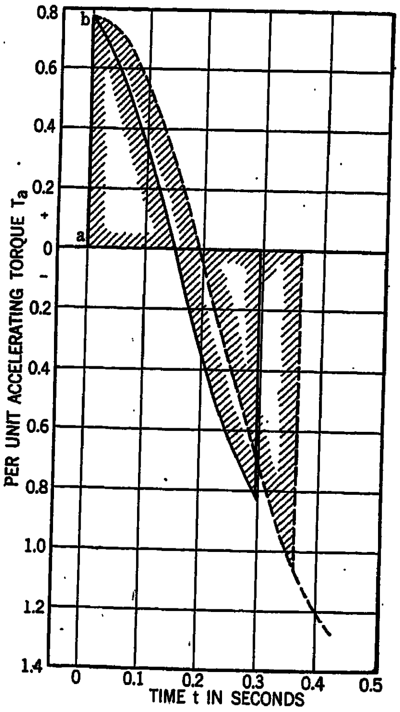

Fig. 5—Difference between Shaft Torque and Electrical Torque on Generator or Motor, shown as a Function of Time

Case I in solid lines. Case II in broken lines

By this means, w is plotted as a function of time in Fig. 6.

An area in Fig. 6 is the product of angular speed and time and is therefore angular displacement, or

$$\delta = \int w \, d t \tag{3}$$

where

δ = angular displacement of the generator rotor from its initial position.

The maximum values of δ occur when the shaded areas in Fig. 6 are positive maximums, and this is seen to occur when w crosses the zero axis at 0.295 sec. and 0.3625 sec. for Cases I and II, respectively.

Performing the integration of (3) by taking the shaded areas in Fig. 6, the angular swings of the rotor are obtained as a function of time, and adding to these angles the initial angle of 35.7 deg. gives the total rotor displacement as a function of time.

The angular swing curves thus obtained are plotted in Fig. 7, where it is seen again that the maximum swings of 53 deg. and 66 deg. occur at 0.295 sec. and 0.3625 sec. for Cases I and II, respectively. It is readily seen that the rotor displacement is greater in Case II than in Case I, and that the excitation for Case I provides the greater stability during the first swing.

The "super-excitation" used in this illustration produces a large and rapid increase in the alternator field flux, and stiffens the machine so that both the magnitude of the rotor swings and the oscillation period are smaller than that which results when less drastic forms of excitation are used.

From the standpoint of system operation and stability after the fault has been removed, the actual use of super-excitation on generators is a mooted question. These calculations include only the first swing and, therefore, do not include any hunting effect. There has been some experimental indication that super-excitation tends to accentuate the hunting, especially on machines not equipped with amortisseur windings, and the calculated improvement in stability may not be fully capable of realization in practise. Moreover, the severity of the fault and the duty on the circuit breakers may be greatly increased, and when the fault is removed, an overvoltage condition may be obtained, that is too great for the connected apparatus.

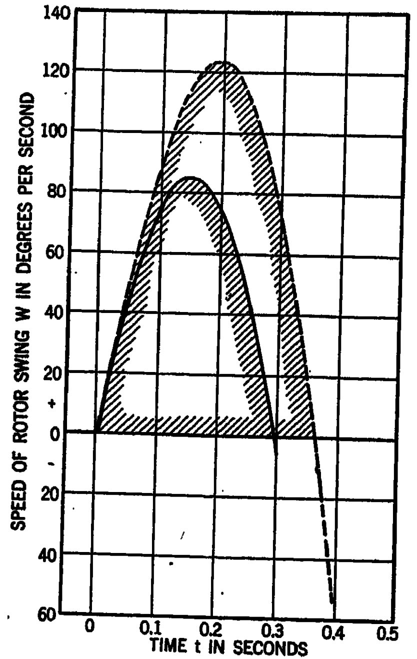

Fig. 6—Speed of Rotor Oscillations, shown as a Function of Time

Case I in solid lines. Case II in broken lines

On the other hand, these considerations may not be found sufficiently important to outweigh the advantages of the scheme, especially when the machine having "super-excitation" is also provided with low resistance amortisseur windings.

Part II

Development of General Swing Curve Formulas

The Impedance Network of the System. A balanced three-phase shunt impedance may be switched across a three-phase circuit at any point where a fault is considered to occur, producing a flow of balanced three-phase power which is exactly equal in magnitude to the flow of power, balanced or unbalanced, resulting from a balanced or unbalanced fault. By the method of "Symmetrical Components"[3,4] any set of three-phase currents, or voltages, balanced or unbalanced, can be divided into three sets of three-phase symmetrical components. One set is balanced and revolves in a positive direction (the positive phase sequence components). The second is balanced and revolves in a positive direction with the sequence of the phases reversed (the negative phase sequence components). In the third set all three vectors are equal in magnitude and lie in the same phase position (the zero phase sequence components). They revolve in a positive direction.

Under balanced conditions the negative and zero phase sequence components are zero.

Each of the three sets of components meets its own particular impedances in the network joining the machines resulting in three distinct types of networks.

The total electrical power is the sum of that produced by the positive phase sequence current flowing in the positive phase sequence network, the negative phase sequence current flowing in the negative phase sequence network, and the zero phase sequence current flowing in the zero phase sequence network.

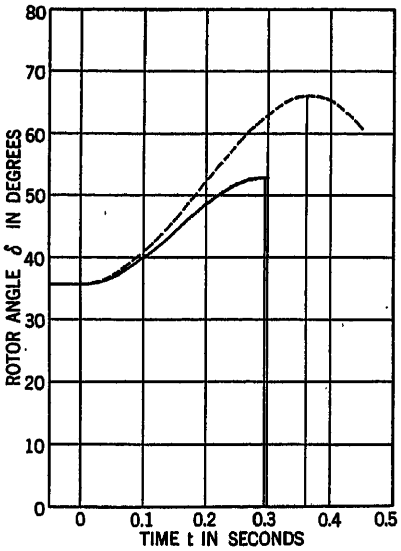

Fig. 7—Electrical Angle between Rotor Axes of Generator and Motor, shown as a Function of Time

Case I in solid lines. Case II in broken lines

The balanced network which gives a flow of power equal in magnitude to that resulting from a fault, balanced or unbalanced, is obtained by placing a balanced three-phase impedance **Z** at the point of fault. Where

$$\mathbf{Z} = \mathbf{Z}_n + \mathbf{Z}_z \qquad (4)$$

if the fault is from one conductor to ground.

$$\mathbf{Z} = \mathbf{Z}_n \qquad (5)$$

if the fault is between two conductors.

$$\mathbf{Z} = \frac{\mathbf{Z}_n\,\mathbf{Z}_z}{\mathbf{Z}_n + \mathbf{Z}_z} \qquad (6)$$

if the fault is from two conductors to ground.

$$\mathbf{Z} = 0 \qquad (7)$$

if the fault is between all three conductors.

$\mathbf{Z}_n$ = Total negative phase sequence impedance of the system, viewed from the point of fault.

$\mathbf{Z}_z$ = Total zero phase sequence impedance of the system viewed from the point of fault.

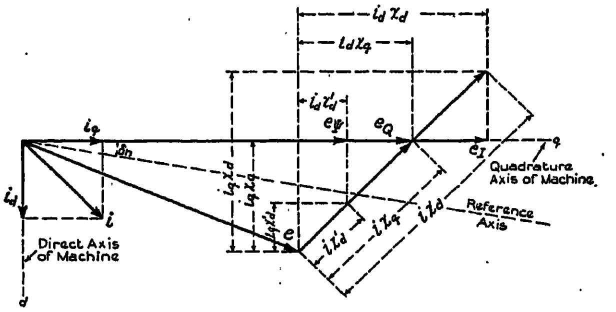

Fig. 8—Vector Diagram of Stator Voltages and Current of a Synchronous Alternator

The Relations between the Electrical Quantities of All Machines on the Network. The excitation voltage $\mathbf{e}_I$ of an alternator is the voltage "behind" the synchronous impedance of the machine projected on the quadrature axis. It is the voltage which would be generated at no load and no saturation when the load field current flows in the rotor winding. It is proportional to the field current I and in percentages it is equal to the field current I. Likewise $\mathbf{e}_Q$ is the voltage "behind" the quadrature impedance projected on the quadrature axis, and $\mathbf{e}_\Psi$ is the voltage "behind" the transient impedance projected on the quadrature axis.

The direct component Ψ_d of the field flux linkages is proportional to $\mathbf{e}_\Psi$.

The slip ring voltage is E.

The stator terminal voltage is $\mathbf{e}$.

The stator current is $\mathbf{i}$. Its projection on the quadrature axis is the quadrature component $\mathbf{i}_q$ and its projection on the direct axis is the direct component $\mathbf{i}_d$.

Unit terminal voltage may be any value decided on but is usually taken as the rated voltage of the machine.

Unit field current is the field current which will produce unit stator voltage at unit speed (taken here as rated speed of the machine) at no saturation and at no load.

Unit field flux linkage is the field flux linkage which will produce unit stator voltage at rated speed and at no load.

Unit slip ring voltage is the slip ring voltage required to give unit field current under steady state conditions.

Unit power is the base power selected.

Fig. 8 is a vector diagram showing voltages and currents for any one machine. Its quadrature axis is designated by q and its direct axis is designated by d. The diagram is a true time diagram of stator voltages and currents, and is sufficient to calculate the flow of power into or out of any of the machines on the network.

The machines are assumed to have zero time constants in their quadrature axes.

It will be seen that this diagram is the same as that derived by Park in his paper on *Two-Reaction Theory*[9] except that resistance in the machine has been neglected, and voltage due to the rate of change of linkages Ψ has been neglected. Practically these quantities are negligible in most cases. A notable example is that of "negative damping," a phenomenon involving cumulative rotor oscillations. This has been described by Nickle and Pierce in the paper, *Effect of Armature Resistance*.[12] Thus the step-by-step procedure given here cannot throw any light on this phenomenon, its causes having been neglected.

The diagram is a time diagram showing voltages. Practically in figuring torque it is easier to do so in terms of linkages, because torque can be expressed in terms of linkages without involving speed. However, the change in speed during a machine swing is small as long as the machine remains in step. Even if it were not small the correct torque could be obtained by using linkages instead of voltages. For purposes of making the formulas consistent with the usual conception of flow of current due to voltages, they are expressed in terms of voltage with the understanding that the magnitude of the voltages used is taken proportional to linkages in order to minimize the errors. When this is done there would be no error due to change in speed if there were no resistance involved. Practically the error due to the presence of resistance is very small. An illustration of this is the familiar phenomenon of the steady short circuit current of an alternator remaining constant as the speed drops to a low value, though the generated voltage is proportional to speed. The linkages are constant, so the current is constant, because the resistance is negligible.

Consider any s number of synchronous machines connected to an impedance network and let the machines be designated by the numbers 1, 2, 3, 4 . . . s.

Currents flowing from a machine will be taken as positive while currents flowing into a machine will be taken as negative.

The subscripts have the following significances:

When one subscript only occurs it designates the machine to which the quantity having the subscript applies.

For stator currents $\mathbf{i}$ the first subscript designates the machine from which the current is flowing while the second subscript designates the machine into which the current is flowing. For impedances $\mathbf{Z}$ two duplicate subscripts designate the driving point impedance of the machine designated by the subscripts, while two unlike subscripts designate the transfer impedance between the machines designated by subscripts.

The driving point impedance is the total impedance of the system from the point taken. The transfer impedance is the impedance between any two machines. Thus $\mathbf{Z}_{11}$ is the driving point impedance of machine No. 1, while $\mathbf{Z}_{21}$ is the transfer impedance between machines 1 and 2.

A rotor angle δ having only one subscript is the angle between the rotor axis of the machine designated by the subscript, and the reference vector. A rotor angle δ having two subscripts is the angle between the rotor axes of the two machines designated by the two subscripts and is mathematically equal to the angle between the rotor of the machine designated by the first subscript and the reference vector, minus the angle between the rotor of the machine designated by the second subscript and the reference vector.

The angle θ is an impedance angle and the subscripts of θ are the same as those of the impedance $\mathbf{Z}$ to which θ applies.

The movement of the rotor of each machine will be measured from some arbitrary reference vector.

Formulas will be written for the nth machine.

From Fig. 8 the following relations may be obtained.

$$\mathbf{e}_{Qn} = \mathbf{e}_{In} - (x_{dn} - x_{qn})\,(\mathbf{i}_n \,.\, \mathbf{d}_n)\,\mathbf{q}_n \tag{8}$$

where $\mathbf{d}_n$ and $\mathbf{q}_n$ are unit vectors along the direct and quadrature axes respectively of machine n.

$$\mathbf{e}_{\Psi n} = \mathbf{e}_{In} - (x_{dn} - x_{dn}')\,(\mathbf{i}_n \,.\, \mathbf{d}_n)\,\mathbf{q}_n \tag{9}$$

Combining (8) and (9)

$$\mathbf{e}_{\Psi n} = \mathbf{e}_{Qn} - (x_{qn} - x_{dn}')\,(\mathbf{i}_n \,.\, \mathbf{d}_n)\,\mathbf{q}_n \tag{10}$$

The stator currents[8] are

$$\mathbf{i}_n = -\,\mathbf{i}_{1n} - \mathbf{i}_{2n} \;.\;.\;.\; + \mathbf{i}_{nn} \;.\;.\;.\; - \mathbf{i}_{sn} \tag{11}$$

where

$$\mathbf{i}_{mn} = \frac{\mathbf{e}_{Qm}}{\mathbf{Z}_{mn}} \tag{12}$$

Since $\mathbf{e}_Q$ exists only along the quadrature axis we may also write

$$\mathbf{e}_{Qn} = (\mathbf{e}_{Qn} \,.\, \mathbf{q}_n)\,\mathbf{q}_n \tag{13}$$

Substituting (11) and (12) in (10)

$$\mathbf{e}_{\Psi n} = \mathbf{e}_{Qn} - \left[(x_{qn} - x_{dn}') \left(\frac{\mathbf{e}_{Qn}}{\mathbf{Z}_{nn}} \;.\;.\;.\; - \frac{\mathbf{e}_{Q1}}{\mathbf{Z}_{1n}} \;.\;.\;.\; - \frac{\mathbf{e}_{Qs}}{\mathbf{Z}_{sn}} \right) .\, \mathbf{d}_n \right] \mathbf{q}_n \tag{14}$$

Let

$$\theta_{mn} = \text{impedance angle of } \mathbf{Z}_{mn} \tag{15}$$

$$\alpha_{mn} = 90° - \theta_{mn} \tag{16}$$

$$\delta_m = \text{angle of } e_{Qm} \text{ with the reference axis} \tag{17}$$

$$\delta_{mn} = \delta_m - \delta_n \tag{18}$$

Then

$$\left[\left(\frac{\mathbf{e}_{Qm}}{\mathbf{Z}_{mn}} \,.\, \mathbf{d}_n \right) \mathbf{q}_n \right] .\, \mathbf{q}_n$$

$$= \frac{e_{Qm}}{Z_{mn}} \operatorname{Cos}(\delta_{mn} + 90° - \theta_{mn})$$

$$= \frac{e_{Qm}}{Z_{mn}} \operatorname{Cos}(\delta_{mn} + \alpha_{mn}) \tag{19}$$

Therefore we may write:

$$\frac{e_{\Psi n}}{x_{qn} - x_{dn}'} = \left[\frac{1}{x_{qn} - x_{dn}'} - \frac{\operatorname{Cos}\alpha_{nn}}{Z_{nn}}\right] e_{Qn} + \sum_{m=1}^{m=n-1} \frac{e_{Qm}}{Z_{mn}} \operatorname{Cos}(\delta_{mn} + \alpha_{mn}) + \sum_{m=n+1}^{m=s} \frac{e_{Qm}}{Z_{nm}} \operatorname{Cos}(\delta_{nm} - \alpha_{nm}) \tag{20}$$

This is the equation for the nth machine out of s, with m representing all the other machines. There is a total of s equations like this, one for each machine.

Now the torque may be taken

$$T_{en} = \mathbf{e}_{Qn} \,.\, \mathbf{i}_n \tag{21}$$

Substituting

$$T_{en} = \frac{e_{Qn}^2}{Z_{nn}} \operatorname{Sin}\alpha_{nn} - \sum_{m=1}^{m=n-1} \frac{e_{Qm}\, e_{Qn}}{Z_{mn}} \operatorname{Sin}(\delta_{mn} + \alpha_{mn}) + \sum_{m=n+1}^{m=s} \frac{e_{Qn}\, e_{Qm}}{Z_{nm}} \operatorname{Sin}(\delta_{nm} - \alpha_{nm}) \tag{22}$$

In order to simplify writing the equations let

$$\frac{e_{\Psi n}}{x_{qn} - x_{dn}'} = W_n \tag{23}$$

$$\frac{1}{x_{qn} - x_{dn}'} - \frac{\operatorname{Cos}\alpha_{nn}}{Z_{nn}} = \mu_n \tag{24}$$

$$\frac{\operatorname{Cos}(\delta_{mn} - \alpha_{mn})}{Z_{mn}} = A_{mn} \tag{25}$$

$$\frac{\operatorname{Cos}(\delta_{mn} + \alpha_{mn})}{Z_{mn}} = B_{mn} \tag{26}$$

Then Equations (27) may be written

$$\left.\begin{array}{l} W_1 = \mu_1 e_{Q1} + A_{12} e_{Q2} + \ldots A_{1s} e_{Qs} \\ W_2 = B_{12} e_{Q1} + \mu_2 e_{Q2} + \ldots A_{2s} e_{Qs} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ W_s = B_{1s} e_{Q1} + B_{2s} e_{Q2} + \ldots \mu_s \,.\, e_{Qs} \end{array}\right\} \tag{27}$$

and solved

$$e_{Qn} = \frac{\begin{vmatrix} \mu_1 & \ldots & W_1 & \ldots & A_{1s} \\ B_{12} & \ldots & W_2 & \ldots & A_{2s} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ B_{1s} & \ldots & W_s & \ldots & \mu_s \end{vmatrix}}{\begin{vmatrix} \mu_1 & A_{12} & A_{13} & \ldots & A_{1s} \\ B_{12} & \mu_2 & A_{23} & \ldots & A_{2s} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ B_{1s} & B_{2s} & B_{3s} & \ldots & \mu_s \end{vmatrix}} \tag{28}$$

where the W column replaces the nth column in the numerator.

The Open Circuit Time Constant of a Synchronous Alternator. This time constant T_o may be calculated from the design constants of the machine, or it may be obtained experimentally by a number of methods. The following is suggested.

With the machine running at normal speed, and with its stator open circuited, build up the stator voltage to about half the rated voltage of the alternator. Let this voltage be e_1.

When e_1 becomes constant, suddenly short circuit the slip rings (a resistance should be placed between the slip rings and exciter armature to prevent short circuiting the exciter) and take an oscillograph record of the change in the stator voltage. This voltage will be:

$$e = e_1\, \epsilon^{-\frac{t}{T_o}} + e_o \tag{29}$$

where e_o is the residual stator voltage.

When $t = T_o$

$$e = \frac{e_1}{\epsilon} + e_o = 0.368\, e_1 + e_o \tag{30}$$

The time in seconds required for e to drop from e_1 to the value given by (30) is T_o.

The Per Unit Inertia Constant. The swing of the rotors is a function of their $W R^2$ s. In the per unit system the $W R^2$ must be converted to an inertia[2] constant M.

Apply unit torque T_u to the rotor when it is at rest, until the machine reaches unit speed.

$$T_u = M \frac{d\, W}{d\, t} \tag{31}$$

where

W is per unit angular speed of the rotor, t is time, and M is a constant.

Integrating

$$t = \frac{M}{T_u} \int_0^1 d\, W = \frac{M}{T_u} \tag{32}$$

but T_u is unity and therefore

$$t = M \tag{33}$$

Therefore M is the time in seconds required to bring the machine from rest to unit speed when unit torque is applied to the shaft. With this definition the value of M in terms of the $W R^2$ may be obtained.

Let

$\mathfrak{T}$ = rotor torque in ft-lb.

ω = rotor speed in radians per second

t = time in seconds

g = acceleration of gravity

$$\mathfrak{T}_u = \frac{W R^2}{g} \frac{d\, \omega}{d\, t} \tag{34}$$

$$t = \frac{W R^2}{\mathfrak{T}_u g} \int_0^{\omega_u} d\,\omega \tag{35}$$

where $\mathfrak{T}_u$ is the torque required to give unit or base power at rated or unit speed, and ω_u is the speed of the rotor in radians per second at rated speed. Under these conditions $t = M$ and integrating (35)

$$M = \frac{W R^2\,\omega_u}{\mathfrak{T}_u g} = \frac{W R^2\,(\omega_u)^2}{g P_u} \tag{36}$$

where P_u is unit or base power in ft-lb. per sec. It is convenient to change P_u to base $K\,W$ and substituting the proper constants (36) becomes

$$M = \frac{(W R^2)\left(\frac{2\,\pi\,n}{60}\right)^2}{(32.2)\ (\text{Base } K\,W)\ (737.6)}$$

$$= \frac{0.462\ (W R^2)\left(\frac{n}{1000}\right)^2}{\text{Base } K\,W} \tag{37}$$

where n is the rated speed of the machine in rev. per min. and $W R^2$ is the weight in pounds of the rotating part times the square of its radius of gyration.

Where two or more machines are assumed to swing together and thus behave as a single unit, the inertia constant M of this fictitious unit is the sum of the inertia constants of the individual units which it represents.

Calculation of the Change in Rotor Angle during Any One of the Step-by-Step[2] *Intervals of Time.* Let the electrical torque T_{en} change suddenly at the instant n from T_{en}' to T_{en}''. It is assumed that T_{en}' is constant from the instant $n - 1/2$ to n, a total time of $\frac{\Delta t}{2}$, and that T_{em}'' is constant from n to $n + 1/2$, a total time of $\frac{\Delta t}{2}$. Thus T_{en}' produces an average change $\Delta\,S_{(n-1/4)}$ in speed during a time $\frac{\Delta t}{2}$, and T_{en}'' produces an average change $\Delta\,S_{(n+1/4)}$ in speed during a time $\frac{\Delta t}{2}$.

Remembering that the increment of time during which T_{en}' or T_{en}'' is applied is $\frac{\Delta t}{2}$ we have

$$\Delta\,S_{(n-1/4)} = \frac{\Delta t}{2\,M}\,(T_1 - T_{en}') \tag{38}$$

and

$$\Delta\,S_{(n+1/4)} = \frac{\Delta t}{2\,M}\,(T_1 - T_{en}'') \tag{39}$$

where T_1 is the mechanical torque applied to the shaft at the instant n.

At the instant $(n + 1/2)$ the speed is:

$$S_{(n+1/2)} = S_{(n-1/2)} + \Delta\,S_{(n-1/4)} + \Delta\,S_{(n+1/4)} \tag{40}$$

Substituting (38) and (39) in (40)

$$S_{(n+1/2)} = S_{(n-1/2)} + \frac{\Delta t}{M}\left(T_1 - \frac{T_{en}' + T_{en}''}{2}\right) \tag{41}$$

The rotor angle δ at the instant $n + 1$ is:

$$\delta_{(n+1)} = \delta_n + \Delta\,\delta_{(n+1)} \tag{42}$$

Where $\Delta\,\delta_{(n+1)}$ is the increase in δ at the instant $(n + 1)$

The average speed between the instants n and $(n + 1)$ is:

$$S_{(n+1/2)} = \frac{\Delta\,\delta_{(n+1)}}{\Delta t} \tag{43}$$

which is the slope of the angle time curve between the instants n and $(n + 1)$.

Rearranging (43)

$$\Delta\,\delta_{(n+1)} = (\Delta\,t)\ S_{(n+1/2)} \tag{44}$$

Substituting (41) in (44)

$$\Delta\,\delta_{(n+1)} = (\Delta\,t)\ S_{(n-1/2)} + \frac{(\Delta\,t)^2}{M}\left(T_1 - \frac{T_{en}' + T_{en}''}{2}\right) \tag{45}$$

From (44)

$$\Delta\,\delta_n = (\Delta\,t)\ S_{(n-1/2)} \tag{35}$$

Substituting (46) in (45)

$$\Delta\,\delta_{(n+1)} = \Delta\,\delta_n + \frac{(\Delta\,t)^2}{M}\left(T_1 - \frac{T_{en}' + T_{en}''}{2}\right) \tag{47}$$

Since all terms in (47) are per unit quantities the term $\frac{(\Delta\,t)^2}{M}\left(T_1 - \frac{T_{en}' + T_{en}''}{2}\right)$ is a per unit angle. Unit angle is the electrical angle through which the rotor moves at normal or unit speed in unit time or one second. Hence:

$$\text{Unit angle} = 360\,f \text{ degrees} \tag{48}$$

where f = normal frequency.

If (47) is to be in degrees we have from (48),

$$\Delta\,\delta_{(n+1)} = \Delta\,\delta_n + \frac{360\,f\,(\Delta\,t)^2}{M}\left(T_1 - \frac{T_{en}' + T_{en}''}{2}\right) \tag{49}$$

Letting

$$\frac{360\,f\,(\Delta\,t)^2}{M} = k \tag{50}$$

(49) becomes:

$$\Delta\,\delta_{(n+1)} = \Delta\,\delta_n + k\left(T_1 - \frac{T_{en}' + T_{en}''}{2}\right) \tag{51}$$

It is convenient to let the accelerating torque $\left(T_1 - \frac{T_{en}' + T_{en}''}{2}\right)$ in (51) be

$$\left(T_1 - \frac{T_{en}' + T_{en}''}{2} \right) = T_{an} \qquad (52)$$

and (51) becomes:

$$\Delta\, \delta_{(n+1)} = \Delta\, \delta_n + k\, T_{an} \qquad (53)$$

When there is no discontinuous change in T_{en} we have $T_{en} = T_{en}' = T_{en}''$ and (52) becomes $(T_1 - T_{en})$, whence (51) becomes:

$$\Delta\, \delta_{(n+1)} = \Delta\, \delta_n + k\, (T_1 - T_{en}) \qquad (54)$$

The swing starts as the result of a discontinuous change in T_{en}. Under these conditions $\Delta\, \delta_n = \Delta\, \delta_o = 0$, $T_{en}' = T_1$, $T_{en}'' = T_{en}$, and $\Delta\, \delta_{(n+1)} = \Delta\, \delta_1$, whence (51) becomes

$$\Delta\, \delta_1 = \frac{k}{2}\, (T_1 - T_{en}) \qquad (55)$$

Any change in the governor position changes T_1 and T_1 may be altered accordingly if the governor characteristics are known. If electrical damping occurs, such as would be produced by an amortisseur winding, it may be included as shown by R. H. Park and E. H. Bancker in Appendix XII of Bibliography 2. The derivations of (54) and (55), which are special forms of (51), are also given in Appendix XII of Bibliography 2.

In calculating the swing curve (55) is used for the first step, (54) is used for subsequent steps until switching or the removal of the fault, or both take place, when (51) is used for the first step following the change in T_{en}, after which (54) is used again in the subsequent steps until another discontinuous change takes place in T_{en} when (51) is again used as described above, and so on.

Procedure for the Calculation of the Swing Curves. The procedure for making a step-by-step calculation may be summed up as follows:

I. Determine $e_{Q1}\, e_{Q2}\, \ldots\, e_{Qs}$ and $\delta_1\, \delta_2\, \ldots\, \delta_s$ from steady state conditions before the fault is applied in the same manner that steady state voltages are usually found, considering that e_Q is the voltage existing behind the quadrature reactance of the machine.

II. Find $e_{\Psi n}$ before the fault is applied from Equation (20).

III. Apply fault and determine new driving point and transfer impedances.

IV. Determine new value of e_{Qn} from (28), letting $e_{\Psi n}$ remain constant and using new values of impedances. Later when this step is repeated after an angle change the new angles are used instead of the new impedances.

V. Find T_{en} from (22) using e_{Qn} just obtained.

VI. Determine e_{In} from

$$e_{In} = e_{Qn} \left(\frac{x_{dn} - x_{dn}'}{x_{qn} - x_{dn}'} \right) - e_{\Psi n} \left(\frac{x_{dn} - x_{qn}}{x_{qn} - x_{dn}'} \right) \qquad (56)$$

This formula is obtained by combining (8) and (9)

VII. Calculate change in $e_{\Psi n}$ from

$$\Delta\, e_{\Psi n} = \frac{(E_n - e_{In})\, \Delta\, t}{T_{on}} \qquad (57)$$

where $\Delta\, t$ is the increment of time used for one step; E_n is the average slip ring volts, during the $\Delta\, t$ considered; e_{In} is the value of the synchronous stator voltage at the beginning of the $\Delta\, t$ considered; T_{on} is the open circuit time constant of the machine; and $\Delta\, e_{\Psi n}$ is the change which takes place in $e_{\Psi n}$ during the $\Delta\, t$ considered.

This formula[9] comes directly from the well known relation that the applied voltage is equal to the resistance drop plus the inductive drop, the quantities being all in per unit values.

VIII. Calculate new value of $e_{\Psi n}$ from $e_{\Psi n}$ found in II and $\Delta\, e_{\Psi n}$ found in VII.

$$\underset{\text{new}}{e_{\Psi n}} = \underset{\text{II.}}{e_{\Psi n}} + \underset{\text{VII.}}{\Delta\, e_{\Psi n}} \qquad (58)$$

IX. Calculate change in angle from (55). The other angular changes are to be calculated as explained in the discussion of (55), (54), and (51).

X. Allow the angle to shift and repeat IV to X.

Appendices I and II of the paper will be devoted to illustrating the procedure by means of a special example. The general procedure is modified in this special case for convenience in illustrating fully each step.

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Appendix I

THE ANALYSIS OF A SIMPLIFIED CASE

The Arrangement of the System to be Studied. The system to be studied usually consists of a network of transmission lines with step-up and step-down transformers at various points where power is supplied to the system or delivered to load centers. While practically all contributing factors affecting the swings of alternator rotors can be included in the step-by-step method, certain reasonable assumptions will greatly simplify and reduce the amount of work required in the calculations.

When the stability of a given generating plant is being considered, it is often possible to reduce the power system to a single generator feeding power through an impedance to a single equivalent motor. These generators may be waterwheel units with quick response excitation. Their salient pole characteristics and the effect of changes in their excitation will be included in the calculations. The single equivalent motor represents the system that is fed by the generating station and may be condensed to one equivalent round rotor machine whose transient reactance is equal to that of the system as viewed from the point of fault and having a constant voltage behind its reactance. When the machines in the generating plant are similar in characteristics they will all behave in a similar manner and the entire generating station may be treated as a single machine.

It is assumed that a hydroelectric generating station feeds power to a system over two paralleled transmission lines. At the terminals of the transmission lines are transformer banks with the neutrals of their high-voltage windings solidly grounded. A single conductor to ground fault is taken on one of the lines and near the high-voltage bus of the generating station. The generators are essentially duplicates and are treated as a single equivalent machine. It is assumed that the resistances are small enough to neglect, and reactances only are considered.

It is not probable that the gate opening of the waterwheels will be able to change much in one or two seconds and the driving torque applied to the shaft of the generators is, therefore, assumed constant for this length of time. The load on the equivalent motor is likewise assumed constant.

The reactances of the "set up" with the shunt x which represents the fault condition, are shown in Fig. 9.

The value of x is obtained from (4).

The voltage (assumed constant) of the motor is $e_{\Psi b}$.

There is no generated voltage in the negative or zero phase sequence networks and since there is no resistance, there is no power in these two networks. The only electrical power, then, is that produced by the positive phase sequence current flowing in the positive phase sequence network in Fig. 9.

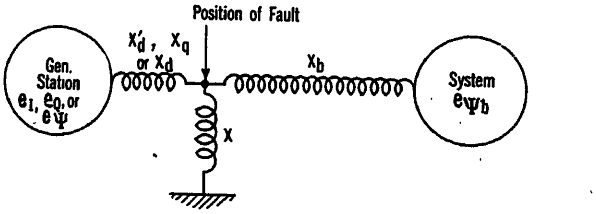

FIG. 9—SIMULATION OF FAULT ON LINE BETWEEN GENERATOR AND MOTOR BY PLACING A BALANCED THREE-PHASE SHUNT AT THE POSITION OF THE FAULT

The currents and voltages in Fig. 9 are perfectly balanced and the resulting balanced three-phase power flow is equal in magnitude to the actual unbalanced power exchange between the generator and motor.

The Vector Diagram[2, 5, 9] *Showing the Electrical Quantities and Angles to be Studied.* The vector diagram of the quantities to be considered is shown in Fig. 10, where V is the voltage at any point between the generator and the motor. For convenience it is taken here at the point where the fault is to be applied.

The generator current i is divided into two components, the direct component i_d in phase with the generator rotor direct axis and the quadrature component i_q at right angles to the generator rotor direct axis.

The angle between the rotors of the generator and motor is δ. The power factor angle is α where V is measured.

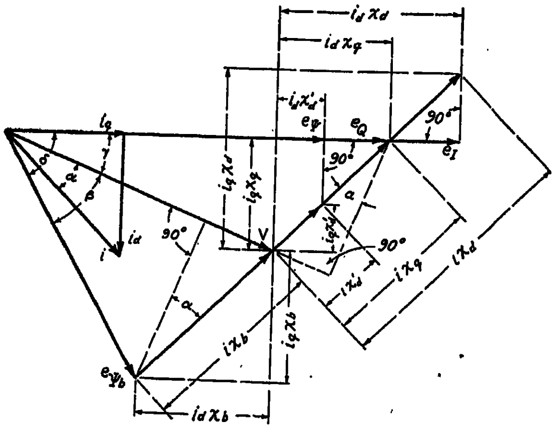

FIG. 10—VECTOR DIAGRAM OF VOLTAGES AND CURRENT WHEN A SALIENT POLE GENERATOR FEEDS A NON-SALIENT POLE MOTOR

The Flow of Power Between the Generating Station and the System Which it Feeds. Since there are only 2 machines $s = 2$ and from (28)

$$e_Q = \frac{\begin{vmatrix} W_1 & A_{12} \\ W_2 & \mu_2 \end{vmatrix}}{\begin{vmatrix} \mu_1 & A_{12} \\ B_{12} & \mu_2 \end{vmatrix}} = \frac{W_1\,\mu_2 - W_2\,A_{12}}{\mu_1\,\mu_2 - A_{12}\,B_{12}} \qquad (59)$$

From (23), (24), (25), and (26) we have before the fault is applied (Fig. 9 without the shunt x)

$$W_1 = \frac{e_\Psi}{(x_q - x_d')} \qquad (60)$$

$$\mu_1 = \left[\frac{1}{(x_q - x_d')} - \frac{1}{(x_q + x_b)}\right] \qquad (61)$$

since $\alpha_{11} = 90 - \theta_{11} = 0$; $\cos\alpha_{11} = 1$; and $Z_{11} = x_q + x_b$.

$$A_{12} = \frac{\cos\delta}{(x_q + x_b)} = B_{12} \qquad (62)$$

$$W_2 = \frac{e_{\Psi b}}{(x_{qb} - x_{db}')} \qquad (63)$$

$$\mu_2 = \left[\frac{1}{(x_{qb} - x_{db}')} - \frac{1}{(x_q + x_b)}\right] \qquad (64)$$

In (63) and (64) the subscript b designates quantities which belong to the motor. Since the motor has non-salient poles $x_{qb} = x_{db}$ and $e_{Ib} = e_{Qb}$. Since the flux linkages behind the transient reactance of the motor are constant $x_{qb} = x_{db}' = x_{db}$ and $e_{Qb} = e_{\Psi b} =$ a constant.

Substituting (**60**) . . (**64**) in (**59**)

$$e_Q = \frac{\left[\dfrac{e_\Psi}{(x_q - x_d')}\right]\left[\dfrac{1}{(x_{qb} - x_{db}')} - \dfrac{1}{(x_q + x_b)}\right] - \left[\dfrac{e_{\Psi b}}{(x_{qb} - x_{db}')}\right]\left[\dfrac{\cos\delta}{(x_q + x_b)}\right]}{\left[\dfrac{1}{(x_q - x_d')} - \dfrac{1}{(x_q + x_b)}\right]\left[\dfrac{1}{(x_{qb} - x_{db}')} - \dfrac{1}{(x_q + x_b)}\right] - \left[\dfrac{\cos\delta}{(x_q + x_b)}\right]^2} \qquad (65)$$

Multiplying numerator and denominator of (**65**) by $(x_{qb} - x_{db}')$, letting this term be zero since $x_{qb} = x_{db}'$, and simplifying.

$$e_Q = e_\Psi \frac{(x_q + x_b)}{(x_d' + x_b)} - e_{\Psi b} \frac{(x_q - x_d')}{(x_d' + x_b)} \cos\delta \qquad (66)$$

Formula (**66**) was developed to demonstrate the application of the general formulas in Part II to any swing curve calculation. It is seen that (**66**) is exactly the same as (**84**).

It happens that the simplifications possible in the particular example that has been selected make it easier to develop the necessary formulas directly from the vector diagram, Fig. 10, and from this point on the formulas will therefore be developed from this diagram. Any one of them, however, can be obtained from the general method given in Part II. The additional method also gives an additional point of view and aids in visualizing the various phenomena.

The power flow in Fig. 10 is the product of V and the projection of i on V. Since there is no resistance, the power is the same at any place along the line and is equal to the product of the voltage at the point taken and the projection of the current on that voltage. Therefore, if the power P_e be measured at e_Q it will be:

$$P_e = e_Q\, i_q \qquad (67)$$

From Fig. 10 we have before the fault is applied

$$e_Q = e_I - i_d\,(x_d - x_q) \qquad (68)$$

$$e_I = e_{\Psi b} \cos\delta + i_d\,(x_d + x_b) \qquad (69)$$

$$i_d = \frac{e_I - e_{\Psi b}\cos\delta}{(x_d + x_b)} \qquad (70)$$

$$e_{\Psi b} \sin\delta = i_q\,(x_q + x_b) \qquad (71)$$

$$i_q = \frac{e_{\Psi b}\sin\delta}{(x_q + x_b)} \qquad (72)$$

Substituting (**70**) in (**68**),

$$e_Q = e_I - \frac{(x_d - x_q)}{(x_d + x_b)}\,(e_I - e_{\Psi b}\cos\delta) \qquad (73)$$

$$e_Q = e_I \frac{(x_q + x_b)}{(x_d + x_b)} + \frac{e_{\Psi b}\,(x_d - x_q)\cos\delta}{(x_d + x_b)} \qquad (74)$$

Substituting (**72**) and (**74**) in (**67**),

$$P_e = \frac{e_I\, e_{\Psi b}}{(x_d + x_b)} \sin\delta + \frac{e_{\Psi b}^2\,(x_d - x_q)\cos\delta\sin\delta}{(x_d + x_b)\,(x_q + x_b)} \qquad (75)$$

$$P_e = \frac{e_I\, e_{\Psi b}}{(x_d + x_b)} \sin\delta + \frac{e_{\Psi b}^2\,(x_d - x_q)}{2\,(x_d + x_b)\,(x_q + x_b)} \sin 2\delta \qquad (76)$$

In (**76**) the power flow is given in terms of the generator synchronous voltage e_I, the system voltage, the rotor angle, and some of the network reactances.

We may conveniently write (**76**) as

$$P_e = e_I H + N \qquad (77)$$

where

$$H = \frac{e_{\Psi b}}{(x_d + x_b)} \sin\delta \qquad (78)$$

$$N = \frac{e_{\Psi b}^2\,(x_d - x_q)}{2\,(x_d + x_b)\,(x_q + x_b)} \sin 2\delta \qquad (79)$$

H and N may be plotted against δ and their values used in (**14**) to obtain the power inasmuch as e_I and δ are calculated in the step-by-step process (see Figs. 15 and 16, curves II). A more detailed explanation is given later.

The power may be also expressed in terms of the e_Ψ in place of the e_I.

From Fig. 10,

$$e_Q = e_\Psi + i_d\,(x_q - x_d') \qquad (80)$$

$$e_\Psi = e_{\Psi b}\cos\delta + i_d\,(x_d' + x_b) \qquad (81)$$

$$i_d = \frac{e_\Psi - e_{\Psi b}\cos\delta}{(x_d' + x_b)} \qquad (82)$$

Substituting (**82**) in (**80**),

$$e_Q = e_\Psi + \frac{(x_q - x_d')}{(x_d' + x_b)}\,(e_\Psi - e_{\Psi b}\cos\delta) \qquad (83)$$

$$e_Q = e_\Psi \frac{(x_q + x_b)}{(x_d' + x_b)} - e_{\Psi b} \frac{(x_q - x_d')}{(x_d' + x_b)} \cos\delta \qquad (84)$$

Substituting (**72**) and (**84**) in (**67**),

$$P_e = \frac{e_\Psi\, e_{\Psi b}}{(x_d' + x_b)} \sin\delta - \frac{e_{\Psi b}^2\,(x_q - x_d')}{2\,(x_q + x_b)\,(x_d' + x_b)} \sin 2\delta \qquad (85)$$

Curve *A*, Figs. 3 and 4, are plotted from (**85**).

To determine δ we have from Fig. 10,

$$\delta = \beta + \gamma \qquad (86)$$

$$\beta = \tan^{-1} \frac{i\, x_b \cos\alpha}{V - i\, x_b \sin\alpha} \qquad (87)$$

$$\gamma = \tan^{-1} \frac{i\, x_q \cos\alpha}{V + i\, x_q \sin\alpha} \qquad (88)$$

$$\delta = \tan^{-1} \frac{i\, x_b \cos\alpha}{V - i\, x_b \sin\alpha} + \tan^{-1} \frac{i\, x_q \cos\alpha}{V + i\, x_q \sin\alpha} \qquad (89)$$

When the shunt x is applied in Fig. 9, the generator current may be found by superposition.

Without the shunt, the current is that produced by the vector difference between $\mathbf{e}_Q$ and $\mathbf{e}_{\Psi b}$ applied to $(\mathbf{x}_q + \mathbf{x}_b)$.

With the shunt

$$\dot{\mathbf{i}} = \dot{\mathbf{i}}_{QQ} - \dot{\mathbf{i}}_{\Psi_b Q} \qquad (90)$$

where $\mathbf{i}_{QQ}$ is the generator current produced by $\mathbf{e}_Q$ only and $\mathbf{i}_{\Psi_b Q}$ is the generator current produced by $\mathbf{e}_{\Psi b}$ only.

Let the current flowing from the generator be positive.

Letting $\mathbf{e}_{\Psi b}$ be zero:

$$\dot{\mathbf{i}}_{QQ} = \frac{\mathbf{e}_Q}{\mathbf{x}_q + \dfrac{\mathbf{x}\,\mathbf{x}_b}{\mathbf{x} + \mathbf{x}_b}} = \frac{\mathbf{e}_Q\,(\mathbf{x} + \mathbf{x}_b)}{\mathbf{x}\,\mathbf{x}_q + \mathbf{x}_q\,\mathbf{x}_b + \mathbf{x}\,\mathbf{x}_b} \qquad (91)$$

Letting $\mathbf{e}_Q$ be zero

$$\dot{\mathbf{i}}_{\Psi_b\Psi_b} = \frac{\mathbf{e}_{\Psi b}}{\mathbf{x}_b + \dfrac{\mathbf{x}\,\mathbf{x}_q}{\mathbf{x} + \mathbf{x}_q}} \qquad (92)$$

Where $\mathbf{i}_{\Psi_b\Psi_b}$ is the motor current produced by $\mathbf{e}_{\Psi b}$ only.

$$\dot{\mathbf{i}}_{\Psi_b Q} = \dot{\mathbf{i}}_{\Psi_b\Psi_b}\,\frac{\mathbf{x}}{\mathbf{x} + \mathbf{x}_q} \qquad (93)$$

Substitute (92) in (93),

$$\dot{\mathbf{i}}_{\Psi_b Q} = \frac{\mathbf{e}_{\Psi b}\,\mathbf{x}}{\mathbf{x}_b\,\mathbf{x} + \mathbf{x}_b\,\mathbf{x}_q + \mathbf{x}\,\mathbf{x}_q} \qquad (94)$$

Substituting (91) and (94) in (90),

$$\dot{\mathbf{i}} = \frac{\mathbf{e}_Q\,(\mathbf{x} + \mathbf{x}_b) - \mathbf{e}_{\Psi b}\,\mathbf{x}}{\mathbf{x}_q\,(\mathbf{x} + \mathbf{x}_b) + \mathbf{x}\,\mathbf{x}_b} \qquad (95)$$

$$\mathbf{i} = \frac{\mathbf{e}_Q - \mathbf{e}_{\Psi b}\left[\dfrac{\mathbf{x}}{\mathbf{x} + \mathbf{x}_b}\right]}{\mathbf{x}_q + \dfrac{\mathbf{x}\,\mathbf{x}_b}{(\mathbf{x} + \mathbf{x}_b)}} \qquad (96)$$

Let the paralleled value of $\mathbf{x}$ and $\mathbf{x}_b$ be $\mathbf{x}_e$ and

$$\mathbf{x}_e = \frac{\mathbf{x}\,\mathbf{x}_b}{\mathbf{x} + \mathbf{x}_b} \qquad (97)$$

Substituting (97) in (96),

$$\mathbf{i} = \frac{\mathbf{e}_Q - \mathbf{e}_{\Psi b}\,(\mathbf{x}_e/\mathbf{x}_b)}{\mathbf{x}_q + \mathbf{x}_e} \qquad (98)$$

Let

$$\mathbf{e}_{\Psi b}\,(\mathbf{x}_e/\mathbf{x}_b) = \mathbf{e}_{\Psi e} \qquad (99)$$

and (98) becomes:

$$\mathbf{i} = \frac{\mathbf{e}_Q - \mathbf{e}_{\Psi e}}{\mathbf{x}_q + \mathbf{x}_e} \qquad (100)$$

Now when there is no shunt it may be seen from Fig. 10 that

$$\dot{\mathbf{i}} = \frac{\mathbf{e}_Q - \mathbf{e}_{\Psi b}}{\mathbf{x}_q + \mathbf{x}_b} \qquad (101)$$

Comparing (101) with (100) and noting the values of $\mathbf{x}_e$ and $\mathbf{e}_{\Psi e}$ in (97) and (99) it is seen that if the shunt $\mathbf{x}$ in Fig. 9, be disconnected from ground and paralleled with the system transient reactance $\mathbf{x}_b$ and the system excitation $\mathbf{e}_{\Psi b}$ be reduced by the ratio $\mathbf{x}_e/\mathbf{x}_b$, the resulting setup shown in Fig. 11 will give exactly the same generator current as that in Fig. 9, and since reactances only are used, the angle between $\mathbf{e}_I$ and $\mathbf{e}_{\Psi e}$ Fig. 11, is the same as the angle between $\mathbf{e}_I$ and $\mathbf{e}_{\Psi b}$ Fig. 9.[7]

It is convenient to let

$$X_d' = x_d' + x_e \qquad (102)$$

$$X_q = x_q + x_e \qquad (103)$$

$$X_d = x_d + x_e \qquad (104)$$

and (100) may be written

$$\dot{i} = \frac{e_Q - e_{\Psi e}\,(\cos\delta - j\sin\delta)}{j\,X_q} \qquad (105)$$

$$\dot{i} = \frac{e_{\Psi e}\sin\delta}{X_q} - j\left(\frac{e_Q - e_{\Psi e}\cos\delta}{X_q}\right) \qquad (106)$$

From (106) we take

$$\dot{i}_q = \frac{e_{\Psi e}\sin\delta}{X_q} \qquad (107)$$

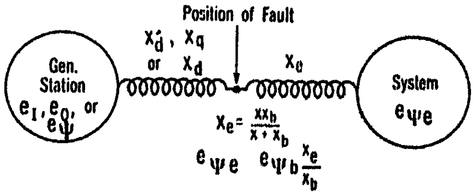
Position of Fault
X_d', X_q or X_d
X_e
Gen. Station
e_I, e_Q, or e_Ψ
System
$e_{\Psi e}$
$X_e = \frac{xx_b}{x+x_b}$
$e_{\Psi e}$ $e_{\Psi b}\frac{x_e}{x_b}$

FIG. 11—SIMPLIFICATION OF CIRCUIT IN FIG. 9, BY ELIMINATION OF THE SHUNT

$$\dot{i}_d = \frac{e_Q - e_{\Psi e}\cos\delta}{X_q} \qquad (108)$$

and from (70) we have during the fault

$$\dot{i}_d = \frac{e_I - e_{\Psi e}\cos\delta}{X_d'} \qquad (109)$$

Fig. 10, is the vector diagram for Fig. 11, when x_e and $e_{\Psi e}$ replace x_b and $e_{\Psi b}$ respectively, and from Fig. 10:

$$e_Q = e_I - i_d\,(x_d - x_q)$$

or

$$e_Q = e_I - i_d\,(X_d - X_q) \qquad (110)$$

Substitute (108) in (110)

$$e_Q = e_I - \frac{(X_d - X_q)}{X_q}\,(e_Q - e_{\Psi e}\cos\delta) \qquad (111)$$

$$e_Q = e_I\,\frac{X_q}{X_d} + \frac{(X_d - X_q)}{X_d}\,e_{\Psi e}\cos\delta \qquad (112)$$

Substitute (107) and (112) in (67).

$$P_e = \frac{e_I\,e_{\Psi e}}{X_d}\sin\delta + \frac{(X_d - X_q)}{X_d\,X_q}\,e_{\Psi e}^2\sin\delta\cos\delta \qquad (113)$$

$$P_e = \frac{e_I\, e_{\Psi e}}{X_d} \sin \delta + \frac{e_{\Psi e}^2\, (X_d - X_q)}{2\, X_d X_q} \sin 2\,\delta \tag{114}$$

Compare (**114**) with (**76**).

In the same manner that (**85**) was obtained it can be shown that:

$$P_e = \frac{e_\Psi\, e_{\Psi e}}{X_d'} \sin \delta - \frac{e_{\Psi e}^2\, (X_q - X_d')}{2\, X_q X_d'} \sin 2\,\delta \tag{115}$$

Both (**114**) and (**115**) are balanced three-phase powers equal in magnitude to the actual unbalanced electrical power on the generator when the short circuit is applied. We thus obtain the power by means of a balanced three-phase calculation.

But the power is a function of the synchronous voltage e_I and it remains to see how e_I is affected by the unbalanced fault. A transient stator current will change the rotor current by armature reaction. The negative phase sequence current has a frequency of 120 cycles with respect to the rotor and induces a 120-cycle harmonic in the rotor current but has no effect upon its d-c. component. The zero phase sequence current produces no armature reaction, for the sum of its three vectors is zero inside the stator winding. The positive phase sequence current i revolves with the rotor, it produces a continuous armature reaction upon the rotor, and it alone produces a net change in the rotor current.

In the per unit system (**114**) may be written:

$$T_e = e_I\, H + N \tag{116}$$

where T_e is the per unit electrical torque and

$$H = \frac{e_{\Psi e}}{X_d} \sin \delta \tag{117}$$

$$N = \frac{e_{\Psi e}^2\, (X_d - X_q)}{2\, X_d X_q} \sin 2\,\delta \tag{118}$$

All quantities in (**117**) and (**118**) are in per units.

Equations (**117**) and (**118**) may be plotted separately as functions of δ, and if at any instant δ and e_I are known, H and N may be read from their curves and T_e obtained conveniently from (**116**). See Curves *I*, Figs. 15 and 16.

In the same manner (**115**) may be written in the per unit system as:

$$T_e = \frac{e_\Psi\, e_{\Psi e}}{X_d'} \sin \delta - \frac{e_{\Psi e}^2\, (X_q - X_d')}{2\, X_q X_d'} \sin 2\,\delta \tag{119}$$

Curve *B*, Figs. 3 and 4 were plotted from (**119**).

If the per unit generator field flux linkages is assumed constant, T_e and δ are the only variables in (**119**) and T_e may be plotted as a function of δ only. For large generators with moderate excitation systems, it has been found that the field flux linkages do not change much during the first 1½ seconds of a single line to ground or a line to line fault and under such conditions, the assumption that e_Ψ is constant introduces only a small error.

The Relation Between the Generator Field Current and the Generator Field Flux Linkages. Let the per unit field current of the generator be I and the direct component of the per unit generator field flux linkages be Ψ_d. In the per unit system the generator synchronous voltage e_I is equal to I and the per unit generator transient voltage e_Ψ is equal to Ψ_d.

The difference between the input and output torques $(T_1 - T_e) = T_a$, will accelerate the generator rotor.

In the step-by-step method it is assumed that T_a remains constant during small and equal increments of time $\Delta\, t$. It has been found that if $\Delta\, t$ be not greater than about one-tenth of the time required for one complete oscillation of the alternator rotor the accuracy of the calculations is very good. This oscillation usually requires from one to two seconds and in nearly all cases we have used a value of $\Delta\, t = 0.05$ second.

At the end of each $\Delta\, t$ it is necessary to find a new T_a for the next increment of time. T_e is a function of the variables e_I and δ, so e_I and δ must be known at the beginning of each $\Delta\, t$. The rotor angle δ is known from calculations made during the preceding $\Delta\, t$. I is a function of the slip ring volts and the angular movement of the rotor. It is assumed that the slip ring volts and rotor position are constant during each $\Delta\, t$, and I approaches, along its exponential curve, the steady state value that the existing slip ring voltage would produce, as determined by the time constant T.

The applied excitation, such as that shown in Fig. 12, is broken up into a number of constant values, each of which is applied during the proper $\Delta\, t$. This gives the change of I due to the slip ring volts E. To this must be added the effect of the change in rotor position during $\Delta\, t$. The way in which these components add is shown in Figs. 13 and 14. The initial value of I at time zero is obtained by the vector diagram, and is shown in Figs. 13 and 14 as I_{02}. It follows its exponential curve to I_{11}, assuming no motion of the rotor until the end of the time $\Delta\, t$ when the rotor suddenly moves to its new position, causing I to change from I_{11} to I_{12}. Knowing I_{12} and δ at the end of $\Delta\, t$, T_a can be found and the movement of the rotor during the next $\Delta\, t$ can be calculated.

Prior to the fault, the load conditions of voltage V, current i, and power factor angle α must be known at some point along the line. Let this position be at V in Fig. 10. Then x'_d, x_q, and x_d are the generator reactances plus the intervening reactances up to this point, and x_b is the reactance from this point into the system.

To obtain I before the fault occurs, the component of V along e_I in Fig. 10 is,

$$V_d = \sqrt{V^2 - (i_q\, x_q)^2} \tag{120}$$

and

$$e_I = \sqrt{V^2 - (i_q\, x_q)^2} + i_d\, x_d \tag{121}$$

Substitute (**72**) and (**70**) in (**121**),

$$e_I = \sqrt{V^2 - \left[\left(\frac{x_q}{x_q + x_b}\right) e_{\Psi b} \sin \delta\right]^2} + \left(\frac{x_d}{x_d + x_b}\right) (e_I - e_{\Psi b} \cos \delta) \qquad (122)$$

$$e_I = \left(1 + \frac{x_d}{x_b}\right) \sqrt{V^2 - \left[\left(\frac{1}{1 + x_b/x_q}\right) e_{\Psi b} \sin \delta\right]^2} - \frac{x_d}{x_b} e_{\Psi b} \cos \delta \qquad (123)$$

To obtain e_Ψ in terms of known load conditions prior to the short circuit, we have from Fig. 10,

$$e_\Psi = V_d + i_d x_d' \qquad (124)$$

Substitute (120) in (124),

$$e_\Psi = \sqrt{V^2 - (x_q i_q)^2} + i_d x_d' \qquad (125)$$

From Fig. 10

$$e_\Psi = e_{\Psi b} \cos \delta + i_d (x_d' + x_b) \qquad (126)$$

$$i_d = \frac{e_\Psi - e_{\Psi b} \cos \delta}{(x_d' + x_b)} \qquad (127)$$

Substitute (72) and (127) in (125),

$$e_\Psi = \sqrt{V^2 - \left[\left(\frac{x_q}{x_q + x_b}\right) e_{\Psi b} \sin \delta\right]^2} + \left(\frac{x_d'}{x_d' + x_b}\right) (e_\Psi - e_{\Psi b} \cos \delta) \qquad (128)$$

$$e_\Psi = \left(1 + \frac{x_d'}{x_b}\right) \sqrt{V^2 - \left[\left(\frac{1}{1 + x_b/x_q}\right) e_{\Psi b} \sin \delta\right]^2} - \frac{x_d'}{x_b} e_{\Psi b} \cos \delta \qquad (129)$$

To obtain e_I in terms of e_Ψ we have from Fig. 10

$$e_I = e_\Psi + i_d (x_d - x_d') \qquad (130)$$

Substitute (127) in (130)

$$e_I = e_\Psi + \frac{(x_d - x_d')}{(x_b + x_d')} (e_\Psi - e_{\Psi b} \cos \delta) \qquad (131)$$

$$e_I = e_\Psi \frac{(x_d + x_b)}{(x_d' + x_b)} - e_{\Psi b} \frac{(x_d - x_d')}{(x_d' + x_b)} \cos \delta \qquad (132)$$

and conversely

$$e_\Psi = e_I \frac{(x_d' + x_b)}{(x_d + x_b)} + e_{\Psi b} \frac{(x_d - x_d')}{(x_d + x_b)} \cos \delta \qquad (133)$$

During the fault x_b becomes x_e and $e_{\Psi b}$ becomes $e_{\Psi e}$ whence (132) and (133) become,

$$e_I = e_\Psi \frac{X_d}{X_d'} - e_{\Psi e} \frac{(X_d - X_d')}{X_d'} \cos \delta \qquad (134)$$

$$e_I = e_\Psi \frac{X_d}{X_d'} - e_{\Psi e} \left(\frac{X_d}{X_d'} - 1\right) \cos \delta \qquad (135)$$

$$e_\Psi = e_I \frac{X_d'}{X_d} + e_{\Psi e} \left(1 - \frac{X_d'}{X_d}\right) \cos \delta \qquad (136)$$

The Short Circuit Time Constant of the Generator and Its Effect upon the Rotor Current. In the per unit system, slip ring volts E are equal to field current I under steady state conditions.

If in any circuit the flow of current i, is controlled by a time constant T, the force which is changing i remains constant and the initial current i_0 and the final current i_∞ are known, the current i at any instant is,

$$i = i_\infty + (i_0 - i_\infty)\, \epsilon^{-\frac{t}{T}} \qquad (137)$$

For the small intervals of time Δt we may use with very good accuracy, the first two terms of the exponential series so that,

$$\epsilon^{-\frac{\Delta t}{T}} = 1 - \frac{\Delta t}{T} \qquad (138)$$

and (137) becomes,

$$i = i_0 - (i_0 - i_\infty) \frac{\Delta t}{T} \qquad (139)$$

As seen in Fig. 13, the value of I at the beginning of the first Δt is I_{02}. During this Δt, Fig. 12 shows that the excitation is E_{01}.

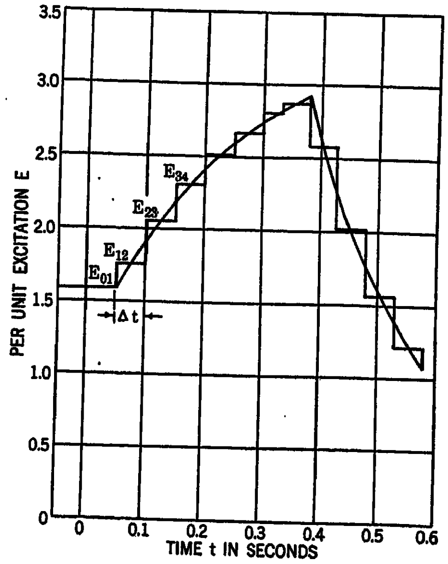

Fig. 12—Slip Ring Voltage of the Generator, Produced by a Quick Response Excitation System, shown as a Function of Time

Therefore from (137) I_{11} in per units is,

$$I_{11} = I_{02} - (I_{02} - E_{01}) \frac{\Delta t}{T} \qquad (140)$$

Since I and e_I are equal in per unit magnitudes, the value of e_I due to the effect of E during Δt is,

$$e_{I(n1)} = e_{I\,(n-1)2} - (e_{I\,(n-1)2} - E_{(n-1)n}) \frac{\Delta t}{T} \qquad (141)$$

The time constant T is a function of the open circuit time constant T_0 and the reactances to which the machine is connected. T_0 is the time constant of the rotor circuit when the stator is open circuited.

The relation[6] is,

$$T = T_0 \frac{(x_d' + x_b)}{(x_d + x_b)} \tag{142}$$

or during the fault

$$T = T_0 \frac{(x_d' + x_e)}{(x_d + x_e)} = T_0 \frac{X_d'}{X_d} \tag{143}$$

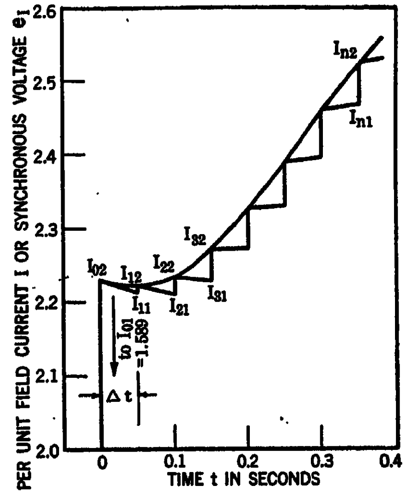

FIG. 13—GENERATOR FIELD CURRENT PRODUCED BY THE EXCITATION IN FIG. 12, SHOWN AS A FUNCTION OF TIME, AND SHOWING THE STEP-BY-STEP CALCULATIONS BY WHICH IT IS OBTAINED

The Effect of the Rotor Swings Upon the Generator Field Currents. To determine the change in I due to the change $\Delta\,\delta$ in the rotor angle during the time $\Delta\,t$ it is assumed that the change in angle takes place instantly at the end of $\Delta\,t$, changing I from I_{11} to I_{12} in Figs. 13 and 14, or at the instant n, I changes from I_{n1} to I_{n2} when the angle changes by $\Delta\,\delta_n$.

In Figs. 13 and 14, and Equations (140) and (141), the first subscript to I (or e_{I}) is the instant at which I (or e_{I}) is taken, the second subscript 1 is for I (or e_{I}) before the rotor changes its angle, and the second subscript 2 is for I (or e_{I}) after the rotor changes its angle.

In Fig. 12 and Equations (140) and (141) the first subscript to E is the instant at which E begins and the second subscript is the instant that E ends. Thus, in Fig. 12, E_{01} is the value of E from instant 0 to instant 1 and E_{12} is the value of E from instant 1 to instant 2. In (141) $E_{(n-1)n}$ is the value of E from instant $(n - 1)$ to instant n.

The subscript to $\Delta\,\delta$ gives the instant at which the change $\Delta\,\delta$ takes place in the angle. The subscript to δ in the following equations gives the instant at which the angle is δ.

The change $(e_{\mathrm{I}(n2)} - e_{\mathrm{I}(n1)})$ in e_{I} due to the change $\Delta\,\delta_n$ in the angle at the instant n is obtained as follows:

From Fig. 10,

$$e_\Psi = e_{\mathrm{I}} - i_d\,(x_d - x_d') \tag{144}$$

and before the instantaneous change $\Delta\,\delta_n$ takes place,

$$e_{\Psi(n)} = e_{\mathrm{I}(n1)} - i_{dn1}\,(x_d - x_d') \tag{145}$$

From (70)

$$i_{dn1} = \frac{e_{\mathrm{I}(n1)} - e_{\Psi b} \cos \delta_{(n-1)}}{(x_d + x_b)} \tag{146}$$

Substituting (146) in (145)

$$e_{\Psi(n)} = e_{\mathrm{I}(n1)} - \frac{(x_d - x_d')}{(x_d + x_b)}\,(e_{\mathrm{I}(n1)} - e_{\Psi b} \cos \delta_{(n-1)}) \tag{147}$$

e_Ψ is produced by the flux linkages of the rotor and cannot change instantly, so that immediately before and immediately after the change $\Delta\,\delta_n$ occurs, e_Ψ has the same value. From (82):

$$i_{dn2} = \frac{e_{\Psi(n)} - e_{\Psi b} \cos \delta_n}{(x_d' + x_b)} \tag{148}$$

Substituting (147) in (148)

$$i_{dn2} = \frac{e_{\mathrm{I}(n1)}}{(x_d' + x_b)} - \frac{(x_d - x_d')}{(x_d' + x_b)\,(x_d + x_b)}\,(e_{\mathrm{I}(n1)}$$

$$- e_{\Psi b} \cos \delta_{(n-1)}) - \frac{e_{\Psi b}}{(x_d' + x_b)} \cos \delta_n \tag{149}$$

From (145)

$$e_{\mathrm{I}(n1)} = e_{\Psi(n)} + i_{dn1}\,(x_d - x_d') \tag{150}$$

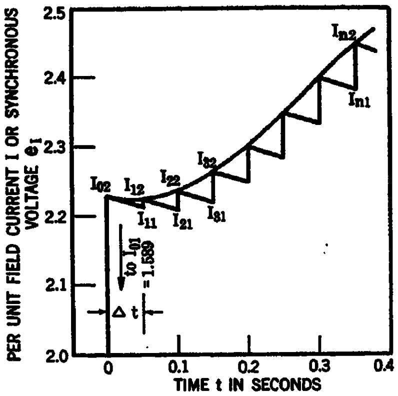

FIG. 14—GENERATOR FIELD CURRENT WHEN THE SLIP RING VOLTS REMAIN CONSTANT, SHOWN AS A FUNCTION OF TIME, AND SHOWING THE STEP-BY-STEP CALCULATIONS BY WHICH IT IS OBTAINED

And since e_Ψ does not change during the instantaneous change $\Delta\,\delta$ in the angle δ,

$$e_{\mathrm{I}(n2)} = e_{\Psi(n)} + i_{dn2}\,(x_d - x_d') \tag{151}$$

Subtracting (150) from (151)

$$e_{\mathrm{I}(n2)} - e_{\mathrm{I}(n1)} = (i_{dn2} - i_{dn1})\,(x_d - x_d') \tag{152}$$

Substituting (149) and (146) in (152)

$$e_{\mathrm{I}(n2)} - e_{\mathrm{I}(n1)} = (x_d - x_d') \left[\frac{e_{\mathrm{I}(n1)}}{(x_d' + x_b)} \right.$$

$$- \frac{(x_d - x_d')}{(x_d' + x_b)\,(x_d + x_b)} (e_{I(n1)} - e_{\Psi b} \cos \delta_{(n-1)})$$

$$- \frac{e_{\Psi b}}{(x_d' + x_b)} \cos \delta_n - \left(\frac{e_{I(n1)} - e_{\Psi b}}{(x_d + x_b)} \cos \delta_{(n-1)} \right) \Bigg]$$

$$= (x_d - x_d') \left[\left(\frac{e_{I(n1)} - e_{\Psi b} \cos \delta_n}{(x_d' + x_b)} \right) \right.$$

$$\left. - \left(1 + \frac{x_d - x_d'}{x_d' + x_b} \right) \left(\frac{e_{I(n1)} - e_{\Psi b} \cos \delta_{(n-1)}}{(x_d + x_b)} \right) \right]$$

$$= \frac{(x_d - x_d')}{(x_d' + x_b)} [e_{I(n1)} - e_{\Psi b} \cos \delta_n - e_{I(n1)} + e_{\Psi b} \cos \delta_{(n-1)}]$$

$$= e_{\Psi b} \left[\frac{(x_d - x_d')}{(x_d' + x_b)} \right] (\cos \delta_{(n-1)} - \cos \delta_n)$$

or

$$e_{I(n2)} = e_{I(n1)} + e_{\Psi b} \left[\frac{x_d - x_d'}{x_d' + x_b} \right] [\cos \delta_{(n-1)} - \cos \delta_n] \tag{153}$$

$$e_{I(n2)} = e_{I(n1)} + (A_{(n-1)} - A_n) \tag{154}$$

where

$$A_{(n-1)} = e_{\Psi b} \left(\frac{x_d - x_d'}{x_d' + x_b} \right) \cos \delta_{(n-1)}$$

$$A_n = e_{\Psi b} \left(\frac{x_d - x_d'}{x_d' + x_b} \right) \cos \delta_n$$

or

$$A = e_{\Psi b} \left(\frac{x_d - x_d'}{x_d' + x_b} \right) \cos \delta \tag{155}$$

Curve *II*, Fig. 17, was plotted from (102).
During the fault (155) becomes:

$$A = e_{\Psi e} \left(\frac{X_d - X_d'}{X_d'} \right) \cos \delta$$

$$A = e_{\Psi e} \left(\frac{X_d}{X_d'} - 1 \right) \cos \delta \tag{156}$$

Curve *I*, Fig. 17, was plotted from (156).
Substituting (156) in (135) we have during the fault:

$$e_I = e_\Psi \frac{X_d}{X_d'} - A \tag{157}$$

Rearranging (157)

$$e_\Psi = \frac{X_d'}{X_d} (e_I + A) \tag{158}$$

Substituting (155) in (132) we have when the fault is not applied:

$$e_I = e_\Psi \frac{(x_d + x_b)}{(x_d' + x_b)} - A \tag{159}$$

Rearranging (159)

$$e_\Psi = \frac{(x_d' + x_b)}{(x_d + x_b)} (e_I + A) \tag{160}$$

Note that A is obtained from (155) when there is no fault, and from (156) when there is a fault.

Modification of the Inertia Constants where Resistances are Neglected and Only Two Machines are Considered. When resistances and machine losses are neglected, the mechanical torque T_1 applied to the shaft of the generator before the disturbance is equal in magnitude but opposite in sign to the mechanical load torque on the shaft of the equivalent motor before the disturbance. The shaft torques are assumed constant on both the generator and motor during the disturbance. Likewise the electrical torque T_e on the generator is equal in magnitude but opposite in sign to the electrical torque on the motor.

Under these conditions the inertia constants M_1 of the generator and M_2 of the motor can be combined in the same manner that resistances are paralleled, to given an equivalent inertia M for the generator which with infinite inertia in the motor, will produce the same total swing between rotors, as that which occurs when the inertia constants are M_1 and M_2 for the generator and motor respectively. This makes it necessary to calculate the swing of one rotor only.

The torques on the generator are:

$$T_1 - T_e - M_1 \frac{d^2 \delta_1}{d t^2} = 0 \tag{161}$$

The torques on the motor are:

$$- T_1 + T_e - M_2 \frac{d^2 \delta_2}{d t^2} = 0 \tag{162}$$

where δ_1 and δ_2 are the increase in the angles of the generator and motor respectively, in per units.

Equating (161) and (162)

$$M_1 \frac{d^2 \delta_1}{d t^2} = - M_2 \frac{d^2 \delta_2}{d t^2} \tag{163}$$

Integrating (163)

$$M_1 \delta_1 = - M_2 \delta_2 \tag{164}$$

The total angular increase δ between rotors is:

$$\delta = \delta_1 - \delta_2 \tag{165}$$

From (164)

$$\delta_2 = - \delta_1 \frac{M_1}{M_2} \tag{166}$$

Substituting (166) in (165)

$$\delta_1 = \left(\frac{M_2}{M_1 + M_2} \right) \delta \tag{167}$$

Substituting (167) in (161)

$$T_1 - T_e = \frac{M_1 M_2}{M_1 + M_2} \frac{d^2 \delta}{d t^2} = T_a \tag{168}$$

or

$$T_a = M \frac{d^2 \delta}{d t^2} \tag{169}$$

where

$$M = \frac{M_1 M_2}{M_1 + M_2} \tag{170}$$

The Manner in which a Quick Response Voltage Regulator Responds when Rapid Changes in A-C. Voltage Take Place. The manner in which the slip ring volts change is a function of the regulator characteristics as well as of the exciter voltage speed.

If the field flux linkages are to be held constant, the field current I will vary as a function of the rotor angle δ.

This relation is obtained directly from (**159**) when there is no fault, and from (**157**) when there is a fault.

In the calculation of the swing curve, $e_I = I$ is determined at each step. If I is less than the value given by (**159**) or (**157**) as the case may be, the voltage regulator will raise the excitation. If I is greater than the value given by (**159**) or (**157**), the regulator will lower the excitation.

The manner in which I must vary with δ to hold constant field flux linkages is shown in Fig. 18.

Tabulation of the Step-by-Step Calculations of the Swing Curve. The calculations of the swing curve may be conveniently tabulated as shown in Table I.

TABLE I
TABULATION OF THE SWING CURVE CALCULATIONS

	Item			
n = number of time instant.............		0	1	2
$n\,(\Delta t)$ = seconds......................		0	$1(\Delta t)$	$2(\Delta t)$
$e_{I(n1)} = e_{I(n-1)2} - [e_{I(n-1)2} - E_{(n-1)n}] \frac{\Delta t}{T} = n_{(n-1)n} + p_{(n-1)}$	a		†	
$A_{(n-1)}$ from curve when $\delta = \delta_{(n-1)}$.......	$b_{(n-1)}$		†	
A_n from curve when $\delta = \delta_n$.............	c_n		†	
$A_{(n-1)} - A_n = b_{(n-1)} - c_n$..............	d_n		†	
$e_{I(n2)} = a_n + d_n$........................	e_n	#	†	
H_n from curve when $\delta = \delta_n$.............	f_n	*	†	
$e_{I(n2)} H_n = (e_n)(f_n)$.....................	g_n	*	†	
N_n from curve when $\delta = \delta_n$.............	h_n	*	†	
$T_{en} = g_n + h_n$.......................	i_n	*	†	
$T_{an} = (T_1 - T_{en}) = T_1 - i_n$...........	j_n	*	†	
$k\,T_{an} = \frac{360 f (\Delta t)^2}{M} T_{an} = k j_n$ except 1st stop switching, etc....................	k_n	*	†	
$\Delta \delta_{(n+1)} = \Delta \delta_n + k_n = l_n + k_n$.........	$l_{(n+1)}$	0	*	†
$\delta_{(n+1)} = \delta_n + \Delta \delta_{(n+1)} = m_n + l_{(n+1)}$...	$m_{(n+1)}$	#	*	†
$E_{n(n+1)}$ from excitation curve............	$n_{n(n+1)}$	n_{01}	n_{12}	n_{23}
$e_{I(n2)} - E_{n(n+1)} = e_n - n_{n(n+1)}$.......	o_n	*	†	
$e_{I(n2)} - E_{n(n+1)}] \frac{\Delta t}{T} = o_n \frac{\Delta t}{T}$	p_n	*	†	

It is a condensed tabulation of the information derived thus far, with the instants at which the various items apply shown by the subscripts n. The preliminary calculations marked # are obtained from (89), (129), and (157). The initial angle δ_0 calculated from (89) cannot change the first instant of the disturbance. Before the disturbance e_Ψ is given by (**129**) but like δ_0, e_Ψ cannot change instantly and (**129**), therefore, gives e_Ψ at the first instant of the disturbance. The value of e_I or I at the first instant of the disturbance is obtained from (**157**).

The first step calculations are marked *, and the second step marked †. Subsequent steps are done in in the same manner as that marked †. It will be noted that δ, and $e_{I(n1)}$ in column 1 are obtained from calculations in column 0. The same conditions holds for all subsequent columns.

Before starting the swing curve, auxiliary curves for A, H, and N should be plotted as functions of δ from (**156**), (**155**), (**117**), (**78**), (**118**), and (**79**). These curves give items b, c, d, f, and h in Table I. Curve H is a fundamental sine wave and when multiplied by e_I it gives the electrical torque that the generator would deliver if it were of non-salient pole construction. Curve N adds the reluctance torque, which is that due to the salient pole construction, and is a second harmonic with reference to the fundamental Curve H. From (**118**) it will be seen that the excitation for N comes from the system.

Item n is simply filled in from the known step-by-step excitation curve, Fig. 12.

Item a is (**141**); Item e is (**154**). Item g is the first term in (**116**). Item i is (**116**). Item j is part of a term in (**51**), (**54**), or (**55**). Item k is a term in (**51**), (**54**), or (**55**). Item l is (**51**), (**54**), or (**55**) depending upon conditions. Item m is (**42**). Item o is part of the second term in (**141**). Item p is the second term in (**141**). And finally the cycle starts over again with item a as (**141**). Note that when a new column is started, the column that was previously $(n + 1)$ becomes column n.

Table I gives, either directly or indirectly, practically all the transient positive phase sequence electrical quantities in the generator. Item e gives the field current. Item i gives the electrical power or torque. Item m gives the angle. The slope of item m against time gives the departure from synchronous speed.

From item c and e the field flux linkages are given by (**158**) or (**160**). The direct component i_d of the stator current i is given by (**70**) or (**109**). The quadrature component i_q of the stator current i is given by (**72**) or (**107**). The stator current is:

$$i = \sqrt{i_d^2 + i_q^2} \tag{171}$$

These values can be plotted not only against time but against each other, such as electrical power against angle.

If the field flux linkages are assumed constant, and this can usually be done during the first swing with only a small error, only items i, j, k, l, and m of Table I are required for the angle time curve. The electrical torque or power is given in per unit values by (**85**) or (**119**) in which T_e and δ are the only variables. The second harmonic is added directly in the equations.

T_e should be plotted against δ, giving item i. The other items are as previously described in Table I.

After the "per unit" quantities have been obtained, they can be expressed in terms of kv., kw., and kv-a., etc., by multiplying by the number of such electrical units in the particular unit used as standard. This was done when (**48**) was used to convert per unit angles in (**47**) to degrees in (**49**).

In other quantities such as kw., if unit kw. is chosen to equal 100,000 kw. and a calculated per unit figure of 0.5 is obtained, then the calculated kw. are 100,000 × 0.5 = 50,000 kw.

Appendix II

EXAMPLE OF A SWING CURVE CALCULATION

Where a Fault is Applied and Removed by Switching Out the Faulty Line in 0.33 Second

The positive phase sequence reactance diagram during the fault is shown in Figs. 9 and 11. Before the fault it is Fig. 9, without the shunt x.

The base chosen is 100,000 kv-a.

The units chosen are:

unit kv-a. = 100,000 kv-a.
unit kw. = 100,000 kw.
unit kv. = 220 kv.

A generating plant is feeding a system over a pair of duplicate paralleled 220-kv. transmission lines at each end of which are solidly grounded Δ Y transformers, with the high-tension windings connected in Y and the low-tension windings connected in Δ.

A single conductor to ground short circuit on one of the lines will be taken near the high-tension generator bus, and the faulty line will be removed in 0.33 second.

Measured at the high-tension terminals of the generating station transformers the load is 238,000 kw. at 0.93 power factor and at a potential of 231 kv. In per units

$$T_1 = \frac{238{,}000}{100{,}000} = 2.38$$

$$V = \frac{231}{220} = 1.05$$

The speed of the generators is 81.8 rev. per min. All the generators are practically duplicates and have duplicate excitation systems. The total $W R^2$ of the generators is 512,654,800 lb. ft.^{2} Adding 12.5 per cent for the waterwheels and their content of water, the total $W R^2$ is 577,000,000 lb. ft.^{2}

From (**37**)

$$M_1 = \frac{0.462 \times 577 \times 10^6 \times (0.0818)^2}{105} = 17.85$$

From the paragraph following (**37**), the inertia constant of the equivalent motor representing the system is:

$$M_2 = 272$$

From (**170**)

$$M = \frac{(17.85)\,(272)}{289.85} = 16.75$$

The frequency is 60 cycles, and $\Delta\, t$ will be taken as 0.05 sec. Therefore from (**50**)

$$k = \frac{360 \times 60\;(0.05)^2}{16.75} = 3.222$$

From the system network

$x_b = 0.0954$ before the fault occurs.
$x_b = 0.1464$ after removing the faulty line.

From the generator reactances the system network and (**4**)

$$x = x_z + z_n = 0.0859$$

From (**97**)

$$x_e = \frac{(0.0859)\,(0.0954)}{0.1813} = 0.0452$$

The generator reactances are:

transient reactance = 0.1035
quadrature reactance = 0.2070
synchronous reactance = 0.3290

The reactance between the generator and the fault is that of the transformer. It is 0.031. Hence:

$$\begin{aligned} x_d' &= 0.1035 + 0.0310 = 0.1345 \\ x_q &= 0.2070 + 0.0310 = 0.2380 \\ x_d &= 0.3290 + 0.0310 = 0.3600 \end{aligned}$$

From (**102**), (**103**), and (**104**)

$$\begin{aligned} X_d' &= 0.1345 + 0.0452 = 0.1797 \\ X_q &= 0.2380 + 0.0452 = 0.2832 \\ X_d &= 0.3600 + 0.0452 = 0.4052 \end{aligned}$$

Before the fault occurs

$$\begin{aligned} x_d' + x_b &= 0.1345 + 0.0954 = 0.2299 \\ x_q + x_b &= 0.2380 + 0.0954 = 0.3334 \\ x_d + x_b &= 0.3600 + 0.0954 = 0.4554 \end{aligned}$$

After removing the faulty line

$$\begin{aligned} x_d' + x_b &= 0.1345 + 0.1464 = 0.2809 \\ x_q + x_b &= 0.2380 + 0.1464 = 0.3844 \\ x_d + x_b &= 0.3600 + 0.1461 = 0.5064 \end{aligned}$$

Before the fault occurs the total a-c. current is

$$i = \frac{\text{Kw.*}}{\text{Kv.}\cos\alpha} = \frac{2.38}{1.05 \times 0.93} = 2.435$$

and using V as the reference vector

$$\begin{aligned} \mathbf{i} &= 2.435\ \underline{/\cos^{-1} 0.93} \\ \mathbf{i} &= 2.435\ \underline{/21.6^\circ} = 2.265 - j\,0.896 \\ \mathbf{e}_{\Psi b} &= \mathbf{V} - \mathbf{i}\,\mathbf{x}_b = 1.05 - \mathrm{j}\,0.0954\,(2.265 - \mathrm{j}\,0.896) \\ &= 1.05 - \mathrm{j}\,0.216 - 0.0855 \\ \mathbf{e}_{\Psi b} &= 0.9645 - \mathrm{j}\,0.216 = 0.988\ \overline{/12.6^\circ} \end{aligned}$$

---

*The use of per units or percentages eliminates the factor $\sqrt{3}$ in three-phase calculations.

The initial angle (before the fault occurs) is obtained from (**89**). It is,

$$\delta_o = \tan^{-1}\frac{(2.435)\ (0.0954)\ (\cos 21.6°)}{1.05 - (2.435)\ (0.0954)\ (\sin 21.6°)}$$

$$+ \tan^{-1}\frac{(2.435)\ (0.2380)\ (\cos 21.6°)}{1.05 + (2.435)\ (0.2380)\ (\sin 21.6°)}$$

$$= \tan^{-1} 0.224 + \tan^{-1} 0.426 = 12.6° + 23.1°$$

$$\delta_o = 35.7°$$

From (**99**)

$$e_{\Psi} = 0.988\ \frac{(0.0452)}{(0.0954)} = 0.468$$

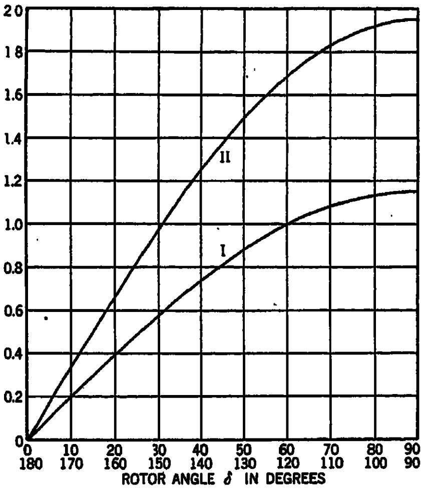

FIG. 15—THE QUANTITY H USED IN CALCULATING THE ELECTRICAL TORQUE BETWEEN THE GENERATOR AND MOTOR, SHOWN AS A FUNCTION OF THE ELECTRICAL ANGLE BETWEEN THE ROTOR AXES

Curve I applies during the fault
Curve II applies after the faulty line has been removed

From (**129**) the initial transient voltage (before the fault occurs) is,

$$e_{\Psi} = \left(1 + \frac{0.1345}{0.0954}\right)$$

$$\sqrt{(1.05)^2 - \left[\left(\frac{1}{1 + 0.0954/0.2380}\right)(0.988)(0.5835)\right]^2}$$

$$- \frac{1.345}{0.0954}\ (0.988)\ (0.8121)$$

$$e_{\Psi} = 1.195$$

After the fault is applied, the H, N, and A curves must be plotted.

The H curves are obtained from (**117**) and (**78**).

During the fault

$$H = \frac{0.468}{0.4052}\sin\delta$$

$$H = 1.154 \sin\delta$$

After the faulty line has been switched out

$$H = \frac{0.988}{0.5064}\sin\delta$$

$$H = 1.95 \sin\delta$$

The H curves are plotted in Fig. 15.

The N curves are obtained from (**118**) and (**79**).

During the fault

$$N = \frac{0.468^2\ (0.4052 - 0.2832)}{2\ (0.4052)\ (0.2832)}\sin 2\delta$$

$$N = 0.1164 \sin 2\delta$$

After the fault

$$N = \frac{0.988^2\ (0.3600 - 0.2380)}{2\ (0.5064)\ (0.3844)}\sin 2\delta$$

$$N = 0.3056 \sin 2\delta$$

The N curves are plotted in Fig. 16.

From (**156**) we have during the fault:

$$A = 0.468\left(\frac{0.4052}{0.1797} - 1\right)\cos\delta$$

$$A = 0.588 \cos\delta$$

After the fault, from (**155**)

$$A = 0.988\left(\frac{0.3600 - 0.1345}{0.2809}\right)\cos\delta$$

$$A = 0.793 \cos\delta$$

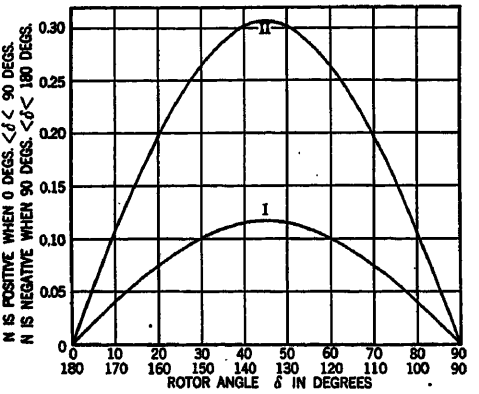

FIG. 16—THE QUANTITY N USED IN CALCULATING THE ELECTRICAL TORQUE BETWEEN THE GENERATOR AND MOTOR, SHOWN AS A FUNCTION OF THE ELECTRICAL ANGLE BETWEEN THE ROTOR AXES

Curve I applies during the fault
Curve II applies after the faulty line has been removed.

The A curves are plotted in Fig. 17.

From (**157**) the synchronous voltage immediately after the fault occurs is,

$$e_{1(02)} = (1.159)\left(\frac{0.4052}{0.1797}\right) - A$$

A is obtained from Curve I of the A Curves when $\delta = \delta_0 = 35.7$ deg. and,

$$e_{1(02)} = 2.695 - 0.478 = 2.217$$

The open circuit time constant was found to be,

$$T_o = 4.8 \text{ seconds}$$

and from (143) we have during the fault,

$$T = (4.8)\,\frac{(0.1797)}{(0.4052)} = 2.13 \text{ sec.}$$

From (142) we have after the fault,

$$T = (4.8)\,\frac{(0.2809)}{(0.5064)} = 2.66 \text{ sec.}$$

The term $\frac{\Delta t}{T}$ in (141) is,

During the fault

$$\frac{\Delta}{T}t = \frac{0.05}{2.13} = 0.0235$$

and after the fault

$$\frac{\Delta t}{T} = \frac{0.05}{2.66} = 0.0188$$

The step-by-step calculations can now be made on the swing curve. Table I can be filled in as shown in Table II. The various values for item n are filled in from Fig. 12. Like all other values Fig. 12 is plotted in per units. From the generator saturation curve it was found that unit I is 363 amperes. The resistance of the field winding is 0.27 ohm, and unit slip ring volts is 363 × 0.27 = 98 volts. To obtain Fig. 12, the ordinates of the curve of excitation volts versus time were divided by 98.

A small time of 0.05 second was allowed for the regulator contacts to operate before the excitation began to rise. It will be seen that the steady state excitations before the disturbance was 1.59, giving 1.59 × 98 = 156 slip ring volts and 1.59 × 363 = 577 field amperes per generator.

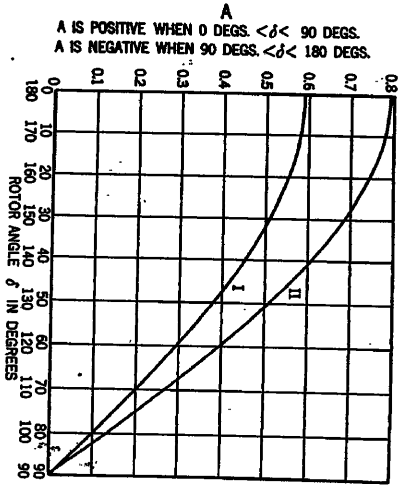

FIG. 17—THE QUANTITY A USED IN CALCULATING THE CHANGE IN FIELD CURRENT PRODUCED BY THE ROTOR OSCILLATIONS, SHOWN AS A FUNCTION OF THE ELECTRICAL ANGLE BETWEEN THE ROTOR AXES

Curve I applies during the fault
Curve II applies after the faulty line has been removed.

TABLE II
SAMPLE SWING CURVE CALCULATION

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
n (0.05)	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.33	0.38	0.43	0.48	0.53	0.58	0.63
$e_{I(n1)}$		2.203	2.198	2.218	2.259	2.319	2.386	2.457		1.905	1.965	1.953	1.874	1.746	
$A_{(n-1)}$		0.478	0.472	0.448	0.408	0.353	0.293	0.230		0.260	0.212	0.225	0.298	0.416	
A_n		0.472	0.448	0.408	0.353	0.293	0.230	0.174		0.212	0.225	0.298	0.416	0.549	
$A_{(n-1)} - A_n$		0.006	0.024	0.040	0.055	0.060	0.063	0.056		0.048	−0.013	−0.073	−0.118	−0.133	
$e_{I(n2)}$	2.217	2.209	2.222	5.258	2.314	2.379	2.449	2.513	1.886	1.953	1.952	1.880	1.756	1.613	
H_n	0.672	0.693	0.751	0.832	0.922	1.001	1.063	1.103	1.842	1.879	1.870	1.804	1.663	1.407	
$e_{I(n2)}\,H_n$	1.490	1.531	1.670	1.880	2.133	2.383	2.608	2.776	3.475	3.670	3.650	3.394	2.920	2.272	
N_n	0.111	0.112	0.115	0.117	0.112	0.101	0.084	0.065	0.190	0.158	0.166	0.213	0.272	0.305	
T_{en}	1.601	1.643	1.785	1.997	2.245	2.484	2.692	2.841	3.665	3.828	3.816	3.607	3.192	2.577	
$2.38 - T_{en}$	0.779	0.737	0.595	0.383	0.135	−0.104	−0.312		−0.85	−1.448	−1.436	−1.227	−0.812	−0.197	
$3.222\,(2.38 - T_{en})$	1.254	2.375	1.918	1.234	0.435	−0.335	−1.005		−2.740	−4.665	−4.627	−3.953	−2.620	−0.635	
$\Delta\,\delta_{(n+1)}$ degrees	0	1.254	3.629	5.547	6.781	7.216	6.881	5.876	6.450	3.710	−0.995	−5.582	−9.535	−12.155	−12.790
$\delta_{(n+1)}$ degrees	35.700°	36.954°	40.583°	46.130°	52.911°	60.127°	67.008°	72.884°	70.750°	74.460°	73.505°	67.923°	58.388°	46.233°	33.443°
$E_{n(n+1)}$	1.590	1.760	2.060	2.310	2.520	2.675	2.810		2.880	2.580	2.020	1.560	1.215		
$e_{I(n2)} - E_{n(n+1)}$	0.627	0.449	0.162	−0.052	−0.206	−0.296	−0.361		−0.994	−0.627	−0.068	0.320	0.541		
$\left[E_{I(n2)} - E_{n(n+1)}\right]\frac{0.05}{T}$	0.014	0.011	0.004	−0.001	−0.005	−0.007	−0.008		−0.019	−0.012	−0.001	0.006	0.010		

To make the transition in the calculations at 0.33 sec. when the fault and faulty line are removed, the calculations are carried on with the fault up to 0.35 sec.

Plotting δ and $e_{I(n2)}$ against time it is seen that at 0.33 sec. $\delta = 70.75$ deg. and $e_{I(n2)} = 2.49$ (the true value of the synchronous voltage or the field current at an instant is $e_{I(n2)}$ and not $e_{I(n1)}$). Immediately before switching the value of A at 70.75 deg. is 0.195 and from (**158**) the transient voltage at this instant is,

$$e_\Psi = \frac{0.1797}{0.4052}(2.49 + 0.195) = 1.19$$

After removing the fault, the number II Curves must be used for A, H, and N. Neither δ nor e_Ψ can change instantly and immediately after removing the fault the value of A at 70.75 deg. is 0.260, the value of e_Ψ is still 1.19, and from (**159**),

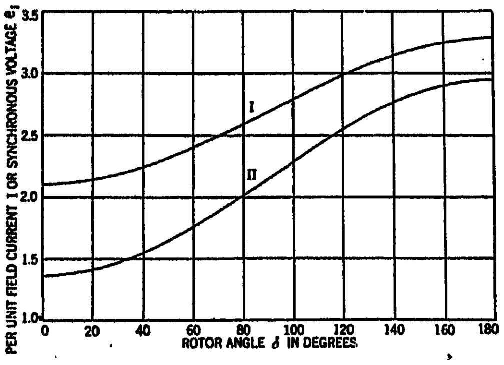

FIG. 18—GENERATOR FIELD CURRENT, OR SYNCHRONOUS VOLTAGE, REQUIRED TO MAINTAIN CONSTANT GENERATOR FIELD FLUX LINKAGES, OR TRANSIENT VOLTAGE, SHOWN AS A FUNCTION OF THE ELECTRICAL ANGLE BETWEEN THE ROTOR AXES

Curve I applies during the fault
Curve II applies after the faulty line has been removed

$$e_I = 1.19\frac{(0.5064)}{(0.2809)} - 0.260 = 1.886$$

From the curve of δ versus time, the increment $\Delta\delta_n$ of δ from 0.28 sec. to 0.33 sec., (the regular Δt internal of 0.05 sec.) is $70.75° - 64.30° = 6.45°$.

These new values of e_I, $\Delta\delta$, and δ are tabulated in column 8 of Table II.

Assuming that the voltage regulator is the quick response type, the regulator will endeavor to hold constant the initial transient voltage of 1.195. To maintain this value of transient voltage during the fault the field current I = the synchronous voltage e_I must be that given by (**157**) which becomes,

$$e_I = 1.195\frac{(.4052)}{(.1797)} - A^*$$

$$e_I = 2.695 - A^*$$

At the moment the fault line is switched out, the field current required to hold the field flux linkages constant at the per unit value of 1.195 is from (**159**):

$$e_I = 1.195\frac{(0.5064)}{(0.2809)} - A^*$$

$$e_I = 2.155 - A^*$$

These values of e_I are plotted in Fig. 18.

By comparing the values of $e_{I(n2)}$ in Table II, which are the per unit synchronous voltages, with the values of the per unit synchronous voltages plotted in Fig. 18, it will be seen that e_I is less than the amount required to hold constant transient voltages until the rotor has swung to an angle of 74.460 deg., which occurs at 0.38 second. Up until this time the voltage regulator forces the excitation voltage to continue to rise, as shown by Fig. 12, in an effort to hold a constant transient voltage, but at the rotor angle of 74.460 deg. the synchronous voltage exceeds the required value as shown by Curve II, Fig. 18, and the voltage regulator reduces the excitation in order to bring the synchronous voltage back down to Curve II. The decrement in slip ring voltage, therefore, starts at 0.38 second, as shown by Fig. 12, and continues until e_I is brought back to Curve II, Fig. 18, at some later time not shown on the curve sheets, and at which time the excitation will rise again in the same manner as it did in the first 0.38 second. This process will continue with the excitation rising and falling as the voltage regulator attempts to hold constant field flux linkages.

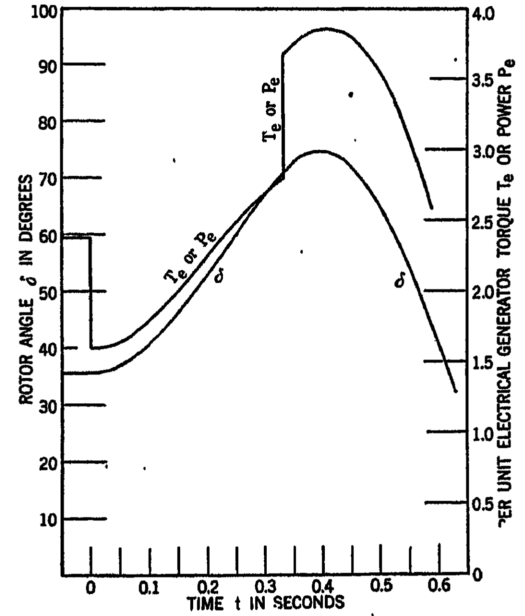

FIG. 19—ELECTRICAL TORQUE OR POWER BETWEEN GENERATOR AND MOTOR, AND ELECTRICAL ANGLE BETWEEN ROTOR AXES OF GENERATOR AND MOTOR, EACH SHOWN AS A FUNCTION OF TIME

By plotting T_e against time it is seen that at 0.33 sec. $T_{en}' = 2.795$. From the calculations in column 8, Table II $T_{en}'' = 3.665$. The average value of T_{en}

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*The values of A, as previously explained, are different during the fault and after the fault.

at 0.33 sec. is $\frac{T_{en}' + T_{en}''}{2} = \frac{2.795 + 3.665}{2} = 3.230$, see (51), and item j becomes $2.38 - 3.23 = -0.85$.

The term $\frac{0.05}{T}$ becomes 0.0188

The swing curves of angle between rotors, and of generator electrical power, as calculated in the above example and tabulated in Table II are plotted in Fig. 19.

The angle is in degrees. The torque or power is plotted in per units. To convert to kilowatts multiply the ordinates of the curve by the number of kilowatts in one unit of power. This, as previously stated, is 100,000 kw. Thus from Fig. 19, the initial power is 2.38 × 100,000 = 238,000 kw. before the fault. When the fault is applied, the power drops instantly to 1.601 × 100,000 = 160,100 kw. and rises during the swing to 2.79 × 100,000 = 279,000 kw. at 0.33 second, when the short circuit and fault line is relayed out and it jumps instantly to 3.665 × 100,000 = 366,500 kw., after which the swing continues as shown.

Appendix III

Accuracy of the Step-by-Step Method of Calculating Alternator Swing Curves

The accuracy of the step-by-step method of calculating the transient phenomena produced in alternators by disturbances such as short circuits, and often referred to as swing curves, is remarkably good.

A high degree of accuracy can be obtained by mechanically integrating the differential equation of the alternator swing. When this is done and the answer compared with that obtained by the step-by-step method, the error will usually be found to come within the limits of slide rule accuracy.

While no method of calculations can give an exact check of test results unless the exact value of all circuit and machine constants are known, good agreements can usually be obtained. This is true of the step-by-step method which was employed to calculate the curves shown in Figs. 10, 11, and 12 of Bibliography 1. The reader can judge for himself the degree of agreement between these calculated and test curves.

In cases where the field flux linkages, the prime mover torque, and the mechanical motor load torque are considered constant, and the damping and resistance losses are neglected, the step-by-step method of calculating the rotor swing gives, as it properly should, practically the same peak value for the second swing as for the first, showing practically no cumulative error between peaks. As an example, in a particular case the rotor angle rose from an initial value of 35.7 deg. to a peak of 80 deg. on the first swing and 80.6 deg. on the second swing. The step-by-step method presented here is a refinement of an earlier step-by-step method which contained a cumulative error. The degree of refinement now attained may be judged by comparing the old and new method. Using the old method on the example described in this paragraph, the rotor angle rose from its initial value of 35.7 deg. to a peak of 85.9 deg. on the first swing and a peak of 119 deg. on the second swing.

In some special cases, such as a three-phase short circuit on the generator bus with a constant prime mover input torque and no losses in the fault or in the machine, there is no error in the step-by-step method of calculating the rotor displacement. The differential equation of the rotor position can be easily integrated for such a condition and it can be shown mathematically that the results thus obtained are a perfect duplicate of those obtained by the step-by-step process.

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Discussion

I. H. Summers: I should like to point out what Mr. Longley has not brought out very clearly perhaps, and that is that the essential contribution of his paper is that he has given a complete outline of a method of calculation, for not only the angular swing of the alternators under transient conditions but also all the other variables in the alternators, such as field current, flux, etc., where any number of alternators is connected to a system network.

In my discussion yesterday of the paper on the network analyzer by Messrs. Hazen and Schurig and Gardner, I mentioned the advantages of using the network analyzer to help in the solution of problems in transient stability when a large number of machines is connected to the network. In fairness to the analytical method, I should like to try to describe some features of the method as described by Mr. Longley.

In the first place, any solution on the network analyzer depends on the analytical solution for the equations on which it is based and for the procedure used. The equations are used in the solution on the network analyzer to make certain auxiliary calculations in carrying out the method. In the second place there is a large class of problems in which a synchronous machine swinging against a bus or two machines swinging against each other are all that are necessary to be investigated. Examples of this are problems relating to high-speed excitation effect of varying field currents, damping, etc. In these cases, it may be that the solution by analytical methods is sufficiently simplified so that it becomes unnecessary to resort to the analyzer, and it may even be that time will be saved by going through the analytical solution rather than making the set-up and going through the checking processes necessary to the analyzer. What I am trying to bring out is that both methods have their place. The analytical method is of great importance because it furnishes a tool available to everyone who wishes to use it, whereas a miniature system such as the network analyzer may not be so readily available to those situated a distance away from places where this kind of apparatus exists.

F. R. Longley: There is just one point I want to bring out in the use of the step-by-step method of calculation. If you will adhere to that method, no matter what you are calculating you will usually eliminate the step-by-step cumulative error. Take for instance an excitation voltage rise curve plotted as a function of time. It may be considered constant for small intervals of time and broken up into small step-by-step intervals accordingly, but the total area under the step-by-step curve must be equal to the area under the actual curve. If you do not keep these areas equal the amount of your cumulated error will be the difference in the areas that you have not taken into account.

Here are a few curves which will demonstrate that there is a real difference in the effect of different types of excitation. These curves (Fig. 1) show the angle between the rotor axes of a synchronous generator and motor during a line to ground fault. The abscissa is in seconds, the ordinate in degrees. Each curve is for a different type of excitation. Otherwise the conditions are the same. Curve *A* is for constant slip ring volts. Curve *B* is for an excitation rise of about 300 volts per second. Curve *C* is for constant rotor flux linkages. Curve *D* is for super-excitation. Curve *A* rides through the first and second swing but appears to be headed for loss of synchronism during the third or fourth swing. Curve *B* has a smaller amplitude in the second swing than in the first, showing that the rise in excitation is slowly getting in its work. Curve *C* has the same amplitudes in each swing, as it properly should, since the flux linkages are held constant. Curve *D* has much smaller amplitudes than *A*, *B*, or *C*, and a higher frequency of oscillation. It can therefore be seen from these curves that the effect of excitation may, in some cases, be an important factor in stability calculations.

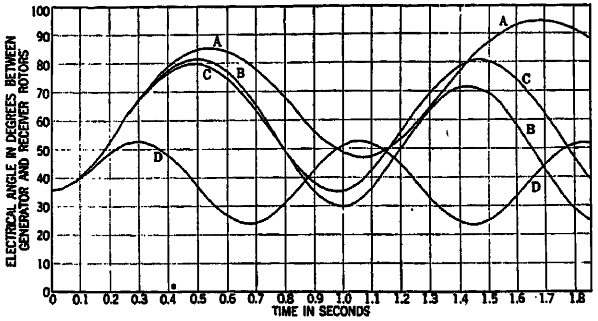

FIG. 1—SWING CURVES BETWEEN GENERATING STATION AND SYSTEM FOR VARIOUS TYPES OF EXCITATION FOLLOWING ONE CONDUCTOR TO GROUND FAULT ON TRANSMISSION LINE. TRANSFORMER NEUTRALS SOLIDLY GROUNDED

Curve *A*—Constant slip ring volts
Curve *B*—Excitation speed of about 300 volts per second
Curve *C*—Constant rotor flux linkages
Curve *D*—Super-excitation

Radio Telephone Service to Ships at Sea

BY WILLIAM WILSON[1]
Member, A. I. E. E.

and

LLOYD ESPENSCHIED[2]
Member, A. I. E. E.

Synopsis.*—The paper discusses the American end of the ship-to-shore radio telephone system and the connecting equipment on board the Leviathan. The most suitable wavelengths for this service are in the short-wave range, but the use of these wavelengths complicates the problem, since different wavelengths are required according to the distance of the ship from shore, the time of day, season of year, etc. The problem on shipboard is further complicated by the fact that the transmitting and receiving systems are necessarily near together and special precautions are necessary to take care of interference from the radio telephone transmitter and the radio telegraphic services. In addition to interference from these sources, there is a background of interference in the ship's electrical equipment, all of which necessitates a much more powerful land station than is necessary on shipboard.*

In the present system, the shore transmitter has a power rating of 15 kw. and the ship transmitter of 500 watts. The shore transmitting station is located at Ocean Gate, N. J., and the receiving station at Forked River, N. J. At both of these locations, directive antennas are employed which cover the ship's lanes. The stations are connected by wire to the Long Lines toll office in New York, and the over-all control of the circuit is carried out from this point. Both the ship and shore transmitters are crystal controlled. The ship's receiver is highly selective and is of the double-detection type. Communication between the ship and the shore is carried out by use of a pair of frequencies, one for transmission in each of the two directions, separated from each other by about three per cent. Ships of a number of nations are being equipped with wireless telephone apparatus and as the service expands, it will undoubtedly be necessary to formulate a plan in which international agreement is reached on the allocations of frequencies for ship-to-shore telephony and telegraphy, in order that undue interference within the services themselves or between the two services shall not ensue.

IN view of the developments which have recently taken place in the field of ship-to-shore radio telephony, it would appear appropriate to review the state of the science and to discuss the problems which have arisen, the facilities which have been installed, and the general results obtained.

The ship-to-shore radio telephone system, which is here described, was opened for public service between the *Leviathan* and the United States on December 8, 1929. This was the first extension of the public telephone service to a ship at sea and enabled calls to be made between the vessel and any Bell System subscriber. The system as set up is intended primarily for giving telephone service to the larger passenger-carrying vessels as an extension to the wire network, and should be distinguished from the more simple uses which have been made of radio telephony in the marine field, such as that of enabling a coastal station operator to talk with coast guard vessels, fishing trawlers, etc.

This paper is concerned with the developments which have been carried out in the United States, including the establishment of transmitting and receiving stations on the New Jersey coast, the equipping of the *Leviathan* and the establishment of service to that ship.

It is significant of the wide-spread interest in this type of service that developments have also gone forward rapidly on the European side where the British, Germans, and French are preparing coastal stations and equipping some of the larger ships for public telephone service. The British have already initiated service to two of the White Star Liners, the *Olympic* and the *Majestic*, and before the summer is over it is likely that half a dozen of the larger transatlantic vessels will be undertaking this service, connecting with both the American and the European networks.

EARLY DEVELOPMENTS

Attempts to apply telephony in the maritime field date back to the pioneer work on radiotelephony itself, but it was not until the application of the vacuum tube were developed that radio telephony for any service became finally practicable.

Following the long distance, point-to-point radio telephone experiments of 1915, there was carried out in the following year what is believed to have been the first trial of two-way radio telephony from the wire telephone system to a vessel at sea. This trial was conducted by Bell System engineers in cooperation with the Navy Department. On that occasion the Secretary of the Navy, in his office in Washington, carried on two-way conversations with the captain of the *U. S. S. New Hampshire* off Hampton Roads.

Following the further development of radio telephony during the War, there was undertaken, in the years 1920-1922, an extensive development of ship-to-shore radio telephony, looking toward the linking of ships at sea with the land line telephone network.[3] At that time there was built a coastal radio telephone station at Deal Beach, N. J., and several ships were equipped on a trial basis. Extensive engineering tests were made and a number of demonstrations carried out which proved the physical feasibility of establishing such connections. While the trials were successful from the technical standpoint, the development was not carried into commercial use because the adverse economic conditions existing in the post-War period did not appear to justify the initiation of the new service at that time. Furthermore, the waves in the range of 300-500 meters, which

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1. Bell Telephone Laboratories, New York, N. Y.

2. American Telephone and Telegraph Company, New York, N. Y.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

3. "Radio Extension of the Telephone System to Ships at Sea," by H. W. Nichols and Lloyd Espenschied, *I. R. E. Proceedings*, Vol. 11, No. 3, 1923.

had been used in these early trials, were soon thereafter assigned for broadcasting.

In the last few years the whole outlook has changed considerably. The development of short-wave radio systems has greatly increased the message carrying capacity of the radio spectrum and has made it feasible to maintain communication over a greater range of distances than was previously practicable for ships. Transoceanic radio telephone services have been inaugurated, and with the large increase in steamship travel there has arisen a renewed interest in the extension of telephone service to ships at sea.

When it became evident that short-wave transmission might be desirable for ship-to-shore telephone service, there was undertaken a program involving the measurement of the strength of the electric fields received aboard ship from a shore transmitter. This work was part of a general program intended to obtain fundamental data upon short-wave transmission, for purposes of point-to-point, as well as for ship-to-shore telephone services. The tests were first made in 1925 on vessels running between New York and Bermuda. Further measurements were made on other ships in subsequent years.

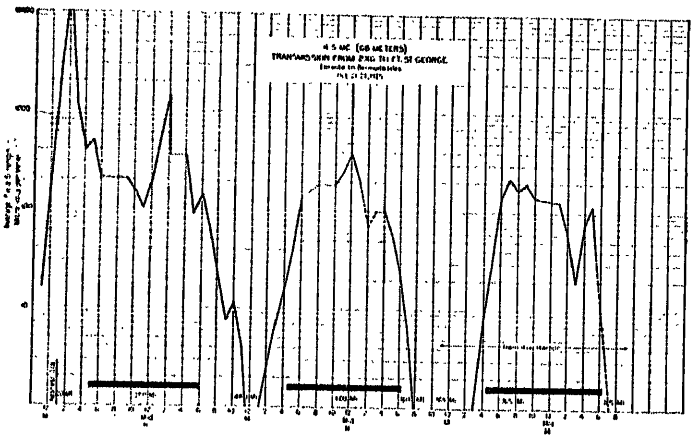

FIG. 1—RECEIVED FIELDS, NEW YORK-BERMUDA RUN 1925

Fig. 1 is an example of the result of these earlier tests. Transmission was from Deal Beach on 4.5 megacycles, (66 meters). The curve shows the relatively weak field which was received as the vessel left dock, due to the considerable stretch of land which intervened in the transmission path, the rise of the field to high values as the ship passed out of the harbor, and the gradual diminution as the vessel continued on her course. It will be observed that transmission on this frequency was effective at night all the way to Bermuda, but that during the daytime the transmission failed for distances greater than a few hundred miles. Corresponding measurements showed that daylight transmission could be secured by means of a higher frequency, such as 9 megacycles (33 meters). Measurements of this kind, supplemented by data obtained for a wide range of distances over land, and for transatlantic distances, have built up a fairly complete set of quantitative data on short-wave transmission over different distances and for various times of the day and year.

Along with this study of transmission conditions, there was carried on the development of short-wave apparatus technique for telephony. The first application was in the field of point-to-point transatlantic operation and the considerable art built up there, including the design of transmitters, receivers, directive antennas, and the working out of two-way operating methods, served as a very useful basis from which to develop the coastal and ship stations for the maritime system.

With this background of development, preparations were made to set up a two-way, short-wave radio telephone system for commercial service. This service was centered upon New York because of the large concentration of ocean-going traffic at that port.

THE TECHNICAL PROBLEM

One of the most important problems to be solved in the design of a short-wave system is that of determining the frequencies necessary for giving the service involved. The frequencies which are best suited to the different distances, time of day, and season of the year for transmission over the North Atlantic are indicated in the curves of Fig. 2. The curves for the greater distances refer to the transmission which appears to take place in the upper regions of the earth's atmosphere and is usually referred to as sky-wave transmission. Each of the sky-wave curves traces the optimum frequency-distance relation for the time of day and season of the year indicated. The curves merely give a general picture of the frequency relation and do not take account of other effects, such as fading, magnetic storms, etc.

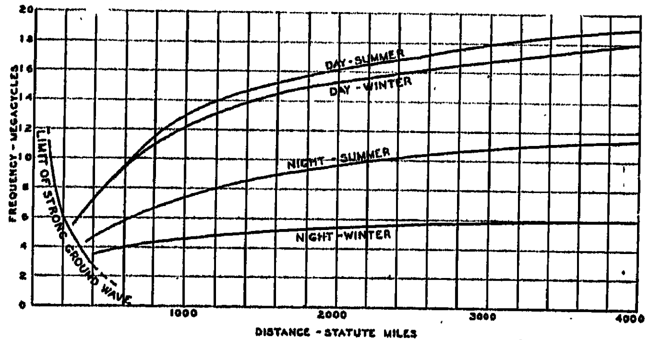

FIG. 2—DISTANCE-FREQUENCY CHARACTERISTIC

The figure brings out very clearly the necessity for using a variety of wavelengths if the ship lanes are to be adequately covered. Fortunately, there is a considerable band on each side of the curves shown, in which good transmission can be obtained, and this enables one to choose a small number of frequencies in the short-wave range which are adequate to cover the conditions. Actually, it is found that a set of about four frequencies will suffice to cover the North Atlantic. For distances greater than a few hundred miles this characteristic obtains irrespective of whether the transmission is over water or over land, by reason of the fact that the transmission appears to take place in the upper regions of the earth's atmosphere.

Closer in to the transmitting station, however, there is the so-called surface component, the attenuation of which is much less over sea water than over land. It will be seen that the surface wave may be relied upon for distances of the order of 200-300 miles, for frequencies of about 4 megacycles. The transmission of this component is much more stable and reliable than is the transmission of the sky wave. It seemed important, therefore, to utilize the surface wave to the maximum extent possible.

With this in mind, a series of transmission measurements was made over a stretch of water between New Jersey, Long Island, and Nantucket for the purpose of more accurately evaluating the effectiveness of the surface wave component, particularly in so far as it bears upon the question of how close to the water front the coastal station need be placed. Transportable transmitting and receiving stations were used in these tests. It was found that as the transmitting or the receiving station was moved away from the water front, the attenuation increased materially. For example, moving either terminal a mile back from the coast line increases the attenuation some 8 db. at 4.5 megacycles. On the other hand, a narrow stretch of land, such as a sand bar, out a few miles from the coast, introduces relatively little loss. These results indicate that if full advantage is to be obtained from the more reliable surface-wave component, the coastal station should be immediately upon the seacoast or a salt-water bay.

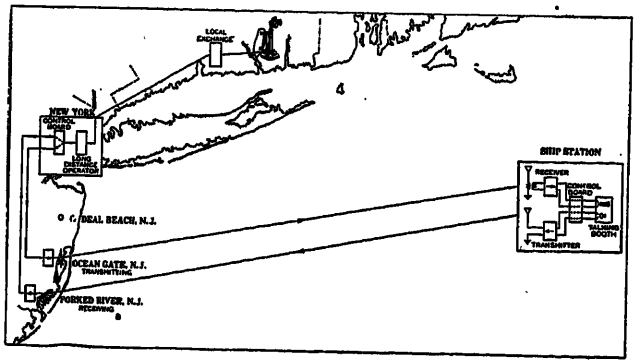

Fig. 3—U. S. Coastal Station Circuit Between New York and Ship

An important factor in connection with radio reception on shipboard is that of electrical interference. The modern steamship requires for its operation and its service a large amount of electrical machinery. In addition to this, it is equipped with various radio telegraphic services. The operation of all of this electrical equipment produces interference in a receiver which is much in excess of that normally encountered in a shore receiving station which can be so located as to be reasonably free from electrical disturbances. Furthermore, there is on the ship another source of disturbance which is due to charging and discharging of various parts of the rigging in the strong electromagnetic fields of the various radio transmitters. These various sources of disturbance were found in the earlier shipboard experiments and the high noise levels are, in general, the predominant factor in limiting the communication range. These factors made it desirable to employ at the shore end as powerful a transmitter as was available and to use whatever benefit could be obtained from antennas designed to be roughly directive along the transatlantic ship lanes. A transmitting set of the type used in transatlantic communication, but adapted for the ship-to-shore wavelengths, was therefore employed.

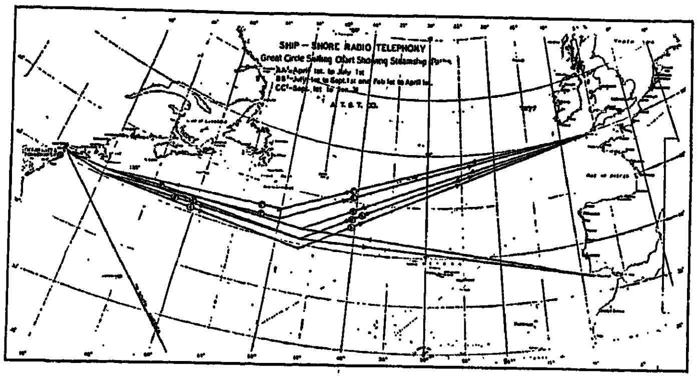
SHIP – SHORE RADIO TELEPHONY
Great Circle Sailing Chart Showing Steamship Routes
AA¹–April 1st. to July 1st
BB¹–July 1st to Sept. 1st and Feb 1st to April 1st
CC¹–Sept. 1st to Jan. 31
A. T. & T. CO.

Fig. 4—North Atlantic Steamship Lanes

Since the shore receiver can be located in a comparatively quiet situation and since use can also be made of roughly directive receiving antennas, there is no advantage in transmitting as large an amount of power from the ship as from the shore. The actual power radiated by the *Leviathan's* transmitter is of the order of 500 watts. The shore receiver is of the type used on the transatlantic radio telephone circuits, working with a directive antenna. The arrangement provides a fairly well proportioned system, the channels being substantially equally effective in the two directions.

The Shore System

The general setup of the system is illustrated in Fig. 3. The coastal stations, sending and receiving, are located about 60 miles south of New York on the New Jersey shore, at Ocean Gate and Forked River. The course followed by the transatlantic ships is indicated on the map of Fig. 4. The directional bearing of this course and the directivity characteristic of the New Jersey shore station antennas are illustrated in Fig. 5. It will be observed that the breadth of the transmitted beam is adequate to take care of the variation of the directional bearing of the course. For steamship routes other than the transatlantic, as for example the coastal route to the South, other antenna arrangements will be required.

In general, the whole coastal station, including the transmitting and receiving units, taken together with the wire line connections and control position in New York, is similar to one end of a transatlantic point-to-point circuit. The transatlantic facilities have been described in previous papers[4] and reference should be made to them for more detail than is given below. The transmitting set has been adapted to cover the frequency range used in the service. It has a power of 15 kw. output of unmodulated carrier and is capable of delivering 60 kw. peak power. A photograph of a similar transmitter at Deal Beach, which is being used for this service pending the completion of the transmitting station at Ocean Gate, is shown in Fig. 6. The antennas are simpler and less directional than those employed in the transatlantic circuit, and give a transmission gain of 8 to 10 db. as compared with a single half-wave antenna.

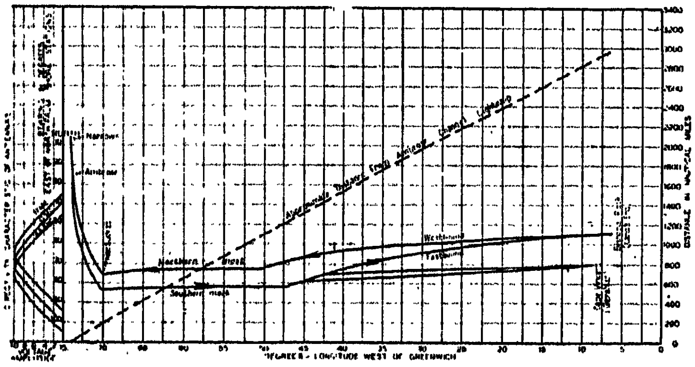

Fig. 5—Directional Bearings

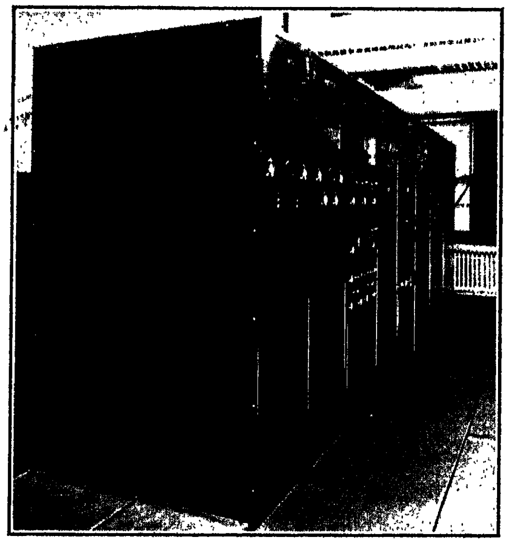

Fig. 6—Illustration Deal Beach Transmitting Set

The receiving station at Forked River has been in operation since the opening of commercial service last December. A photograph of the receiving set is shown in Fig. 7. The receiver is of the double-detection type, of high gain and selectivity, and employs screen-grid tubes. It is provided with automatic gain control. The apparatus shown includes not only the receiving set proper but also the equipment which is required for monitoring the circuit and for connecting with the wire line into New York. The receiving antennas are of the same general type as those used in the transatlantic system, which consist of a row of quarter-wavelength verticals connected alternately top and bottom by quarter-wavelength conductors. In the case of the longer wavelengths used in the ship-to-shore service, the vertical conductors are reduced in height and the horizontal links correspondingly elongated. A photograph of the station at Forked River and two of the antennas is shown in Fig. 8.

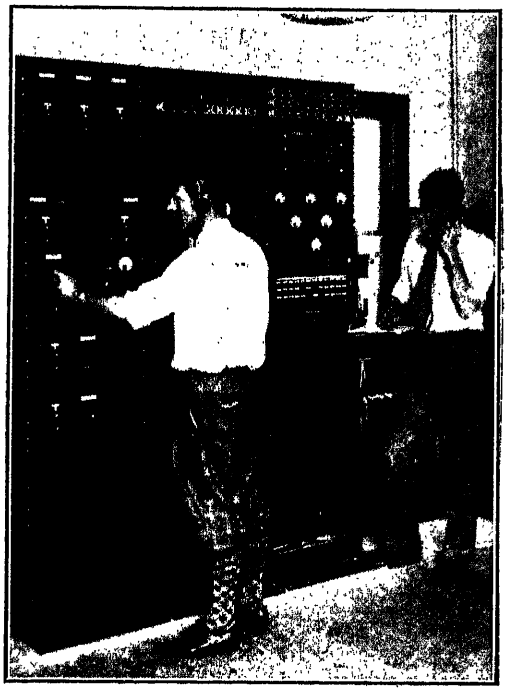

Fig. 7—Illustration Forked River Receiving Set

The control and operating terminal equipment in New York is identical with that in use on the transoceanic radio telephone circuits. The control positions, as they exist in the New York long-distance telephone building for both transatlantic and ship-to-shore circuits, are pictured in Fig. 9. These control positions have associated with them such things as voice-frequency repeaters, indicators of the volume being transmitted and received over the circuit, gain controls, monitoring and testing facilities, and voice-current operated switching devices. The latter prevent the speech received from the ship from being reradiated from the shore transmitting station and permit independent adjustment of amplification in the circuits leading to the transmitting and receiving stations. Thus, the volume sent to the transmitting station may be kept substantially constant, despite variations in speech volume received from different land line subscribers and full modulation of the transmitter may be obtained for over-riding noise on the ship. The function of the technical control operator is that of maintaining the circuit in the correct technical condition for talking. In general, it is the intention that the shore transmitting and receiving stations should function, as far as possible, merely as repeater stations, with the control of the over-all circuit from New York to the ship resting in the New York technical operator.

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[4] Papers on Transatlantic Telephone Service by Messrs. Miller, Bown, Oswald, and Cowan, A. I. E. E. Quarterly Trans., Vol. 49, April 1930, p. 621.

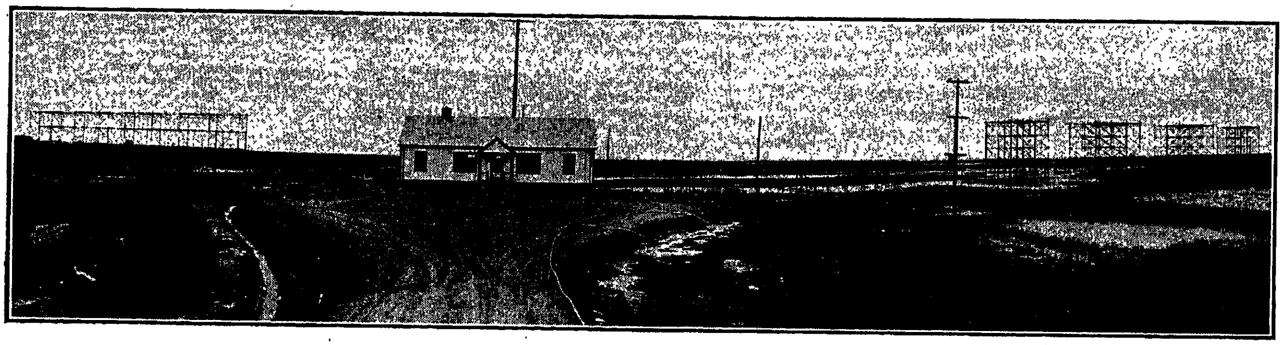

FIG. 8—ILLUSTRATION FORKED RIVER STATION WITH ANTENNA

FIG. 9—NEW YORK TECHNICAL CONTROL POSITIONS

The circuit terminates as an operating facility before a traffic operator at one of the long-distance telephone boards. In Fig. 10 is shown an illustration of the traffic positions for the transatlantic radio telephone circuits, including, at the right, two positions devoted to the ship-to-shore service. The duty of one of these two girls is confined to the radio circuit itself in that she talks to the ship operator, passes and receives information as to calls, and is generally responsible for completing the connection between the ship circuit and the land line subscriber. The adjacent operator is concerned more particularly with the land line subscribers, answering inquiries and recording calls outbound to ships and, in turn, getting in touch with and holding land line subscribers for inbound calls.

THE SHIP STATION

The Leviathan's radio transmitter was designed to supply about 500 watts, 80 per cent modulated radio frequency power to an antenna at frequencies from 3 to 17 megacycles. To insure satisfactory operating conditions, the carrier frequency stability was made as good as that required for point-to-point service and the transmitter has been designed with the object of holding the frequency within 0.01 per cent. This facilitates the establishment of contact between the ship and shore and obviates the necessity for frequent retuning of the shore receiver. The background noise on the unmodulated carrier, due to commutator ripple, etc., is inappreciable and the audio-frequency characteristics from 200 to 2750 cycles is flat to within ± 2 db.

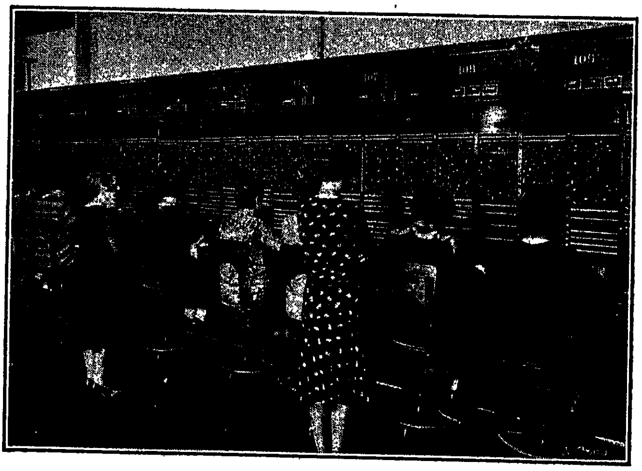

FIG. 10—NEW YORK TRAFFIC POSITIONS

In addition to these electrical requirements, the mechanical design must be such as to withstand ship's vibration, permit easy access to the interior so as to facilitate wave change, and at the same time protect the operators from electrical shock.

The transmitter consists of a crystal oscillator and associated amplifiers. The crystal provides the necessary carrier frequency accuracy and stability and the amplifiers step up the power of the carrier to the desired level. Audio-frequency filters are placed in all voltage supply circuits to eliminate background noise. The modulation system with its associated transformers is designed to produce the requisite audio-frequency quality. A diagram of the circuit is shown in Fig. 11.

Very thorough electrical shielding is necessary between amplifier stages to prevent singing. This shielding makes the changing of coils, which is necessary for the changing of wavelengths, very unhandy, and hence the crystal control and amplifier system, except the last stage, is provided in duplicate, one side being used for the longer and the other for the two shorter waves. Wave changing, except for the output circuit of the power stage, is then accomplished by connecting the proper amplifier to the power stage.

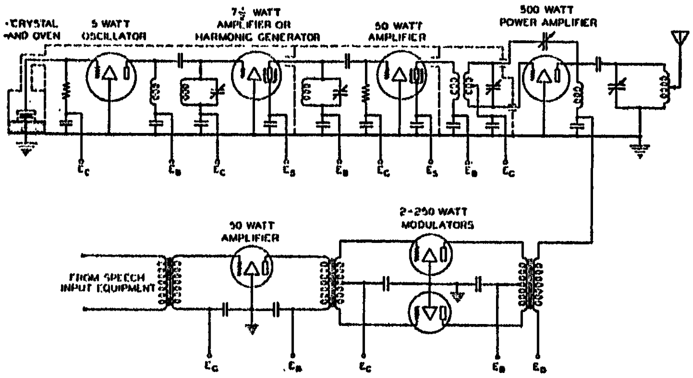

FIG. 11—SHIP TRANSMITTER'S SCHEMATIC DIAGRAM

The quartz plates used in the crystal control system are circular, being approximately one inch in diameter, and are clamped rigidly in the holder. This clamping serves to prevent any change of frequency with mechanical vibration. The holder with its crystal is mounted in a small oven, the temperature of which is held constant at 50 deg. cent. to better than ± 0.1 deg. cent. The thermal system of this oven is so designed that the change of internal oven temperature with temperature changes of the surrounding air is negligible.

As shown in the figure, the crystal is connected between the grid and filament of a 5-watt vacuum tube which, together with the parallel resonant circuit connected to the output of this tube, forms the crystal oscillator. The radio-frequency voltage developed by the crystal oscillator is applied directly to the grid of a 7½-watt screen-grid tube which can be used either as an amplifier or a frequency doubler. The output of this tube, except in the case of the higher frequencies, is applied directly to the grid of a 50-watt screen-grid amplifier. For the higher frequencies a second frequency doubler can be switched into the circuit. The output of the 50-watt tube is coupled through a balanced transformer to the final amplifier stage. The amplifier or frequency doubler stages are separately shielded and radio-frequency filters are provided in all power supply leads.

The power amplifier consists of an air-cooled, three-element, one-kilowatt tube. Neutralization is accomplished by the familiar balancing arrangement shown in the figure. The output circuit of this stage consists of a parallel resonant circuit with provision for tapping in the connection to the antenna.

Modulation takes place in the plate circuit of the final amplifier stage, the plate current supply being fed through a special transformer, the secondary of which is connected to two 250-watt tubes connected push-pull and fed by a 50-watt amplifier.

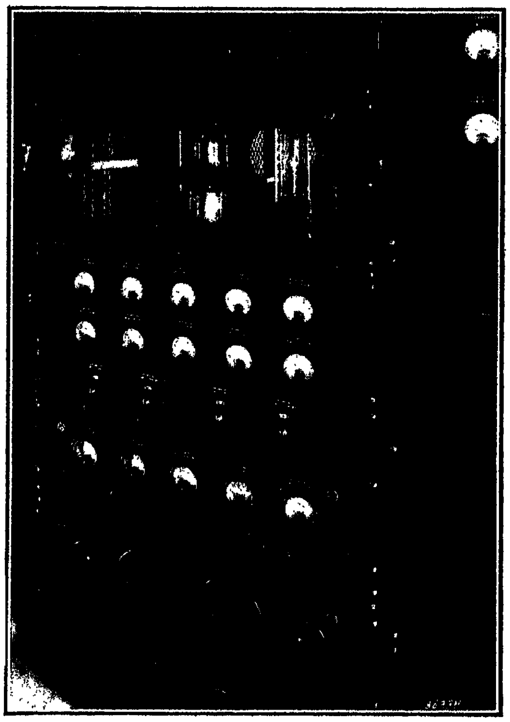

FIG. 12—ILLUSTRATION LEVIATHAN TRANSMITTER

The power supply is obtained from motor-generator sets operated from the 110-volt, d-c. ship supply. Protection of the operators and apparatus is provided by means of relays and contactors in the high-voltage supply circuits which prevent the high voltages from being applied if the filament or grid circuits are not closed or if the doors of the transmitter are open.

An illustration of the ship's transmitter is shown in Fig. 12. The picture is somewhat out of perspective owing to difficulty in taking the photograph in the limited space available on shipboard.

The receiving problem on shipboard is complicated by a number of factors. The transmitting and receiving frequencies must be within a few per cent of each other, if the best transmission conditions for the time and place are to be utilized and if the frequencies are to remain in the bands assigned internationally to the mobile services. This requirement, as well as the noise conditions on shipboard, calls for a receiver of high selectivity, which is obtained, in the present instance, by the use of a double-detection set. The over-all selectivity is accomplished both by having a number of highly selective circuits ahead of the first detector and by using tuned circuits in the intermediate frequency stages, the high-frequency selectivity being used primarily to prevent overloading of the first tube and the intermediate frequency circuits being used to obtain the final selectivity required.

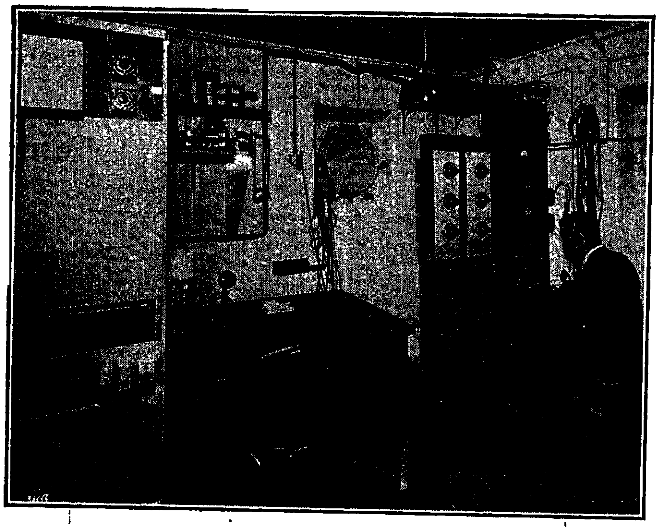

Fig. 13—Receiver on SS. Leviathan

A reduction of the disturbances due to stay noises and better discrimination against the transmitted carrier is obtained if the transmitting and receiving antennas are widely spaced. On the other hand, for operating reasons, it is desirable to have the transmitter and receiver located in the same room. In the case of the *Leviathan* installation, the transmitting antenna is located directly above the radio room, between the second and third stacks, and the receiving antenna is placed as far as possible behind the third stack. The receiving antenna is connected through a suitable step-down circuit to a shielded transmission line, the other end of which is connected to the receiver; the receiver itself being very thoroughly shielded to avoid direct interference from the transmitter. On account of limited space, only two antennas are provided to handle the four frequencies, each antenna representing a compromise between the most efficient antennas which could be put up to handle the separate wavelengths.

As stated above, the receiver itself is of the double-detection type, using heater type tubes throughout. Screen-grid tubes are used for the first detector and intermediate frequency amplifiers and three-element tubes in the remaining positions. A photograph of the receiver and associated voice-frequency equipment, as it is installed on the *Leviathan*, is shown in Fig. 13, and a diagrammatic representation of the receiver is shown in Fig. 14. The high-frequency selective system consists of four separately shielded tuned circuits, coupled by small capacities. The use of a screen-grid tube in the detector circuit gives a twofold advantage over the use of a three-element tube in that a higher input impedance is maintained at the higher frequencies and the necessity for neutralizing against the reaction of the beating oscillator on the input circuit is eliminated. The beating voltage is made of the order of 75 to 100 volts for the purpose of reducing the effective tube noise in the detector plate circuit. With this arrangement no d-c. plate voltage is ordinarily required The screen voltage is 22 volts. The output circuit is tuned to the intermediate frequency of 300,000 cycles and connection with the first intermediate amplifier is effected by means of a low impedance transmission line. The intermediate frequency amplifier stages are coupled by means of double tuned circuits. The use of properly designed circuits of this type makes it possible to obtain a high degree of selectivity against undesired frequencies while obtaining sufficient band width to maintain ease of tuning and to pass the desired frequencies. The second detector is of the conventional grid bias type. Automatic gain control is provided in which a certain amount of the carrier is taken at the end of the intermediate frequency stages, amplified and rectified. The resulting d-c. current produces a voltage drop across a resistance, which is applied to the grid of the first detector in such a manner that an increase in the intermediate frequency output brings about a reduction in the total set gain and vice versa. Manual gain control for following wide changes in the received fields is accomplished by variation of the

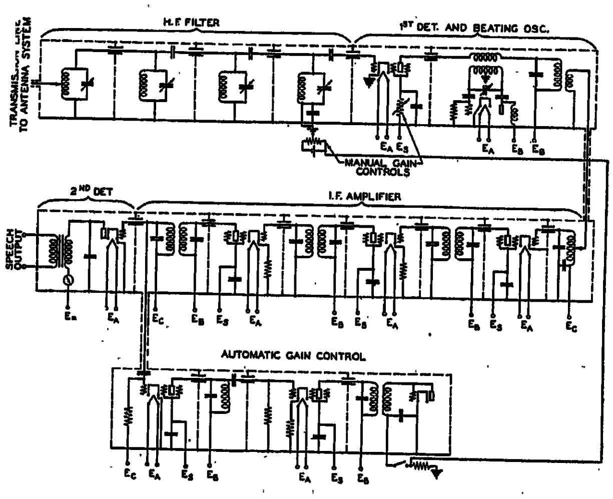
TRANSMISSION LINE TO ANTENNA SYSTEM
H.F FILTER
1ST DET. AND BEATING OSC.
E_A E_S
E_A E_B E_B
MANUAL GAIN CONTROLS
2ND DET
I.F. AMPLIFIER
SPEECH OUTPUT
E_B E_A E_C E_B E_S E_A E_B E_S E_A E_B E_S E_A E_C
AUTOMATIC GAIN CONTROL
E_C E_A E_S E_B E_A E_S E_B

Fig. 14—Ship Receiver Schematic Diagram

voltages applied to the grid and the screen of the first detector.

The voice-frequency equipment, in addition to the desk telephone set located in the subscriber's booth, comprises a technical operator's position located adjacent to the ship's receiver, and an attendant's desk located on a lower deck in a room adjacent to the subscriber's booth. The control equipment consists of repeaters, volume control devices, and volume indicators, by means of which the levels of the incoming and outgoing signals can be properly adjusted. Keys are provided which enable the technical operator to talk either over the radio circuit or to the ship subscriber. The booth attendant has facilities by which he can talk either to the subscriber or to the control operator and has a connection with the ship's telephone system for the purpose of locating persons on the ship and calling them to the radio telephone booth. The subscriber's booth is provided with a desk telephone set having a high-grade transmitter. The outgoing and incoming circuits are shielded from each other and brought separately to the transmitter and receiver of the subscriber's set. An illustration of the subscriber's booth on the *Leviathan* is shown in Fig. 15.

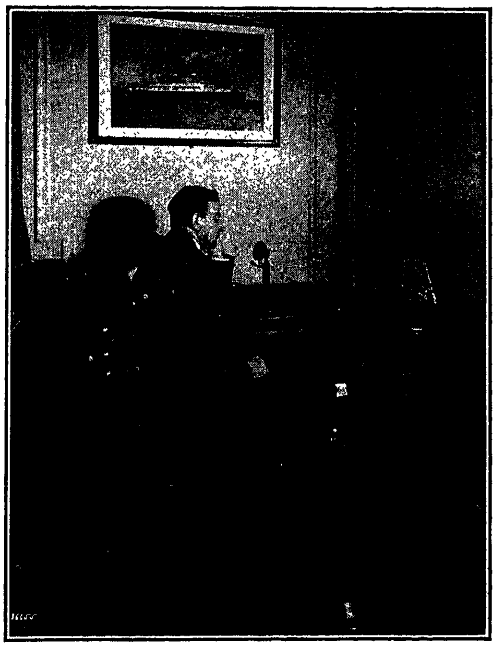

FIG. 15—SUBSCRIBER'S BOOTH ON LEVIATHAN

THE WAVELENGTH SITUATION AND SIMULTANEOUS TELEPHONE AND TELEGRAPH OPERATION

Communication between ship and shore is carried out by the use of a pair of frequencies, one for transmission in each of the two directions, separated from each other by about 3 per cent. The specific frequencies which were first assigned by the Federal Radio Commission to the shore station and the *Leviathan* were necessarily chosen on more or less of a makeshift basis, in the absence of any comprehensive wavelength plan for this new service. The Commission has recently had under study the setting up of more adequate provisions for ship-to-shore telephone channels, whereby it is hoped a series of frequencies may be designated for telephone service exclusively and whereby there may be established the relation between the telephone and the telegraph frequencies necessary for the avoidance of interference between the two services. Especially is coordination of the two sets of frequencies necessary on the larger vessels, in order that simultaneous telegraph and telephone service may be given without mutual interference. On the larger liners simultaneous use of the radiotelephone and radiotelegraph service must be provided for. This means that the transmitters of both services must keep accurately on their frequencies and be free of spurious components, and that the receivers must be highly selective. It further entails that the transmitting and receiving frequencies in each of the two cases be so coordinated that the transmission frequency of one service does not lie too near the receiving frequency of the other, and bespeaks a considerable amount of mutual cooperation between the operating agencies involved. Difficulties of fitting in the two services were encountered in the early work on the

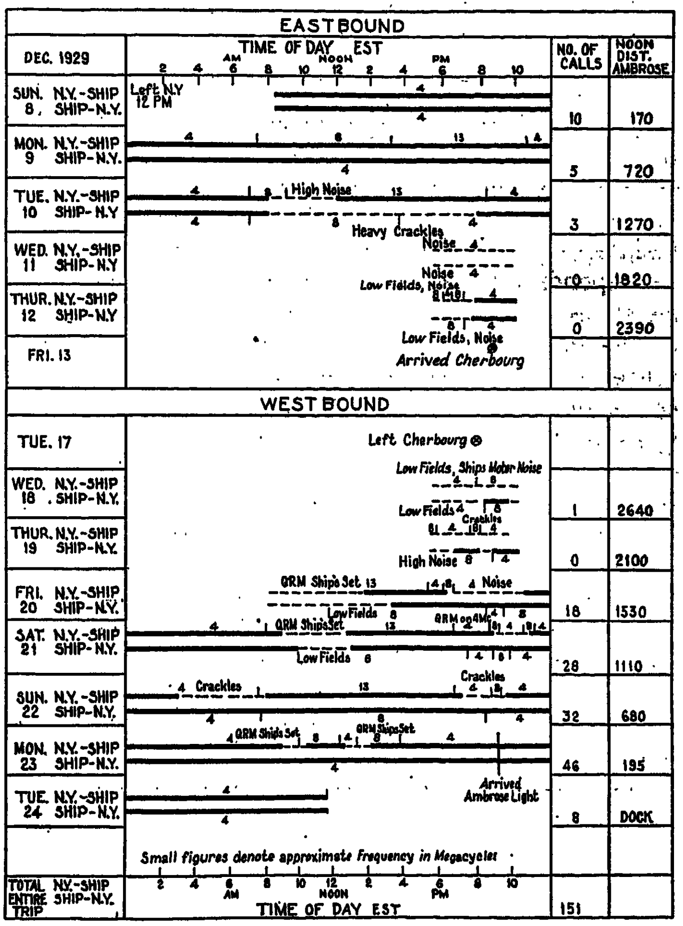

FIG. 16—TRANSMISSION RESULTS BETWEEN SS. LEVIATHAN AND NEW YORK

Leviathan and, although the problem has not been worked out to final solution, sufficient progress has been made, in cooperation with the engineers of the Radio Corporation of America, to enable the telegraph and telephone services to be conducted simultaneously without undue interference.

In view of the fact that ships of a number of nations are already preparing to give radio telephone service on the transatlantic routes and with the probability of this service also extending to other parts of the world, it would appear to be a matter of importance that the whole question of marine frequency allocations be worked out in the near future not merely on a national but also on an international basis.

TRANSMISSION RESULTS

The transmission results which have been obtained with the *Leviathan* on her first trip of commercial service are summarized in Fig. 16. It will be noted that practically continuous 24-hour communication was maintained for distances within 1000 miles of the shore, corresponding to two days out. The service at greater distances was more intermittent. This was largely due to the fact that during this first trip the effort was concentrated on covering reliably the more important nearer-in distances, and the ship was not prepared to transmit on frequencies above 8 megacycles. The service proved to be much in demand by the passengers, as is indicated by the number of calls completed each day, particularly on the return trip. A similar number of test and demonstration calls was made during the voyage. The calls were completed without undue delay, there being only one ship involved, and a fairly high grade of communication was obtained.

In conclusion it will be realized that the solution of the technical problem of ship-to-shore telephony is now well in hand and has been carried to the point of having proved the practicability of giving this service. Further problems are naturally arising in carrying the development into more general effect, particularly operating problems and those concerned with the international coordination of the service. The indications are that the larger transoceanic ships will be rather generally equipped for telephony and that the service will become one of permanent value in the maritime field.

Discussion

W. B. Kirke: I should like to ask the authors if there are other insulations beside the *Leviathan* installed?

Lloyd Espenschied: Yes, in addition to the *Leviathan* there are the *Majestic* and *Olympic* which have been equipped in England. All three ships are now giving service to both the English and the American sides of the Atlantic.

In reference to the suggestion which was just made, that a record be kept of the radio transmission effects for the purpose of establishing a possible correlation with natural phenomena, such as solar activity and magnetic storms, I might say that we do keep quite a careful record of quantitative measurements of radio transmission, made from day to day, and that we attempt to correlate these results with the occurrence of sunspots and magnetic storms. In general, there appears to be quite a definite correlation between variations in radio transmission and the 11-year cycle of solar activity. There is as yet no very consistent correlation between the individual sunspots and radio transmission. However, we do find that magnetic storms, *i. e.*, at abnormal conditions in the earth's magnetic field, are accompanied almost invariably by a reduced effectiveness of short-wave transmission across the North Atlantic. It may be of interest to know this: As a convenient means for quickly detecting the occurrence of a magnetic storm, we maintain a pair of wires, grounded at each end and each one hundred miles or so long, one wire running north and south, the other, east and west. We have current recorders continually in operation on these two circuits and find that the failure of short-wave transmission is almost invariably accompanied by abnormal fluctuations in ground current. This whole matter of the relationship between solar disturbances, magnetic storms, and radio transmission is, of course, a large and interesting field for exploration, and is being studied by a number of people. It is hoped that radio will prove to be a useful tool in helping to disclose the natural phenomena which are at work.

Shunt Resistors for Reactors

BY F. H. KIERSTEAD,* H. L. RORDEN,* and L. V. BEWLEY*
Member, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*The object of this paper is to present the results of an investigation to determine the effect of resistors shunting current limiting reactors, on the impulse behavior of the typical system employing them. The typical system is reduced, without loss of essential generality, to a comparatively simple analytic network; and the impulse behavior of this network is then calculated mathematically. These calculations are adequately supported by test results with impulse generator and cathode ray oscillograph, both in the field and in the laboratory, and accurate agreement established. Methods are developed and illustrated for taking into account incident waves, such as those due to lightning, of arbitrary shape. Under the most unfavorable condition of wave shape and circuit conditions, voltages approaching four times that of the incident wave may be built up at the bus by reflections and oscillations. In general, through the use of a shunting resistor, of proper value, three beneficial results are obtained: (a) Oscillations are completely eliminated, thereby limiting the maximum voltage which can occur on the bus to never more than twice that of the incident wave. (b) The initial rise in voltage on the line side of the reactor, due to positive reflections, is reduced and can be held down to the final value corresponding to the tail of an infinite rectangular incident wave. (c) The internal oscillations in the reactor itself can be eliminated, and the voltage distribution improved. The extent to which these improvements can be realized, under a wide range of possible operating conditions, is discussed and illustrated in this paper. With some combinations, the improvements are very great, while for others, there is no appreciable gain. The criterion for utilization of shunt resistors is,—will the elimination of oscillations and the reduction of positive reflections, under the most unfavorable possible operating condition which can be experienced on a given system, be worth the cost?*

* * * * *

INTRODUCTION

THIS paper is part of an extensive investigation of lightning waves on transmission lines, which has been under way for a number of years under the general direction of Mr. F. W. Peek, Jr.

There has been a wide divergence in the opinion of engineers as to the relative merits of resistors shunting current limiting reactors. Some have ascribed to them the property of reducing transient voltages, which apparatus has to stand, while others have felt they do not reduce these transients and, in some cases, may actually increase the voltage stresses in connected apparatus. The purpose of the investigation described in this paper was to determine by an analytical treatment of the apparatus and circuits involved, what benefits or detriments should be expected from the use of resistors.

The presentation of this investigation takes the following form:

I. The typical system using current limiting reactors is considered.

II. The general analytic network of the typical system is resolved into its generalized impedances and the progress of an incoming or incident wave through these impedances is outlined.

III. The equivalent circuit of the network is described.

IV. The shape of the incident waves used in the calculations is defined and the mathematical methods of dealing with them are described.

V. Formulas for calculating the effects of the incident wave on the typical system are given, together with the results of calculations on this system.

VI. Experimental results are given which show that the effect of waves upon such systems may be accurately calculated.

VII. The effect of a resistor on the internal distribution of voltage in a reactor is discussed and experimental results are shown.

VIII. General conclusions are drawn.

I. TYPICAL CIRCUIT USING CURRENT LIMITING REACTORS

In a typical system in which reactors are used, as shown in Fig. 1, the apparatus and circuits may be represented for mathematical treatment of impulse voltages as follows:

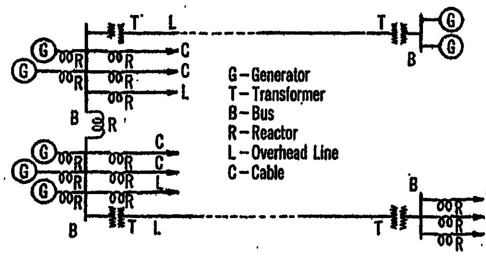

FIG. 1—SINGLE LINE DIAGRAM OF A TYPICAL SYSTEM USING CURRENT LIMITING REACTORS

1. Overhead lines by surge impedances of 300 to 600 ohms.
2. Underground cables by surge impedances of 30 to 90 ohms.
3. Station busses by capacitances to ground.
4. Terminal equipment by the equivalent circuits shown in Fig. 2. The use of a capacitance as the equivalent circuit of a transformer, has been established theoretically and experimentally.$^{1-7-10}$ The equivalent circuit for a generator was determined by Mr. E. W. Boehne and will be discussed in a paper by

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*General Transformer Engg. Dept., General Electric Company, Pittsfield, Mass.

Presented at the North Eastern District Meeting of the A. I. E. E., Springfield, Mass., May 7-10, 1930.

[1] For references see Bibliography.

him to be presented at the summer convention. Reactors behave practically as pure inductances in their external effect on traveling waves, as will be shown by the experimental results recorded in this paper. They range in value from 0.0002 to 0.010 henries.

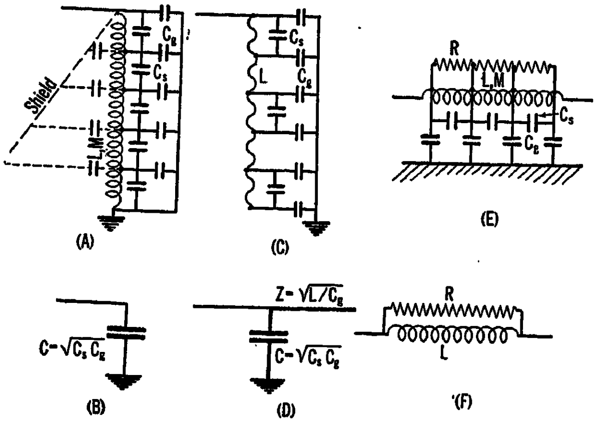

FIG. 2—EQUIVALENT CIRCUITS OF TERMINAL APPARATUS TO LIGHTNING SURGES

- A. Transformer, ideal.
- B. Transformer, approx.
- C. Rotating machine, ideal.
- D. Rotating machine, approx.
- E. Reactor with shunt resistor, ideal.
- F. Reactor with shunt resistor, approx.

The following table gives the range of effective capacitance to ground of apparatus and station busses.

Apparatus	Capacitance in m. f.		
	Max.	Min.	Ave. for all apparatus
Generators (salient poles)	0.003	0.001	0.002
Turbo alternators	0.010	0.002	0.005
Transformers (distribution)	0.0020	0.0004	0.0010
Transformers (power)	0.0010	0.0002	0.0005
Transformers (non-resonating)	0.0050	0.002	0.003
Bus systems	0.015	0.002	0.005

II. THE GENERAL ANALYTIC NETWORK OF A TYPICAL SYSTEM

The general analytic network of a typical system is shown in Fig. 3. It consists of the following generalized impedances:

1. Surge impedances of lines and cables—z_1, z_2 . . . . . z_n.
2. Series impedance (reactors, choke coils, series capacitors, etc.) $Z_1(p)$, $Z_2(p)$ . . . . . $Z_n(p)$.
3. Shunt impedances to ground (transformers, generators, busses, etc.) $Z_g(p)$.
4. Impedances $Z_o(p)$ and $Z(p)$ which consist of several impedances in series and in parallel, included together to simplify mathematical treatment.

An incident wave e_1 traveling along line z_1 and striking impedance $Z_o(p)$ causes:

1. A reflected wave e_1' which combines with e_1 to give a total potential e_1'' at the transition point.
2. Refracted waves transmitted to the other lines, e_2'', e_3'' . . . . . e_n''.
3. A potential e across the shunt impedance $Z_g(p)$ to ground.
4. Potentials E_1, E_2 . . . . . E_n across the series impedances $Z_1(p)$, $Z_2(p)$ . . . . . $Z_n(p)$.

The general mathematical solution of this circuit is given in Appendix I.

III. EQUIVALENT CIRCUIT OF THE NETWORK

The circuit that has been used (as the equivalent of the network of a typical system in which reactors are placed) in the calculations of this investigation is shown in Fig. 4 and its complete solution is given in Appendix II. In this figure, the surge impedance z_1 is the feeder (either overhead line or underground cable) which carries the oncoming incident wave e_1. This feeder terminates at the bus in a reactor unit of inductance L, shunt resistance R, and an effective series resistance r (due primarily to the transient skin effect of the lightning wave). Tests with a typical impulse on a reactor of 0.002 henrys inductance indicated an effective series resistance of about 100 ohms. The total equivalent capacitance to ground, due to all terminal equipment (including transformers, generators, and busses) has been lumped as a single capacitance C. All outgoing feeders as well as the effective surge impedances of rotating apparatus, have been lumped as a single outgoing feeder of surge impedance z_2. Attention is called to the fact that in a typical power system, there would also be reactors in the outgoing feeders, but including them in the analysis increases the order of the differential equations so that their explicit solution becomes too involved. The error introduced by omitting these inductances is not great, and will be

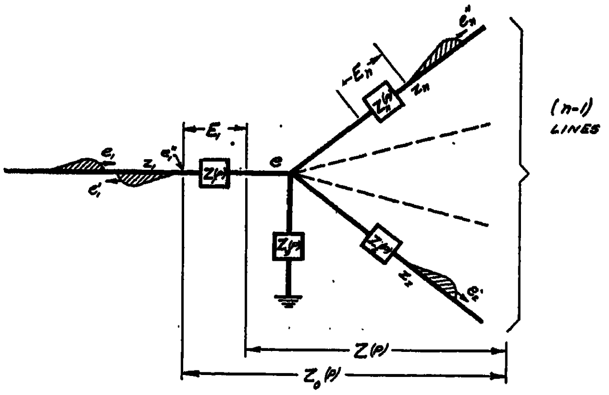

FIG. 3—GENERAL NETWORK

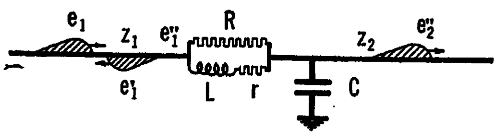

FIG. 4—EQUIVALENT CIRCUIT OF LINES, REACTOR AND TERMINAL APPARATUS

discussed later on. When the incident wave e_1, strikes the junction, a part e_1' is reflected back, and a part e_2'' passes on to the outgoing feeders as a transmitted wave.

IV. SHAPES OF THE INCIDENT WAVE USED IN CALCULATIONS AND MATHEMATICAL METHOD OF DEALING WITH THEM

There are usually three ways in which the effect of wave shape may be computed:

(a) Substitute in the circuit equations of the variable, the operational equivalent for the analytical expression of the wave and solve the resultant operational equations. This practise raises the degree of the differential equation and may therefore complicate it too much to allow of a rational solution.

(b) Find the solution for an infinite rectangular wave, and then through the application of Duhamel's theorem, find the solution corresponding to the time function of the applied wave. This practise necessitates making a complicated integration which may prove impossible except in very simple cases.

(c) By means of Heaviside's shifting theorem, find solutions for simple exponential applied waves. Then the solution corresponding to more complicated waves, which are compounded of exponentials, is found by superposition of solutions.

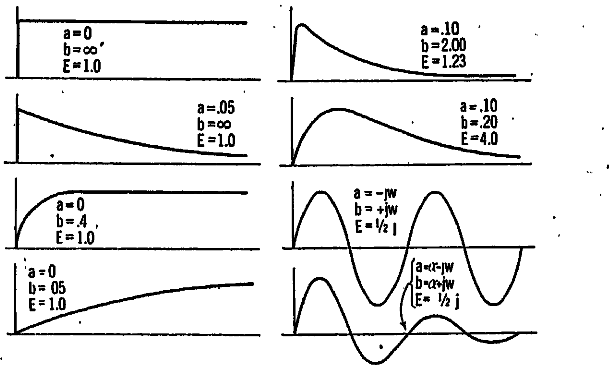

FIG. 5—EMPIRICAL WAVE SHAPES GIVEN BY $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

The computations in this paper have been based on this third method which is more fully described in Appendix I.

Examples of waves which may be built up out of a pair of exponential waves are shown in Fig. 5. These also may be compounded. For instance, if it is required to find the solution corresponding to a "typical lightning surge" having a damped high-frequency oscillation, a superposition of the 6th and 8th waves of Fig. 5 (with the constants adjusted, of course, to give the proper wave-fronts, crests, tails, frequencies, and damping) will prove quite satisfactory.

The formulas and methods given in this paper for the calculation of reflected and transmitted voltages apply for an incident wave having any of the shapes given in Fig. 5. The computations of reflected and transmitted waves have been based on incident waves of shapes similar to the 1st, 2nd, and 6th waves in Fig. 5.

V. FORMULAS FOR CALCULATING THE EFFECTS OF AN INCIDENT WAVE ON THE TYPICAL SYSTEM, AND RESULTS OF CALCULATIONS

In this section are given the calculated effects of impulses on a network having an equivalent circuit as shown in Fig. 4.

The derivation of the equations which were used in making these calculations is given in Appendix II. It is based on an incident wave of the shape given by the equation $e_1 = E_1\,\epsilon^{-at}$.

The equation of the reflected wave is:

$$\begin{aligned} e_1' &= x\,E_1\epsilon^{-at} + y\,E_1\,\epsilon^{-\alpha t}\sin(\omega\,t + \phi) \\ &= x\,E_1\,\epsilon^{-at} + Y\,E_1\,\epsilon^{-\alpha t}\sinh(\Omega\,t + \Phi) \\ &= x\,E_1\,\epsilon^{-at} + E_1\,\epsilon^{-\alpha t}(W_1\,\epsilon^{\Omega t} - W_2\ {}^{-\Omega t}) \end{aligned}$$

The equation of the transmitted wave is:

$$\begin{aligned} e_2'' &= u\,E_1\,\epsilon^{-at} + v\,E_1\,\epsilon^{-\alpha t}\sin(\omega\,t + \psi) \\ &= u\,E_1\,\epsilon^{-at} + V\,E_1\,\epsilon^{-\alpha t}\sinh(\Omega\,t + \Psi) \\ &= u\,E_1\,\epsilon^{-at} + E_1\,\epsilon^{-\alpha t}(J_1\,\epsilon^{\Omega t} - J_2\,\epsilon^{-\Omega t}) \end{aligned}$$

The definition of each of the symbols has been given in Appendix II and for the sake of brevity is not repeated here.

Values of α, $\omega = j\,\Omega$, x, $y = Y/j$, $\phi = .\,j\,\Phi$, W_1, W_2, u, $v = V/j$, $\psi = j\,\Psi$, J_1 and J_2 are tabulated for various combinations of r, L, R, C, z_1 and z_2 as follows:

1. Table I for $a = 0$ in $e_1 = (\epsilon^{-at}$ rectangular wave of unit height).
2. Table II for $a = 0.1 \times 10^6$.
3. Table II for $a = 0.2 \times 10^6$.

Curves drawn from values in Table I are plotted in Figs. 6 to 17 inclusive. They give the total voltage on each side of the reactor ($e_1'' = e_1 + e_1'$ and e_2'') resulting from the incident wave $e = \epsilon^{-at}$ where $a = 0$. Also on Figs. 6, 7, 8, 12, and 13, are drawn for comparison e_1'' and e_2'' corresponding to the incident wave $e_1 = 4\,(\epsilon^{-0.1t} - \epsilon^{-0.2t})$. This wave has a seven microsecond front, a crest value of unity, and the tail decreases to ½ of the crest value in 20 microseconds. The data used in plotting these latter curves were taken from Tables II and III. They are plotted below the zero axis to avoid interference with the waves corresponding to the rectangular incident wave.

The waves plotted on the left hand side of the figures give the voltages obtained with no resistor ($R = \infty$) shunting the reactor while those on the right hand side correspond to a 1000-ohm resistor shunting the reactor.

Fig. 6. Referring to Fig. 6 and Fig. 4 it will be noted that the curves apply to the case of an impulse coming from an overhead line and passing through a reactor into a capacitance to ground. This case represents a transformer at the end of a line with a reactor between it and the line. It also represents the case where all the feeders except one have been disconnected from the bus with the wave entering on this remaining

TABLE I

(Calculated) $a = 0,\ r = 100,\ L = 0.002$

Z_1	Z_2	$C\ 10^{-6}$	R	$\alpha\ 10^6$	$\omega\ 10^6$	x	y	ϕ	W_1	W_2	e_1'' max.	u	v	ψ	J_1	J_2	e_2'' max.
400	∞	0.0050	∞	0.125	0.291	1.00	1.37	180	..	..	2.200	2.00	2.17	−66.8	..	..	2.735
“	“	“	1000	0.168	0.224	1.00	1.13	210.4	..	..	2.064	2.00	2.01	263.7	..	..	2.227
“	“	0.0005	∞	0.125	0.992	1.00	0.40	180	..	..	2.225	2.00	2.02	−82.8	..	..	3.392
“	“	“	1000	0.810	0.362	1.00	0.71	306	..	..	2.014	2.00	3.95	−30.4	..	..	2.194
400	400	0.0050	∞	0.375	0.291	0.111	0.916	104.4	..	..	2.000	0.889	1.46	217.8	..	..	1.051
“	“	“	1000	0.418	0.020	0.102	0.780	24.5	..	..	1.438	0.898	4.60	191.2	..	..	0.898
“	“	0.0005	∞	2.625	$j2.154$	0.111	$-j0.124$	$j2.66$	..	..	2.000	0.889	$+j0.62$	$-j1.16$	−0.985	−0.097	0.942
“	“	“	1000	3.310	$j\ 3.035$	0.102	$-j0.452$	$j0.67$	0.442	0.116	1.492	0.898	..	..	−0.468	0.430	0.898
400	50	0.0050	∞	2.125	$j1.85$	−0.455	$-j0.065$	$j3.77$	1.445	0.001	2.000	0.182	$-j0.104$	$-j1.33$	−0.195	−0.014	0.191
“	“	“	1000	2.168	$j1.96$	−0.479	$-j0.246$	$j2.02$	0.924	0.016	1.431	0.185	..	..	−0.122	−0.063	0.185
“	“	0.0005	∞	20.13	$j19.85$	−0.455	0	$j5.99$	1.455	0.000	2.000	0.182	$-j0.030$	$-j2.48$	−0.183	−0.001	0.182
“	“	“	1000	20.81	$j20.61$	−0.479	$-j0.268$	$j1.94$	0.927	0.019	1.431	0.185	..	..	−0.117	−0.068	0.185
50	∞	0.0050	∞	0.038	0.314	1.00	0.159	180	..	..	2.091	2.00	2.015	263.2	..	..	3.370
“	“	“	1000	0.132	0.296	1.00	0.165	215.3	..	..	2.029	2.00	2.040	281.0	..	..	2.601
“	“	0.0005	∞	0.038	0.999	1.00	0.05	180	..	..	2.042	2.00	2.001	267.9	..	..	3.774
“	“	“	1000	0.989	0.263	1.00	0.19	210.7	..	..	2.000	2.00	7.25	196.0	..	..	2.000
50	400	0.0050	∞	0.288	0.234	0.818	0.183	86.9	...	..	2.000	1.455	2.303	219.1	..	..	1.726
“	“	“	1000	0.382	$j0.066$	0.815	$-j0.108$	$j0.76$	..	..	1.903	1.479	$-j2.364$	$-j0.59$	..	..	1.429
“	“	0.0005	∞	2.538	$j2.25$	0.818	$-j0.019$	$j2.96$	..	..	2.000	1.455	$+j0.758$	$-j1.40$	..	..	1.517
“	“	“	1000	3.489	$j3.28$	0.815	$-j0.102$	$j0.79$	0.113	0.024	1.918	1.479	..	..	−0.945	0.534	1.479
50	50	0.0050	∞	2.038	$j1.94$	0.500	$-j0.013$	$j4.33$	0.500	0.000	2.000	0.500	$-j0.163$	$-j1.84$	−0.513	0.013	0.511
“	“	“	1000	2.132	$j2.04$	0.476	$-j0.077$	$j2.42$	0.432	0.003	1.912	0.524	..	..	−0.443	0.081	0.522
“	“	0.0005	∞	20.04	$j19.94$	0.500	0	$j5.99$	0.500	0.000	2.000	0.500	$-j0.050$	$-j2.99$	−0.501	0.001	0.501
“	“	“	1000	20.99	$j20.89$	0.476	$-j0.105$	$j2.12$	0.435	0.006	1.908	0.524	..	..	−0.434	0.090	0.522

feeder. It seems probable that such cases rarely occur in practise.

It is to be noted that with a rectangular wave, the maximum voltage on the line side of the reactor is 2.2 times the incident wave and that the transmitted voltage is 2.74 times the incident wave. When the reactor is shunted by a 1000-ohm resistor, the voltage on the line side is reduced to 94 per cent of the voltage without resistors and on the other side to 81½ per cent. With the wave having a 7-microsecond front, the voltage on the line side is practically unchanged by the shunt resistor while on the other side the voltage is reduced to 90 per cent of the voltage without resistors. A greater improvement would have resulted if the shunt resistance had been of approximately the same magnitude as the surge impedance Z_1.

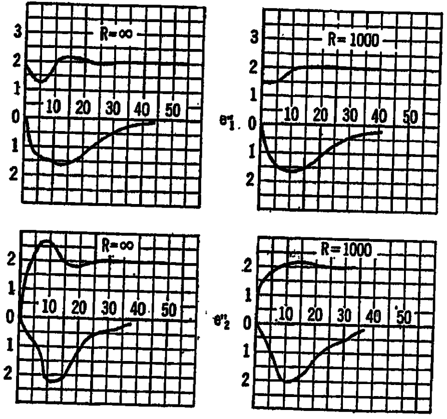

FIG. 6—$r = 100,\ L = 0.002,\ Z_1 = 400,\ Z_2 = \infty,$ $C = 0.005 \times 10^{-6}$

Fig. 7. This case is very similar to that of Fig. 6. The capacitance to ground is only 1/10 of that of Fig. 6 and consequently, the voltages are higher and the improvement due to the resistor is greater, the line voltage and the transmitted voltage being reduced to 73 per cent and 64 per cent respectively of that without the resistor.

Figs. 8 and 9. These cases are similar to that of a bus system with overhead feeders. In such a system, z_2 would have reactors in it and would represent several feeders in parallel. It is believed that the inaccuracy due to not having reactors is partially offset by the use of the surge impedance of only one line. The fact that the transmitted voltage is low and that its wave front is not as steep as that of the incident wave (due to the shunt capacitance), in a measure justifies this simplification of the circuit.

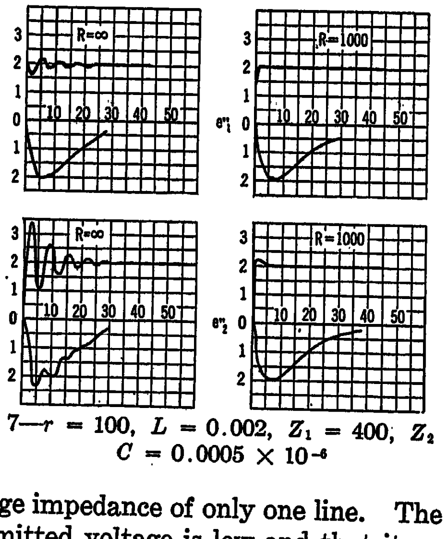

FIG. 7—$r = 100,\ L = 0.002,\ Z_1 = 400,\ Z_2 = \infty,$ $C = 0.0005 \times 10^{-6}$

The improvement due to the resistor is chiefly on the line side of the reactor. The voltage at this point is

TABLE II—(Calculated) $a = 0.1 \times 10^6, r = 100, L = 0.002$

Z_1	Z_2	$C\ 10^{-6}$	R	$\alpha\ 10^6$	$\omega\ 10^6$	x	y	ϕ	W_1	W_2	u	v	ψ	J_1	J_2
400	∞	0.0050	∞	0.125	0.2905	1.471	1.494	198.4°			2.353	2.362	265.1°		
"	"	"	1000	0.1679	0.2245	1.466	1.341	230.4°			2.34	2.41	283.6°		
"	"	0.0005	∞	0.125	0.9922	1.041	0.4054	185.7°			2.030	2.031	268.6°		
"	"	"	1000	0.811	0.358	1.022	1.38	335.3°			2.03	4.44	333.8°		
400	400	0.0050	∞	0.3750	0.2905	0	1.088	113.3°			1.250	1.721	226.6°		
"	"	"	1000	0.418	0.01982	−0.01407	1.012	25.0°			1.268	6.03	192.1°		
"	"	0.0005	∞	2.625	−j2.154	−0.1297	−j0.490	j1.57			1.153	j0.705	j1.64		
"	"	"	1000	3.3107	j3.034	−0.1479	−j0.570	j0.89	0.694	0.1177	1.171			−0.734	0.437
400	50	0.0050	∞	2.125	j1.848	−1.321	−j0.8996	j1.68			0.2920	−j0.1308	j1.54		
"	"	"	1000	2.168	j1.962	−1.1352	−j0.3485	j2.33	1.797	0.0169	0.3015			−0.2369	0.0646
"	"	0.0005	∞	20.125	j19.85	−1.289	−j0.2638	j2.85			0.2863	j0.03812	j2.71		
"	"	"	1000	20.81	j20.61	−1.360	−j0.3750	j2.26	1.807	0.0194	0.2957			−0.2272	0.0686
50	∞	0.0050	∞	0.0375	0.3140	1.049	0.1573	198.1°			1.951	1.990	281.3°		
"	"	"	1000	0.1321	0.2955	1.049	0.1793	233.2°			1.94	2.22	299.1°		
"	"	0.0005	∞	0.0375	0.9993	1.005	0.04997	185.7°			1.995	1.999	266.4°		
"	"	"	1000	0.9893	0.2626	1.004	0.205	330.7°			1.993	8.40	193.8°		
50	400	0.0050	∞	0.2875	0.2342	0.7778	0.2250	99.1°			2.222	2.846	231.3°		
"	"	"	1000	0.3821	j0.06608	0.7722	−j0.07912	j1.29			2.278	−j3.243	j0.65		
"	"	0.0005	∞	2.5375	j2.250	0.7208	−j0.02374	j3.16			2.278	−j9488	j1.61		
"	"	"	1000	3.489	j0.328	0.7119	−j0.1441	j1.10	0.2169	0.0239	2.352			−1.811	0.542
50	50	0.0050	∞	2.0375	j1.9369	77.0	0		78	0	80	−j2.065	j4.33	−80.012	−0.012
"	"	"	1000	2.132	j20.36	11.17			−10.27	0.0035	−10.43			11.71	1.28
"	"	0.0005	∞	20.0375	j19.937	−797	0		798	0	800	−j2.006		−800	0
"	"	"	1000	20.99	j20.89	10.10			−9.193	0.0042	−9.125			9.215	0.090

approximately 75 per cent of the voltage without the resistor when the incident wave is rectangular. The improvement is less for sloping waves, being inappreciable for the 7-microsecond wave as is shown by the waves below the zero axis in Fig. 8. The actual improvement depends on the steepness of the waves which occur in practise.

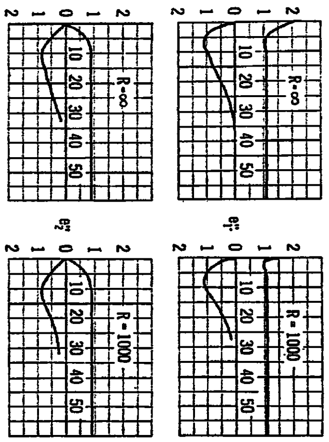

FIG. 8—$r = 100$, $L = 0.002$, $Z_1 = 400$, $Z_2 = 400$, $C = 0.005 \times 10^{-6}$

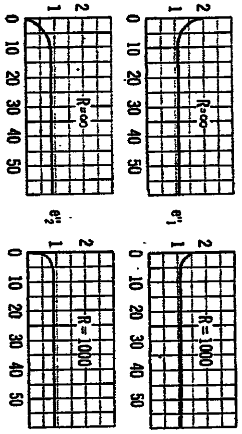

FIG. 9—$r = 100$, $L = 0.002$, $Z_1 = 400$, $Z_2 = 400$, $C = 0.0005 \times 10^{-6}$

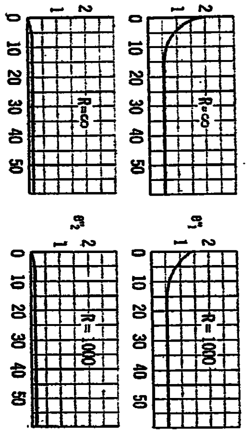

FIG. 10—$r = 100$, $L = 0.002$, $Z_1 = 400$, $Z_2 = 50$, $C = 0.005 \times 10^{-6}$

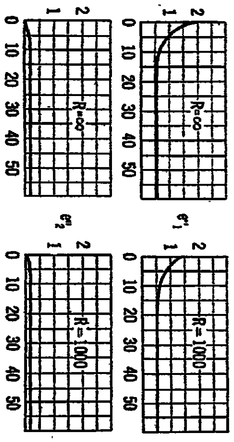

FIG. 11—$r = 100$, $L = 0.002$, $Z_1 = 400$, $Z_2 = 50$, $C = 0.0005 \times 10^{-6}$

Figs. 10 and 11. These cases are very similar to those of Figs. 8 and 9, the difference being in the change of the impedance z_2 from 400 ohms to 50 ohms. The cases of Figs. 8 and 9 merge into these as the number of feeders is increased. Here, also, the improvement

TABLE III

(Calculated) $a = 0.2 \times 10^6$, $r = 100$, $L = 0.002$

Z_1	Z_2	$C\ 10^{-6}$	R	$\alpha\ 10^6$	$\omega\ 10^6$	x	y	ϕ	W_1	W_2	u	v	ψ	J_1	J_2
400	∞	0.005	∞	0.125	0.2905	1.889	1.452	217.8°			2.222	2.295	284.5°		
"	"	"	1000	0.1679	0.2245	1.778	1.394	255.4°			1.944	2.487	308.6°		
"	"	0.0005	∞	0.125	0.9922	1.081	0.4052	191.5°			2.020	2.026	274.3°		
"	"	"	1000	0.8107	0.3584	0.9829	1.544	339.0°			1.994	4.989	336.4°		
400	400	0.005	∞	0.375	0.2905	−0.04348	1.2843	125.7°			1.739	2.0304	238.9°		
"	"	"	1000	0.4179	0.01989	−0.2537	1.4828	27.4°			2.089	8.775	193.8°		
"	"	0.0005	∞	2.625	$j2.154$	−0.5484	$-j0.1667$	$j2.93$			1.613	$j0.8337$	$j1.41$		
"	"	"	1000	3.3107	$j3.035$	−1.0614	$-j0.8770$	$j1.30$	1.6094	0.1195	2.1473			−1.7033	0.4439
400	50	0.005	∞	2.125	$j1.848$	−4.448	$-j1.382$	$j2.08$	5.5345	0.0862	0.6897	$j0.2010$	$j1.95$	−0.7040	−0.0143
"	"	"	1000	2.168	$j1.962$	−32.25	$-j1.502$	$j3.76$	32.696	0.0172	4.375			−4.3088	0.0663
"	"	0.0005	∞	20.125	$j19.85$	−4.344	$-j0.4035$	$j3.28$	5.3521	0.0076	0.6689	$j0.05827$	$j\ 3.13$	−0.6702	−0.0012
"	"	"	1000	20.81	20.61	−35.9			36.348	0.020	4.636			−4.5670	0.0687
50	∞	0.005	∞	0.0375	0.314	1.080	0.1319	214.2°			1.600	1.802	297.4°		
"	"	"	1000	0.1321	0.2955	1.073	0.1762	252.3°			1.451	2.177	318.2°		
"	"	0.0005	∞	0.0375	0.9993	1.010	0.04942	191.4°			1.951	1.977	279.2°		
"	"	"	1000	0.9893	0.2626	1.009	0.2430	334.7°			1.927	8.926	347.5°		
50	400	0.005	∞	0.2875	.2342	0.76	2.701	117.3°			3.2	3.416	249.5°		
"	"	"	1000	0.3821	$j0.06608$	0.6529	$-j0.1280$	$j1.42$			4.628	$j5.240$	$j0.80$		
"	"	0.0005	∞	2.5375	$j2.2503$	0.4	$-j0.0352$	$j3.52$	0.6005	0.0005	5.000	$j1.405$	$j1.98$	−5.094	−0.094
"	"	"	1000	3.489	$j3.280$	1.625	$-j0.4984$	$j.4984$	2.5540	0.0243	21.87			−21.32	−0.56
50	50	0.005	∞	2.0375	$j1.937$	1.5067			−0.5066	0.0001	−0.5333			0.5197	−0.0137
"	"	"	1000	2.1321	$j1.036$	−2.845			3.6477	−0.1025	−0.3229			0.4083	0.0853
"	"	0.0005	∞	20.038	$j19.94$	1.5006			−0.5006	0	−0.5031			0.5019	−0.0013
"	"	"	1000	20.989	$j20.89$	1.350			−0.3957	0.0042	−0.3061			0.3965	0.0904

due to the resistor is on the line side of the reactor and is of substantially the same amount as in the case of Figs. 8 and 9.

Figs. 12 and 13. These cases are similar to those of Figs. 6 and 7, differing only in that z_1 corresponds to a cable of 50 ohms surge impedance. Substantial oscillations occur in the transmitted wave causing it in the case of Fig. 12 to rise to 3.37 times the incident voltage and 3.77 times for Fig. 13. The reductions effected by the resistor for Figs. 12 and 13 are 23 per cent and 47 per cent respectively when the incident wave is rectangular and a much smaller amount for the 7-microsecond incident wave.

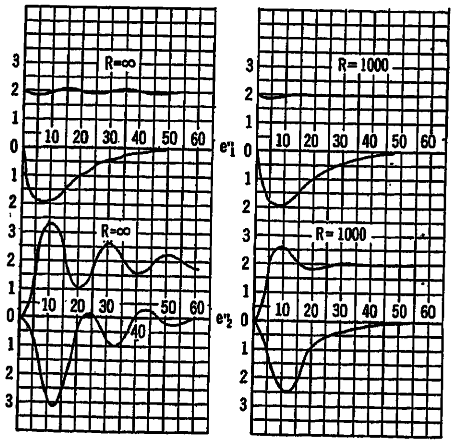

FIG. 12—$r = 100$, $L = 0.002$, $Z_1 = 50$, $Z_2 = \infty$
$C = 0.005 \times 10^{-6}$

Figs. 14 and 15. These cases are similar to that of a bus system with underground cables. Here, the gain due to the resistor is not appreciable, due to the very large difference between the resistance of the resistor (1000 ohms) and the surge impedance of the cable (50 ohms). This does not represent a very practical case because z_2 would in practise, be considerably less than z_1, due to the large number of cables that would ordinarily be represented by z_2.

Figs. 16 and 17. This case represents the usual bus system with underground cable. Here again, the gain due to the resistor is negligible due to its high resistance. With a resistor of 50 ohms, the gain would be large as there would be no appreciable reflection and consequently, the line voltage would be very little higher than the incident wave. In other words, the voltage on the line side of the reactor would be decreased by the resistor from 2 times the incident voltage to not more than 1.1 times the incident voltage.

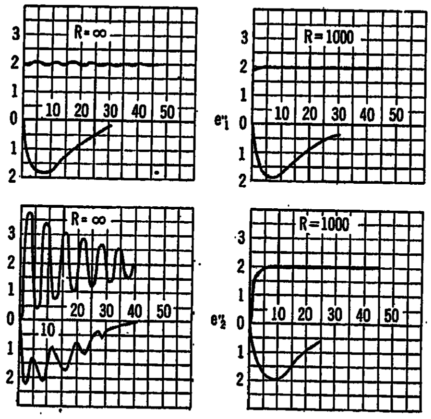

FIG. 13—$r = 100$, $L = 0.002$, $Z_1 = 50$, $Z_2 = \infty$,
$C = 0.0005 \times 10^{-6}$

Attention is called to the fact that the resistance of the conductor (r) of the reactor is a variable depending on the steepness of the wave or the frequency of the oscillation. For lightning waves, it is chiefly due to skin effect and its value for the reactor used in this investigation is of the order of 100 ohms. To treat the resistance as a variable in the equations makes their solution extremely difficult, if possible at all, due to the complicated manner in which the resistance changes with frequency or wave shape. For this reason, the calculations and curves have been made with (r) as a constant value of 100 ohms. This has not led to any large error, except where z_2 is 50 ohms. Figs. 16 and 17 show cases where this applies. If (r) is considered as a constant of 100 ohms e_1'' approaches 1.5 times the incident wave and e_2'', 0.5 times as shown by the curves. Actually, (r) reduces down to much less than one ohm when the slope of the wave is zero. Therefore, e_1'' and e_2'' both approach the incident wave as a limit. One is chiefly interested in the wave at the time the voltages are maximum which occurs at the front of the wave. At this time, the resistance is of the order of 100 ohms and the voltages calculated are correct.

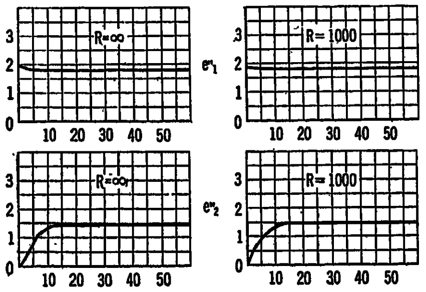

FIG. 14—r = 100, L = 0.002, Z_1 = 50, Z_2 = 400, C = 0.005 × 10^{-6}

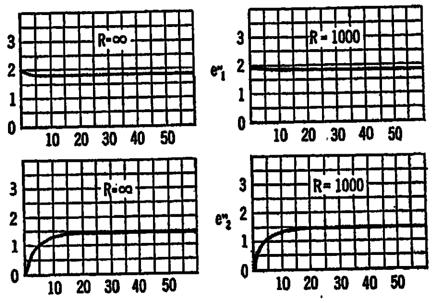

FIG. 15—r = 100, L = 0.002, Z_1 = 50, Z_2 = 400, C = 0.0005 × 10^{-6}

VI. EXPERIMENTAL RESULTS

In order to check the accuracy with which the transmitted wave, due to a given impulse may be calculated, tests were made, using the circuit shown at the top of Fig. 18. The results of these tests are tabulated in Table IV. The tested waves are shown by the oscillograms in Figs. 19, 20, 21, and 22. These waves are reproduced by curves below the oscillograms with the calculated voltages shown by points plotted on the curves.

The tests shown in Fig. 19 represent an impulse striking a reactor with the transmitted wave passing into a line of 600 ohms surge impedance. The tests shown by Fig. 20 are of the same nature except that the reactor is shunted by a resistor of 1200 ohms. These two tests are tabulated as Tests 1 and 2 in Table IV.

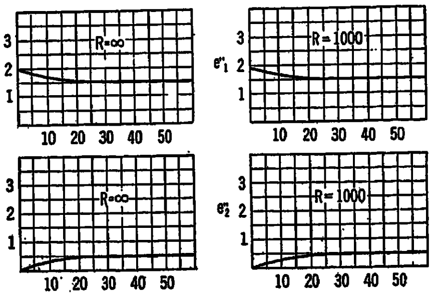

FIG. 16—r = 100, L = 0.002, Z_1 = 50, Z_2 = 50, C = 0.005 × 10^{-6}

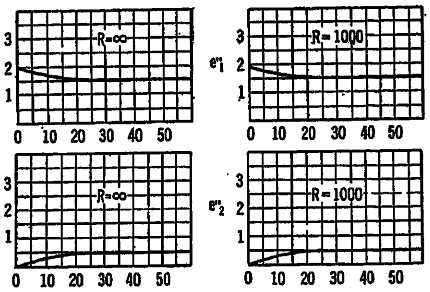

FIG. 17—r = 100, L = 0.002, Z_1 = 50, Z_2 = 50, C = 0.0005 × 10^{-6}

TABLE IV
Tests with L = 0.002 henry Reactor

				Crests kv.			Fronts microseconds			Frequency kilocycles		Wavelengths		
Test	R_1	R_2	C_2	e_1	e_2	e_2/e_1	e_1	e_2	e_2/e_1	e_1	e_2	e_1	e_2	e_2/e_1
1	∞	600	0	47	28.4	0.603	0.25	4	16.0	0	0	8	16	2.00
2	1200	600	0	47	25.6	0.545	0.25	3	12.0	0	0	8	9	1.12
3	∞	∞	0	47	86.	1.83	0.25	1.8	7.2	0	275	11	12	1.09
4	1200	∞	0	47	52.	1.105	0.25	0.25	1.00	0	0	13	13	1.00
5	∞	∞	0.00054	47	76.5	1.630	0.25	3.5	14.0	0	159	13	14	1.08
6	1200	∞	0.00054	47	53.2	1.130	0.25	2	8.0	0	0	11	10	0.91
7	Thyrite	∞	0.00054	47	51.1	1.086	0.25	0.25	1.00	0	150	13	13	1.00
8	∞	∞	0.0052	47	65.8	1.400	0.25	6	24.0	61.5	61.5	14	25	1.79
9	1200	∞	0.0052	47	51.1	1.086	0.25	6	24.0	0	48.5	9	8	0.89
10	Thyrite	∞	0.0052	47	52.7	1.122	0.25	0.25	1.00	0	50	13	13	1.00
11	Thyrite	0	..	47	0	0	0.25	..	..	15.4	0	1.5	..	..
12	22	0	..	47	0	0	0.25	..	..	0	0	1.5	..	..

They do not show any substantial difference in the transmitted wave although the resistor reduces the voltage 10 per cent. The chief value of the test is in showing how well the test checks with the calculation. Tests 3 and 4 represent an impulse striking a reactor at the end of an open line. For the sake of brevity, the oscillograms are not shown. The wave transmitted through the reactor without a resistor is oscillatory and is 1.83 times the wave on the other side of the reactor. When the reactor is shunted with a 1200-ohm resistor, the transmitted wave is not oscillatory and is only 1.10 times that on the other side of the reactor. The circuit of this test represents a feeder disconnected at the generating station. It may or may not be closed at the substation.

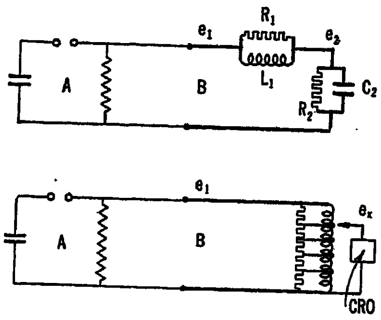

Fig. 18—Test Circuits

A. Impulse generator
B. Circuit under test

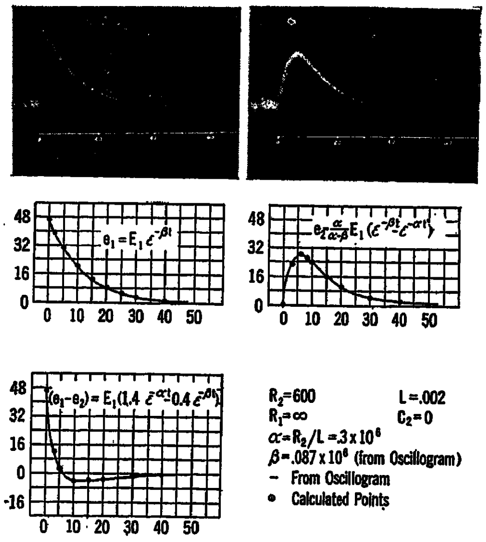

Fig. 19—Impulse Tests on Reactor—No Shunt Resistor

Tests 5, 6, and 7 represent an impulse striking a reactor in series with a transformer at the end of a line. The results of Tests 5 and 6 are shown in Figs. 21 and 22. When the reactor is not shunted by a resistor the transmitted wave is oscillatory and is 63 per cent greater than the wave on the other side of the reactor, while with a 1200-ohm resistor, the wave is not oscillatory and is only 13 per cent greater than on the other side. It will be seen from the calculated points plotted on Figs. 21 and 22, that the calculations agree closely with the tests. In Test 7, a thyrite resistor was used, having a resistance characteristic similar to that shown on Fig. 24. The transmitted wave was only 9 per cent greater than the wave on the other side.

Tests 8, 9, and 10 are similar to Tests 5, 6, and 7 except that they represent several banks of transformers of the non-resonating type. The results of the test are very similar to those of Tests 5, 6, and 7.

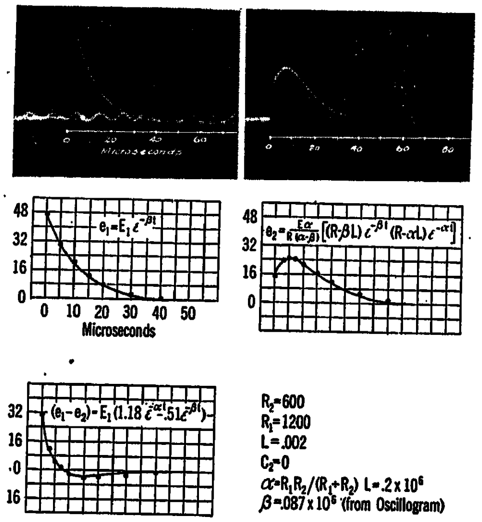

Fig. 20—Impulse Tests on Reactor—With Shunt Resistor

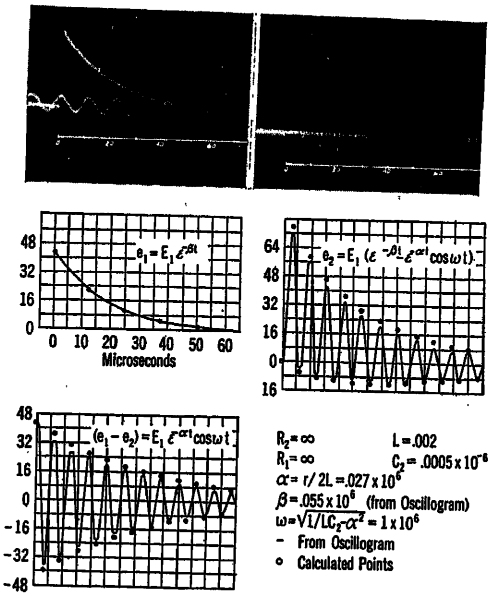

Fig. 21—Impulse Tests on Reactor—No Shunt Resistor

The purpose of the tests described above was to check the general method of calculating the voltage transmitted through a reactor, due to an impulse. A definite total voltage ($e_1 = E_1 \epsilon^{-\beta t}$) was applied to the test circuit shown in the upper diagram of Fig. 18 and the transmitted voltage was calculated and measured.

Transmitted voltages may be computed either in terms of the incident voltage (if it is known) or in terms of the total voltage at the reactor. These equivalent relationships are given by Equation (5), Appendix I. In the case of these tests, the impulse did not enter the reactor from a line, but was discharged from a capacitor close to the reactor, so that there was no incident wave (in the sense that the word is usually used). For this reason, the computed voltages are given in terms of the total voltage at the reactor. The close agreement between these tests and calculations shows that the general method of calculation is correct.

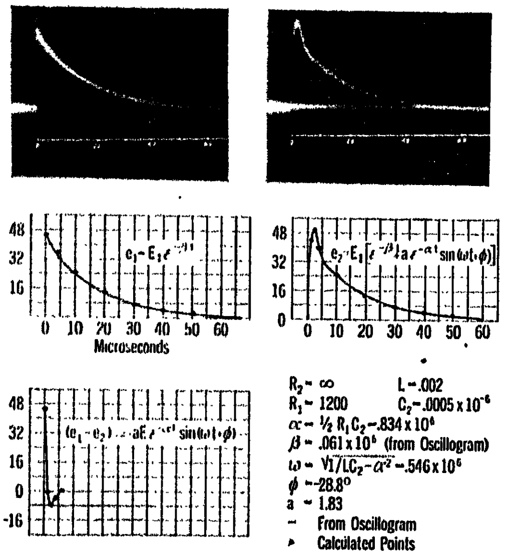

FIG. 22—IMPULSE TESTS ON REACTOR—WITH SHUNT RESISTOR

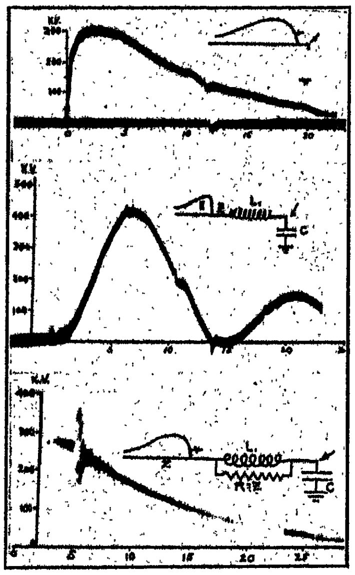

FIG. 23—IMPULSE TESTS WITH TRANSMISSION LINE TOP

Top: Open-circuited line
Middle: Reactor in series with transformer
Bottom: Reactor shunted by a resistor

Tests* were made on a 5-mile length of a 66,000-volt transmission line under conditions similar to those for which Figs. 6, 7, 12, and 13 apply. The results of these tests are shown by the oscillograms in Fig. 23. The points of interest in these oscillograms are:

1. When the wave strikes the open end of the line, there is no oscillation.
2. When it passes, at the end of the line, through a reactor into a capacitance to ground an oscillation of greater crest voltage than in the first test, occurs.
3. When all the conditions are the same as in the second test, except that the reactor is shunted by a resistor, the oscillation is eliminated and the crest voltage is approximately the same as in the first test.

VII. THE EFFECT OF A RESISTOR ON THE INTERNAL DISTRIBUTION OF VOLTAGE IN A REACTOR

In Appendix III are given formulas for calculating the internal distribution of voltage in a reactor both with and without a resistor in shunt. These formulas show that with no resistor, the distribution consists of a very complex oscillation superimposed on the linear distribution which represents the final state, but that with a properly chosen resistor, the oscillation is eliminated.

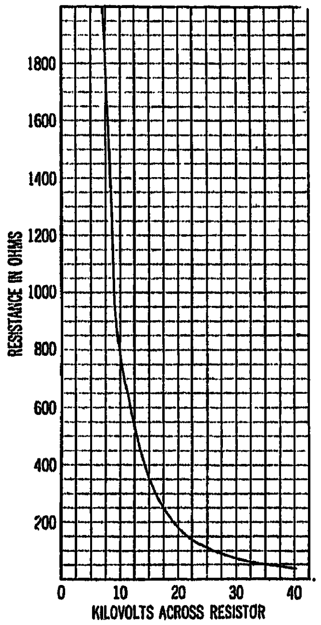

FIG. 24—VOLTAGE—RESISTANCE CHARACTERISTIC OF THYRITE

Tests were made to determine the internal distribution of voltage in a reactor of 0.002 henrys inductance with an applied wave of 50 kv. as follows:

1. Reactor not shunted by a resistor.
2. Reactor shunted by a 1000-ohm resistor with six intermediate points connected to the corresponding intermediate points of the reactor.

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*These tests were made by Messrs. McEachron and Goodwin on the Turners Falls transmission line in 1929. The line is described in their paper, *Cathode Ray Oscillograph Study of Artificial Lightning Surges on the Turners Falls Transmission Line,* A. I. E. E. TRANS., Vol. 48, July 1929, p. 953.

3. Reactor shunted by a resistor having the characteristics shown in Fig. 24 and connected to intermediate points as noted under 2.

The circuit used in these tests is shown by the lower diagram of Fig. 18.

The results of the tests are shown in Figs. 25 and 26.

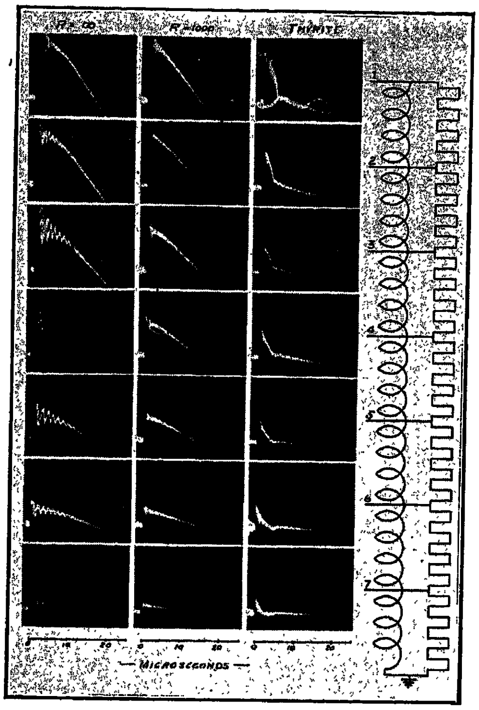

FIG. 25—OSCILLOGRAMS SHOWING INTERNAL VOLTAGE DISTRIBUTION IN REACTORS

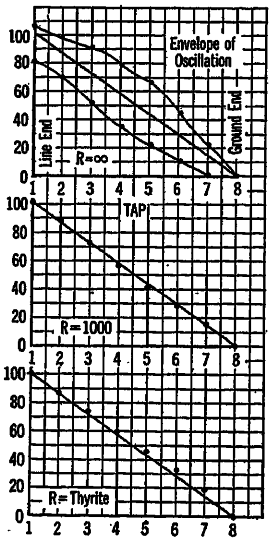

FIG. 26—EFFECT OF TIED-IN SHUNT RESISTOR ON INTERNAL VOLTAGE DISTRIBUTION

The oscillograms in Fig. 25 give the voltage across the reactor and also the voltage from the bottom terminal of the reactor to each of the points where the resistor was connected to the reactor. They show that there is quite an appreciable internal oscillation when the reactor is not shunted by the resistor but that the oscillation is eliminated when the reactor is shunted by the resistor.

The curves shown in Fig. 26 have been plotted from points taken from these oscillograms. They show that the distribution of voltage across the reactor is much more uniform than it is in a non-shielded transformer. The reason for this is that the ratio of ground capacitance to series capacitance in a reactor is relatively small. The absence of a secondary winding and a core make the ground capacitance small while the series capacitance is comparatively large. The curves show that the distribution is more nearly uniform with the reactor shunted by a resistor than it is with the reactor not shunted. The resistor thus reduces the internal voltage stresses in the reactor by the elimination of the oscillations and making the voltage distribution more uniform.

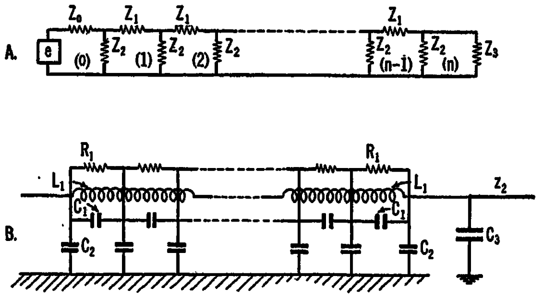

FIG. 27—NETWORKS OF PERIODIC SECTIONS

A. General impedances
B. Reactor with tied-in resistor units

VIII. CONCLUSIONS

The following general conclusions can be drawn from the calculations and tests:

1. If an impulse passes into a circuit as shown in Fig. 4, where z_2 is very high or infinite, an oscillation results which gives rise to voltages which approach four times the incident voltage as a limit. These oscillations are entirely eliminated by shunting the reactor with a resistance approximately equal to the surge impedance z_1. Then the voltage rise is due to reflection and in the limit approaches two times the incident voltage.

2. If z_2 is not higher than z_1, no oscillations will occur, and the transmitted voltage will be equal to or lower than the incident wave. Here the advantage of the resistor is in lowering the reflected voltage on the line side of the reactor. In order to do this effectively, the resistance should be equal to or lower than the surge impedance (z_1) of the line. Then the reflected voltage will be very low and the maximum voltage on either side of the reactor will not be substantially greater than the incident voltage.

3. If a substantial voltage occurs across a reactor (due to an oscillation or to the reactor acting as a reflection point), the transient voltage that the line or bus has to stand as a result of the disturbance, is always reduced by shunting the reactor with a resistance of the proper value. Conversely, if shunting the reactor with the proper resistance does not reduce the transient voltage

that the line or bus has to stand, then it is because these transients are of such low frequency or such sloping wave fronts that they do not produce an appreciable voltage across the reactor.

SUMMARY

In general, through the use of a shunting resistor of proper value, three beneficial results are obtained: (a) Oscillations are completely eliminated, thereby limiting the maximum voltage which can occur on the bus to never more than twice that of the incident wave. (b) The initial rise in voltage on the line side of the reactor, due to positive reflections, is reduced and can be held down to the final value corresponding to the tail of an infinite rectangular incident wave. (c) The internal oscillations in the reactor itself can be eliminated, and the voltage distribution improved. The extent to which these improvements can be realized under a wide range of possible operating conditions is discussed and illustrated in this paper. With some combinations the improvements are very great, while for others, there is no appreciable gain. The criterion for utilization of shunt resistors is,—will the elimination of oscillations and the reduction of positive reflections, under the most unfavorable possible operating condition which can be experienced on a given system, be worth the cost?

GENERAL DISCUSSION

There are two essential conditions to be met in the design of shunt resistors for reactors. First, in order to give effective protection against lightning or other high-potential surges, the resistance should not be more than 400 ohms for overhead lines or 50 ohms for underground cables. Second, the resistor must not overheat when the reactor it is used with is undergoing short-circuit.

To meet the latter condition, a high resistance is necessary. The rate at which energy is absorbed by the resistor is $P = E^2/R$ where E is the voltage across the resistor and R is its resistance. Thus in a 13,800-volt circuit, the voltage across the reactor during short circuit is $13{,}800/\sqrt{3} = 7980$ volts, and if the reactor is for use with an underground cable, the resistor (in order to have maximum effectiveness in reducing transient voltages) should have a resistance of not over 50 ohms. The energy absorbed would then be at the rate of $\frac{7980^2}{50 \times 1000} = 1274$ kw. A resistor capable of absorbing so much energy without over-heating would be prohibitive in cost. For these reasons, a lower resistance for high voltages than for low voltages is essential. For instance, if the resistance is 600 ohms with 7980 volts r. m. s. across it and 50 ohms for 35 kv. crest (which is approaching the danger zone for 13.8 kv. circuits) it has the proper characteristic for the above case. In Fig. 24 is given the voltage-resistance curve of the type of resistor used. It will be noted that it has the required characteristics.

This investigation has treated only the effect of resistors upon voltages caused by impulses originating on a line and entering a bus system. High transient voltages may also originate on the bus itself, due to the interruption of a short-circuit. Such transients are trapped in the bus by the reactors in the outgoing feeders. By shunting each of these reactors with a resistor, a non-inductive path is provided for the escape of the energy of these voltages into the capacity of the feeders. The tendency for such a rise in voltage is thus materially reduced.

Appendix I

RELATIONSHIPS AT A GENERAL TRANSITION POINT

One of the most general types of transition points met with in transmission systems consists of a junction at which there is a general impedance network to ground $Z_g(p)$; and n transmission lines of surge impedances $(z_1, \ldots, z_n)$ joined through networks $Z_1(p) \ldots Z_n(p)$ respectively. Such a system is shown in Fig. 3. When an incident wave e_1 approaching along the line z_1 reaches the junction it will give rise to a wave e_1' reflected back on line z_1; transmitted waves $e_2'', \ldots, e_n''$ on lines z_2 to z_n respectively; and a potential e at the junction. Now the total impedance at the end of z_1 is

$$Z_o(p) = Z_1(p) + Z(p) = Z_1(p) + \frac{1}{\frac{1}{Z_g(p)} + \sum_{2}^{n} \frac{1}{z_k + Z_k(p)}} \qquad (1)$$

and the reflected potential wave on z_1 therefore is[4]

$$e_1' = \frac{Z_o(p) - z_1}{Z_o(p) + z_1}\, e_1 \qquad (2)$$

The total potential at the reflection point is

$$e_1'' = (e_1 + e_1') = \frac{2\, Z_o(p)}{Z_o(p) + z_1}\, e_1 \qquad (3)$$

The current transmitted across $Z_1(p)$ is

$$i_1'' = \frac{e_1''}{Z_o(p)} = \frac{2}{Z_o(p) + z_1}\, e_1 \qquad (4)$$

The potential at the grounding impedance network is

$$e = \frac{Z(p)}{Z_o(p)}\, e_1'' = \frac{2\, Z(p)}{Z_o(p) + z_1}\, e_1 \qquad (5)$$

and the current through $Z_g(p)$ therefore is

$$i_g = \frac{e}{Z_g(p)} = \frac{Z(p)}{Z_o(p)} \frac{e_1''}{Z_g(p)} = \frac{2\, Z(p)}{Z_o(p) + z_1} \frac{e_1}{Z_g(p)} \qquad (6)$$

The current and voltage waves transmitted to any line k (where $2 \leq k \leq n$) are

$$i_k'' = \frac{e}{Z_k(p) + z_k} = \frac{Z(p)}{Z_o(p)} \frac{e_1''}{Z_k(p) + z_k} = \frac{2\, Z(p)}{Z_o(p) + z_1} \frac{e_1}{Z_k(p) + z_k} \qquad (7)$$

$$e_k'' = z_k i_k'' = \frac{Z(p)}{Z_o(p)} \frac{z_k e_1''}{Z_k(p) + z_k}$$
$$= \frac{2\,Z(p)}{Z_o(p) + z_1} \frac{z_k e_1}{Z_k(p) + z_k} \qquad (8)$$

The potential drops across the lumped impedance networks are

$$E_1 = e_1'' - e = \frac{Z_1(p)}{Z_o(p)} e_1'' = \frac{2\,Z_1(p)}{Z_o(p) + z_1} e_1 \qquad (9)$$

$$E_k = e - e_k'' = \frac{Z_k(p)}{Z_k(p) + z_k} \frac{Z(p)}{Z_o(p)} e_1''$$
$$= \frac{Z_k(p)}{Z_k(p) + z_k} \frac{2\,Z(p)}{Z_o(p) + z_1} e_1 \qquad (10)$$

The division of energy during the transition period at the junction furnishes a valid check in any specific case on the above relationships, and is of interest on its own account. At any time t, counting from the instant that the incident wave e_1 arrives at the junction, there is (dropping the p's for simplicity)

$$\int_t^\infty e_1\left(\frac{e_1}{z_1}\right) dt = \text{energy remaining in the incident wave} \qquad (11)$$

$$\int_0^t e_1'\left(\frac{e_1'}{z_1}\right) dt = \text{energy in the reflected wave} \qquad (12)$$

$$\int_0^t e_k''\left(\frac{e_k''}{z_k}\right) dt = \text{energy in wave transmitted to line } k \qquad (13)$$

$$\int_0^t E_k\left(\frac{E_k}{Z_k}\right) dt = \text{energy absorbed by impedance network } Z_k(p) \qquad (14)$$

$$\int_0^t e\left(\frac{e}{Z_0}\right) dt = \text{energy absorbed by impedance network } Z_0(p) \qquad (15)$$

Equating to the energy of the original incident wave, by the conservation of energy,

$$\int_0^\infty e_1\left(\frac{e_1}{z_1}\right) dt = \int_t^\infty e_1\left(\frac{e_1}{z_1}\right) dt + \int_0^t e_1'\left(\frac{e_1'}{z_1}\right) dt$$
$$+ \sum_2^n \int_0^t \left\{ e_k''\left(\frac{e_k''}{z_k}\right) + E_k\left(\frac{E_k}{Z_k}\right) \right\} dt$$
$$+ \int_0^t E_1\left(\frac{E_1}{Z_1}\right) dt + \int_0^t e\left(\frac{e}{Z_0}\right) dt \qquad (16)$$

But the two terms involving e_1 may be combined as

$$\int_0^\infty e_1\left(\frac{e_1}{z_1}\right) dt - \int_t^\infty e_1\left(\frac{e_1}{z_1}\right) dt$$
$$= \int_0^t e_1\left(\frac{e_1}{z_1}\right) dt \qquad (17)$$

Then, discarding the integrals, there is the general relationship

$$\frac{e_1^2}{z_1} = \frac{e_1'^2}{z_1} + \sum_2^n \left\{ e_k''\left(\frac{e_k''}{z_k}\right) + E_k\left(\frac{E_k}{Z_k}\right) \right\}$$
$$+ E_1\left(\frac{E_1}{Z_1}\right) + e\left(\frac{e}{Z_0}\right) \qquad (18)$$

Substituting throughout, in terms of e_1, from the previous equations of reflection and refraction, this expression reduces to an identity, thus proving that the reflection and refraction operators are consistent with the energy relationships.

The above operational equations, when applied to an infinite rectangular incident wave e_1, may be solved by the expansion theorem, or series expansion, or reference to a table of operational solutions. But when e_1 is any arbitrary time function, it must either be replaced by its operational equivalent, or else recourse made to Duhamel's theorem. Practially, the operational equivalents of only a relatively few simple functions have been tabulated; and the integrations required in an application of Duhamel's theorem also limit the arbitrary functions e_1 to just about the same simple cases.

However, the solution corresponding to

$$e_1 = E_1 \epsilon^{-at} \qquad (19)$$

is usually easily effected by means of Heaviside's shifting theorem,

$$f(p)\, \epsilon^{-at} 1 = \epsilon^{-at} f(p - a)\, 1 \qquad (20)$$

and then an application of the expansion theorem.

Now just as a curve may be approximated by a power series, or in the case of periodic functions by a trigonometric series, so the possibility of an expansion in terms of exponentials may be assumed, and e_1 fitted by

$$e_1 = f(t) = \sum_0^n A_s \epsilon^{a_s t} \qquad (21)$$

Then if the solution corresponding to Equation (19) is $\xi_1(t)$, that corresponding to Equation (21) is

$$\xi(t) = \sum_0^n \xi_s(t) \qquad (22)$$

Since the parameters A_s and a_s may be real, imaginary, or complex, a damped oscillation component in the incident wave may be represented as

$$A\,\epsilon^{-at} \sin(b\,t + c) = \left(\frac{A\,\epsilon^{jc}}{2j}\right) \epsilon^{(-a+jb)t} - \left(\frac{A\,\epsilon^{-jc}}{2j}\right) \epsilon^{(-a-jb)t}$$
$$= A'\,\epsilon^{-a't} - A''\,\epsilon^{-a''t}. \qquad (23)$$

Usually, two terms of (21) suffice for the adequate description of the incident wave. The range is indicated in Fig. 5. More complex waves may be built up by the superposition of such solutions.

Appendix II

Equivalent Circuit of Lines, Reactors, and Terminal Apparatus

It was pointed out in the main body of the paper that a transformer acts like a capacitance to ground as far as its effect on the behavior of traveling waves is concerned. The reactor unit consists of an inductance L having a series resistance r (due principally to skin effect) and a shunt resistance R. In addition there are the distributed capacitances along the stack and to ground, leakage currents to ground, and mutual inductance; but the comparison of calculations and tests show that these latter circuit constants may be neglected in computing phenomena external to the reactor unit itself. However, the distribution of potential inside the coil stack of the reactor is greatly influenced by the distributed capacitances, in the same way as in transformers, and is the subject of Appendix III.

The equivalent circuit is shown in Fig. 4, where the incident wave e_1 is approaching the junction along line z_1. When this incident wave reaches the junction a part will be reflected back to the line z_1, as a reflected wave e_1' and a part will pass through the protecting reactor as a transmitted or refracted wave e_2''. The analysis is a special case of that in Appendix I.

The generalized terminal impedance at the junction is

$$Z_o(p) = Z_1(p) + Z(p) = \frac{R(r + Lp)}{r+R+pL} + \frac{z_2}{1 + z_2 C p} \tag{1}$$

By Equations (2) and (6) of Appendix I,

$$\frac{Z_o(p) - z_1}{Z_o(p) + z_1} = \frac{(R - z_1)}{(R + z_1)} \, \frac{p^2 + A p + B}{p^2 + 2\alpha p + \omega_o^2}$$

$$= \text{reflection operator} \tag{2}$$

$$\frac{2 Z(p)}{Z_o(p) + z_1} = \frac{2}{C(R + z_1)} \, \frac{p + F}{p^2 + 2\alpha p + \omega_o^2}$$

$$= \text{refraction operator} \tag{3}$$

where

$$\alpha = \frac{(R + z_1 + z_2) L + (r R + r z_1 + R z_1) z_2 C}{2 (R + z_1) z_2 L C}$$

$$\omega_o^2 = \frac{(z_1 + z_2)(r + R) + r R}{(R + z_1) z_2 L C}$$

$$\omega = j \Omega = \sqrt{\omega_o^2 - \alpha^2}$$

$$A = \frac{(R - z_1 + z_2) L + (r R - r z_1 - R z_1) z_2 C}{(R - z_1) z_2 L C}$$

$$B = \frac{(z_2 - z_1)(r + R) + r R}{(R - z_1) z_2 L C}$$

$$F = \frac{r + R}{L}$$

In Appendix I it was shown that solutions corresponding to a wide range of incident wave shape can be built up from the solution corresponding to an exponential wave. Let this incident wave be given by

$$e_1 = E_1 \epsilon^{-at} \tag{4}$$

where time is counted from the instant when the wave arrives at the junction. Applying the *reflection* and *refraction* operators of Equations (2) and (3) to (4) and using Heaviside's shifting theorem, there is, after some simplification,

$$e_1' = x E_1 \epsilon^{-at} + y E_1 \epsilon^{-\alpha t} \sin(\omega t + \phi) \tag{5a}$$

$$= x E_1 \epsilon^{-at} + Y E_1 \epsilon^{-\alpha t} \sinh(\Omega t + \Phi) \tag{5b}$$

$$= x E_1 \epsilon^{-at} + E_1 \epsilon^{-\alpha t} (W_1 \epsilon^{\Omega t} - W_2 \epsilon^{-\Omega t}) \tag{5c}$$

$$= \text{reflected wave}$$

$$e_2'' = u E_1 \epsilon^{-at} + v E_1 \epsilon^{-\alpha t} \sin(\omega t + \psi) \tag{6a}$$

$$= u E_1 \epsilon^{-at} + V E_1 \epsilon^{-\alpha t} \sinh(\Omega t + \Psi) \tag{6b}$$

$$= u E_1 \epsilon^{-at} + E_1 \epsilon^{-\alpha t} (J_1 \epsilon^{\Omega t} - J_2 \epsilon^{-\Omega t}) \tag{6c}$$

$$= \text{refracted wave}$$

where

$$x = \left(\frac{R - z_1}{R + z_1}\right)\left(\frac{a^2 - a A + B}{\omega_o^2 - 2\alpha a + a^2}\right),$$

$$y = \frac{Y}{j} = \left(\frac{R - z_1}{R + z_1}\right) \frac{\sqrt{m^2 + n^2}}{(\omega_o^2 - 2\alpha a + a^2)\,\omega},$$

$$\phi = j\Phi = \tan^{-1}\left(\frac{n}{m}\right) = j \tanh^{-1}\left(\frac{-j n}{m}\right)$$

$$m = [(A - a - \alpha)(\omega_o^2 - 2\alpha a + a^2) - (a^2 - a A + B)(\alpha - a)]$$

$$n = \omega(\omega_o^2 - 2\alpha a + a A - B)$$

$$u = \frac{2}{C(R + z_1)} \, \frac{(F - a)}{(\omega_o^2 - 2\alpha a + a^2)},$$

$$v = \frac{V}{j} = \frac{2}{C(R + z_1)} \, \frac{\sqrt{k^2 + l^2}}{(\omega_o^2 - 2\alpha a + a^2)\,\omega},$$

$$\psi = j\Psi = \tan^{-1}\left(\frac{l}{k}\right) = j \tanh^{-1}\left(\frac{-j l}{k}\right)$$

$$k = [\omega_o^2 - \alpha a - (\alpha - a) F]$$

$$l = (a - F)\,\omega$$

$$W_1 = \frac{R - z_1}{R + z_1} \, \frac{(m - j n)}{2\Omega(\omega_o^2 - 2\alpha a + a^2)}$$

$$W_2 = \frac{R - z_1}{R + z_1} \, \frac{(m + j m)}{2\Omega(\omega_o^2 - 2\alpha a + a^2)}$$

$$J_1 = \frac{1}{C(R + z_1)} \, \frac{(k - j l)}{\Omega(\omega_o^2 - 2\alpha a + a^2)}$$

$$J_2 = \frac{1}{C(R + z_1)} \, \frac{(k + j l)}{\Omega(\omega_o^2 - 2\alpha a + a^2)}$$

The total voltage at the reactor is

$$e_1'' = e_1 + e_1' \tag{7}$$

The three separate expressions given in both Equations (5) and (6) are mutually convertible through the relationships between the trigonometric, hyperbolic, and exponential functions of complex arguments. For computation, that expression is used for which the quantities are entirely real. Now when the quantities are entirely real in (**5b**) they are also entirely real in (**5c**), but the latter is the more convenient formula to use when m and n are close together. In fact, some of the calculations of this paper were made with values of m and n so near together as to make the use of Equation (**5b**) quite impossible. The same argument applies to Equations (**6b**) and (**6c**).

From the above equations it is evident that a simple exponential incident wave striking the junction gives rise, in both the reflected and transmitted waves, to an exactly similar wave plus a damped oscillation. This damped oscillation has a frequency and decrement factor depending on every circuit constant present. In the case that $(\alpha^2 > \omega_o^2)$ the oscillation ceases to exist, and the component degenerates to simple exponential decay.

The amplitude factors (x, y, u, v), the instantaneous maximums, the frequency $f = \omega/2\,\pi$, and the displacement angles ϕ and ψ have been tabulated for different conditions in Tables II, III, and IV of the paper. As a check on such computations it is worth observing from the previous equation, that (making $t = 0$)

$$x + y \sin \phi = x + Y \sinh \Phi = \frac{R - z_1}{R + z_1} \tag{8}$$

$$u + v \sin \psi = u + V \sinh \Psi = 0 \tag{9}$$

These conditions are also evident from physical considerations; for at the first instant the inductance acts as an open circuit and the capacitance as a short circuit to ground. Therefore at this first instant the potential at the capacitance must be zero, and e_1' reflects as from a line grounded through a resistor R.

Owing to their transcendental character, the instantaneous maxima of Equations (5) or (6) cannot be found except by trial or graphically. However, the maximum amplitude of the damped oscillations are readily obtained by differentiating with respect to t and equating to zero. These maxima occur at

$$t_1 = \frac{1}{\omega}\left(\tan^{-1}\frac{\omega}{\alpha} - \phi\right) \tag{10}$$

$$t_2 = \frac{1}{\omega}\left(\tan^{-1}\frac{\omega}{\alpha} - \psi\right) \tag{11}$$

and the maxima of the oscillations are

$$y\left(\frac{\omega}{\omega_o}\right) E_1\, \epsilon^{-\alpha t_1} \tag{12}$$

$$v\left(\frac{\omega}{\omega_o}\right) E_1\, \epsilon^{-\alpha t_2} \tag{13}$$

Obviously, they cannot exceed $(y\,E_1)$ and $(v\,E_1)$ respectively, but approach these limiting values as $\alpha \to 0$.

From the energy relationships of Appendix I the location of the energy at any time t is found to be (using the solution for the oscillatory case):

$$w_o = \frac{1}{z_1}\int_o^\infty (e_1)^2\,d\,t = \frac{E_1^2}{2\,a\,z_1}$$

$$= \text{energy in incident wave} \tag{14}$$

$$w_1 = \frac{1}{z_1}\int_t^\infty (e_1)^2\,d\,t = \frac{E_1^2}{2\,a\,z_1}\,\epsilon^{-2at}$$

$$= \text{energy remaining at time } t \tag{15}$$

$$w_1' = \frac{1}{z_1}\int_o^t (e_1')^2\,d\,t = \frac{E_1^2}{z_1}\left\{ \frac{x^2}{2\,a}(1 - \epsilon^{-2at}) \right.$$

$$+\frac{y^2}{4\,\alpha}(1-\epsilon^{-2\alpha t}) + \frac{y^2}{4\sqrt{\alpha^2+\omega^2}}\left[\sin\left(2\,\phi - \tan^{-1}\frac{\alpha}{\omega}\right)\right.$$

$$\left. - \epsilon^{-2\alpha t}\sin\left(2\,\omega\,t + 2\,\phi - \tan^{-1}\frac{\alpha}{\omega}\right)\right]$$

$$-\frac{2\,x\,y}{\sqrt{(a+\alpha)^2+\omega^2}}\left[\sin\left(\phi - \tan^{-1}\frac{\alpha + a}{\omega}\right)\right.$$

$$\left.\left. - \epsilon^{-(a+\alpha)t}\sin\left(\omega\,t + \phi - \tan^{-1}\frac{a+\alpha}{\omega}\right)\right]\right\}$$

$$= \text{energy in the reflected wave at time } t \tag{16}$$

$$w_2'' = \frac{1}{z_2}\int_o^t (e_2'')^2\,d\,t = \frac{E_1^2}{z_2}\left\{ \frac{u^2}{2\,a}(1 - \epsilon^{-2at}) \right.$$

$$+\frac{v^2}{4\,\alpha}(1-\epsilon^{-2\alpha t}) + \frac{v^2}{4\sqrt{\alpha^2+\omega^2}}\left[\sin\left(2\,\psi - \tan^{-1}\frac{\alpha}{\omega}\right)\right.$$

$$\left. - \epsilon^{-2\alpha t}\sin\left(2\,\omega\,t + 2\,\psi - \tan^{-1}\frac{\alpha}{\omega}\right)\right]$$

$$-\frac{2\,u\,v}{\sqrt{(a+\alpha)^2+\omega^2}}\left[\sin\left(\psi - \tan^{-1}\frac{\alpha + a}{\omega}\right)\right.$$

$$\left.\left. - \epsilon^{-(a+\alpha)t}\sin\left(\omega\,t + \psi - \tan^{-1}\frac{a+\alpha}{\omega}\right)\right]\right\}$$

$$= \text{energy in the transmitted wave at time } t \tag{17}$$

$$W_o = \frac{C}{2}(e_2'')^2 = E_1^2\,\frac{C}{2}\,[u\epsilon^{-at} + v\;\epsilon^{-\alpha t}\sin(\omega\;t + \psi)]^2$$

$$= \text{energy stored in the capacitor at time } t \tag{18}$$

The difference between the energy in the initial wave and the sum of the above is the energy absorbed by the reactor unit,

$$W_1 = w - (w_1 + w_1' + w_2'' + W_o)$$

$$= \text{energy absorbed by reactor} \tag{19}$$

Appendix III

DISTRIBUTION OF POTENTIAL IN A REACTOR

Consider the circuit of Fig. 27A consisting of $(n - 1)$ similar sections of series elements Z_1 and shunt elements Z_2, closed by an impedance Z_3, and supplied by a potential e through an impedance Z_o.

Applying Kirchoff's Laws to the kth section there is

$$Z_1 i_k + Z_2 (i_k - i_{k+1}) - Z_2 (i_{k-1} - i_k) = 0 \qquad (1)$$

This is a "difference" equation as distinguished from the differential equation to which it reduces when the elements are taken infinitesimally close together. The general exponential may be assumed to satisfy either, so that

$$i_k = A\, \epsilon^{k\lambda} + B\, \epsilon^{-k\lambda} \qquad (2)$$

Substituting (2) in (1) and simplifying

$$(Z_1 + 2 Z_2 - Z_2 \epsilon^{\lambda} - Z_2 \epsilon^{-\lambda})(A\, \epsilon^{k\lambda} + B\, \epsilon^{-k\lambda}) = 0 \qquad (3)$$

$$\therefore \quad (\epsilon^{\lambda} - \epsilon^{-\lambda}) = 2 \cosh \lambda = \left(2 + \frac{Z_1}{Z_2}\right) \qquad (4)$$

For the 0th section

$$\begin{aligned} e = Z_o i_o + Z_2 (i_o - i_1) &= A\,(Z_o + Z_2 - Z_2 \epsilon^{\lambda}) \\ &\quad + B\,(Z_o + Z_2 - Z\, \epsilon^{-\lambda}) \qquad (5) \end{aligned}$$

and for the last, or nth section

$$\begin{aligned} 0 = Z_3 i_n + Z_2 (i_n - i_{n-1}) &= A\,(Z_3 + Z_2 - Z_2 \epsilon^{-\lambda})\, \epsilon^{n\lambda} \\ &\quad + B\,(Z_3 + Z_2 - Z_2 \epsilon^{\lambda})\, \epsilon^{-n\lambda} \qquad (6) \end{aligned}$$

Solving (5) and (6) simultaneously for the integration constants A and B, and simplifying

$$A = \frac{0.5\,[Z_3 + Z_2 (1 - \epsilon^{\lambda})]\, \epsilon^{-n\lambda}\, e}{\begin{array}{c} -Z_2^2 \sinh (n-2)\lambda + (2 Z_2^2 + Z_2 Z_o + Z_2 Z_3) \sinh (n-1)\lambda \\ - (Z_2^2 + Z_2 Z_o + Z_2 Z_3 + Z_o Z_3) \sinh n \lambda \end{array}} \qquad (7)$$

$$B = \frac{-0.5\,[Z_3 + Z_2 (1 - \epsilon^{-\lambda})]\, \epsilon^{+n\lambda}\, e}{\begin{array}{c} -Z_2^2 \sinh (n-2)\lambda + (2 Z_2^2 + Z_2 Z_o + Z_2 Z_3) \sinh (n-1)\lambda \\ - (Z_2^2 + Z_2 Z_o + Z_2 Z_3 + Z_o Z_3) \sinh n \lambda \end{array}} \qquad (8)$$

The potential to ground at any Z_2 between sections $(k - 1)$ and (k) is

$$e_k = Z_2 (i_{k-1} - i_k) = A\, Z_2 (\epsilon^{-\lambda} - 1)\, \epsilon^{k\lambda} + B\, Z_2 (\epsilon^{\lambda} - 1)\, \epsilon^{-k\lambda} \qquad (9)$$

A reactor shunted by a resistor tied in at regular intervals is a circuit consisting of the following constants:

(a) Uniformly distributed self inductance $L_1 = L/(n - 1)$

(b) Uniformly distributed capacitance to ground $C_2 = C_o/n$

(c) Uniformly distributed capacitance along the coil stack $C_1 = (n - 1)\, C_s$

(d) Uniformly distributed series resistance r

(e) Lumped resistor units tied in at regular intervals $R_1 = R/(n - 1)$

(f) Non-uniform distribution of mutual inductance M.

Of these, the series resistance r is principally due to skin effect, and changes with the steepness of the wave front and tail, or the frequency of oscillation. Tests indicate that it may be taken as of the order of 100 ohms for surges similar to those due to lighting. But when the shunt resistance is of the order used with the reactors described in this paper, the effect of the series resistance may be neglected in comparison therewith. It will therefore be taken as $r = 0$ in this discussion. While the mutual inductance M has a definite influence on the distribution, its effect is not so pronounced as to justify its inclusion in the mathematics; for to do so, even under the most simplifying assumptions as to the flux paths, changes the order of the differential equation by two, and introduces a variable coefficient. These complications prohibit a solution except for open and grounded terminal conditions. For these reasons the mutual inductance between coils will be neglected. Finally, on the assumption that the number of sections is large, either the distributed inductance and capacitances may be lumped between tie-in points, or the shunt resistance may be taken as a uniform distribution tied in at differential intervals. Fig. 27B applies, and e is taken as the actual voltage at the terminals, as sum of the incident and reflected waves. As previously discussed in Appendix II, this may be computed by considering the entire reactor unit as a lumped network. Then

$$Z_o = 0, \qquad\qquad Z_2 = \frac{1}{p\, C_2}$$

$$Z_1 = \frac{R_1 L_1 p}{R_1 L_1 C_1 p^2 + L_1 p + R_1}, \quad Z_3 = \frac{z_3}{1 + z_3 C_3 p}$$

$$\begin{aligned} \cosh \lambda &= 1 + \frac{R_1 L_1 C_2 p^2}{2 R_1 L_1 C_1 p^2 + 2 L_1 p + 2 R_1} \\ &= \left(1 + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^4}{\lfloor 4} + \ldots\right) \qquad (10) \end{aligned}$$

Substituting Equations (7) and (8) into (9) for $Z_o = 0$ there is

$$e_k = \frac{(2 Z_2 + Z_3) \sinh (n - k)\lambda \,.\, (1 - \cosh \lambda)}{(2 Z_2 + Z_3) \sinh (n - 1)\lambda\, (1 - \cosh \lambda)}$$

$$\frac{- Z_3 \cosh (n - k)\lambda \sinh \lambda}{- Z_3 \cosh (n - 1)\lambda \sinh \lambda}\, e\,(t) \qquad (11)$$

where λ, Z_2, and Z_3 have the values previously assigned. The initial distribution at $t = 0$ may be obtained from the above operational solution by substituting $p = \infty$. Then from Equation (10)

$$\begin{aligned} \lambda = \lambda' &= \cosh^{-1}\left(1 + \frac{C_2}{2 C_1}\right) \\ &= \cosh^{-1}\left[1 + \frac{C_o}{2 n (n - 1)\, C_s}\right] \qquad (12) \end{aligned}$$

$$e_k\Big]_{t=0} = \frac{(2\,C_3 + C_2)\sinh(n-k)\,\lambda' + 2\,C_1\cosh(n-k)\,\lambda' \,.\, \sinh\lambda'}{(2\,C_3 + C_2)\sinh(n-1)\,\lambda' + 2\,C_1\cosh(n-1)\,\lambda' \,.\, \sinh\lambda'} \,.\, e\,(o) \tag{13}$$

This equation shows that the *initial distribution* depends only on the capacitances of the circuit.

The *final distribution*, if $e\,(t)$ becomes constant, is given by substituting $p = 0$. In this case, by Equation (10), $\lambda = \sqrt{L_1 C_2}\, p$, and therefore

$$e_k\Big]_{t=\infty} = \frac{(n-k)\sqrt{L_1/C_2} + z_3}{(n-1)\sqrt{L_1/C_2} + z_3} \,.\, e\,(\infty) \tag{14}$$

This equation shows that the final distribution is also independent of the shunt resistance R_1, and is a straight line. The only influence of the shunt resistance R_1 is in controlling the transient between the initial and final states.

In order to study the effect of the shunt resistance on the nature of the transition from the initial to the final states, it will be assumed that the number of sections is very large, and that the voltage is applied at one end with the other end grounded. It is then evident from Equation (10) that $\lambda \to 0$ as $n \to 0$, so that

$$\lambda^2 = \frac{1}{n^2}\,\frac{(R\,L\,C_g\,p^2)}{(R\,L\,C_s\,p^2 + L\,p + R)} = \frac{\sigma^2}{n^2} \tag{15}$$

and Equation (11) reduces to

$$e_k = e_x = \frac{\sinh(n-k)\,\lambda}{\sinh(n-1)\,\lambda}\,e = \frac{\sinh\sigma\,x}{\sinh\sigma}\,e \tag{16}$$

where x is the distance from the grounded end measured as a fraction of the total winding length.

If $e = 1$, Equation (16) is solved by the expansion theorem

$$e_x = \frac{Y\,(p)}{Z\,(p)}\,1 = \frac{Y\,(o)}{Z\,(o)} + \sum \frac{Y\,(p)\,\epsilon^{pt}}{p\,Z'\,(p)} \tag{17}$$

$$Z\,(p) = \sinh\sigma = \sigma\prod_{1}^{\infty}\left(1 + \frac{\sigma^2}{\pi^2 s^2}\right) \tag{18}$$

from which the roots of $Z\,(p) = 0$ are

$$\sigma = \pm j\,\pi\,s \qquad \text{where } s = (1, 2, \ldots\ldots) \tag{19}$$

From (15) and (19)

$$p = \frac{-\pi^2 s^2 \mp j\,\pi\,s\sqrt{4\,R^2\,(C_s\,\pi^2 s^2 + C_g)/L - \pi^2 s^2}}{2\,R\,(C_s\,\pi^2 s^2 + C_g)} = -\alpha_s \mp j\,\omega_s \tag{20}$$

From (18), (19), and (20)

$$p\,Z'\,(p) = p\,\frac{d\,Z\,(p)}{d\,\sigma}\;\frac{d\,\sigma}{d\,p}$$

$$= \pm j\,\frac{\pi\,s}{C_g}\,(C_s\,\pi^2 s^2 + C_g)\,.\,\cosh j\,\pi\,s$$

$$\left\{\frac{(C_s\,\pi^2 s^2 + C_g)\,(2\,R^2 \mp j\,R\,\omega_s) - L\,\pi^2 s^2)}{(C_s\,\pi^2 s^2 + C_g)\,(R^2 \mp j\,R\,\omega_s) - L\,\pi^2 s^2)}\right\} \tag{21}$$

in which the outside (±) sign originated from (19) and is independent of the inside (±) sign, so that there are four combinations involved. Also

$$\cosh j\,\pi\,s = \cos\pi\,s = (-1)^s \tag{22}$$

$$Y\,(p) = \sinh\sigma\,x = \sinh(\pm j\,\pi\,s\,x) = \pm j\sin(s\,\pi\,x) \tag{23}$$

$$\frac{Y\,(o)}{Z\,(o)} = \frac{\sinh\sigma\,x}{\sinh\sigma}\Big]_{p=o} = \frac{\sinh\sigma\,x}{\sinh\sigma}\Big]_{\sigma=o} = x \tag{24}$$

Substituting (20) to (24) in (17) and simplifying, there is

$$e_x = x + \sum_{s=1}^{\infty}\frac{2\,(-1)^s\,C_g\sin s\,\pi\,x}{\pi\,s\,(C_s\,\pi^2 s^2 + C_g)}\,\epsilon^{-\alpha_s t}\left\{\cos\omega_s\,t + \frac{\pi^2 s^2\sin\omega_s\,t}{2\,R\,(C_s\,\pi^2 s^2 + C_g)\,\omega_s}\right\} \tag{25}$$

when there is no resistor in shunt ($R = \infty$) this reduces to

$$e_x = x + \sum_{s=1}^{\infty}\frac{2\,(-1)^s\,C_g\sin s\,\pi\,x}{\pi\,s\,(C_s\,\pi^2 s^2 + C_g)}\cos\omega_s\,t \tag{26}$$

Thus the distribution consists of a very complex oscillation, superimposed on the linear distribution representing the final steady state. If R is large and C_s is very small compared to C_g, the frequencies of the first few time harmonics are

$$f_s = \frac{s}{2\sqrt{L\,C_g}} \tag{27}$$

and the circuit behaves as though the series capacitances were absent. But the higher harmonics approach the limiting frequency

$$f = \frac{1}{2\,\pi\sqrt{L\,C_s}} \tag{28}$$

so that the series capacitance predominates at higher frequencies. Any group of frequencies in the oscillation may be suppressed by making

$$R^2 \le \frac{\pi^2 s^2\,L}{4\,(C_s\,\pi^2 s^2 + C_g)} \tag{29}$$

for that group.

Comparing (25) and (26) it is evident that the effect of the shunt resistance is two-fold. First, it introduces a decrement factor α_s which damps out any oscillations which are present; and second, it prohibits all frequencies for which (29) holds, so that the initial distribution tends to diffuse into the final distribution without oscillation.

If the applied wave is other than infinite rectangular, the corresponding voltage distribution may be found from Duhamel's theorem

$$e_x = E\,(o)\,.\,\phi\,(t) + \int_o^t \phi\,(\tau)\,\frac{\partial}{\partial t}\,E\,(t-\tau)\,d\,\tau \qquad (30)$$

where $\phi\,(t)$ is the solution given by Equation (25) corresponding to an infinite d-c. voltage suddenly applied (Heaviside's "unit function"); and $E\,(t)$ is any arbitrary applied voltage function.

If the applied wave is one of those of Fig. 5

$$E\,(t) = E\,(\epsilon^{-at} - \epsilon^{-bt}) \qquad (31)$$

then by Equations (25) and (30)

$$e_x = x\,E\,(\epsilon^{-at} - \epsilon^{-bt}) + E \sum_{s=1}^{\infty} \frac{4(-1)^s}{\pi^3\, s^3} R\, C_g\, \alpha_s \sin s\,\pi\, x$$

$$\left\{ \frac{a\,(a - 2\alpha_s)}{(a - \alpha_s)^2 + \omega_s^2}\,\epsilon^{-at} - \frac{b\,(b - 2\,\alpha_s)}{(b - \alpha_s)^2 + \omega_s^2}\,\epsilon^{-bt} \right.$$

$$+ \frac{\epsilon^{-\alpha_s t}\,(\alpha_s^2 + \omega_s^2)}{\omega_s\,[\omega_s^2 + (a - \alpha_s)^2]^{1/2}} \cos\left(\omega_s\, t + \tan^{-1}\frac{a - \alpha_s}{\omega_s}\right)$$

$$\left. - \frac{\epsilon^{-\alpha_s t}\,(\alpha_s^2 + \omega_s^2)}{\omega_s\,[\omega_s^2 + (b - \alpha_s)^2]^{1/2}} \cos\left(\omega_s\, t + \tan^{-1}\frac{b - \alpha_s}{\omega_s}\right) \right\} \qquad (32)$$

where

$$\alpha_s = \pi^2\, s^2 / 2\, R\,(C_s\, \pi^2\, s^2 + C_g)$$

$$\omega_s = \sqrt{(2\,\alpha_s\, R/L - \alpha_s^2)}$$

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Discussion

F. J. Vogel: This paper is on a subject of great interest to us and it is interesting to see that its conclusions are generally similar to our own. The application of an ordinary resistance is very difficult. The example given by the author illustrates this point very forcibly. In practically all cases we have found that the use of an ordinary resistance was prohibited from the standpoint of its energy loss. However, it has also been realized that there was a great possibility of the reactor entering into oscillation when at the end of a line and between the line and transformers and other equipment. Under these circumstances we have felt it desirable and have recommended the use of lightning arresters which have served to practically short circuit the reactor when voltages above a predetermined value were reached.

The resistor which is described in the paper has characteristics similarly tending to reduce the resistance as the voltage across the reactor goes up. We believe the two applications are very similar in character in so far as the net result obtained is concerned. We consider that the valve type lightning arrester has many decided advantages. It is not subject to any energy losses under any circumstances except when the voltage is in its discharge range. It is easy to apply since the arrester may be picked to have its cut off voltage at a reasonable value above the maximum voltage across the reactor under short circuit conditions. We believe these advantages make it preferable to the use of a resistance material which is permanently connected into the circuit.

Where reactors are used as ties between systems and in many other applications, there is no tendency for the reactor to enter into oscillation with any other equipment in the system. In this case the application of either a resistance or a lightning arrester is unwarranted. These considerations have been the rule and guide which we have followed in the application of reactors.

S. I. Oesterreicher: Shunting a reactor by resistance is a very live topic whenever or wherever such protective equipment is discussed. The authors of the paper who for years were the chief advocates of shunt resistors, leave us in a somewhat doubtful position by their criterion, which is, will the utilization of shunt resistors under the most unfavorable possible operating conditions be worth the cost?

About fifteen years ago a number of reactors began to act in a rather embarrassing manner. Naturally we all were somewhat annoyed and tried to diagnose the symptoms as well as to apply a remedy.

The remedy was most gratifying. We have produced a piece of apparatus which under the most severe operating conditions may be relied upon. However, this reliability does not depend upon shunt resistors but rather upon strong mechanical construction, large conductor cross section, and sufficient insulation, which are paramount. However, even after the reading of this paper, there is some optimism required to be convinced about the usefulness of shunt resistors. . . . .

With this statement I do not wish to reflect upon the merits of the paper. I believe the analytical part as to the manner in which the wave travels through the various inductances, capacitances, and resistances involved in the circuits through and around the reactors is a fine piece of work and the authors should be thanked for its publication.

A claim is made that thyrite has all the required characteristics which a shunt resistor for a reactor should have. Why? Is there anything wrong with the present resistors? It is stated that thyrite is a material having characteristics somewhat similar to dry process porcelain. If that is so, and if such a shunt resistor is able to protect a reactor from dangerous oscillations, then we wish to thank the authors for telling us about this. We did not know that the 16 dry process porcelain shields we put in every one of our reactors act as shunt resistors.

K. K. Palueff: In my paper *Effect of Transient Voltage on Power Transformer Design*[1] presented to the Institute in January, 1929, and in the later communication[2] presented in May of the same year, an attempt was made to direct the attention of the reader to the dangerous overvoltages that may be produced in transformers due to the use of a small inductance, such as choke coil, reactor, or current transformer, when connected in series with power transformer and not shunted with resistance. (See Fig. 1.) The physical and mathematical relations governing the phenomena were established in the paper.

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[1] A. I. E. E. Quarterly TRANS., Vol. 48, July 1929, p. 681.

[2] *Ibid*, p. 998.

It is gratifying to find that other investigators not only arrived at the same conclusions, but have developed a suitable resistor which eliminates the above mentioned oscillations, and thereby not only protects the reactor, but also the station apparatus. Since a reactor gives satisfactory service without resistor, the resistor should be looked upon primarily as a device protecting the more expensive part of the station equipment, as power transformers, busses, or generators, when the latter are connected directly to the lines.

The authors should be complimented on the excellent mathematical analysis of the phenomena: Many formulas derived in the paper are of such general scope that they could be used in solving many other allied problems.

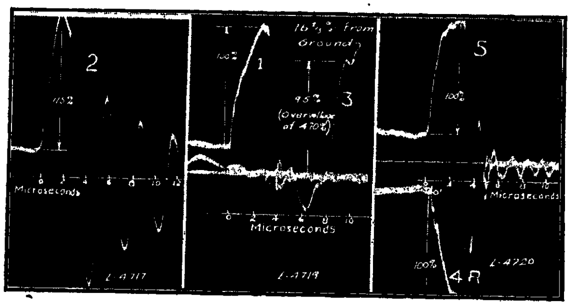

FIG. 1

I should like to call your attention to the fact that the equivalent circuit of an ordinary transformer, when the latter is subjected to steep front long wave excitation, undergoes a very radical evolution during the first few hundred microseconds. Three stages can be recognized in this evolution.

For the first few microseconds, the transformer can be represented by a pure capacitance. For the following period of one or two hundred microseconds, or even more, it acts as a complex network, consisting of distributed capacitances and inductances, with mutual inductance between various elementary inductances playing a prominent role in the phenomenon. After the oscillation of this network dies out, the transformer equivalent circuit reaches its last stage in the evolution and becomes a pure inductance connected in series with resistance.

R. B. George: In order to obtain effective protection against lightning or other high potential surges, relatively low shunting resistance is required which will result in power losses causing heating in the resistor. With constant value of resistance the losses in the shunting resistor are directly proportional to the square of the voltage across the reactor. On sustained short circuits of comparatively long duration, the shunting resistor becomes a serious hazard instead of a protection for the reactor.

It is a well recognized principle that any protective device should be self-protecting in order to avoid the complications of auxiliary devices to protect the protective device. For general applications such as bus tie reactors and feeder reactors, it is often more economical to use additional clearances and high grade insulation to obtain reactors which will give an assurance of a high factor of safety against flashover.

If the reactor is connected in series with expensive terminal apparatus which justifies the expense of a shunting device, a lightning arrester fulfills all requirements.

When a reactor shunting device is required, it is very important that it possess the characteristics of a valve type arrester which acts as a shunt for high voltage only.

F. H. Kierstead: Referring to Mr. F. J. Vogel's discussion of our paper we agree that there are isolated cases where it is more advantageous to shunt the reactor with a lightning arrester than with a resistor. Such cases occur when the system voltage is sustained across the reactor for a relatively long period. In general, however, this voltage is sustained during the period of a short circuit lasting for only a few seconds. For this reason, we have found resistors having the characteristic described in the paper (higher resistance for low voltage than for high voltage) are better suited to use in shunting reactors than arresters.

We believe that Mr. Vogel advocates a too limited use of resistors. We also believe that Mr. S. Oesterreicher and Mr. R. B. George in assuming that the resistor is only a means of protecting the reactor from high voltage have overlooked its very broad field of usefulness. Therefore, a discussion of its field of application may be useful. Since it is relatively easy to design as simple a piece of apparatus as the reactor is, to stand the voltages it is subjected to in service; there is no justification for the use of resistors to protect the reactor from overvoltage. However, by shunting the reactor with a resistor the overvoltages that generators, transformers, station busses, current transformers, and other station equipment have to stand is materially reduced. In short, the system as a whole is protected by the resistor.

Resistors are most necessary where the feeders entering the station are overhead, for the reason that lightning disturbances are most dangerous to station equipment. Underground feeders are not only shielded from lightning disturbances but also overvoltages which may arise on them are more rapidly damped out. Resistors are therefore not so necessary for use with underground feeders. Another feature which may determine whether or not resistors are to be used is whether or not a large number of disturbances is occurring on the system. By the use of resistors the voltage of the disturbances is reduced and therefore the number of failures occurring on the system will also be reduced.

Effect of Transient Voltage on Power Transformer Design

The Behavior of Transformers with Neutral Isolated or Grounded through an Impedance

BY K. K. PALUEFF*

Member, A. I. E. E.

Synopsis.—The results of theoretical and oscillographic study of transient voltage phenomena in shell type and core type transformers with neutral isolated from ground, and also with neutral grounded through impedances of different characteristics are briefly described.

In the case of isolated neutral, the majority of lightning traveling waves cause the potential of the entire winding, including neutral, to rise above ground to a value at least equal to the applied voltage.

In the case of switching surges, the voltage of the neutral, as well as of the rest of the winding, is apt to rise to a value considerably in excess of the applied voltage. In the case of transformers excited with damped oscillations similar to switching surges, oscillographic records of both shell and core type transformers indicate internal voltages to ground approximately four times the applied.

Grounding of a transformer through resistance, inductance, or capacitance does not reduce the above internal transient voltages, unless a certain relation is established between these constants, the constants of the transformer circuit, and the wave shape of the line surge.

If the proper relation of these constants is established, then internal transient voltages are reduced practically to those of a solidly grounded transformer.

A grounding device, called an "impedor," can be designed to have impedance at operating frequency equal to any desired value and to act at transient voltage frequencies as if its impedance were practically zero. An impedor can be designed for each transformer bank, or for a number of banks in parallel.

A non-resonating transformer, if grounded through a suitable impedor, retains its uniform transient voltage distribution.

During the last year and a half nearly 800,000 kv-a. of transformers of non-resonating type, have been built so that they can be operated with impedor in the neutral.

INTRODUCTION

THE effect of transients on design of power transformers operated with solidly grounded neutral was described by the author in a preceding paper of the same general title.[3]

The present paper describes in a non-mathematical way the behavior of a transformer with neutral isolated or grounded through any impedance, when subjected to transient voltages, and shows the effect of this behavior on transformer design.

While the majority of existing, very high voltage power transformers are operated with the neutral solidly grounded, there appeared recently a demand for grounding neutrals through inductance or resistance. This demand is stimulated by the desire to limit the harmful effect of excessive current flowing through the neutral in case of accidental one-wire or two-wire ground.

It has been shown[1] that the harmful effect of neutral current can be more effectively minimized by quickening switching operations and by improving relaying methods, than by insertion of an impedance between neutral and ground. However, these better methods are in a state of development, while grounding through a resistance or an inductance can be done successfully at the present time. It is expected therefore that for the time being a certain use will be made of impedance in the neutral of some strategically located banks in systems suffering from excessive short circuit neutral currents.

Part 1. Principles Governing Transient Phenomena

It is assumed that the reader is already familiar with the preceding paper[3] of the author; therefore the principal laws governing transient voltage phenomena will be described here very briefly.

EQUIVALENT CIRCUIT

At operating frequency many pieces of electrical apparatus act practically as pure inductance, capacitance, or resistance, but at high frequency, such as that produced by lightning, switching, and other disturbances, they behave as complex networks composed of elementary inductances, capacitances, and resistances distributed in a manner characteristic of the given piece of apparatus.

As has been shown in previous publications [2,3,4] the character of internal oscillations depends principally on values of $\frac{C_s}{C_g}$ and $\frac{M}{L}$ or $\left(1 - \frac{M}{L}\right)$ where C_s is capacitance between adjacent elements, C_g is capacitance between the elements and ground, M is mutual inductance between adjacent elements, and L their self-inductance.

$\frac{C_s}{C_g}$ and $\frac{M}{L}$ have minimum values in a transmission line and maximum in a transformer.

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*General Electric Company, Pittsfield, Mass.

3. For references see Bibliography.

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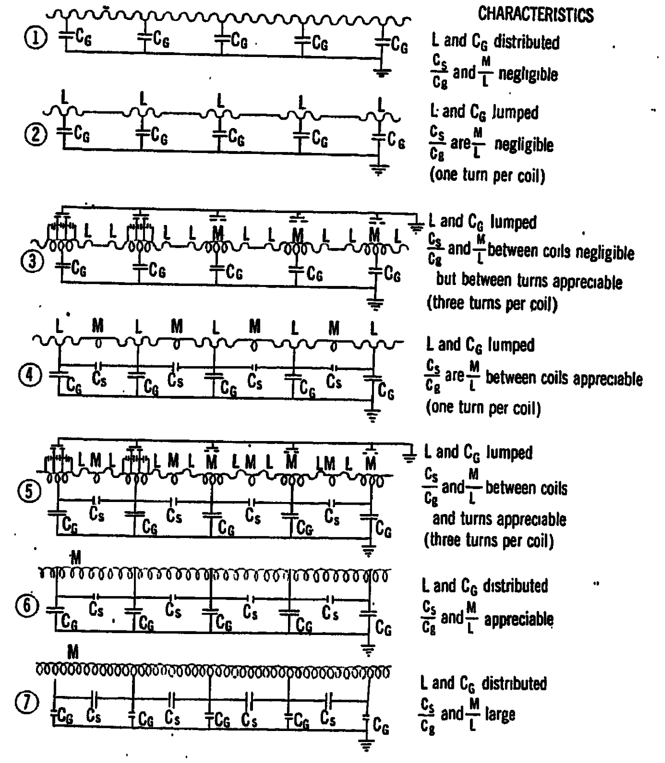
Examples of various equivalent circuits are given in Fig. 1. Resistance is neglected. The wavy line stands for self-inductance of an element of the winding. The loop stands for the mutual inductance between adjacent elements of the winding. For example, circuit No. 5 represents a winding having three turns per coil with high mutual inductance between the turns, and rather small mutual inductance between coils.

Generally a circuit exactly equivalent to a given piece of apparatus is extremely complicated and is employed only where an accurate numerical relation of various phases of the transient phenomena cannot be established without it. However, if it is desired to describe only some particular transient voltage characteristics a simplified equivalent circuit may be of assistance. To give a general idea of the behavior of transformers under certain transient conditions, the simplified circuit (Fig. 1) is found sufficiently accurate. This simplified circuit is quite unsatisfactory, however, for numerical analysis of transient phenomena because in actual transformers each coil acts as a chain of condensers, made up of capacitances of adjacent turns, while Fig. 1 is based on the assumption that coils act as solid metal plates.



CHARACTERISTICS
①
L and C_G distributed
$\frac{C_s}{C_g}$ and $\frac{M}{L}$ negligible
②
L and C_G lumped
$\frac{C_s}{C_g}$ are $\frac{M}{L}$ negligible
(one turn per coil)
③
L and C_G lumped
$\frac{C_s}{C_g}$ and $\frac{M}{L}$ between coils negligible
but between turns appreciable
(three turns per coil)
④
L and C_G lumped
$\frac{C_s}{C_g}$ are $\frac{M}{L}$ between coils appreciable
(one turn per coil)
⑤
L and C_G lumped
$\frac{C_s}{C_g}$ and $\frac{M}{L}$ between coils
and turns appreciable
(three turns per coil)
⑥
L and C_G distributed
$\frac{C_s}{C_g}$ and $\frac{M}{L}$ appreciable
⑦
L and C_G distributed
$\frac{C_s}{C_g}$ and $\frac{M}{L}$ large
L
M
C_G
C_s

FIG. 1—SPECTRUM OF HIGH-FREQUENCY EQUIVALENT CIRCUIT

CONDITIONS NECESSARY FOR A TRANSIENT

A circuit will pass through a transient state if the distribution of voltage in this circuit at the moment immediately following a sudden application of potential E is different from one that finally takes place after E has been maintained for an infinite length of time. Therefore the transient is the readjustment of local potentials from their initial to their final values.

INITIAL AND FINAL VOLTAGE DISTRIBUTION PRODUCED BY INFINITELY LONG TRAVELING WAVE OF RECTANGULAR FRONT

Case I. When the equivalent circuit consists of pure inductances connected in series with pure capacitances in any sequence, then at the instant of sudden voltage application the terminal voltage distributes between inductances in direct proportion to their values, while voltage across all capacitances is zero. Since an infinitely long traveling wave acts as a constant voltage, the terminal voltage finally distributes itself between capacitances inversely to their values, and the voltage across inductances becomes zero.

Case II. When the equivalent circuit consists of an unbroken chain of condensers in parallel with an unbroken chain of inductances, both extending from one terminal of the circuit to the other and interconnected at different points either directly or through condensers or inductances, then the initial voltage distribution is controlled by the capacitance part of the circuit and final voltage distribution by the inductance part of the circuit, (see Figs. 1 and 2).

TYPE OF TRANSIENT

As was pointed out before, all circuits met with in practise consist of inductance, capacitance, and resistance. If, however, the effect of inductance or capacitance is small enough to be neglected, then the transient is unidirectional and non-oscillatory. During the transient the potential of any point of the circuit with respect to either terminal changes by an amount equal to the difference between its initial and final value; the rate of change depends on the time constant of the circuit $\left(\frac{L}{R} \text{ or } R\,C\right)$.

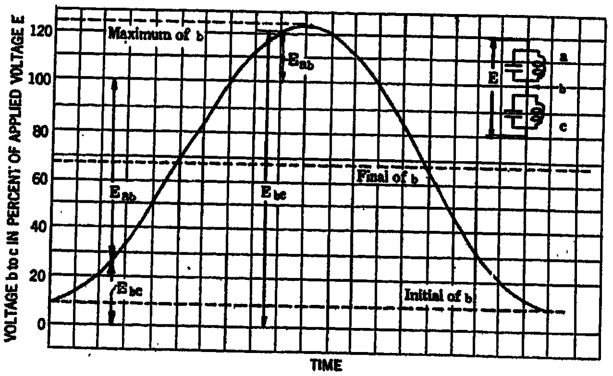


FIG. 2—OSCILLATION OF TWO ELEMENTARY CIRCUITS CONNECTED IN SERIES DUE TO INFINITE RECTANGULAR WAVE EXCITATION

In case the resistance can be neglected and the circuit contains both inductance and capacitance, the transient will be oscillatory and two conditions will be possible.

Condition I. When the oscillation takes place as if all the inductances of the system were lumped together and all capacitances were lumped together, the oscillation is of a single frequency and is determined by the product $L\,C$, where L and C are the effective lumped inductance and capacitance respectively (Fig. 2).

The amplitude of oscillation of various points is equal to the difference between their initial and final potential. Close examination of the figure reveals all the above characteristics.

Condition II. When inductance and capacitance cannot be separated and then lumped in "effective" L and C, but must remain distributed, the oscillation of various points of the circuit is complex and may consist of a number of frequencies which depend on distributed constants. The amplitude of oscillation of each point depends, as before, on the difference between the initial and final voltage distribution but is not necessarily equal to it, following a more complex law, due to the fact that the difference between the initial and final voltage of any given point is a vector sum of the amplitudes of oscillation at that point of all frequencies composing the oscillation.[2,3]

Part 2. Transformer with Neutral Isolated

LIGHTNING SURGES

Traveling waves due to lightning are produced on the three phases of a transmission line simultaneously but are usually of different amplitude. Were the amplitudes of these waves equal, each transformer would act as if it were disconnected from the other two and the oscillation itself would become simpler. In such a case the internal oscillation of the transformers will be a maximum and higher than that which usually occurs. For these reasons we shall discuss the phenomena on the assumption that a transformer bank is subjected to three equal traveling waves.

In ordinary power transformers, either of shell or of core type, the initial voltage distribution produced by a steep front of a traveling wave is practically the same whether the neutral is solidly grounded or isolated. This occurs because at the first instant voltage concentrates across the line end of the winding, and the drop across a considerable part of the winding near the neutral end is only a few per cent of the total applied voltage. Obviously, grounding or isolating the neutral can have only negligible effect on the initial distribution.

The final voltage distribution in a transformer with isolated neutral, however, is radically different from that of a transformer with grounded neutral. The effect of the traveling wave on the transformer, after the crest value of voltage is reached and maintained, is similar to direct current. Therefore all points of an isolated winding finally acquire a potential above ground equal to the terminal voltage, (*A*, Fig. 3), while if the neutral of a winding were grounded the final voltage of various points would be proportional to turns between these points and the neutral. (*F*, Fig. 3.)

Referring to *A* of Fig. 3, since the initial and final voltage distributions are different, the voltage throughout the winding will readjust itself from the initial to the final value through a complex oscillation. The line of final voltage distribution will serve as line of equilibrium or axis for the oscillation. The amplitude of the oscillation of each point will depend on the difference between the initial and final potential of this point.

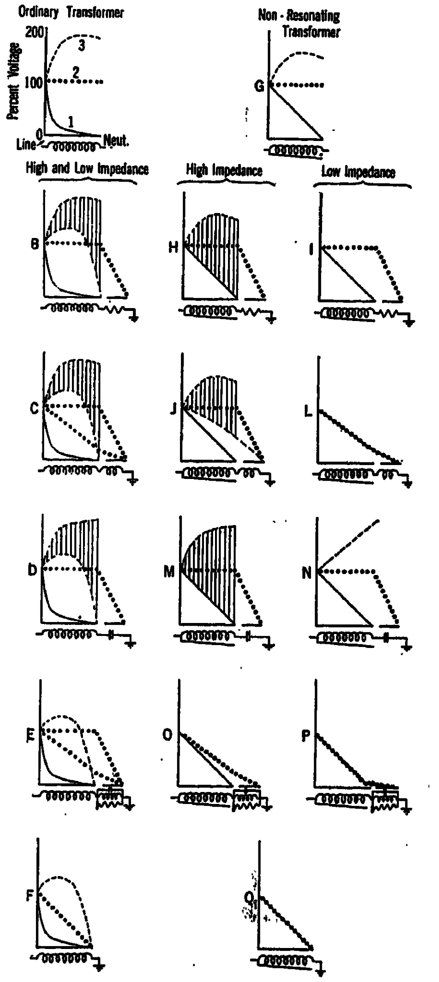


FIG. 3—SUMMARY OF EFFECT OF VARIATION OF NEUTRAL IMPEDANCE FROM ZERO TO INFINITY ON INTERNAL TRANSIENT VOLTAGES IN AN ORDINARY AND A NON-RESONATING TRANSFORMER PRODUCED BY TRAVELING WAVES MET IN SERVICE

———— initial voltage distribution
- - - - envelope of maximum voltage to ground
• • • • • • final voltage distribution

Shaded areas indicate the range in variation of voltages due to internal oscillation with increase of neutral impedance from zero to infinity

A & *G*—neutral isolated
F & *Q*—neutral grounded directly
E, *O*, *P*—neutral grounded through impedor
The remainder grounded as shown

The initial voltage distribution curve, when referred to the line of equilibrium, gives the initial voltage distortion curve, which consists of a number of space harmonics (in terms of the length of the winding stack).

These space harmonics are of various time frequencies so that the composite amplitude of oscillation below the line of equilibrium is generally not equal to maximum composite amplitude of the same point above the line of equilibrium.

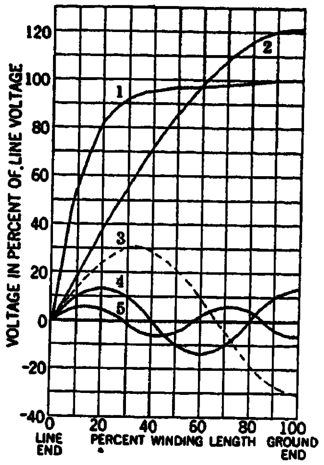


FIG. 4—WINDING SPACE HARMONICS OF TRANSFORMER WITH ISOLATED NEUTRAL AT THE INSTANT OF IMPACT OF THE TRAVELING WAVE

1. Initial voltage distortion curve (obtained from Curve 1, Fig. 2 of *Lightning Studies of Transformers by the Cathode Ray Oscillograph*, by F. F. Brand and K. K. Palueff, by referring it to its line of equilibrium of 100 per cent)

2, 3, 4, and 5 are first, third, fifth, and seventh space harmonics of curve 1

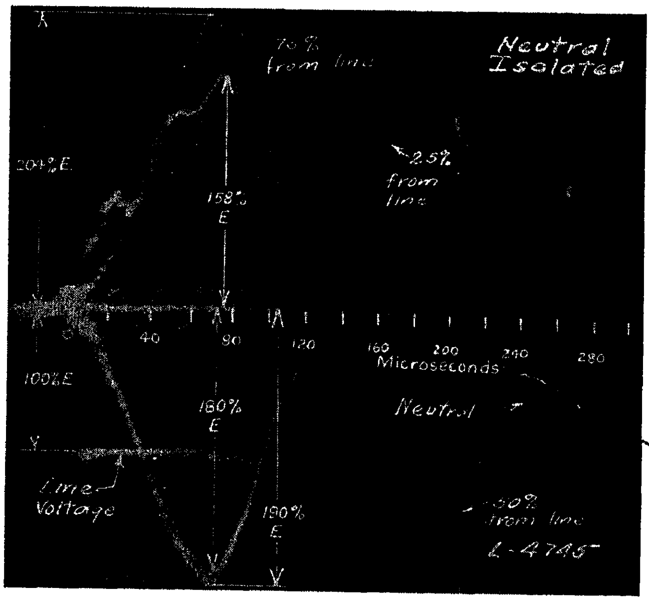


FIG. 5—OSCILLATIONS IN A SHELL TYPE TRANSFORMER WITH ISOLATED NEUTRAL

1—Voltage at line terminal

2, 3, 4, and 5 are voltages-to-ground at points 75, 50, 25, 0 per cent away from neutral terminal with crest values of 158, 190, 204, 180 per cent of applied voltage respectively. Note all voltages recorded below zero line are taken with reversed polarity

On Fig. 4 the initial voltage distortion curve is shown as well as the first few space harmonics composing it.[3]

Referring once again to *A* of Fig. 3, imagine a very elastic solenoidal spring laid along line *L N*. If end *L*, firmly gripped, is moved sufficiently suddenly along the voltage axis to the hundred per cent point and held there, then at the initial moment, the spring would bend in a curve similar to the initial voltage distribution, (curve No. 1). A moment later it would begin to move toward the line of final state (curve No. 2) and on account of inertia would overshoot it. After it reaches a certain position beyond the final, somewhat similar to curve No. 3, it would start again toward the final position. The oscillation finally would die out and position of the spring would coincide with line 2.

Similarity between the oscillation of a spring and of simultaneous voltages along a winding, while superficial, is sufficient to assist in the description of the phenomena. (The relationship between space harmonics and their frequencies is so radically different in a spring and in a transformer that mathematical representation of the oscillation in the two cases is essentially different.)

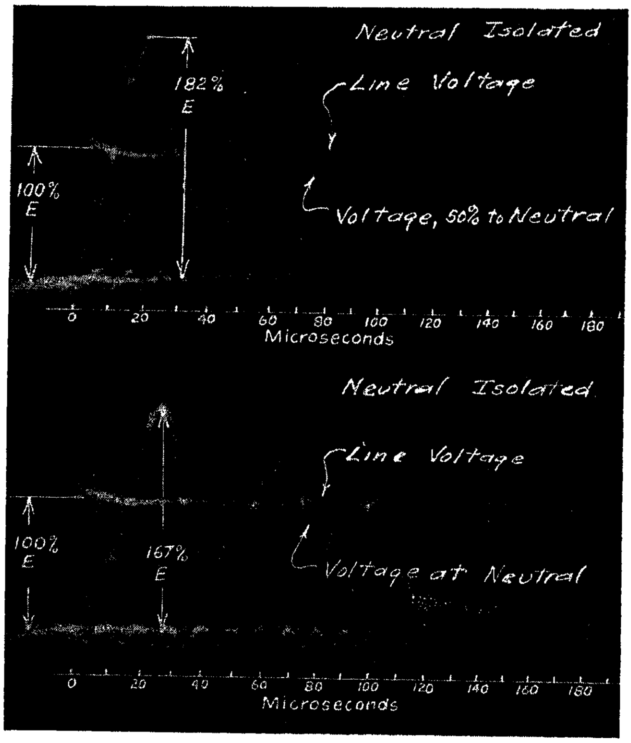


FIG. 6—OSCILLATION IN A CORE TYPE TRANSFORMER WITH ISOLATED NEUTRAL

1—Voltage at line terminal

2—Voltage to ground at 50 per cent point with crest value of 182 per cent

3—Voltage to ground at neutral with crest value of 167 per cent

On Fig. 5 voltage oscillations at four points of a shell type transformer winding are shown. Note that the line voltage (that is, the final voltage) serves as an axis for all points, as was predicted. This is particularly evident on the bottom half of the oscillogram where line voltage, the voltage of the 50 per cent point, and that of the neutral are recorded with reversed polarity and properly superimposed. On Fig. 6 similar voltage oscillations at two points of a core type transformer winding are shown.

Curves 3 and 4 of Fig. 7 give the maximum voltage to ground at various points of windings of these transformers obtained from oscillograms of Figs. 5 and 6 and of some others not shown in the paper.

SWITCHING SURGES

The importance of switching transients is often underestimated due to the fact that the amplitude of the average switching transient is about half of that of the lightning wave but, as was pointed out in the previous paper, the switching transient is a damped oscillation (see Fig. 8) and produces forced or cumulative oscillations in transformer windings. This means that amplitudes of internal oscillations increase with each succeeding half of a cycle of the switching transient (see Fig. 9). Thus while the terminal potential is reduced to half in comparison with lightning disturbances, the internal voltages, if expressed in per cent of terminal voltage, are at least doubled (curves 5 and 6 of Fig. 7). The result is that the absolute values of internal stresses produced by an average lightning or switching transient are very nearly the same. (Curves 7 and 8 of Fig. 7.)

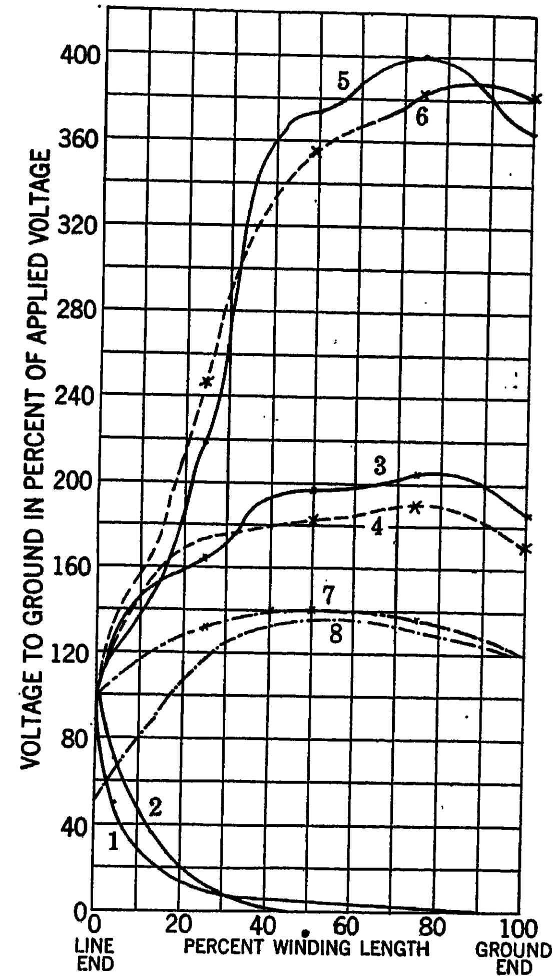


FIG. 7—VOLTAGE-TO-GROUND IN SHELL AND CORE TYPE TRANSFORMERS DUE TO LIGHTNING AND SWITCHING TRANSIENTS. (TEST)

1 & 2—Initial voltage distribution
3 & 4—Maximum voltage-to-ground due to lightning
5 & 6—Maximum voltage-to-ground due to switching
(1, 3, & 5 refer to shell type—2, 4, & 6 refer to core type transformers)
7—Estimated voltages met in practise, due to lightning
8—Estimated voltages met in practise, due to switching

The high ratio between applied and internal voltages, as shown by curves 5 and 6 of Fig. 7, is a result of resonance between the frequency of the switching surge applied and the natural frequency (or one of its harmonics) of the transformer winding. It may appear that the probability of such a coincidence is too small to be considered as one of service conditions. However, elementary calculations show that a very high ratio between applied and internal voltages can be created with the frequency of the applied surge some 40 per cent higher or lower than the natural frequency (or one of its harmonics) of the winding. Curve 5, for instance, was obtained with 20 per cent difference between the applied frequency and the fundamental natural frequency of the transformer. As an ordinary transformer may have as many as eleven natural frequencies (and these frequencies are different for different transformers) it appears more probable than not, that with transmission systems being as complicated as they are, many switching operations produce surges of dangerous frequencies.

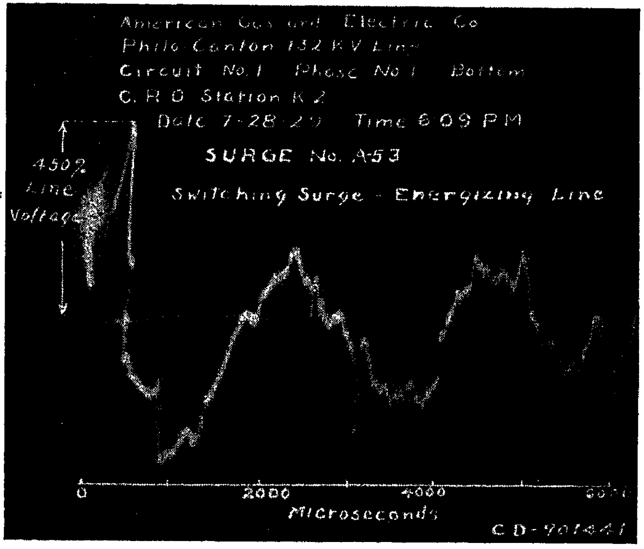


FIG. 8—OSCILLOGRAM SHOWING ENERGIZING A 132-KV. LINE OF 70 MI.

Oscillograph was stationed at approximately midpoint of line. Note complexity of the wave shape

In actual service, the traveling waves due to a given lightning discharge are very rarely of the same amplitude on all three phases, and the switching surges are never simultaneous. The effect is that two out of three transformers of a star connected bank tend to bring down the potential of the neutral of the transformer subjected to the highest surge. Curves 7 and 8 of Fig. 7 were drawn on this basis with average applied lightning voltage taken as 100 per cent and average switching voltage as 50 per cent.

EFFECT ON TRANSFORMER DESIGN

The obvious conclusion is that in the case of transformers operating with isolated neutral, the entire winding must be insulated for line voltage. This means that the winding must be capable of withstanding a high potential test of at least 3.46 times normal operating voltage from line to neutral (so-called "leg voltage").

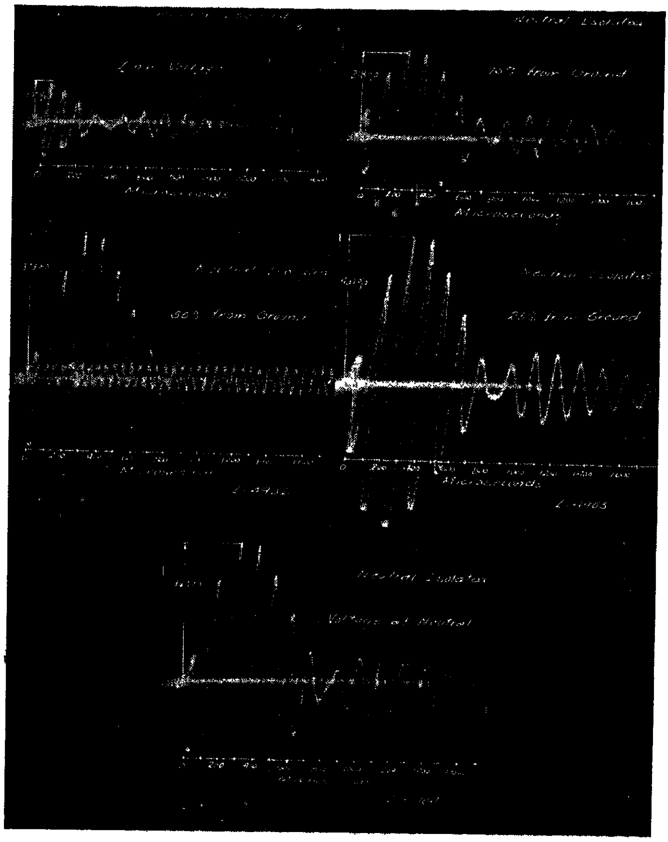


FIG. 9—OSCILLOGRAMS OF VOLTAGES IN A SHELL TYPE TRANSFORMER PRODUCED BY DAMPED OSCILLATIONS SIMILAR TO SWITCHING

1—Voltage at the line terminal
2, 3, 4, and 5 are voltages to ground at points 75, 50, 25, and 0 per cent away from neutral terminal, with crest values of 218, 373, 400, 364 per cent of applied voltage, respectively

Part 3. Neutral Grounded through Impedance

A. Neutral Grounded through Resistance

INFINITELY LONG TRAVELING WAVE

Two essentially different transients take place when a transformer grounded through resistance is subjected to a traveling wave. One transient is exponential and is governed by the transformer acting as a pure inductance while the neutral resistance and the surge impedance of the line act as pure resistances. The other transient is the oscillation of the transformer winding as described for the circuit of Fig. 1.

EXPONENTIAL TRANSIENT

The equivalent circuit for the exponential transient is shown on *B* of Fig. 10. Z is the surge impedance of the transmission line, L_T the inductance of the transformer, and R_N the neutral resistance.

At the instant immediately following the impact of the traveling wave of E volts, $2\,E$ appears across L_T. The final voltage across L_T is zero. The final voltage across R_N is $E_N = 2\,E \times \dfrac{R_N}{Z + R_N}$. Thus with sufficiently long traveling wave, transient voltage across R_N rises exponentially from zero to $2\,E \times \dfrac{R_N}{Z + R_N}$ at a rate depending on $\dfrac{L_T}{Z + R_N}$.

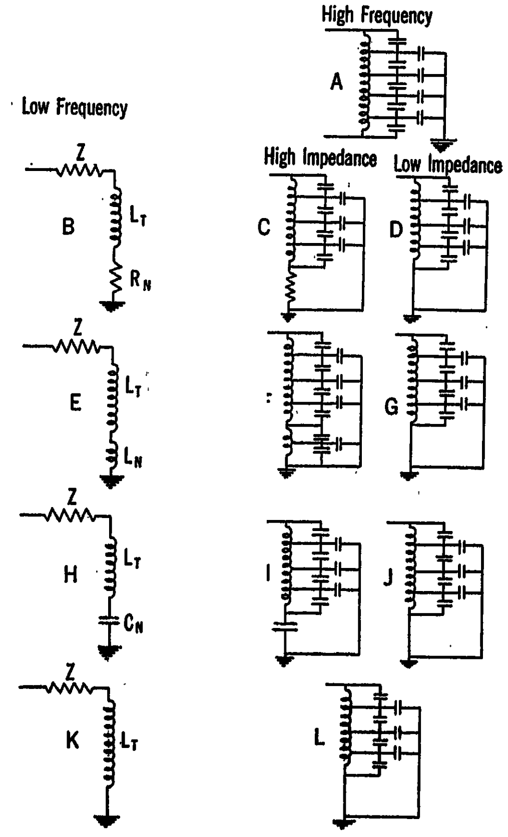


FIG. 10—APPROXIMATE EQUIVALENT CIRCUITS FOR HIGH AND LOW FREQUENCY COMPONENTS OF TRANSFORMER OSCILLATIONS FOR TRANSFORMERS GROUNDED THROUGH DIFFERENT IMPEDANCES

A—Isolated neutral
K, L—Solidly grounded neutral

INTERNAL WINDING TRANSIENT

It was shown in a preceding paper that since the surge impedance of transmission lines is of the order of a few hundred ohms its effect on internal transformer oscillations can be neglected. The equivalent circuit for internal oscillations therefore is as shown on Fig. 10.

The initial distribution in a transformer grounded through resistance is the same as in a solidly grounded transformer.

With the neutral isolated, the winding potential rises, under the influence of a traveling wave, above applied voltage. The rate of this rise depends on natural frequencies of the winding. With neutral grounded through resistance the rate of rise of its potential depends on $\frac{L_T}{Z + R_N}$. If this rate is higher than that for isolated neutral the presence of resistance has practically no effect on maximum voltages to ground throughout the winding created by the oscillation. If it is lower, the presence of resistance tends to lower these voltages. The effect is greatest at the neutral point; the effect near line end is small. Thus the slower the exponential transient the more the transformer acts as if its neutral were solidly grounded.

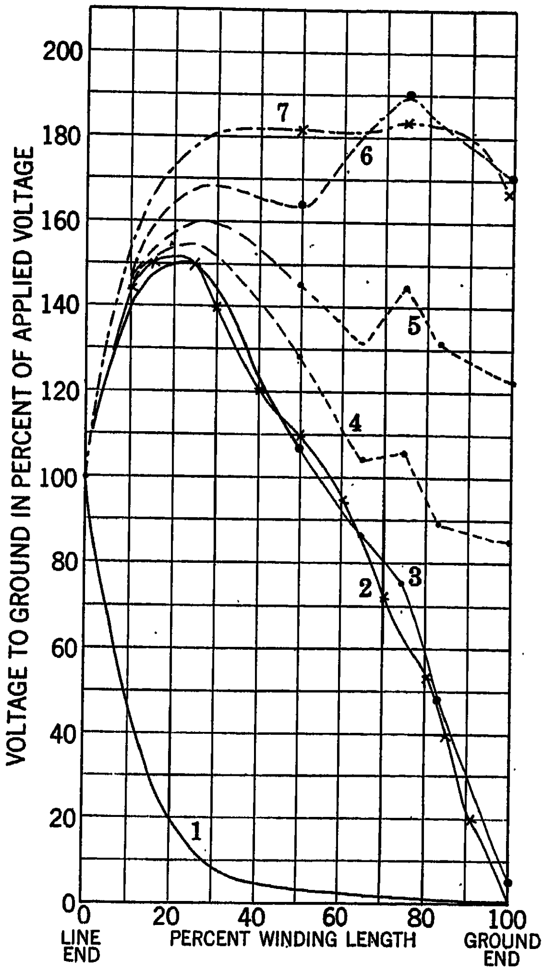


FIG. 11—EFFECT OF MAGNITUDE OF NEUTRAL RESISTANCE ON INTERNAL TRANSIENTS OF TRANSFORMERS. (TEST)

1—Initial voltage distribution

2, 3, 4, 5, 6, 7—Maximum voltages to ground for different resistances in the neutral

2—R = 0 (Solidly grounded neutral)

3—R = 500 ohm

4—R = 5,000 ohm

5—R = 25,000 ohm

6—R = 100,000 ohm

7—R = Infinite (Neutral isolated)

The maximum voltages to ground produced by waves of the same length but with transformer grounded through different neutral resistances were studied with the oscillograph on a 6600-kv-a. core type transformer and results are shown in Fig. 11. Note that voltage distribution with 500-ohm resistance in the neutral is the same as with solidly grounded neutral, while with 100,000-ohm resistance the voltage is practically the same as with neutral isolated.

FINITE TRAVELING WAVE

As the final voltage of the exponential transient is $E_N = 2\,E \times \frac{R_N}{Z + R_N}$, with Z of the order of 300 ohms, an appreciable voltage may finally appear at the neutral if the traveling wave is sufficiently long. Just what fraction of $2\,E\,\frac{R_N}{Z + R_N}$ a given wave will produce depends on the ratio of the length of the wave and the rate of the transient $\frac{L_T}{Z + R_N}$. Thus with a given length of the wave it is possible to limit the voltage at the neutral to any desired fraction of $2\,E\,\frac{R_N}{Z + R_N}$ by choosing R_N of the proper value.

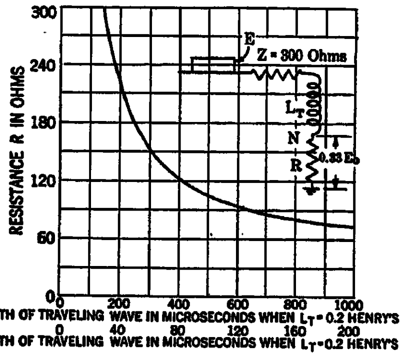


FIG. 12—RELATION BETWEEN NEUTRAL RESISTANCE AND LENGTH OF TRAVELING WAVE NECESSARY TO RAISE NEUTRAL VOLTAGE TO 33 PER CENT OF THE VOLTAGE OF THE TRAVELING WAVE. (CALCULATED)

Curve of Fig. 12 shows the relation between the length of the traveling wave and the neutral resistance, which gives an ultimate neutral voltage of $0.33\,E$ for the case of Z = 300 ohms, and L_T = 0.2 henry or 0.04 henry. (The inductance of a large power transformer will usually have a value between 0.04 and 0.2 henry.) The length of the wave causing an ultimate voltage of 0.33 E with a given resistance varies in direct proportion to inductance. The principal features of the transient, with resistance in the neutral, are shown by *B* of Fig. 3.

In practise the conclusions given above must be modified where ground resistors are metallic, as they may consist of many hundreds and sometimes thousands of feet of conductor. While the inductance of such devices can perhaps be neglected at operating frequencies, it cannot be neglected under transient conditions caused by lightning or switching surges and consideration of grounding through such resistances must take into account the conclusions reached in the next part of the paper.

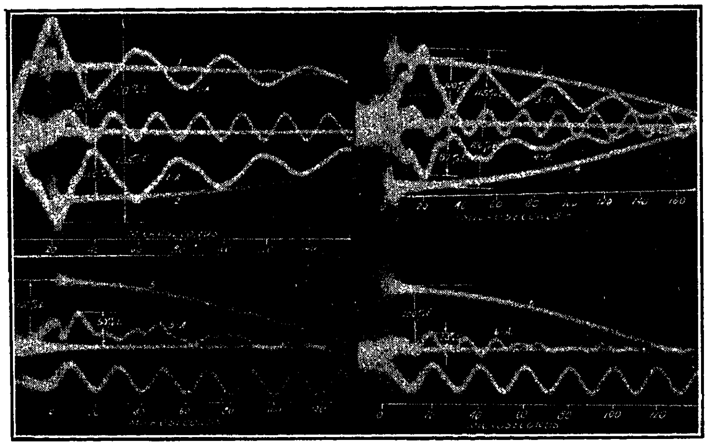


FIG. 13—EFFECT OF INDUCTANCE ON NEUTRAL OSCILLATIONS

1, 2, 3, 4, 5, & 6—Voltages at line terminal

1 *A*, 2 *A*, 3 *A*, 4 *A*, 5 *A*, 6 *A*—Voltages neutral to ground with different ratios of neutral inductance to transformer inductance

$$\text{Ratio } \frac{L_N}{L_N + L_T}$$

L A	2 *A*	3 *A*	4 *A*	5 *A*	6 *A*
1	0.725	0.67	0.48	0.26	0.097

B. Neutral Grounded through Inductance

INFINITELY LONG TRAVELING WAVE

With a transformer grounded through an inductance the initial voltage distribution will be the same as in the case of a solidly grounded transformer and the final voltage at the neutral will be $E_N = E \times \frac{L_N}{L_N + L_T}$ (E is the voltage applied between line terminal and ground, L_N and L_T are neutral and transformer inductances respectively.) Thus the axis of transformer internal oscillation is a straight line connecting the potential of the line terminal with the final potential E_N of the neutral terminal. (See *C* of Fig. 3.) On Fig. 14 curve 1 gives maximum voltage E_N max between neutral and ground as a function of $\frac{L_N}{L_T}$ obtained by scaling oscillograms of Fig. 13. Curve 2 gives the final voltage E_N at the neutral calculated by the above formula. Dots on this curve are the points of equilibrium about which the oscillations of the neutral were actually taking place, as shown by oscillograms of Fig. 13.

The coincidence of the dots with the curve confirms the theory of oscillation of the neutral when grounded through inductance.

It must be realized that L_T, the transformer inductance, depends on three conditions: first, the number of transformers of a given bank that are simultaneously subjected to a given surge; second, whether other windings are connected in delta or star; third, the high-frequency characteristics of the network connected to all windings. For example, when all three transformers of star delta bank are struck, say by lightning, L_T is equal to one-third of the short circuit inductance of one transformer. In case one transformer is struck, say by a switching surge, L_T is the open circuit inductance if other windings are disconnected from their networks. In the first case, a neutral inductance of 5 per cent (based on rated kv-a. of one transformer of the bank) will cause transient voltage at the neutral of the order of the applied voltage, if the transformer short circuit inductance is, say 10.5 per cent, because then $\frac{L_N}{L_N + L_T} = \frac{5}{5 + 3.5} = 0.59$ and from Fig. 14 E_N max. equals 1.18 E. Still higher voltage will appear with greater neutral inductance. For instance with inductances of 100 per cent (based

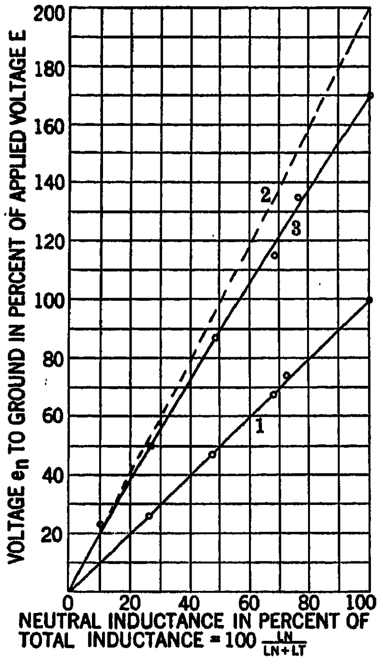


FIG. 14—EFFECT OF NEUTRAL INDUCTANCE ON TRANSIENT VOLTAGE FROM NEUTRAL TO GROUND

1—Line of equilibrium for voltage oscillations at the neutral. (Calculated—dots represent tested data obtained from oscillograms Fig. 13)

2—Maximum voltage of neutral to ground that could be expected in service

3—Maximum voltage of neutral to ground (Test data) again on rated kv-a. of one transformer of the bank), which is the order of magnitude of the Peterson coil, $\frac{L_N}{L_N + L_T} = \frac{100}{100 + 3.5} = 0.97$ and E_N max. equals $1.94\,E$.

FINITE TRAVELING WAVE

The neutral voltage will not reach the above values in case the length of the wave is less than half of the period of oscillation of the neutral.[3]

C. Neutral Grounded through Capacitance

INFINITELY LONG TRAVELING WAVE

The initial voltage distribution along a transformer winding grounded through capacitance is the same as in a solidly grounded transformer.

The final voltage at the neutral is equal to the applied voltage E, because with a constant voltage suddenly applied and maintained across the terminals of the circuit, the voltage across the capacitance will start to oscillate in very much the same manner as if the transformer winding were a pure inductance. That is the voltage across the capacitance beginning with zero, will oscillate with amplitude equal to the applied voltage E about a point of equilibrium E volts above ground (see Fig. 15). Neglecting the surge impedance of the line, the frequency of the oscillation is $f = \frac{1}{2\,\pi\,\sqrt{L_T\,C_N}}$ (L_T is transformer inductance, and C_N is neutral capacitance provided C_N is big enough in comparison with transformer electrostatic capacitance.) After the oscillation dies out the voltage across the capacitance becomes E.

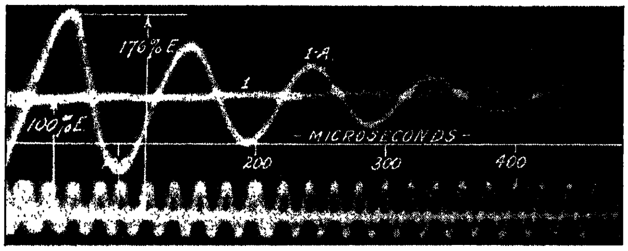


FIG. 15—OSCILLATION OF NEUTRAL VOLTAGE WITH CONDENSER IN THE NEUTRAL OF THE TRANSFORMER

1 —Voltage at line terminal
1 A—Voltage from neutral to ground

In addition to this oscillation the entire winding will oscillate similarly to previous cases, at its natural frequencies. As in the case of a transformer grounded through resistance, the line end of the axis of the internal oscillation will remain at the hundred per cent point. But contrary to the case of neutral resistance the neutral end of the axis will tend to oscillate at frequency f. Whether it will succeed or not depends on the ratio of the fundamental natural frequency of the transformer with neutral isolated and the frequency, f, of the neutral grounded through capacitance. The smaller of the two dominates the phenomena. Since the internal transient is a damped oscillation the position of its axis is of importance (as far as the effect of this oscillation is concerned) only for the period t microseconds, at the end of which the amplitude of internal oscillation reduces below dangerous value. In case C_N is big enough to control the frequency at the neutral, and this frequency is such that at the end of t microseconds the neutral voltage rises to only a small fraction of the applied voltage, the internal oscillation will be practically identical with that of a solidly grounded transformer. If this frequency is so high that at a small fraction of t microseconds the voltage of the neutral reaches a value comparable with the applied voltage, the transformer oscillation will approach that of a transformer with isolated neutral. The envelopes of maximum voltages to ground occurring during t microseconds for various values of neutral capacitance are similar in appearance to curves of Fig. 11.

In case the neutral frequency f is so low that the transformer winding oscillates practically as if it were grounded for the period of t microseconds, the voltages of all points of the winding will oscillate after t microseconds, at frequency f. The axis of the latter oscillation will be a horizontal line drawn through the point corresponding to the applied voltage E, and the entire winding will oscillate as a perfectly rigid rod with the line end remaining stationary at E volts above ground and the neutral end oscillating between zero and $2\,E$ volts.

FINITE TRAVELING WAVE

In practise the traveling waves are finite and the voltages at different points of the transformer winding depend not only on the just mentioned relations but also on the relation of the length of the wave, the fundamental natural frequency of the transformer, and the frequency f of the neutral.

The relation between the length of the wave and the internal transformer oscillations was described in a preceding paper.[3] Fig. 16 shows the relation between maximum neutral voltage and the ratio of the length of traveling wave to period of f. Therefore it is possible by a proper choice of the neutral capacitance to make transformers behave under finite traveling wave excitation as if they were solidly grounded.

D. Neutral Grounded through Impedor

It was found by various investigators that the short circuit neutral current can be reduced to a safe value by grounding some of the transformer banks of a system through either resistance or reactance.

The ohmic values of the grounding resistance or reactance depend on system characteristics and, naturally, vary over quite a wide range. The order of magnitude of maximum resistance, however, is not expected to exceed 300 to 400 ohms. In most of the cases, values below a hundred ohms were found to be satisfactory at large substations; the maximum value of neutral reactance (Peterson Coil) is of order of 100 per cent and the minimum is probably 5 per cent (both based on capacity of one phase of the sub-station). These values if expressed in henrys are somewhere between 0.01 and 2.0 henrys.

With resistance of the above order of magnitude only the very long traveling waves and switching transients are capable of raising the voltage at the neutral to a considerable fraction of the applied voltage. Fig. 17 indicates that waves of many hundreds of microseconds in length must be of comparatively low amplitude to be able to propagate along a transmission line without arcing over its insulators. In such a case, the neutral potential, while it may rise to an appreciable fraction of the applied voltage, is proportionately lower in its absolute value. Since voltage resonance between transformer and the neutral resistance is physically impossible, it appears that with resistance of not more than some 30 or 40 ohms the neutral potential under lightning or switching transient excitation will not exceed some 20 per cent of the impulse arc-over of a given transmission line insulation.

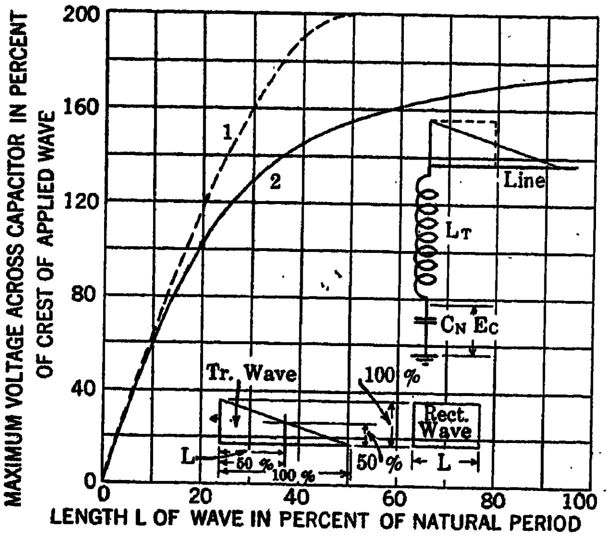


FIG. 16—RELATION BETWEEN MAXIMUM VOLTAGE FROM NEUTRAL TO GROUND, AND THE RATIO OF LENGTH OF THE APPLIED WAVE TO THE NATURAL PERIOD OF THE NEUTRAL OSCILLATION. (SINGLE FREQUENCY OSCILLATION)

1—For rectangular wave
2—For triangular wave

It was found that in most of the cases reactance in the neutral is preferable to resistance. Unfortunately, the desirable values of the neutral reactance are generally such that the transient neutral potential would necessarily rise to dangerous magnitudes under most voltage disturbances.

Furthermore, during switching out operations, when breaking single- or two-phase ground shorts, sudden rupturing of the neutral current may produce an "inductive kick" of a very destructive magnitude. The inductive kick of a neutral reactor is usually more severe than that of a transformer because the neutral reactor is generally of air core type and the residual magnetism, reducing the magnetic kick in a transformer, is absent.

As was shown, the neutral transient voltage is proportional to $\frac{L_N}{L_N + L_T}$. The selection of the values of L_N and L_T cannot, however, be governed by transient voltage considerations, but must depend entirely on operating frequency requirements. This makes special measures necessary when it is desired to keep the rise of neutral transient voltage below values which would be attained with only inductance L_N between the neutral and ground.

Careful consideration of all aspects of the problem makes it appear that the most satisfactory means of controlling neutral transient voltage is an addition of capacitance and resistance in shunt with the inductance. The capacitance and resistance are so selected that on one hand, the neutral voltage does not rise above a specified fraction of the applied voltage regardless of the type of the line transient, and on the other hand, the impedance of such a grounding device at operating frequency is equal to the desired value. In addition the rate of rise of neutral potential, produced by any surge, is made so slow in comparison with the natural frequency of the transformer that the transformer winding oscillates as if it were solidly grounded. It is suggested to call the above device an impedor.

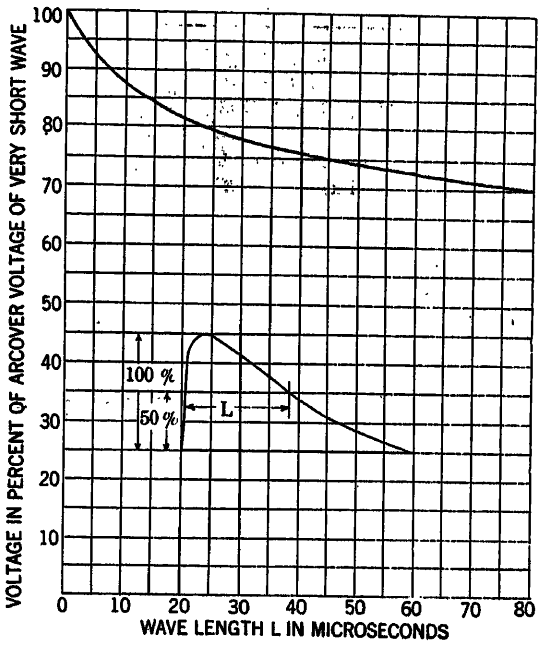


FIG. 17—RELATION BETWEEN ARC-OVER VOLTAGE OF A STRING OF INSULATORS AND LENGTH OF TRAVELING WAVE

Under short circuit the operating frequency neutral voltage rises to a certain value e_1 depending on the ratio of the neutral reactance to the transformer reactance and the reactance of the rest of the system. For this reason e_1 is chosen as a criterion for the selection of the constants of the impedor. They are chosen so that equivalent transient stresses do not exceed 2 e_1, which is the value of the high potential test that the impedor and the neutral end of the transformer must be subjected to, if their operating voltage is e_1. Fig. 18 shows the relation between the operating frequency voltage and current of a neutral impedor as function of its reactance for various ratios of kv-a. generating capacity of the station to the kv-a. capacity of the transformer bank involved in a single-phase short-circuit.

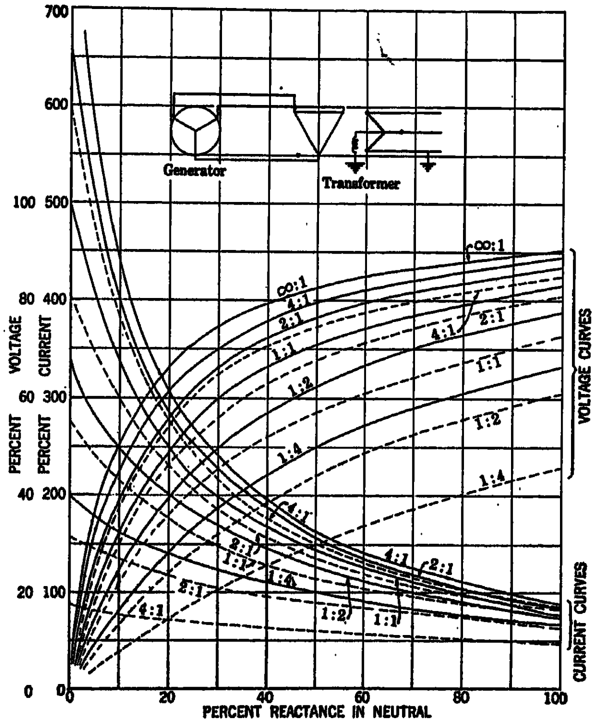


FIG. 18—OPERATING FREQUENCY VOLTAGE AND CURRENT OF NEUTRAL REACTORS FOR LINE TO GROUND FAULT, WITH VARIOUS RATIOS OF GENERATOR TO TRANSFORMER CAPACITY

Solid lines—generator reactance equals 20 per cent (Steam turbine)
Dotted lines—generator reactance equals 35 per cent (Hydro turbine)
Transformer reactance in both cases equals 10 per cent
Ratios given on the curves are ratios of generator capacity to transformer capacity

Part 4. Impedor with Non-Resonating Transformer

A non-resonating transformer which is designed for solidly grounded neutral when grounded through ordinary resistance or capacitance oscillates very much like an ordinary transformer, as shown diagrammatically on Fig. 3. It does so because, while its "initial" voltage distribution is a straight line, similar to the voltage distribution of a non-resonating transformer with solidly grounded neutral, it does not coincide with its "final" voltage distribution which is the same horizontal line as that of an ordinary transformer. To design a shield which would give initial distribution as a straight horizontal line would mean to secure infinite capacitance between the shield and the winding, which, of course, is impossible. However, the neutral capacitance may be so chosen that for the finite waves which are found in practise the neutral will remain substantially at ground potential, and thus the initial and final voltage distributions will coincide and the transformer will not oscillate. To prevent the possibility of resonance between the neutral capacitance and the inductance of the transformer as a whole under oscillatory transient, a resistance is placed in shunt with the capacitance. Under such conditions the transient voltage distribution along the winding will be uniform. Since impedance of such an impedor at operating frequency can be made very high, the transformer may be made to act at operating frequency as if its neutral were isolated.

A bank of non-resonating transformers, if designed with insulation of the neutral sufficient to withstand the dynamic voltage rise of the neutral, could, in most cases, be grounded through a pure resistance (*i. e.*, non-inductive) of 30 to 40 ohms and act even under severe transient voltages practically as if they were solidly grounded.

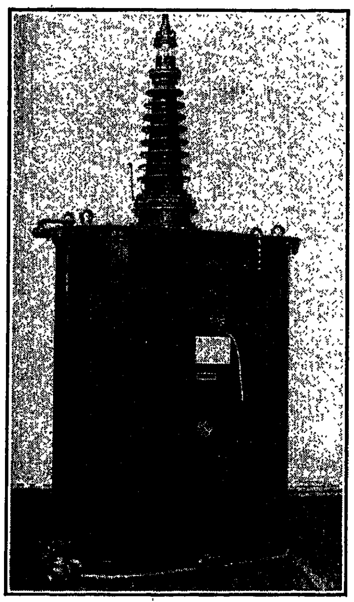


FIG. 19—IMPEDOR FOR BANK OF (3) 40,000-KV-A. 220-KV. TRANSFORMERS

When it is desired not only to retain the non-resonating characteristics but also to keep the neutral voltage below $E \dfrac{L_N}{L_N + L_T}$, under transient conditions, an impedor is designed to hold the neutral voltage and its rate of rise below a specified value for the longest traveling wave possible in practise.

During the last year and a half nearly 800,000 kv-a.

of transformers of non-resonating type, have been built so that they can be operated with impedor in the neutral.

Conclusions

Ordinary Transformers

It has been shown in previous papers that the insulation of ordinary transformers, with solidly grounded neutral, should not be graded in the order of the normal frequency voltage stress to ground, due to the presence under transient voltage stress of high voltages to ground along a major portion of the winding from line toward the neutral. The insulation immediately adjacent to the neutral terminal (say the last 10 per cent) and the neutral bushing itself may be of lower insulation strength.

In this paper it is shown that:

1. Where the neutral is not solidly grounded but is grounded through inductance, the insulation of the whole of the winding and neutral bushing must be equal to that of the line terminal.

2. Where the neutral is grounded through resistance, if this resistance exceeds 300 to 400 ohms per transformer bank, the insulation of the whole of the winding and neutral bushing should be equal to that of the line terminal. For lower values of resistance, the insulation of the neutral bushing may be reduced depending on the dynamic rise of neutral voltage and the transformer high-frequency characteristics.

3. Where the neutral is grounded through an impedor of proper characteristics, the neutral bushing and the insulation between neutral end of the winding and ground may be reduced depending on the dynamic rise of the neutral voltage, but the insulation between the rest of the winding and ground and low voltage winding cannot be graded.

Non-Resonating Transformers

1. Where the neutral is solidly grounded the insulation from line terminal to neutral can be graded in the order of the normal frequency voltage stress.

2. Grounding the neutral through an inductance is not permissible with a non-resonating transformer of the construction described in the papers referred to.

3. Grounding the neutral through a resistance not exceeding 30 or 40 ohms is permissible with non-resonating transformers. The insulation of the winding may be graded in the order of the fall of voltage from line terminal to the neutral under the maximum value of dynamic neutral voltage rise. For higher values of resistance, a capacitance shunt across the ground resistance must be used.

4. Grounding the neutral through an impedor of proper characteristics is permissible with non-resonating transformers. The insulation of the winding may be graded in the order of the fall of voltage from line terminal to the neutral under the maximum value of dynamic neutral voltage rise. The impedor may consist of:

(a) A combination of resistance and capacitance.

(b) A combination of resistance, capacitance, and inductance, when the latter is required by short circuit considerations.

The author wishes to express his indebtedness to Mr. J. H. Hagenguth for his assistance with tests and calculation of curves, and to Messrs. A. N. Garin and S. T. Martin for editing the paper during the author's absence.

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3. *Effect of Transient Voltage on Power Transformer Design*, by K. K. Palueff, A. I. E. E. Quarterly Trans., Vol. 48, July 1929, pp. 681-706.

4. *Abnormal Voltages within Transformers*, by L. F. Blume and A. Boyajian, A. I. E. E. Trans. ,Vol. 38, 1919, p. 577.

Discussion

J. K. Hodnette: In an investigation of the nature of that conducted by Mr. Palueff, it is necessary to make certain assumptions. Some of the assumptions made by Mr. Palueff do not appear correct and it is desired to substitute more logical ones and compare the results obtained.

This refers particularly to Part 2 of the author's text, *Transformer with Neutral Isolated.* It is evident from the descriptive matter and from the oscillograms reproduced in the paper that the author has assumed an extremely long wave as being representative of a lightning wave. The statement is made that the effect of such a wave is similar to that of direct current, which in itself is correct, but it is incorrect to construe such a wave as being representative of a lightning wave. Recently published data[1] giving the result of many tests in the field indicate that the maximum length of lightning surges expected in service probably would not exceed 60 microseconds to half value and the most rapid rise was about one microsecond to maximum value. Such a wave, which could easily be reproduced in the laboratory, would be representative of the most severe conditions with regard to stresses in transformers expected from any lightning surges. A wave of this character was used in obtaining the following test results.

Fig. 1 of this discussion compares the initial voltage distribution obtained by Mr. Palueff in a shell type and a core type transformer with the distribution in 30,000-kv-a. shell type transformer of modern design. The initial distribution is a function of electrostatic circuit of the transformer, or a function of proportions of the transformer. The author intimates that these curves are representative of the initial distribution in shell and core type transformers in general. Fig. 1 illustrates the error that would be introduced by making such a generalization.

Fig. 2 of this discussion shows the maximum voltage to ground in the transformers by oscillation. Curves 1 and 2 are reproduced from the author's curves Fig. 7, which were taken from the oscillograms of Figs. 5 and 6. It is to be noted that the author has used the extremely long wave in producing these oscillations. The amplitude of the oscillations depends upon the initial distortion, the length of the wave, and the natural period of the transformer. Under these conditions high percentage of overvoltage would naturally be expected. Curve 3 of Fig. 2 shows the maximum voltage to ground in a 25,000-kv-a. isolated neutral transformer produced by the 60-microsecond surge. Note that the maximum voltage is 74 per cent as compared to 240 per cent by Curve 2. Had the author made his experiments with a wave comparable to a lightning wave his results would have been more of the order of Curve 3. In view of these facts I cannot see how the author justifies Curves 5, 6, and 7 of Fig. 7 as test values of the maximum voltage to ground due to lightning.

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[1] *Symposium on Lightning Investigations*, see p. 857.

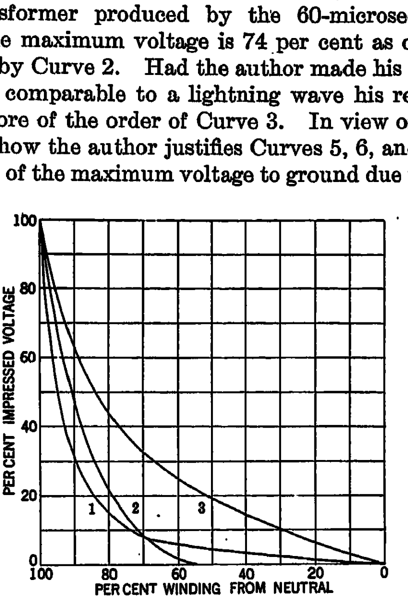

Fig. 1—Initial Voltage Distribution

1. Mr. Palueff's shell type transformer
2. Mr. Palueff's core type transformer
3. 30,000-kv-a. shell type transformer (test)

In the preliminary conclusions for ordinary transformers the statement is made that the grading of insulation in a solidly grounded neutral transformer in the order of the normal frequency stress is not permissible. The author gives no data to substantiate that conclusion but refers to previous papers for proof. In these papers he has proceeded with the same assumptions made here, that is, that a lightning wave is extremely long, an obviously incorrect assumption. On the contrary, an analysis of the stresses in transformer windings[2] based on lightning surges, both in magnitude and shape, and coordinated with field and laboratory tests on the transmission insulation, has shown clearly that the grading of insulation is a permissible practise.

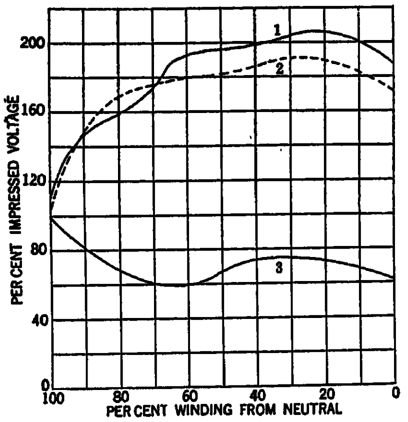

Fig. 2—Internal Voltages to Ground in Transformers Due to Lightning Transients

1. Mr. Palueff's shell type transformer (extremely long rectangular wave)
2. Mr. Palueff's core type transformer (extremely long rectangular wave)
3. 25,000-kv-a. shell type transformer with 60-microsecond impressed surge (test)

Referring to the author's text on switching surges, it would be of interest to know the source of information for the statement that the amplitude of the average switching transient is about half of that of the lightning wave. Many data have been obtained in recent years on the amplitude of switching surges.[3] These data taken on many systems indicate that surges as high as 4.5 times normal are rare and occur on but few of the systems. The average switching surge based on these data would probably not exceed one and one-half times normal. Also most switching transients were recorded unidirectional, or the attenuation was so rapid that only one crest occurred above normal voltage. This is illustrated by the author's Fig. 8 where the decay is from 4.5 times normal to about 1.5 times normal in the first cycle or a decrease of 65 to 70 per cent.

The author, however, disregarded this fact in representing a switching transient for his experiments by a damped oscillation which has a decrement of about 25 per cent. Fig. 3 of this discussion shows the forced oscillations in the 25,000-kv-a. shell type transformer above produced by a damped oscillation having a 45 per cent decrement. Practically exact resonance exists between the applied voltage and the transformer oscillations whereas the author's oscillograms, Fig. 9, were taken with a 20 per cent discrepancy from resonance. Even with this discrepancy the maximum voltage in the first case is less than one-half that of the latter. Had the author used a more rapidly decaying oscillation, such as Fig. 8, the internal voltages would have been greatly reduced.

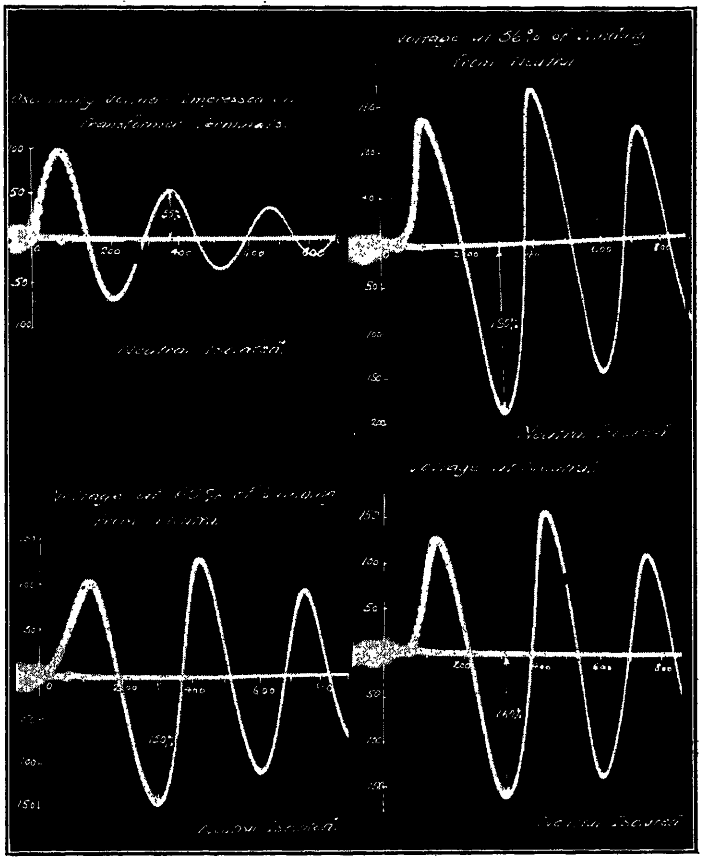

Fig. 3

Sufficient data are not available at present for drawing conclusions as to the intensity of stress setup by switching transients. Failures of transformers from this type of transient are practically unknown, which is the true criterion for the sufficiency of the design or the relative intensity of the stress. Therefore, until more conclusive data are presented, it appears that the author's conclusions are drawn from insufficient evidence.

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[2] *The Effect of Surges on Transformer Windings*, J. K. Hodnette, A. I. E. E. Quarterly Trans., Jan. 1930, p. 68.

[3] *Klydonograph Surge Investigations*, J. H. Cox, P. H. McAuley, and L. G. Huggins, A. I. E. E. Trans., Vol. 46, 1927, p. 315. *Symposium on Surge Voltage Investigation*, A. I. E. E. Quarterly Trans., Oct. 1928, pp. 1111-1154

F. J. Vogel: I should like to refer to Mr. Palueff's Fig. 11. In common with his other curves he has assumed a definite kind of lightning surge. This has been a surge which has been of infinite length. We also have taken test data using the surge which we have assumed to be more representative of actual lightning conditions than the surge which he has picked. Fig. 4 shows oscillations with different values of resistance at the neutral. Curve 6 shows practically the same results that Mr. Palueff has given. Comparison of the other values would be 62½ per cent versus 171 per cent for approximately 100,000 ohms; and also, 40 per cent versus 125 per cent, for 25,000 ohms.

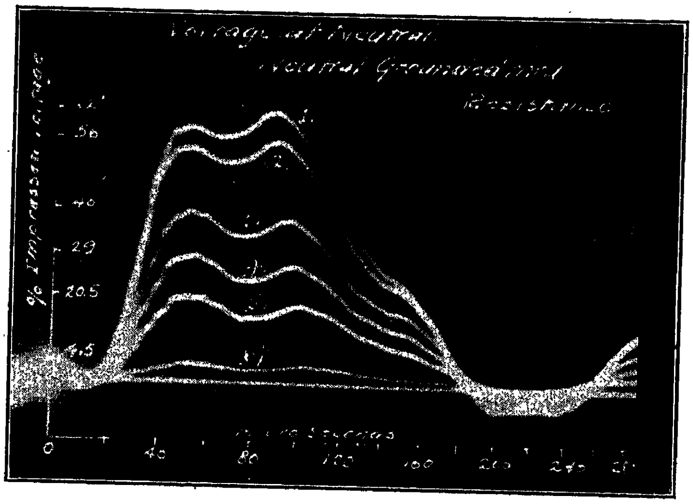

FIG. 4

1—R = ∞
2—R = 100,000 ohms
3—R = 25,000 ohms
4—R = 11,000 ohms
5—R = 5,000 ohms
6—R = 460 ohms

Continuing, we note on page 1185, the first paragraph, second column, that the "voltage distribution with 500 ohm resistance in the neutral is the same as with solidly grounded neutral." We are very much inclined to agree. However, on page 1188 and in the conclusions, he makes quite different statements. I believe that he cannot justify these latter statements on the basis on which they are calculated. They are calculated from the data given under the heading "Finite Traveling Wave." He has assumed a certain value of L_T. He estimated the time necessary to reach the final distribution, using this value of L_T. If this is followed to its conclusion, you will find the current large, which means there would be iron losses and magnetization. The effect of the iron on L_T cannot be neglected. L_T will be far from a constant during the first part of the surge and at times very large. For that reason, his test data are much more valuable than what he has obtained from his calculations.

Figs. 13 and 14 are again made by assuming infinitely long waves which the author was not actually able to obtain. Further, there is not an exact comparison between Figs. 13 and 14 because the data given under 2A, 4A, and 5A and the points on the curves do not correspond. Good correspondence with the calculated values was not actually obtained.

There is, in the literature published by different Westinghouse and General Electric Company engineers, a discrepancy in the impulse voltage flashovers for insulators. It is unnecessary to go into that except to state that there are considerable data published by Mr. J. J. Torok for a 60-microsecond wave. That is the only point we can check on the author's curve, Fig. 17. Torok's data practically show no impulse factor whatever, whereas the author shows a considerable impulse factor for a 60-microsecond wave. This means that the author is estimating on a higher voltage due to long waves than we are. This difference is quite considerable.

Mr. Hodnette has discussed the preface to the conclusions. The second conclusion is also already covered. I wish to make an addition to the third conclusion and that is, that either an impedor such as the author has worked out or other protective means will give very nearly the same results as the solidly grounded neutral.

I wish also to state that under these conditions graded insulation is permissible, which is a direct contradiction to this part of the conclusion.

J. H. Hagenguth: It probably will be of interest to show the agreement between calculation and test results of oscillations in an ordinary transformer winding, the neutral of which was directly grounded. In a paper by Messrs. F. F. Brand and K. K. Palueff (A. I. E. E. Quarterly TRANS. Vol. 48, 1929, p. 1005), Fig. 15, was shown a series of curves which were obtained from oscillograms. Fig. 5 of this discussion shows similar curves which represent calculated values. The abscissa represents per cent winding, and ordinate the voltage at different points in per cent of applied voltage. The calculation is relatively simple but quite laborious. Curve *O* represents the initial voltage distribution due to impact of traveling wave and was obtained by test. It can also be calculated as has been shown before. This curve has been analyzed into its space harmonics with respect to the line of final voltage distribution. As has been shown previously, these harmonics do not oscillate at the frequency which their order indicates but at a higher order. For instance, the second harmonic may oscillate at a frequency between 2½ to 4 times the fundamental or even higher. By superimposing the amplitudes of these harmonics at different points of the winding, when oscillating at their proper frequencies, the volt-time relations, as well as simultaneous voltage distributions, can be obtained. Simultaneous voltage distributions

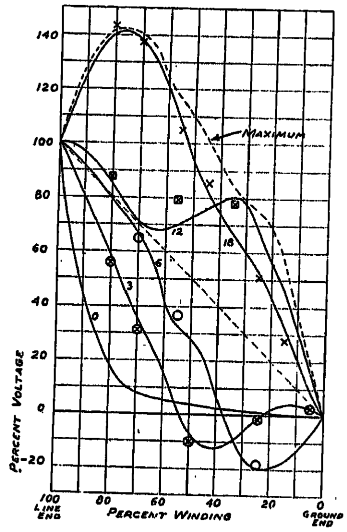

FIG. 5—VOLTAGE DISTRIBUTION PRODUCED IN AN ORDINARY TRANSFORMER BY A SINGLE TRAVELING WAVE. (CALCULATED)

0 Initial voltage distribution at impact of traveling wave
3, 6, 12, 18 simultaneous voltage distribution throughout the winding after 3, 6, 12, and 18 microseconds. Dotted line represents envelope of maximum voltages. x indicates test value

are shown at 0, 3, 6, 12, and 18 microseconds. It should be noted how rapidly the change of voltage takes place. For instance, at the 30 per cent point the voltage increases from 15 to + 78 per cent in six microseconds. The dotted curve represents the envelope of maximum voltages which is not simultaneous. In order to show the accuracy of the calculations a few points are plotted from actual oscillograph records. The greatest error is only 10 per cent, which is not much considering the complexity of the transformer circuit for high-frequency phenomena. On Fig. 6 the volt-time characteristic for impulse waves of a single point is shown. Curve 1 is the oscillographic record. Curve 2 represents calculated values. The tested curve is slightly lower, which may be attributed to damping, but the close resemblance in almost every detail can be easily seen. The damping up to the first half cycle is almost negligible. It has not been taken care of at all in the calculations. The maximum rise at this point, 58 per cent from neutral, is 115 per cent. On Fig. 7 the voltage rise of the transformer neutral with neutral isolated and neutral grounded through capacitor is shown. The great difference in time at which maximum occurs should be noted. When the neutral is isolated the maximum is reached in 25 microseconds; when grounded through the capacitor, in 180 microseconds. It has been shown in the paper that for all practical purposes the frequency of oscillation of the neutral can be calculated from the relations $f = \dfrac{1}{2\pi\sqrt{LC}}$, where L is the equivalent reactance of transformer and neutral grounding device and C grounding capacitor.

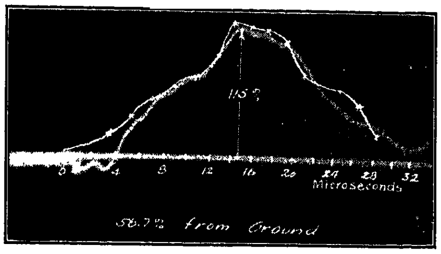

FIG. 6—VOLT-TIME CHARACTERISTICS OF A POINT 56.7% FROM NEUTRAL DUE TO A SINGLE TRAVELING WAVE

Curve 1 oscillographic record
Curve 2 calculated values

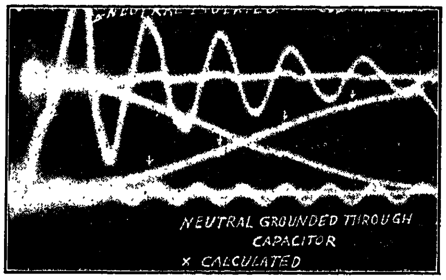

FIG. 7—VOLTAGE RISE OF THE NEUTRAL DUE TO A SINGLE TRAVELING WAVE

1. Neutral isolated
2. Neutral grounded through capacitor

1 *A* and 2 *A* are applied waves
Frequency of the oscillator wave is 50,000 cycles

To prove this point the rise of neutral voltage has been calculated, taking in this case the short circuit reactance of the transformer, which by the way represents the reactance of a bank of three 40,000-kv-a. transformers 220,000 *Y*, and the capacitance in the neutral. The crosses on the film indicate calculated values and they check very closely the test results.

It may be mentioned that there is a difference in the applied waves for the two conditions shown on Fig. 7, but the resulting voltage rises at the neutral can be compared. For Condition 1, *i. e.*, neutral isolated, the maximum voltage at the neutral would be reached if the applied wave has a length of at least half of the natural period, which in this case is a rectangular wave of 25 microseconds. It can be seen that up to 25 microseconds the two applied waves almost coincide, and therefore the comparison of the maximums is correct for the applied wave of condition 2. For a rectangular wave of 25 microseconds duration, the maximum voltage at the neutral under condition 1 would be the same as for the very long wave shown on the figure, *i. e.*, 164 per cent, while for condition 2 the maximum voltage would be 27 per cent instead of 92 per cent as for the applied wave 2*A*. The capacitor used for this test was much smaller than would be used in service to permit the superposition of both conditions on one film, by decreasing the natural period and thereby increasing the neutral rise.

W. A. McMorris: In the design of these impedors, the neutral grounding resistor or reactor used is determined by the operating frequency requirements. The problem is then to modify the high-frequency characteristics of the installation, so that the transient voltage appearing at the neutral will be limited to a predetermined value and wave shape.

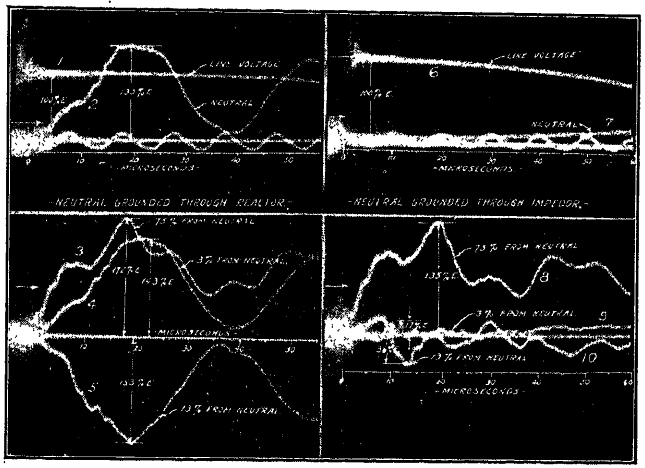

FIG. 8—EFFECT OF IMPEDOR ON IMPULSE VOLTAGES AT NEUTRAL AND INTERNAL POINTS OF TRANSFORMER WINDING

1 to 5—Neutral grounded through reactor
6 to 10—Neutral grounded through impedor
1 and 6—Applied wave
2 and 7—Voltage at neutral
3 and 8—Voltage at 75 per cent from neutral
4 and 9—3 per cent from neutral
5 and 10—13 per cent from neutral (obtained with reversed polarity)

The oscillograms of Fig. 8 herewith show the voltages produced by a single traveling wave at the neutral and at several points of a transformer winding. The oscillograms on the left half of the figure were made with the neutral grounded through a reactor. The neutral voltage rises rapidly to a value even in excess of the applied voltage, and severe oscillations occur within the winding.

The oscillograms on the right half of the figure were made with conditions the same, except that a capacitor was connected in parallel with the reactor. The neutral voltage rises gradually, and reaches only a small fraction of the applied voltage. No internal oscillations are forced in the windings of the transformer or reactor. The transformer behaves substantially the same as if it were solidly grounded.

Calculation of the capacitance required is a simple matter, and may be accomplished in a short time by the use of a set of design curves. Using the curves of Fig. 9 herewith, the required natural period can be found when the ratio of neutral inductance to total inductance, and the allowable voltage rise at the neutral are given. From the ratio $\frac{L_N}{L_T + L_N}$ read vertically to the per cent voltage rise, horizontally to the length of the traveling wave, then vertically to the required natural period.

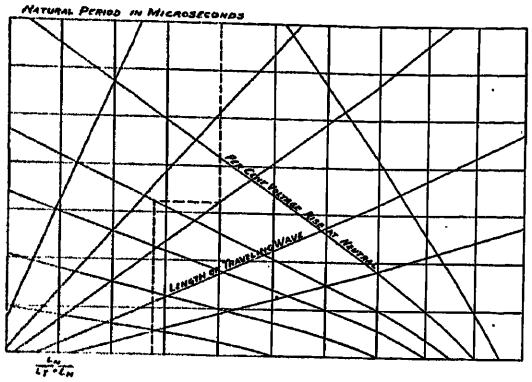

FIG. 9—CHART FOR DETERMINING REQUIRED NATURAL PERIOD

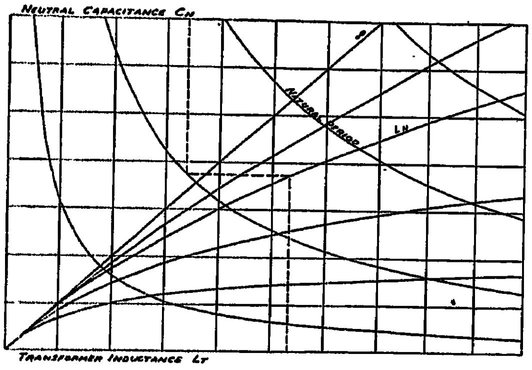

FIG. 10—CHART FOR DETERMINING REQUIRED CAPACITANCE

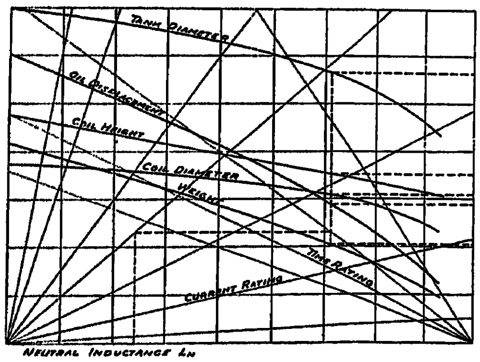

FIG. 11—CHART FOR DETERMINING DIMENSIONS AND WEIGHT OF REACTOR

Using the curves of Fig. 10 herewith, the required capacitance can be found when the transformer inductance, neutral inductance, and natural period are given. From the transformer inductance L_T read vertically to L_N, horizontally to natural period, then vertically to neutral capacitance C_N.

After the circuit constants have been determined, the weights, costs, and dimensions of the apparatus can be determined from other curve sheets. Fig. 11 herewith shows such a sheet for neutral reactors designed for a given high-potential test. From L_N read vertically to current rating, horizontally to time rating, then vertically to curves showing dimensions and weights.

In Fig. 19 of Mr. Palueff's paper is shown a grounding impedor for a bank of three 40,000-kv-a., 220-kv. transformers. This unit is entirely self-contained. The only external leads are the high-voltage bushing, which is to be connected to the transformer neutral, and a grounding terminal, which is to be connected to the station ground bus.

K. K. Palueff: I wish to answer first Mr. J. H. Hodnette's criticisms on the following points.

A: "More logical" shape of wave should have been used for some of the data given.

B: Comparison of initial or electrostatic distribution in core and shell type transformers.

C: Maximum voltage to ground produced by traveling wave in a transformer with isolated neutral.

D: Amplitude and the rate of decay of damped oscillations and "average surge."

E: Internal voltages produced by damped oscillations.

F: Experience with graded insulation transformers.

A: In the paper presented by the author to the Institute in January, 1929, considerable space was given to the description of the influence of the shape of the incident traveling wave and of the natural frequency of the transformer on the magnitude of voltages produced in the winding.

I am glad to see Mr. Hodnette quote this relation without challenge.

In the present paper, as well as in the preceding papers, an attempt was made, first to present separately the effect on the phenomenon of various factors involved, then to combine them in a manner that would represent conditions met in practise, so as to be able to draw conclusions of practical interest. For this reason very long waves have been applied to the transformer to record its free oscillations fully developed and undistorted by the shape of the incident wave. After such records are obtained, the amplitude and the shape of the voltage wave of any given point in the winding, for any shape of the incident wave, can be computed with a high degree of precision, as was shown in the papers referred to above.

In the above criticism, Mr. Hodnette refers to Part II of the paper. The section entitled "Finite Traveling Wave," deals with the effect of wavelengths found in practise.

B: The fact that in a shell type transformer the initial or electrostatic distribution causes at the line end at least as high a concentration of voltage as in a core type transformer, was proved by Mr. Hodnette's and the author's tests. Those interested in this phase of the phenomenon, I should like to refer to Mr. Hodnette's paper and my discussion of it. (A. I. E. E. Quarterly TRANS., Vol. 49, Jan. 1930, p. 68.) If Curve No. 3, Fig. 1 of his present discussion was obtained in the same manner, as some of the similar data given in the paper referred to, then it is as much in error as were those and for the same reason. More than eighteen years of experimenting with high-frequency transients, taught us that exceptional care should be exercised in all details of the experiment.

I again suggest that Mr. Hodnette present an analytical proof, accompanied by a numerical example, of his statement that shell type transformers by virtue of their very wide coils and relatively short stack have more uniform initial voltage distribution than is found in longer stacks of narrow coils.

I should like to repeat that it would be of general interest should he find it possible to publish data on *sphere-gap* measurements of impulse voltage between the line terminal and turns not only on outside, but also on *inside edges* of coils. Such a test, if properly conducted, gives a truer picture of initial voltage distribution.

C. This question for the solidly grounded transformers was also answered very completely in my discussion of Mr. Hodnette's paper just referred to.

I pointed out, in that discussion, a very substantial discrepancy existing between Mr. Hodnette's oscillographic records, and the curves plotted from them.

I have pointed out in a previous paper the simple relation existing between the ratio of natural period of oscillation to the length of the applied wave and the magnitude of maximum voltage produced inside of the transformer winding. This relation can also be derived from Fig. 16. For example, with natural frequency of 12,500 cycles, the voltage to ground of the half of the winding next to neutral produced by a 60 microsecond wave will be approximately 70 per cent higher than in a transformer whose natural frequency is 3300 cycles.

The latter frequency is approximately equal to the natural frequency of the 25,000-kv-a. transformer referred to by Mr. Hodnette in his present discussion. The natural frequency of 12,500 was found by Mr. Hodnette in some other shell type transformer he tested and referred to in the closing discussion of his paper. Curve 3 of Fig. 2 of this discussion therefore becomes Curve 3*A* for the transformer with natural frequency of 12,500 cycles. (See Fig. 12 of this discussion.) Curve 7 of Fig. 7 of the present paper is reproduced as Curve 7 on Fig. 12 herewith. Curve 3 of Fig. 12 is a reproduction of Curve 3 of Fig. 2 of Mr. Hodnette. The natural frequency of transformers, of both shell and core type, varies over a wide range (between say 2000 and some 40,000 cycles), so that there is bound to be a difference in voltage produced in various transformers by a given wave. Therefore, the agreement between Curve 7 and 3*A* is indeed as satisfactory as could be expected.

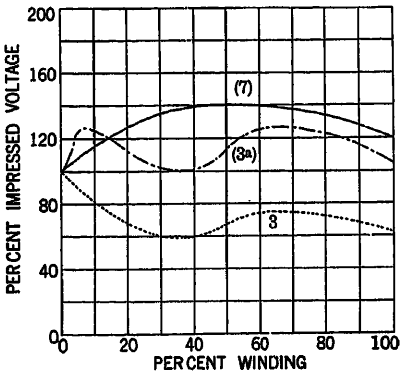

Fig. 12

It is Curve 7 and not 3 and 4 of Fig. 7 that represents the author's idea of voltages found in practise. Curves 3 and 4 give voltage to ground found in transformers when an extremely long wave is applied, while Curve 7 is described as "estimated voltage found in practise."

On this account, in Fig. 2 of this discussion, in place of Curves 1 and 2, Curve 7 of Fig. 7 should have been reproduced for comparison with Mr. Hodnette's Curve 3.

More than 95 per cent of total published oscillographic data on the shape of the natural lightning wave, referred to by Mr. Hodnette, has been obtained by my associates, and I therefore had the unusual opportunity of studying these data in the original. In spite of this, I am of the opinion that it is too early to speak with authority of the length and shape of the most dangerous lightning waves that are encountered in practise.

E: Switching surges above five times normal are on record. Some of these records have been obtained by Mr. Hodnette's associates.

Since most of the records were secured by means of surge voltage recorder and klydonograph, the data on the shape of the wave are not sufficient to allow anyone to speak with certainty of the damping factor.

Mr. Hodnette, in his closing statement, admits that his knowledge of switching surge characteristics is not sufficient. At the same time he definitely states that I am not justified in using a damped oscillation with 25 per cent damping factor for Fig. 9, while he considers his tests with damping factor of 40 per cent to be representative of the worst conditions found in service.

It is generally recognized that the corona loss is responsible for the high damping factor of voltage transients on overhead lines. For this reason, the damping effect reduces with successive cycles of the oscillation as the corona loss decreases very rapidly with reduction in magnitude of the voltage. Therefore, the damping effect becomes very small when the transient voltage is only two or three times normal.

Voltage within winding resonating to a surge increases with each successive half cycle. This increase is proportional to the amplitude of the respective half cycle of the applied wave. Thus, a highly damped surge (decrement 75 per cent) consisting of three half cycles of, say, 5, 2.5, and 1.25 times normal (total 8.75) will produce internal oscillations approximately 20 per cent lower than those produced by a damped oscillation consisting of 9½ cycles (decrement 36 per cent), with amplitude as follows: 2.5, 2, 1.6, 1.3, 1.04, 0.83, 0.67, 0.56, 0.47 times normal (total 10.97).

All investigators, including those associated with Mr. Hodnette, agreed that the damped oscillations of two and one-half times normal are very common in all systems.

For these reasons, the rarity of oscillatory transients of extremely high amplitude, to which Mr. Hodnette attaches so much importance, is not important at all, in the case of cumulative oscillations.

From the above it appears that should Mr. Hodnette's tendency to regard switching surges as of little practical importance, because they seldom attain extremely high magnitude, be made use of in transformer design, a difficulty may arise in practise.

On Fig. 8 of the present paper appears the only cathode ray oscillogram of switching surge so far published. It should be noted that the damping factor drops after the first cycle from 50 per cent to 20 per cent, and the oscillation persists for many cycles.

F: It is my conviction that data given in Mr. Hodnette's paper and several papers published by me, as well as my discussion of Mr. Hodnette's paper, prove conclusively that the use of graded major insulation, even in solidly grounded transformers, is a hazardous practise. Mr. Hodnette refers to service record as being the final proof of suitability of such a design. It is difficult to use service records as an argument in a discussion like this, because there are no published data on failures of transformers. My knowledge of operating experience with this type of transformer is therefore very limited. I understand that in Canada, some 220-kv. transformers with graded insulation have failed during lightning, in spite of the fact that the line insulation adjacent to the transformers was even weaker than the rule of coordination of line and transformer insulation calls for.

I also know that in Germany, at first, graded transformers were used in conjunction with the Petersen coil. This practise caused so much trouble that it was abandoned. That such a practise should cause trouble follows directly from the conclusions arrived at in the present paper.

I should now like to answer Mr. Vogel's discussion. He, like Mr. Hodnette, wants to see in my use of an extremely long wave for some of the test data, a sign that, to my mind, such waves are representative of natural lightning waves. As I have explained before, this is not true.

To derive sufficiently general conclusions from tests made with a 60-microsecond wave or any other short wave, it is necessary to test a great many transformers and perform a very laborious calculation. The use of very long waves saves all this time, as it simplifies the calculations and tests.

Mr. Vogel and Mr. Hodnette invite us to approve gradings of major insulation on the ground that one of the three transformers they have tested happened to have a natural frequency of about 3300 cycles, and therefore it gave, under most favorable conditions of single traveling wave excitation, results satisfactory to them. It would be of interest to see the results of their tests on the two other transformers which Mr. Hodnette mentioned in his paper and his closing discussion, particularly on the transformer whose natural frequency was 12,500, according to his measurements.

Mr. Vogel speaks of the effect of iron loss and magnetization of the core on magnitude of L_T. Should he analyze the flux distribution produced in the transformer when lightning waves simultaneously strike all three transformers of a given bank, he would discover that for the period of time that we are interested in, the effect of the iron is absolutely negligible. Conditions under which it is not negligible are stated in the paper on page 1186.

Oscillograms of Fig. 13 and curves of Fig. 14 of the paper were obtained, as stated, under the assumption of simultaneous excitation of all three transformers of the bank. L_T, therefore, was measured and calculated. Of course, Mr. Vogel realizes that under such conditions the accuracy of calculation is a matter of a few per cent, which is ample for the purpose.

The discrepancy between the amplitude of oscillograms 2A, 4A, and 5A, as marked on Fig. 13, and position of corresponding dots on Fig. 14 is due to error in marking the oscillograms (on the advance copies). There is a complete agreement between actual maximum amplitude of voltages as recorded by these oscillograms, and the dots of Fig. 14. This can be verified by scaling the oscillograms of Fig. 13. However, the correctness of the law suggested in the paper and demonstrated by Figs. 13 and 14 is obvious. (The values on Fig. 13 are now correct.)

The controversy on impulse ratio is too involved to be answered here. Mr. Peek's data shown on Fig. 17 were obtained with waves similar to those found in service. Mr. Torok's data were secured with flat top waves which so far were not observed in service except of extremely low amplitude. It is hoped that in due time different investigators will agree on the interpretation of the data secured under different test conditions.

The neutral impedor may consist of either special resistance connected from the neutral to ground (with or without gap) or of special resistor and condensor connected in shunt with the resistor. The values of the constants and the choice between the two types of impedor are determined by the characteristics of the transformer and the character of the impedance that it is desired to use for the limitation of the short-circuit current.

As was observed in the paper, the internal high-frequency voltages in transformer grounded directly or through a resistance of some 500 ohms, is very nearly the same. However, when the transformer is struck by a very long traveling wave, in addition to the high-frequency internal oscillation, the entire winding begins to rise above ground potential, as was shown on page 1184. The conclusions on page 1185 refer to voltages due to high-frequency internal oscillations. The conclusions on the last page are general, and refer to voltages that can be produced by any type of transient mentioned in the paper. They also refer to any size of power transformer. There is, therefore, no contradiction between the two conclusions. This is illustrated by Fig. 10.

Mr. Hodnette remarked that I am exaggerating the danger of the transient voltages, because the average transient voltage waves are not as high as those I mentioned. I believe that by this remark he emphasized the principal difference between his and our attitude toward the problem of transformer design. While he proposes to base transformer design on the average or the most probable conditions, we believe that conditions that can reasonably be considered the very worst that can be experienced, should be the basis for design.

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1930

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Forty-sixth Annual Report, for the fiscal year ending April 30, 1930. A general balance sheet showing the condition of the Institute's finances on April 30, 1930, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year, more detailed information having been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held seven meetings during the year, five in New York, one at Swampscott, Mass., and one at Chicago, Ill. The Executive Committee acted upon various matters during intervals between Board Meetings.

Information regarding the more important activities of the Institute which have been under consideration by the Board of Directors, the committees, and the various officers is published each month in the section of the JOURNAL devoted to "Institute and Related Activities."

President's Visits.—President Smith attended the two national conventions and one district meeting held since the beginning of his administration. He has visited a large majority of the Sections, also groups of members in several centers having no Sections. Members of Branches located near Sections have been invited to attend the meetings, and in addition he visited some of the neighboring Branches.

The following is a list of places visited in addition to many trips to Institute headquarters in New York: Winnipeg, Can.; Regina, Sask., Can.; Vancouver, Can.; Seattle, Wash.; Spokane, Wash.; Portland, Ore.; San Francisco, Calif.; Santa Monica, Calif. (Pacific Coast Convention); Salt Lake City, Utah; Denver, Colo.; Omaha, Nebr.; Des Moines, Iowa; Kansas City, Mo.; St. Louis, Mo.; Sharon, Pa.; Syracuse, N. Y.; Fort Wayne, Ind.; Milwaukee, Wis.; Minneapolis, Minn.; Madison, Wis.; Chicago, Ill. (District Meeting); Urbana, Ill.; Lafayette, Ind.; Washington, D. C. (A. E. C. Meeting); Pittsburgh, Pa.; Buffalo, N. Y.; Toledo, Ohio; Erie, Pa.; New York, N. Y. (including Winter Convention); Cincinnati, Ohio; Louisville, Ky.; Memphis, Tenn.; Oklahoma City, Okla.; Dallas, Houston, and San Antonio, Texas; Mexico City, Mexico; Havana, Cuba; New Orleans, La.; Birmingham, Ala.; Atlanta, Ga.; Raleigh, N. C.; Norfolk, Va.; Washington, D. C.; Baltimore, Md.; Philadelphia, Pa.; Rochester, N. Y.; Ithaca, N. Y.

On April 25, he addressed a joint meeting of the Boston, Connecticut, Lynn, Pittsfield, Providence, Springfield, and Worcester Sections at Worcester.

President Smith served as one of the Institute's representatives at the Fiftieth Anniversary Celebration of the American Society of Mechanical Engineers held in New York, Hoboken, and Washington in April.

During May and June, the president will attend the Springfield District Meeting, the Summer Convention in Toronto, Canada, and will visit several Sections.

Meetings.—Three national conventions of the Institute were held during the year: Summer, Pacific Coast, and Winter. District meetings under the auspices of the Geographical Districts were held in Dallas, Texas, District No. 7, in May, and in Chicago, Ill., District No. 5, in December, 1929.

Annual Meeting.—The Annual Business Meeting was held at the New Ocean House, Swampscott, Mass., on Tuesday morning, June 25, 1929, during the annual Summer Convention. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1929, was presented, and the Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1929.

Summer Convention.—The forty-fifth annual Summer Convention was held at Swampscott, Mass., June 24-28, 1929. Twenty-six papers were presented at five technical sessions, and eighteen technical committee reports were presented at two additional sessions. Other sessions were the Annual Business Meeting mentioned above, and one at which an address on Astronomy was given by Dr. Harlowe Shapley. The Annual Conference of Officers and Delegates, under the auspices of the Sections Committee and the Committee on Student Branches, was held on Monday, June 24, and forty-nine Sections were represented. The entertainment features included golf and tennis tournaments, a dinner, and an all-day trip to Rye Beach, N. H. The Lamme Medal, awarded by the Institute, and the John Scott Medal, awarded by the Board of City Trusts of Philadelphia, were presented to Allan B. Field at the dinner. More than 1000 members and guests attended the convention.

Pacific Coast Convention.—The eighteenth Pacific Coast Convention was held at Santa Monica, Calif., September 3-6, 1929. At four technical sessions, eighteen papers were presented. Eleven technical papers by students were presented at two additional sessions. An illustrated address was given by Dr. W. V. Houston on Wednesday evening. The program included various social events and trips. The attendance was about 440.

Winter Convention.—The eighteenth Winter Convention was held in New York, January 27-31, 1930. Fifty technical papers were presented at ten sessions. At an evening session, the Edison Medal was presented to Professor Charles F. Scott, and a lecture on "Discovery and Excavation at Olynthos" was given by Dr. David M. Robinson. Numerous inspection trips, a smoker, and a dinner-dance were held. Attendance about 2,000.

District Meetings.—The first District meeting of the year was held at Dallas, Texas, May 7-9, 1929. The program included fifteen papers presented at four sessions and eleven papers by students presented at two other sessions. Attendance about 540. The second was held at Chicago, Ill., District No. 5, December 2-4, 1929. At four of the sessions, nineteen papers were presented, and thirteen papers by students were presented at a fifth session. Attendance about 600.

World Engineering Congress.—Many members of the Institute, including four past-presidents and the National Secretary, participated in the World Engineering Congress in Tokyo, Japan, in October-November, 1929. The official delegates of the Institute were: F. C. Hanker, F. L. Hutchinson, P. M. Lincoln, L. A. Osborne, R. F. Schuchardt, Elmer A. Sperry, and Charles W. Stone.

Twenty-six countries were represented at the Congress and about 800 papers were presented. The total attendance exceeded 1,000, including about 500 from foreign countries, of whom about one-half were from the United States.

The Tokyo Sectional Meeting of the World Power Conference was held concurrently, and about 150 papers were presented during its sessions.

Honorary Members.—Three distinguished American engineers were elected Honorary Members of the Institute by the Board of Directors on June 25, 1929: Herbert Hoover, Charles F. Scott, and Charles F. Brush (posthumously). Dr. Motoji Shibusawa, Dean of Engineering at Tokyo Imperial University, Japan, was elected an Honorary Member by the Board of Directors on August 6, 1929. He was installed at a special meeting of the Institute held in Tokyo, November 7, 1929, during the World Engineering Congress.

Sections.—The Sections have, with very few exceptions, been active during the year. Many excellent technical programs have been presented, and there has been a good variety of programs of other types. Considerable attention has been given to the desirability of the individual participation of engineers in public affairs.

The New York Section organized four technical groups, Communication, Illumination, Power, and Transportation, and each held two or more meetings during the year, with a total of twelve, in addition to the regular Section meetings. The Power Group of the Chicago Section held three meetings. The group meetings in both cities have been very successful.

The Chicago Section and the Western Society of Engineers cooperated in arranging for two lecture courses for practicing engineers who wish to supplement their technical education: "Engineering Economics" at Armour Institute of Technology and "Recent Developments in Electron Physics and Chemistry" at the University of Chicago.

Many joint meetings were held with neighboring Branches, and the programs were supplied partly or entirely by students at twenty-two or more meetings. Many Sections held joint meetings with other local groups.

Three meetings were held by the Pittsfield, Schenectady, and Seattle Sections, at which their younger members presented papers in competition for prizes.

Nearly all of the District Executive Committees held meetings during the year.

President Smith has visited a large majority of the Sections, and will visit others before the end of the administrative year.

President Smith's message in the September issue of the JOURNAL was devoted to the activities of the Sections.

The Birmingham, Iowa, and North Carolina Sections were organized during the year, and all have been active. The existence of the Panama Section, which has been inactive for several years, was terminated by the Board of Directors at its meeting held March 14, 1930.

In April, each Section was supplied with the names of its Associates whose dues were to become $15.00 on May 1, and it was recommended that each Section appoint a small committee to review the names and remind all such Associates who are clearly qualified for the higher grades to submit their applications promptly.

Student Activities.—Nearly all of the Branches have been very active during the year both in their individual meetings and in coöperative relations with other Branches and with Sections.

Eight of the Districts held Conferences on Student Activities.

Ten Student Conventions were held, six sponsored by District Committees on Student Activities, three by Sections, and one by a Branch.

The increased interest of the students has been shown by the larger number of talks by them, a greater number of Branch meetings at which special types of programs were presented, and a considerable number of statements by Branch officers that the student programs usually produce better attendance and greater interest than do other kinds.

Meetings of some of the Branches were devoted to general discussions on particular subjects (one subject per meeting), debates, and mock trials. Students supplied the programs at twelve joint meetings with

Sections, and parts of the programs at ten or more such meetings.

At its meeting on January 29, 1930, the Board of Directors approved the recommendations of a special committee that Enrolled Students be permitted to enter as Associates without paying the usual entrance fee. The necessary Constitutional amendments were submitted to a vote of the membership, and the results will be announced at the Annual Business Meeting on June 23.

The message of President Schuchardt in the May issue and that of President Smith in the October issue of the JOURNAL were directed especially to Branch members.

The Board of Directors authorized the payment of traveling expenses at the rate of ten cents per mile one way for an annual visit of each Vice-President to each Branch within his District.

As a result of recommendations made the Counselor Delegates at the Summer Convention at Swampscott, Mass., a set of cards was sent to each Counselor in February, giving the names of Enrolled Students of the Institute who had been at his institution at the time of enrollment, with the names of those who had not paid their fees for 1930 on cards of a different color.

New Branches were organized at Michigan College of Mining and Technology, University of New Mexico, North Dakota Agricultural College, Pratt Institute, Southern Methodist University, and Texas Technological College. All of them have been active.

Section and Branch Statistics.—

	For Fiscal Year Ending			
	April 30 1924	April 30 1926	April 30 1928	April 30 1930
SECTIONS				
Number of Sections......	47	51	52	56
Number of Section meetings held..............	381	405	431	480
Total Attendance..... .	58,945	58,959	64,276	84,615
BRANCHES				
Number of Branches... .	77	86	96	106
Number of Branch meetings held.......... . .	530	714	915	1,009
Total Attendance..... .	25,674	35,270	44,334	50,401

Meetings and Papers Committee.—A major phase of the work of the Meetings and Papers Committee consisted of arranging the technical sessions of conventions. The Committee has also coöperated with Local District Meeting Committees and has assisted with the arranging of technical sessions and supplying the local committees with lists of technical papers available.

Three national conventions and two District meetings were held during the year. The number of papers presented and the attendance at each convention or meeting are given above under appropriate headings. The total attendance for national conventions was about 3400. The total attendance for District meetings was about 1150, making a grand total attendance of approximately 4550. A total of 146 papers were presented, including 18 technical committee reports. During the year, over 200 papers were reviewed by technical committees; of these 118 were passed on for the TRANSACTIONS.

Due to (1) the increase of technical subject matter submitted for Institute presentation and publication, (2) a tendency toward a higher degree of specialization, and (3) the President's call for suggestions, a careful study of Institute Policies in regard to Meetings and Papers was made in order to determine the best way of providing for the above changes and needs. With a view toward meeting these needs, the Committee has decided to present at the coming Summer Convention the Technical Committee Reports by title only and provide opportunity for their discussion together with the discussion on the session on selected subjects. More sessions have been arranged in parallel and on Wednesday morning three sessions will be in parallel. These changes are to be tried at the Summer Convention, in an effort to provide a greater outlet for technical subject matter submitted for Institute presentation by the membership.

A subcommittee on Technical Program and Publication Policies was formed to draw up definite recommendations. A report of this subcommittee on "Proposed Changes in the A. I. E. E. Policies in Regard to Meetings and Papers," dated January 30, 1930, was submitted to the Board of Directors, with a copy to the Publication Committee for its information.

At a meeting held September 20, 1929, a budget for the fiscal year beginning October 1st, based on the proposed number of papers, 139, at an estimated expense of $23,000, was accepted. The estimate was based on the cost of meetings of previous years 1926-1929 and cost for the year 1928-1929.

The reviewing of technical papers and recommending of technical papers for publication has comprised another important phase of the Committee's work throughtout the year. Furthermore, Committee members have assisted the Committee on Award of Institute Prizes by recommending papers worthy of final consideration for national prize awards. The Committee has rendered other assistance upon request such as recommending lecturers to National Convention Committees and suggesting the names of authors best qualified to prepare papers for special occasions.

The technical program for the 1930 Toronto Summer Convention Program has been completed. The programs for the Pacific Coast Convention and the Middle Eastern District Meeting are taking definite form. Consideration is also being given to appropriate subjects for the 1931 Winter Convention.

Publication Committee.—The question of Institute Publications has been growing in importance in recent years, and it has been the subject of probably more discussion than any other activity, particularly during the past year. The Publication Committee in its endeavor to make this Institute activity as useful as possible to all the membership, began about a year ago a thorough systematic survey for the purpose of obtaining the viewpoints of the members all over the country, as to what changes, if any, should be made in Institute Publications. This nation-wide survey has now been completed, the replies analyzed, and the results discussed at considerable length at the meeting of the Committee held on March 7.

After studying the results of the survey the Committee agreed on a general plan which is now being drawn up in final form, together with definite recommendations for submission to the Board of Directors at the May meeting. It is proposed to include in this plan the broad general policies only, leaving many of the details to be worked out by the Publication Committee in coöperation with the editorial staff, it being understood that some of the details will necessarily be modified from time to time. An estimate will also accompany the report, showing approximately what the additional cost will be to carry out such a plan.

Since changing the policy of Institute publications is a matter of such importance, the Committee suggested that before action is taken by the Board, the report be presented for discussion at the Conference of Officers, Delegates, and Members at the Toronto Convention in June. The Board concurred and arrangements have been made with the Sections Committee for the inclusion of this discussion in its program.

Coordination Committee.—The Committee corresponded with the officers of all Districts and Sections, and obtained their views regarding national and District meetings during the year 1931, resulting in recommendations to and adoption by the Board of Directors, in January and March, 1930, of a complete schedule of national and District meetings to be held during 1931.

The Committee has considered the coördination of the work of the A. I. E. E. technical committees with that of similar committees of other societies, and expects to recommend to the Board of Directors the adoption of an amendment to Section 82 of the By-laws, which states the functions of the technical committees.

Standards Committee.—During the past year, the Standards Committee's work has been largely confined to the consideration of revisions of existing Standards and the development of proposed standards. The work of revision has been carried on generally through subcommittees of two of the technical committees, *i. e.*, the Electrical Machinery Committee and the Instruments and Measurements Committee. No new Standards have been published, but revisions of six existing Standards have been approved and five reports on proposed standards have become available. The Sectional Committees of the American Standards Association, of which the Institute is sponsor or joint sponsor, have been active, especially the Sectional Committee on Scientific and Engineering Symbols and Abbreviations. This committee is rapidly completing the work originally laid out for it, and its accomplishments should prove of great value to industry, particularly to those who deal with publications and drawings.

The Institute is now sole sponsor for eight projects and joint sponsor for fifteen, and is represented on thirty-one Sectional Committees which are under the sponsorship of other organizations. One development of interest has been the agreement reached with the National Electrical Manufacturers' Association to consolidate under a joint sponsorship of A. I. E. E. and N. E. M. A. the Sectional Committees dealing with Rotating Electrical Machinery. These projects have been making but little progress for several years and it is hoped that the new arrangement will not only reduce the total number of man-hours required to carry on the work, but will also expedite the final approval by the electrical industry of several Standards which are vital. At a special meeting of the Standards Committee of January 7, 1930, there was approved, in principle, a proposition for the formation of a joint agency of the electrical industry for the purpose of carrying out the necessary standardization activities of that industry. It is contemplated that this agency will have three distinct functions: (a) The Standards Committee of the electrical industry in the United States; (b) The Electrical Advisory Committee of the American Standards Association; (c) The U. S. National Committee of the I. E. C., and of any other international organizations for the handling of electrical matters. The Standards Committee's recommendations were transmitted to the Board of Directors, which approved and transmitted them to the present Electrical Advisory Committee, where they are now under consideration. This project was described more fully in the President's message published in the March, 1930, JOURNAL.

U. S. National Committee of the I. E. C.—During the past year a number of important meetings of advisory committees have been held abroad as follows:

The Advisory Committee on Nomenclature has held a number of meetings throughout the year and the international vocabulary which is in preparation has been advanced very greatly thereby. This work is under the direction of Dr. C. O. Mailloux.

An international subcommittee under the chairmanship of Dr. A. E. Kennelly has been considering the question of Magnetic Units. This subcommittee was unable to find solutions for the questions referred to it; but decided to invite and elicit discussions on them, prior to the next plenary meeting, among the various national committees. In this connection a paper was presented by Dr. Kennelly at the Winter Convention of the

Institute with the idea of eliciting expressions of opinion in this country. It is hoped that some decision on this very difficult and contentious matter may be reached at the next Plenary Meeting of the Commission.

The International Advisory Committee on Rating of Electrical Machinery met in London on July 10th, 11th, and 12th and took certain important action in regard to dielectric tests and methods of testing; but because of lack of time was unable to bring matters to as satisfactory a conclusion as was hoped for in the direction of rating temperature rise. The difficult matter of temperature rises was left in the hands of a subcommittee consisting of Messrs. Everest, Kloss, Robinson, and Roth. This subcommittee has been actively engaged ever since in an endeavor to agree on all points which will be acceptable to the Advisory Committee when next it meets. The United States was represented by Dr. A. E. Kennelly, Dr. C. O. Mailloux, Dr. L. T. Robinson, Mr. L. L. Elden, Mr. R. E. Ferris, Prof. W. L. Upson, Prof. H. Pender, Mr. A. Wright, and Mr. W. W. Lewis.

At a meeting of the Advisory Committee on Steam Turbines held in London from July 1st to July 8th, the United States was represented by Prof. W. F. Durand as chief delegate and Mr. F. Hodgkinson and Mr. Irving E. Moultrop as advisory delegates. Mr. C. B. Le Page was present as representing the Secretariat. The Advisory Committee brought the specifications and test code for Steam Turbines to a point where adoption in large part can be anticipated at the Stockholm meeting.

The Advisory Committee on Aluminum met in Berlin on November 6th, the United States being represented by Mr. W. C. Binz, Mr. L. Braasch, and Dr. C. O. Mailloux. Among other things a value was tentatively agreed to for the conductivity of hard-drawn aluminum, which is to be submitted to the forthcoming Plenary Meeting.

The Advisory Committee on Electric Traction Equipment met in London on July 8th and 9th, the United States being represented by Mr. R. E. Ferris, official delegate, and Dr. A. E. Kennelly, Dr. L. T. Robinson, and Prof. W. L. Upson in an advisory capacity. This work proceeded in a satisfactory manner.

The Advisory Committee on Measuring Instruments met in Berlin on the 6th and 7th of November, 1929, the United States not being represented by any official delegate. The Committee made great progress in the formulation of a Code for meters of the less accurate or Class 2 type. This entire matter will come up for consideration at the Plenary Meeting.

A Plenary Meeting of the Commission is to be held this year in Scandinavia. The preliminary program calls for the first assembly in Copenhagen on June 27th, departure for Sweden on June 28th, Technical Sessions in Stockholm from June 30th to July 5th, arrival at Oslo on July 6th, excursions to power plants on July 7th and 8th, and final Plenary Meeting on July 9th. It is expected that the United States will be adequately represented at this meeting. Preparations looking forward to it have been going on throughout the year and have occupied the USNC and its various groups of Advisors continuously during this time.

A joint committee has discussed the proper relationships between the USNC and the American Standards Association and a report has been prepared giving a basis on which closer and more satisfactory contacts and interconnections might be worked out with advantage to both bodies. The further discussion of this matter has been postponed by mutual consent pending the outcome of the recent movement looking to the establishment of a standardizing committee of the electrical industry with which the USNC might, in due course, be affiliated or amalgamated.

U. S. National Committee of the International Commission on Illumination.—During the year, various matters in connection with the International Illumination Congress and Plenary Meeting of the International Commission on Illumination held in Saranac in September, 1928, were completed. The final accounting of funds raised for the purposes of this meeting through the Committee of the Illuminating Engineering Society showed a surplus which was distributed pro rata among the contributors.

The volume of Proceedings of the Congress and Plenary Meeting, amounting to 1443 pages and constituting a very valuable collection of information on the art and science of illumination as of to-day, was distributed by the Illuminating Engineering Society.

The National Committee was able to make effective use of its contacts with the National Committees which constitute the International Commission in connection with the celebration of Light's Golden Jubliee. It was desired in this country that other nations should join in the observance of this important epoch, and the United States National Committee approached the other National Committees with information as to what was being planned in this country and the suggestion that some form of observance adapted to national conditions might be desirable in other countries. Following this the other National Committees were kept informed of the progress and plans in this country. In a number of countries, notably Italy, Japan, and the Netherlands and, on a less pretentious scale, in France, Austria, Germany, and Sweden, effective observances of this Jubliee were held.

The next Plenary Meeting of the Commission is to be held in Great Britain in 1932. In connection with this, it has been the privilege of the United States National Committee to submit to the Central Office and to the British National Committee the results of the experience gained here in connection with the Saranac meeting with a view to giving whatever assistance might be given toward making the next meeting the great success which the British Committee is endeavoring to achieve

The Committee, as Secretariat for Aviation Lighting, has coöperated in a meeting held in London, having as its object the unification of practise in order to facilitate international flying. Another meeting on this subject was held in Berlin in April.

The Committee has been otherwise engaged in routine matters including the exchange of documents and other information with other National Committees and in preparations looking to effective participation in the next Plenary Meeting.

The International Commission on Illumination has recently been accorded a very definite recognition of its status in that the International Committee on Weights and Measures in requesting of its Advisory Committee on Electricity a report on the procedure whereby the International Bureau can take over the supervision of the matter of an international standard of light gave to that Committee definite authority to coöperate with the International Commission on Illumination. At the present stage this project is largely in the hands of the several national laboratories, but the machinery of the International Commission on Illumination gives a channel through which the organized interests concerned with illumination can be informed regarding developments and can be given an opportunity to make their influence felt through guidance and stimulation.

Technical Committees.—Reports of Technical Committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the annual Summer Convention and printed in the TRANSACTIONS.

Membership.—The fundamental duty of the Membership Committee, as defined by the By-laws, is to "bring the advantages of membership to the attention of desirable candidates for admission to the Institute." The advantages of membership can be presented to them most effectively by carefully selected Section Membership Committees.

In order to appeal to their Section loyalty and thus most effectively stimulate the activity of the Section Membership Committees, a slight change was made this year in the method of carrying out the work of the Membership Committee. This change consisted in establishing contact of the candidate directly with the Section instead of with National Headquarters. This was effected by having the names of candidates forwarded by the suggesting member directly to his own Section Membership Committee Chairman instead of to National Headquarters. It is believed that the direct personal contacts thus locally established have been somewhat more successful than the more impersonal method formerly employed.

The additions to, and deductions from, the membership are shown in the following table. In accordance with the established policy of the Institute, emphasis has continued to be placed on the qualifications of candidates. The total membership as of April 30, is 18,003.

	Honorary	Fellow	Member	Associate	Total
Membership on April 30, 1929..	8	687	3,377	14,061	18,133
Additions:					
Transferred....	2	24	227		
New Members qualified.....	1	4	73	1,139	
Reinstated.....		1	7	56	
Total............	11	716	3,684	15,256	19,667
Deductions:					
Died...........	1	12	29	59	
Resigned.......			27	365	
Transferred....		1	14	238	
Dropped.......		1	56	861	
Membership on April 30, 1930..	10	702	3,558	13,733	18,003

Deaths.—The following deaths have occurred during the year.

Honorary Members: Charles F. Brush, Sebastian Ziani de Ferranti.

Fellows: William S. Andrews, Alfred H. Cowles, Edward B. Craft, Walter C. Fish, Arthur H. Ford, David L. Huntington, Peter Junkersfeld, Charles S. Lawson, John W. Lieb, Walter H. Millan, Thomas E. Murray, Harry W. Weller.

Members: Eugene H. Abadie, Armistead K. Baylor, John R. Benton, Henry W. Blake, John J. Barger, Wallace W. Briggs, Bedford J. Brown, Sidney Hand Browne, Henry A. Coles, John A. Cressey, Frederic Cutts, Edmund P. Edwards, William L. Fey, John H. Finney, F. A. T. FitzGerald, Ralph W. Goddard, Everett D. Hall, Joseph A. Hebert, James T. Hutchings, Harry W. Jeannin, Samuel M. Kennedy, Charles B. Larzelere, Maurice A. Oudin, Arthur C. Scott, Everett H. Smith, Harry L. Tanner, Arthur E. Walden, James T. Watson.

Associates: Charles S. Allen, Isaac M. Beatty, Nicholas F. Brady, Charles C. Chatley, Wallace S. Clark, Herbert A. Clarke, John L. Curtiss, J. Maurice De Vean, Clark E. Diehl, Andrew E. Dobson, William D. Forster, N. Govindraj, John H. Graybill, Clarence G. Hadley, Fred C. Hamer, Shimpei Hayase, James F. Hillock, Tsuyoshi Hirayama, Harold J. Horn, Henry C. Houck, Noboru Inouye, Allan F. Jacobson, John M. Jolly, Frederick L. Jones, William E. Keily, Fred M. Kimball, Charles W. Kincaid, Ralph V. S. Kirkwood, Leo R. Lake, Paul Liunge, Calvin O. Loewenberg, Oscar S. Major, Allen M. Manning, Frederick M. Martinego, Marshall J. McMuran, Frank R. Mistersky, Percy N. Moore, Max G. Newman, Joseph H. Owsley, Edward F. Peck, George A. Plank, Ralph W. Pope, Francis C. Pratt, J. Lloyd Prince, Robert M. Searle, Guido Semenza, Charles A. Tripp, Carl E. Trube, Ralph E. Tuttle, Miner H. Vallas, Ferdinand Voelker, Jr., Roy A. Walker, J. Wallace, Ralph L. Werden, Herbert B. Whyte, Albert E. Wing, William O. Winston, Jr., Charles T. Wright, John W. Wright.

Board of Examiners.—The Board of Examiners held nine meetings during the past year, averaging about three and one-quarter hours. A total of 3796 cases was considered, divided as shown in the following table:

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate......	989	
Not recommended........................	21	1010
Recommended for grade of Member.......	82	
Not recommended........................	23	105
Recommended for grade of Fellow.........	5	
Not recommended........................	0	5
Recommended for enrolment as Students...		2406

APPLICATIONS FOR TRANSFER

Recommended for grade of Member.......	211	
Not recommended........................	29	240
Recommended for grade of Fellow.........	28	
Not recommended........................	2	30
		3796

Institute Prizes.—Four National Prizes and eleven District Prizes for papers were awarded during the year, as announced through the Institute JOURNAL. These prizes were for papers presented at Institute meetings during 1928. The National Prizes were presented at the 1929 Summer Convention and the District Prizes at various meetings in the respective Districts. The National Prizes consist of $100.00 each and a certificate, while the District Prizes consist of $25.00 cash and a certificate.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute each year a scholarship in electrical engineering for each class. The awards are made annually by an Institute committee. The scholarships pay $350.00 toward annual tuition with provision for reappointment.

Complete details governing prizes and scholarships may be obtained by applying to the National Secretary of the Institute.

Edison Medal.—The Edison Medal, founded by associates and friends of Thomas A. Edison, is awarded annually by a committee consisting of twenty-four members of the Institute "for meritorious achievement in electrical science, electrical engineering, or the electrical arts." The medal for 1929 was awarded to Professor Charles F. Scott, "for his contributions to the science and art of polyphase transmission of electrical energy." The medal was presented at the Winter Convention of the Institute, January 29, 1930.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical, and Electrical Engineers, awarded the twenty-sixth medal to Ralph Modjeski for "notable achievement as an engineer of great bridges combining the principles of strength and beauty." The medal was presented by the Board of Award at the annual banquet and reception of the A. S. C. E., New York, January 15, 1930.

Hoover Medal.—The Hoover Medal, established through a trust fund created by the gift of Conrad N. Lauer, is to be awarded periodically by a Board of Award, composed of representatives of the national societies of Civil, Mining, Mechanical, and Electrical Engineers, "to a fellow engineer for distinguished public service." The first award was made to President Herbert Hoover to commemorate his civic and humanitarian achievements, and the medal was presented at the A. S. M. E. Anniversary Dinner in Washington, D. C., April 9, 1930.

Lamme Medal.—The Lamme Medal was founded as a result of a bequest of the late Benjamin G. Lamme, Chief Engineer of the Westinghouse Electric and Manufacturing Company, who died on July 8, 1924. The bequest provides for the award by the Institute of a gold medal (together with a bronze replica thereof) annually to a member of the A. I. E. E. "who has shown meritorious achievement in the development of electrical apparatus or machinery" and for the award of two such medals in some years if the accumulation from the funds warrants.

The Lamme Medal Committee of the Institute awarded the second (1929) Lamme Medal to Rudolf E. Hellmund, "for his contributions to the design and development of rotating electrical machinery." Arrangements are being made for the presentation of the medal at the annual Summer Convention at Toronto, Canada, June 23-27, 1930.

Commission of Washington Award.—The Washington Award for 1930 was made to Dr. Mortimer E. Cooley, Dean Emeritus of the Colleges of Engineering and Architecture, University of Michigan, "for vision and constructive leadership in the education of the engineer."

The award is made annually "to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good," by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E., and the A. I. E. E.

Employment Service.—The employment service which the Institute has maintained for many years is conducted as a cooperative bureau in conjunction with a similar service maintained by the National Societies of Civil, Mining, and Mechanical Engineers under the title, "Engineering Societies Employment Service." In addition to the main office in the Engineering Societies Building, New York, offices have been opened in Chicago in cooperation with the Western Society of Engineers and in San Francisco under similar arrangements with the California Section, American Chemical Society, and Engineers Club of San Francisco. It is hoped to continue this development from year to year as conditions warrant. The service is supported by the joint contributions of the societies and their individual members who are benefited. As in the past it consists principally in acting as a medium for bringing together the employer and the employee. In addition to the publication of the Employment Service announcements in the monthly JOURNAL, weekly subscription bulletins are issued for those seeking positions.

American Engineering Council.—Since its organization nearly ten years ago, American Engineering Council has experienced a steady, normal growth. It now includes in its membership twenty-four national, state, and local engineering societies which have a total membership of more than 57,000 professional engineers.

The Council represents its constituents at conferences, conventions, and congressional hearings upon matters involving engineering and on important national committees. It also extends valuable individual service to its member organizations. Several notable surveys on matters of concern to the entire engineering profession have been conducted.

The Fourth Conference of Secretaries of Engineering Societies, sponsored by the American Engineering Council, was held in Chicago, June 6 and 7, 1929.

A standing Committee on Engineering and Allied Technical Professions has been organized to enable the Council to enter upon an extensive program designed to secure information upon and to improve the general status of the engineering profession.

Other standing committees are Constitution and By-laws, Finance, Membership and Representation, Public Affairs, Publicity and Publications, and Regional Activities.

The Council has nineteen special committees including those on Airports, Bridges, Communications, Flood Control, Government Reorganization, Patents, Power, Radio Broadcasting, Reforestation, Water Resources, etc.

Approximately 150 bills introduced in the present session of Congress have been considered by the Council. A few of the important bills which have received specific attention are: Boulder Canyon Dam, Communications Commission, Uniform Government Contracts, Pay Status of Government Employees, National Hydraulic Laboratory, Inter-oceanic Canal Commission, Patent System, Radio Commission, Reforestation, Topographic Surveys, and Water Resources.

The Council has continued the publication of a quarterly known as the A. E. C. Bulletin.

Engineering Foundation, Inc.—This organization which succeeded United Engineering Society, by change of name only, was set up by the four national societies of Civil, Mining, Mechanical, and Electrical Engineers to hold in trust and to administer for them the Engineering Societies Building, in which their headquarters are located, certain funds, and the Library. Its charter gives it broad powers for the advancement of the engineering arts and sciences.

Extracts from the annual financial report of Engineering Foundation, Inc., were published in the March, 1930, JOURNAL.

Engineering Societies Research Board.—This Board, a department of Engineering Foundation, Inc., is the successor to Engineering Foundation which was established in 1914 by the national societies of Civil, Mining, Mechanical, and Electrical Engineers "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." It was conceived by Ambrose Swasey, of Cleveland, Ohio, and he has made three gifts toward its endowment. The fund has been generously increased through the gifts of Edward D. Adams and others, and also through a bequest of the late Henry R. Towne.

Appropriations have been made for various research projects, and cooperation has been extended in others.

Engineering Societies Library.—The Library is administered as a free public library under the direction of the Library Board of Engineering Foundation, Inc., this Board being composed of representatives of the national societies of Civil, Mining, Mechanical, and Electrical Engineers.

The Library contains about 125,000 books and pamphlets, and receives regularly about 1700 technical periodicals in 18 languages.

A staff of technically trained searchers and translators is maintained. The staff is prepared to furnish the following types of service: photoprints, abstracts, translations, bibliographies, searches, etc. Special arrangements have been made for lending books.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has accepted sponsorship and appointed representatives upon a number of new

Sectional Committees of the American Standards Association. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins & Sells, certified public accountants, have audited the books, and their report follows:

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON,

National Secretary.

New York, May 23, 1930.

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS

OFFICES IN THE PRINCIPAL CITIES OF
THE UNITED STATES OF AMERICA
—AND IN—
LONDON, PARIS, BERLIN, SHANGHAI,
MANILA, MONTREAL, HAVANA,
MEXICO CITY

CABLE ADDRESS "HASKSELLS"

NEW YORK CENTRAL BUILDING
75 EAST 45TH STREET
NEW YORK

May 16, 1930.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

We have audited your accounts for the year ended April 30, 1930, and submit the following exhibits and schedule:

Exhibit

A—Balance Sheet, April 30, 1930.
  Schedule 1—Reserve Capital Fund—Securities.
B—Summary of Income and Profit and Loss for the Year Ended April 30, 1930.

WE HEREBY CERTIFY that in our opinion Exhibits A and B set forth the financial conditions of the Institute at April 30, 1930, and the results of its operations for the year ended that date.

Yours truly,
HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

EXHIBIT A.

BALANCE SHEET, APRIL 30, 1930

ASSETS

REAL ESTATE:			
One-fourth interest in Land, Building, and Building Equipment of Engineering Foundation, Inc., 25 to 33 West 39th Street (depreciation carried on books of Engineering Foundation, Inc.)			$493,352.60
EQUIPMENT:			
Library—Volumes and fixtures		$ 41,562.23	
Works of art, paintings, etc		3,001.35	
Office furniture and fixtures	$30,285.41		
Less reserve for depreciation (including $8,931.79 funded)	13,419.85	16,865.56	
Total equipment			61,429.14
WORKING ASSETS:			
"Transactions," etc		$ 5,596.00	
Text and cover paper		3,393.98	
Badges		1,430.84	
Total working assets			10,420.82
CURRENT ASSETS:			
Cash		$ 9,195.82	
Accounts receivable:			
Members—for dues		20,891.26	
Advertisers		1,579.45	
Miscellaneous		1,804.75	
Accrued interest on investments		3,351.82	
Total current assets			36,823.10
FUNDS:			
Reserve Capital Fund:			
Securities—Schedule 1		$210,226.88	
Life Membership Fund:			
Cash	$ 6,837.74		
Chicago, Burlington & Quincy Railroad Company 4% bonds, 1958, registered, face value, $5,000.00	4,868.75		
Accrued interest	33.33	11,739.82	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 1,185.30		
New York City 4½% corporate stock, 1957, par value, $2,000.00	2,204.05		
New York Telephone Company 4½% bond, 1939, registered, face value, $1,000.00	878.75		
Accrued interest	67.50	4,335.60	
Mailloux Fund:			
Cash	$ 2.30		
New York Telephone Company 4½% bond, 1939, registered, face value, $1,000.00	1,000.00		
Accrued interest	22.50	1,024.80	
Lamme Medal Fund:			
Cash	$ 267.32		
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series "C" bonds, 1995, face value, $4,000.00	4,330.00		
Accrued interest	100.00	4,697.32	
Depreciation of Furniture and Fixtures Fund:			
Cash	$ 3,921.79		
Cleveland Union Terminals Company 5% sinking fund series "B" gold bonds, 1973, registered, face value, $4,000.00	4,010.00		
Fidelity Union Title and Mortgage Guaranty Company first mortgage certificates (on property, Nos. 75-79 Prospect Street, East Orange, N. J.) 5½%, due 1933.	1,000.00	8,931.79	
Total funds			240,956.21
TOTAL			$842,981.87

LIABILITIES

CURRENT LIABILITIES:		
Accounts payable	$ 11,296.69	
Dues received in advance	2,740.84	
Entrance fees and dues advanced by applicants for membership	498.50	
Subscriptions for "Transactions" received in advance	539.00	
Total current liabilities		$ 15,075.03
FUND RESERVES (NOT INCLUDING DEPRECIATION RESERVE):		
Reserve Capital Fund	$210,226.88	
Life Membership Fund	11,739.82	
International Electrical Congress of St. Louis—Library Fund	4,335.60	
Mailloux Fund	1,024.80	
Lamme Medal Fund	4,697.32	
Total fund reserves (not including depreciation reserve)		232,024.42
SURPLUS, Per Exhibit "B"		$595,882.42
Total		$842,981.87

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT AND LOSS FOR THE YEAR ENDED APRIL 30, 1930

EXHIBIT B.

INCOME:			
Dues		*$228,251.20	
Students' fees		12,166.00	
Entrance fees		11,955.00	
Transfer fees		1,295.00	
Advertising		88,272.58	
"Journal" subscriptions		11,464.11	
"Transactions" subscriptions		1,141.30	
Miscellaneous sales		10,363.92	
Badges sold	$ 3,635.50		
Less cost	3,404.78	230.72	
Interest on securities in reserve capital fund		10,213.32	
Interest on securities in depreciation reserve fund		244.16	
Interest on bank balances		1,655.62	
Total			$377,252.93
EXPENSES:			
Publications:			
"Journal"	$87,463.37		
"Transactions"	34,859.65		
Year book	9,328.66		
Miscellaneous	4,266.50	$135,918.18	
Institute meetings		24,625.08	
Administrative expenses		64,005.19	
Presidents' special appropriation		3,738.20	
Board of Directors, traveling expenses		4,643.54	
National Nominating Committee, traveling expenses		1,003.35	
Institute representatives, traveling expenses		132.24	
Sections		37,539.15	
Geographical Districts' expenses:			
Traveling expense:			
Executive Committee	$ 2,790.00		
Vice-Presidents	895.97		
District best paper prize	63.00		
District prize for initial paper	63.00	3,811.97	
Branches		12,421.98	
Membership		7,632.49	
Finance		400.00	
Code Committee		60.00	
Technical Committees		235.38	
Law Committee		1,046.16	
Headquarters Committee		181.50	
Forward		$295,871.37	$377,252.93

TOTAL INCOME—FORWARD		$377,252.93
EXPENSES—FORWARD	$295,871.37	
Standards	9,959.01	
American Standards Association	1,500.00	
International Electrotechnical Commission	1,453.61	
Engineering Societies Library, maintenance	10,196.56	
Engineering Foundation, Inc., assessment	5,713.56	
American Engineering Council	17,596.00	
Engineering Societies Employment Service	1,635.00	
Institute prizes	440.00	
Edison Medal Committee	194.26	
United States National Committee of International Commission on Illumination	300.00	
Honorary Secretary	2,000.00	
Total		348,382.41
NET INCOME		$ 28,870.52
PROFIT AND LOSS CREDITS:		
Adjustment of value of library volumes and fixtures	$ 109.93	
Unused balance of Winter Convention Fund	26.70	
Profit on dinner dance	120.20	
Profit on furniture and fixtures sold—Net	11.50	
Adjustment of Life Membership Fund Reserve—Excess of fund over required amount as determined by the Institute in accordance with the by-laws	739.92	
Total		1,008.25
GROSS SURPLUS FOR THE YEAR		$ 29,878.77
PROFIT AND LOSS CHARGES:		
Uncollectible dues and members charges written off	$ 9,302.64	
Provision for depreciation of furniture and fixtures	2,156.66	
Total		11,459.30
SURPLUS FOR THE YEAR		$ 18,419.47
SURPLUS, MAY 1, 1929	$616,319.20	
Less transferred to Reserve Capital Fund in accordance with resolution of the Board of Directors.	38,856.25	577,462.95
SURPLUS, APRIL 30, 1930		$595,882.42

*Includes $90,665.00 allocated to JOURNAL subscriptions.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Reserve Capital Fund—Securities

April 30, 1930

Exhibit A.
Schedule No. 1.

	Face Value	Book Value
Railroad Bonds:		
Baltimore and Ohio Railroad Company 4½% convertible gold bonds, due 1933, registered	$ 10,000.00	$ 9,387.50
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series "C" bonds, due 1995, registered	8,000.00	8,940.00
Central of Georgia Railway Company 5½% refunding and general mortgage series "B" bonds, due 1959, registered	5,000.00	5,283.75
Chicago and Erie Railroad Company 5% first mortgage gold bonds, due 1982, registered	1,000.00	1,105.00
Chicago, Burlington & Quincy Railroad Company 5% first and refunding mortgage series "A" gold bond, due 1971, registered	1,000.00	1,010.00
Chicago, Terre Haute & Southeastern Railway Company 5% first and refunding mortgage gold bonds, due 1960	8,000.00	7,910.00
Florida East Coast Railway Company 5% first and refunding mortgage series "A" gold bonds, due 1974, registered	10,000.00	9,818.75
Great Northern Railroad Company 5½% general mortgage series "B" gold bonds, due 1952, registered	10,000.00	9,847.50
New York Central Railroad Company 5% refunding and improvement mortgage series "C" bonds, due 2013, registered	6,000.00	5,742.50
Pennsylvania Railroad Company 4½% general mortgage series "A" gold bonds, due 1965, registered	5,000.00	5,130.00
St. Louis, San Francisco Railway Company 5% prior lien mortgage series "B" bonds, due 1950, registered	6,000.00	5,497.50
Southern Railway Company 5% first consolidated mortgage gold bonds, due 1994, registered	1,000.00	980.00
Total (Forward)	$ 71,000.00	$ 70,682.50
Total (Forward)	$71,000.00	$ 70,682.50
Public Utility Bonds:		
American Telephone and Telegraph Company 5% sinking fund gold debentures, due 1960, registered	$ 15,000.00	$ 14,625.00
Consolidated Gas Company of New York 5½% gold debentures, due 1945, registered	5,000.00	5,187.50
Detroit Edison Company 6% first and refunding mortgage series "B" gold bonds, due 1940, registered	5,000.00	5,178.13
Duquesne Light Company 4½% first mortgage series "A" gold bonds, due 1967, registered	3,000.00	2,970.00
New York Telephone Company 6% gold debenture bond, due 1949	5,000.00	5,525.00
Northern States Power Company 5½% first lien and general mortgage gold bond series "B", due 1950	10,000.00	10,400.00
Pacific Gas & Electric Company 5½% first and refunding mortgage gold bonds series "C", due 1952, registered	5,000.00	5,137.50
Philadelphia Company secured 5% gold bond series "A", due 1967	10,000.00	10,000.00
Shawinigan Water and Power Company first mortgage and collateral trust sinking fund gold bond series "A" 4½%, due 1967	5,000.00	4,581.25
United Light and Power Company 5½% first lien and consolidated mortgage gold bond, due 1959	5,000.00	4,975.00
Total	$ 68,000.00	$ 68,579.38
Industrial Bonds:		
American Smelting and Refining Company 5% first mortgage gold bonds series "A," due 1947, registered	$ 9,000.00	$ 9,085.00
Bethlehem Steel Company 5% purchase money and improvement mortgage sinking fund gold bonds, due 1936, registered	5,000.00	5,033.75
Fidelity Union Title and Mortgage Guaranty Company first mortgage certificates (on property Nos. 75-79 Prospect Street, East Orange, N. J.) 5½%, due 1933	14,000.00	14,000.00
New York Steam Corporation 6% first mortgage gold bonds, due 1947, registered	10,000.00	10,837.50
United States Rubber Company 5% first and refunding mortgage series "A" bonds, due 1947, registered	2,000.00	1,915.00
Western Electric Company 5% debenture bonds, due 1944, registered	10,000.00	9,818.75
Youngstown Sheet and Tube Company first mortgage sinking fund gold bonds series "A" 5%, due 1978, registered	20,000.00	20,275.00
Total	$ 70,000.00	$ 70,965.00
Total	$209,000.00	$210,226.88

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